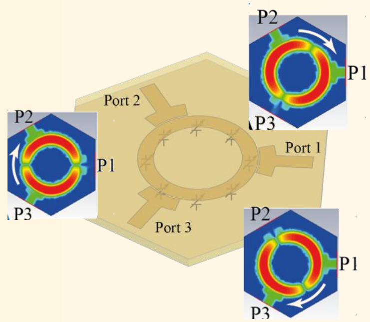
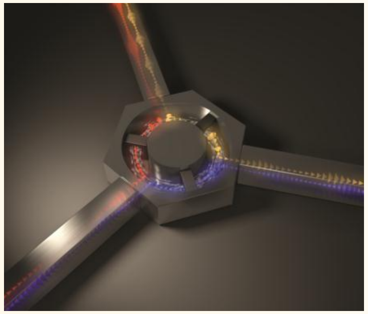
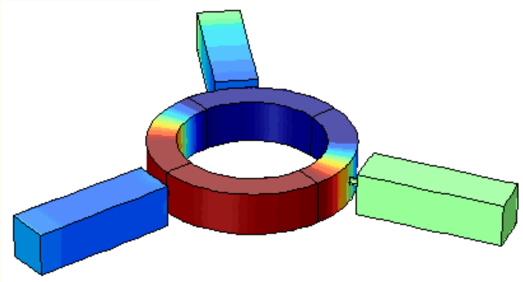






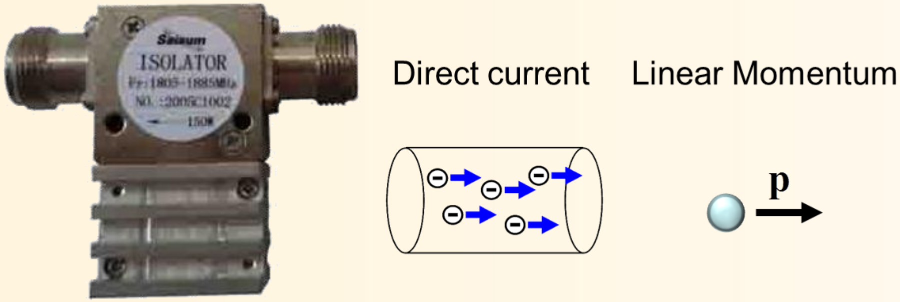
Nonreciprocity without Magnetism: Angular-Momentum Biased Metamaterials

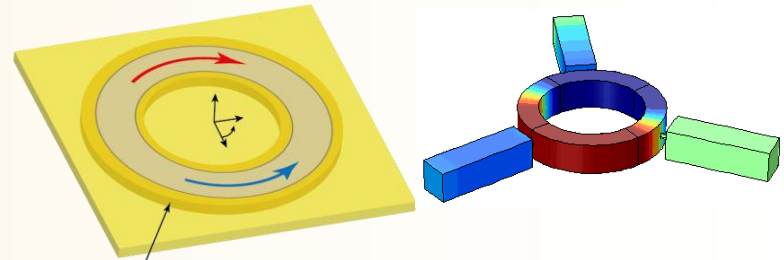
Andrea Alù

Contributors: Dimitrios Sounas, Romain Fleury, Michael Haberman, Caleb F. Sieck

Department of Electrical and Computer Engineering
The University of Texas at Austin
http://users.ece.utexas.edu/~aalu, alu@mail.utexas.edu

Overview

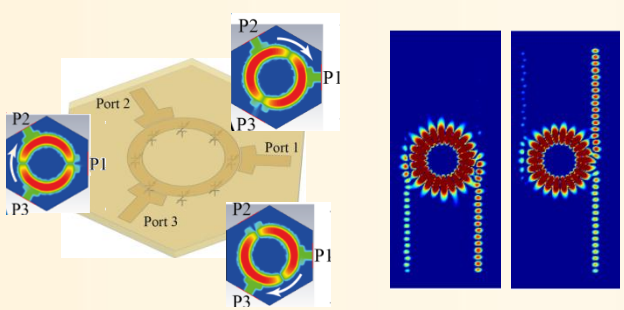


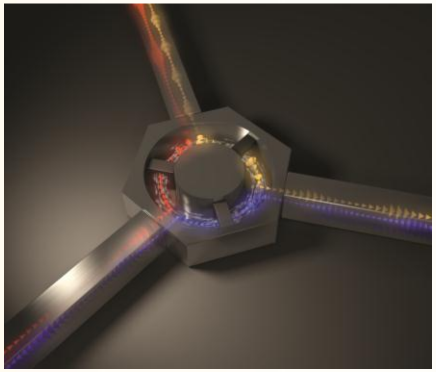


$$\Delta\varepsilon = \Delta\varepsilon_m \cos(\omega_m t - 2\varphi)$$

Available approaches to break reciprocity

Angular-momentum biased 'Zeeman' meta-molecules



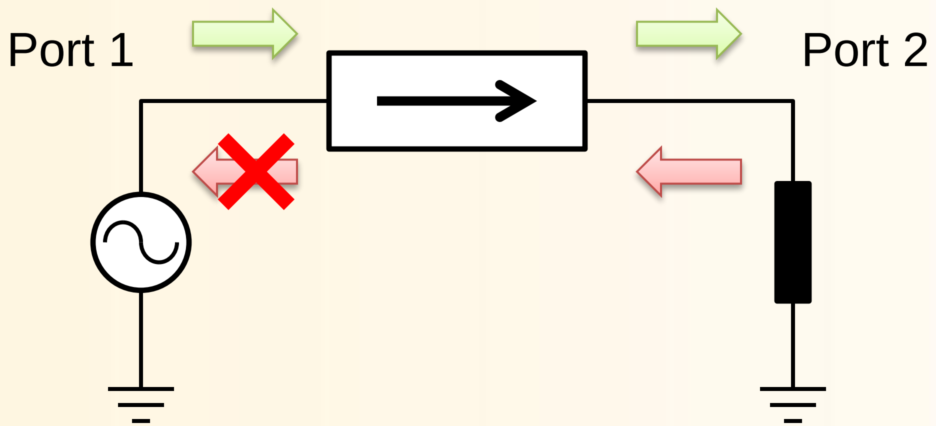


Proposed electromagnetic devices: from RF to optics

A first-off-its-kind acoustic circulator

Conventional Nonreciprocal Components

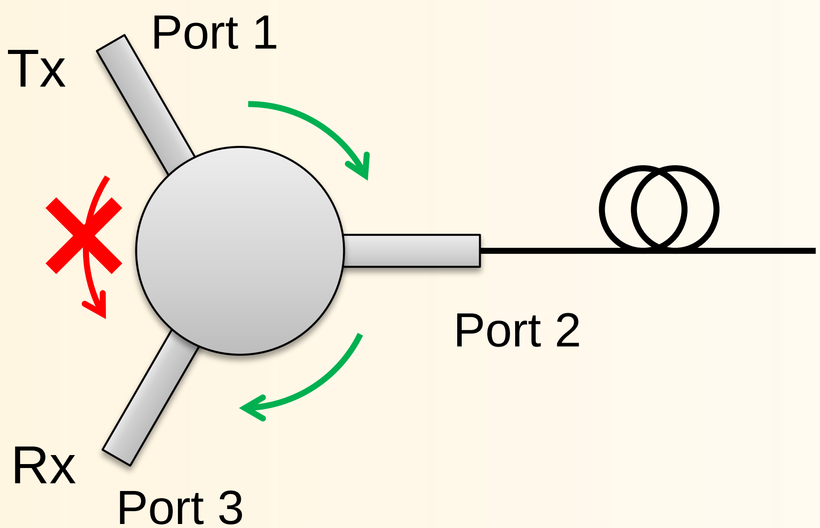
Isolator



$$S_{21} \neq S_{12}$$

$$\bar{\bar{S}} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$

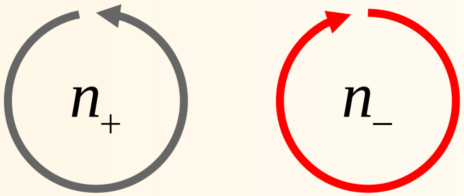
Circulator



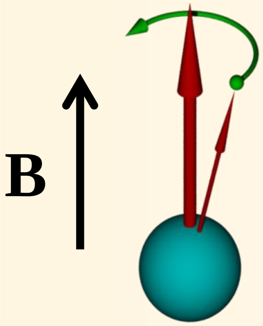
$$S_{ij} \neq S_{ji}$$

$$\bar{\bar{S}} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

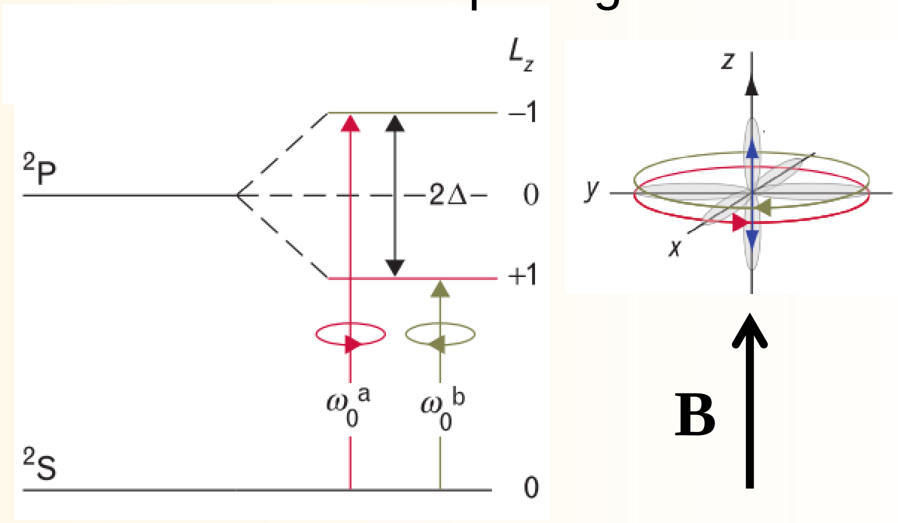
Circular Birefringence



Larmor precession



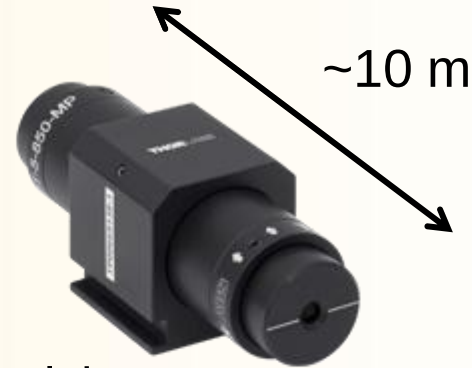
Zeeman splitting



Magnetically Biased Nonreciprocal Devices

Massive devices at microwaves, IR, optics

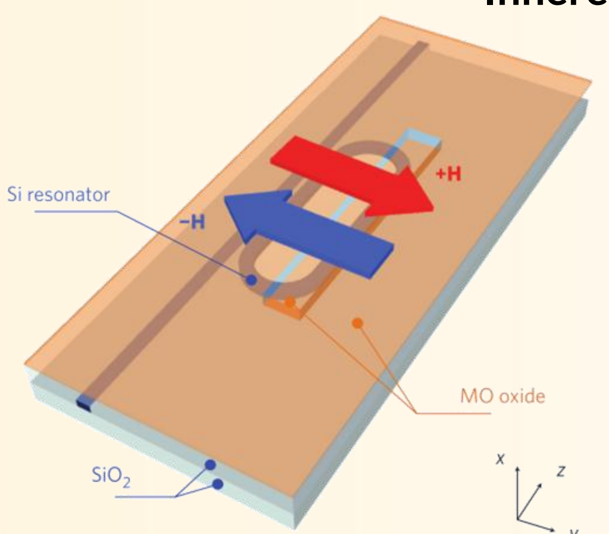
~10 mm

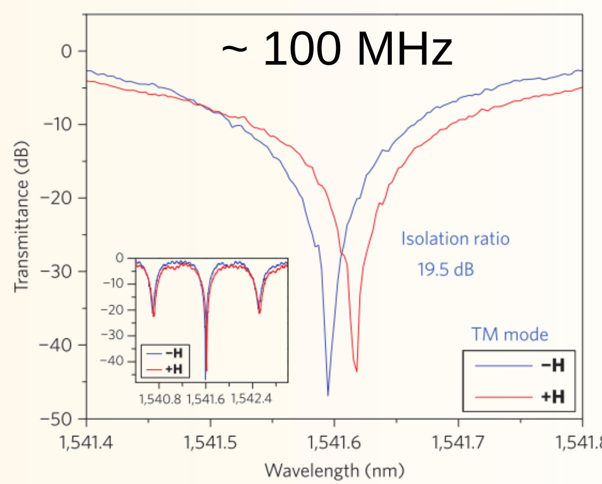


Thorlabs

λ~1 μm

Inherently narrow bandwidths



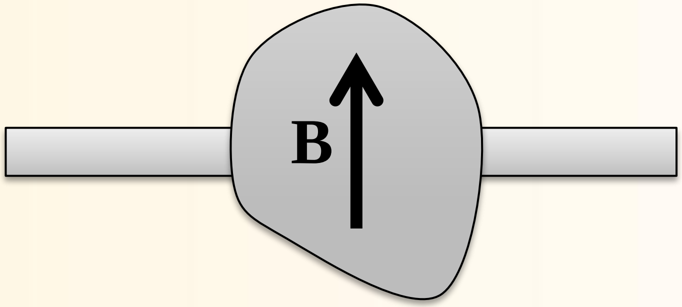


Bi, L. *et* al. *Nature Photonics* 5, 758–762 (2011)

Do We Need Magnetic Bias?

Onsager-Casimir Principle

Port i

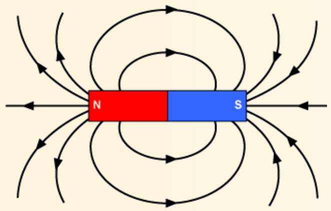


Port j

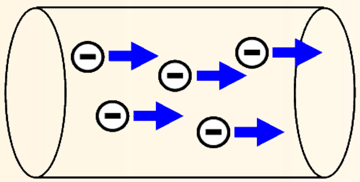
$$S_{ij}(\mathbf{B}) = S_{ji}(-\mathbf{B})$$

Odd vector under time reversal

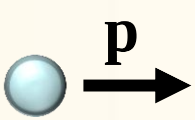
Magnetic Field



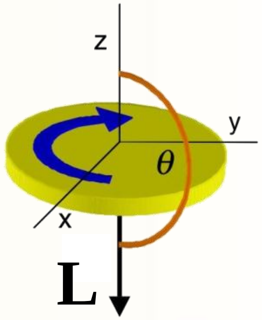
Direct current



Linear Momentum

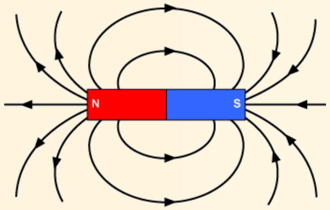


Angular Momentum

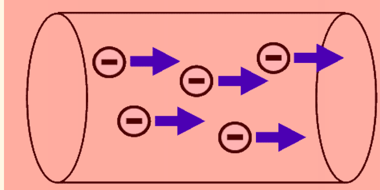


Other Available Approaches

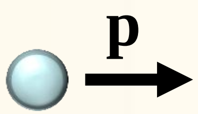
Magnetic Field



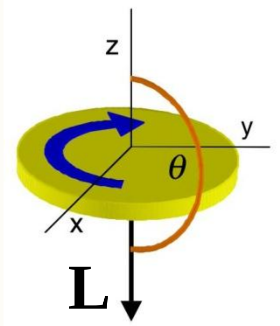
Direct current

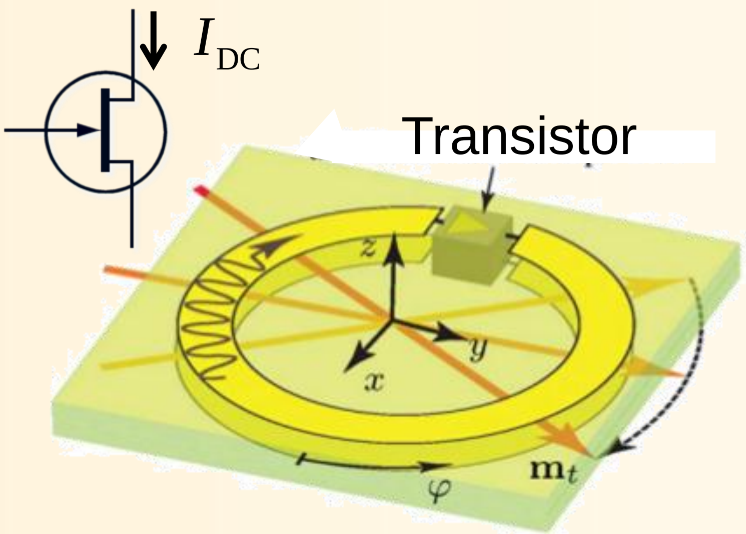


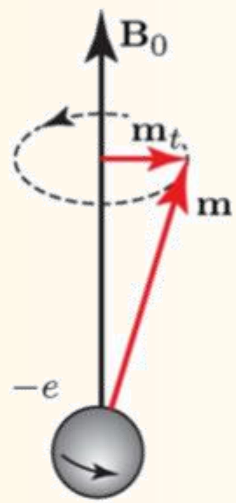
Linear Momentum



Angular Momentum





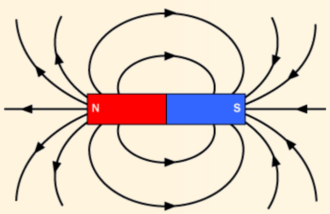


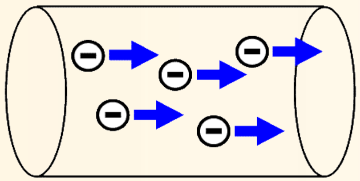
- ❑ **Only at low microwaves**
- ❑ **Significant power consumption**
- ❑ **Limitations on technology**

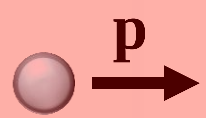
Kodera, T., Sounas, D. L. & Caloz, C. *Appl. Phys. Lett.* 99, 03114 (2011)

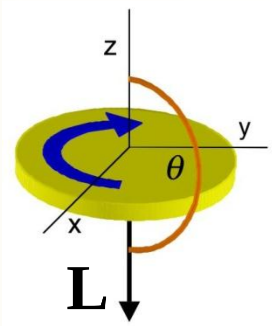
Other Available Approaches

Magnetic Field | Direct current | Linear Momentum | Angular Momentum

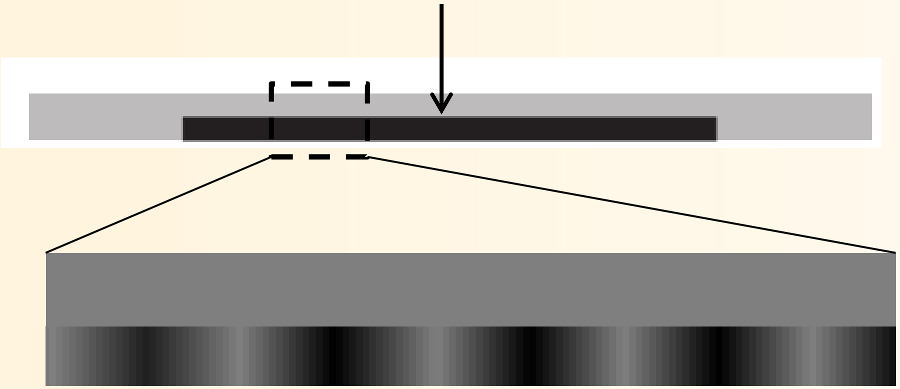


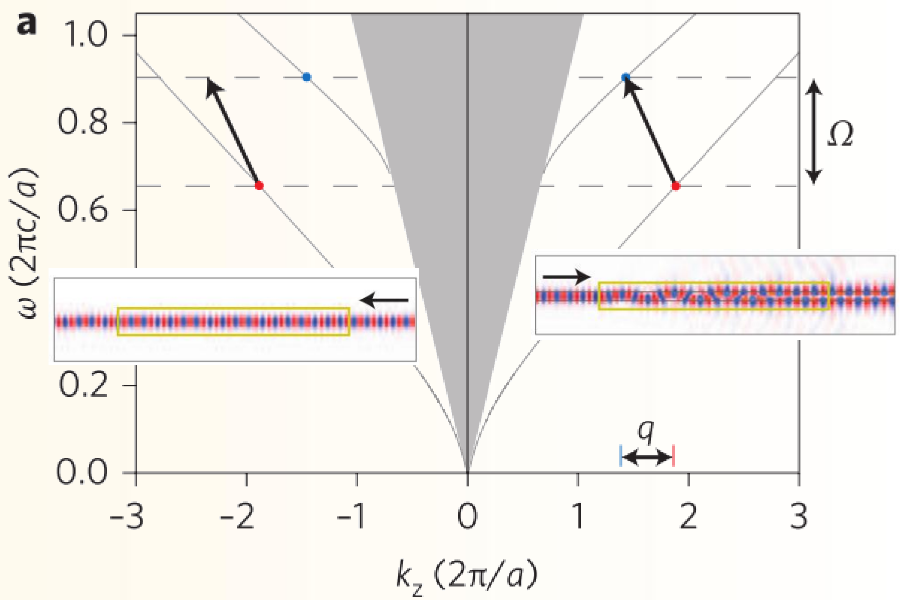






$$\Delta\varepsilon = \Delta\varepsilon_m \cos(\Omega t + qz)$$



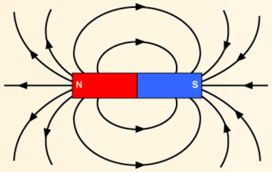


- ❑ **Very large volumes**
- ❑ **Non-uniform modulation across the waveguide cross-section**

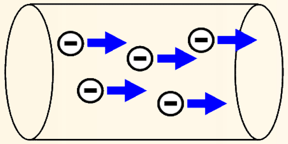
Yu, Z. & Fan, S. *Nature Photonics* 3, 91-94 (2009)

ANGULAR-MOMENTUM BIASING?

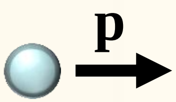
Magnetic Field



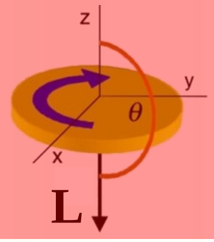
Direct current



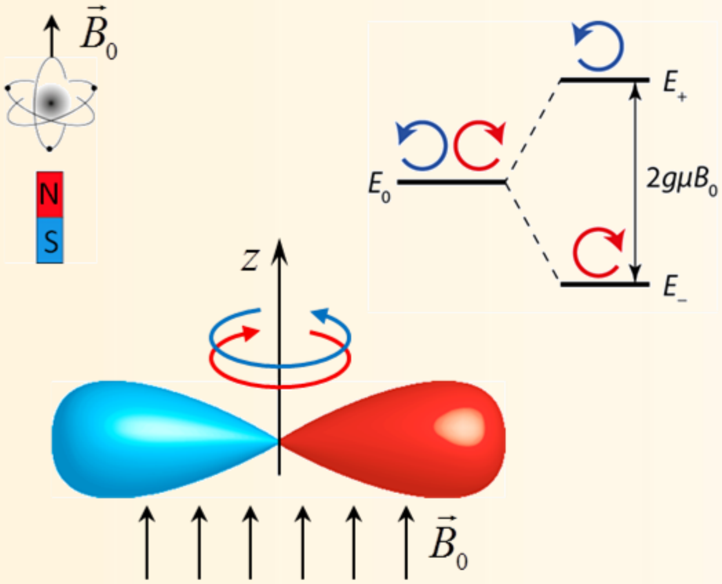
Linear Momentum

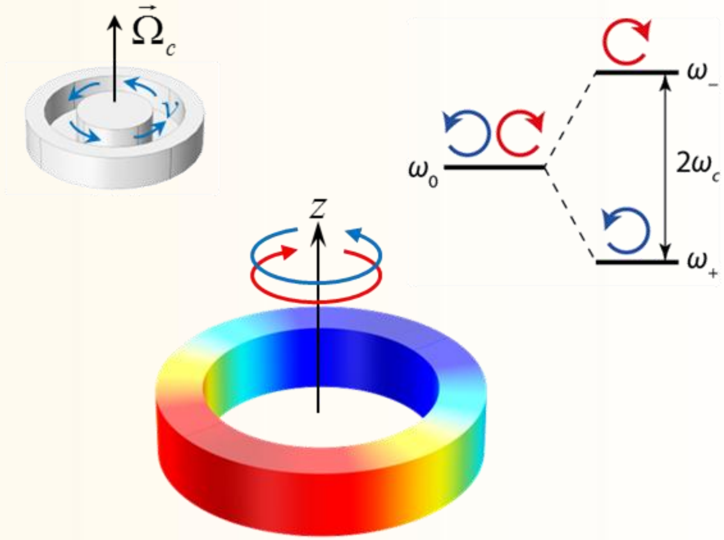


Angular Momentum



Angular-Momentum Biasing: a Zeeman Meta-Molecule





An Acoustic Ring Cavity: Mode Splitting

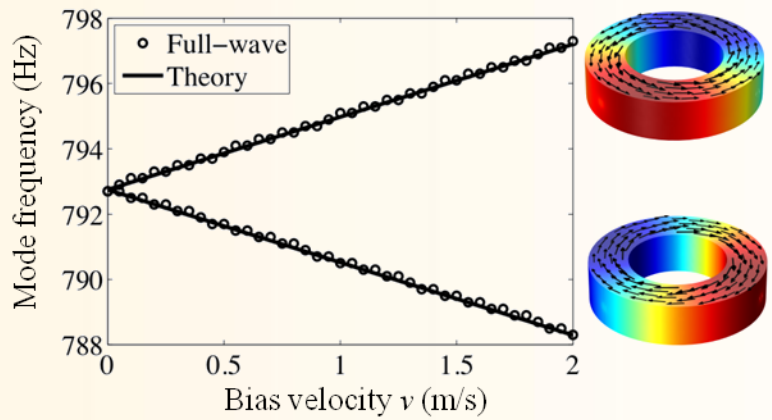
$$|\psi\rangle = \begin{pmatrix} \hat{p} \\ -i\hat{\phi} \end{pmatrix}$$

$$(H_0 + P)|\psi\rangle = \omega|\psi\rangle$$

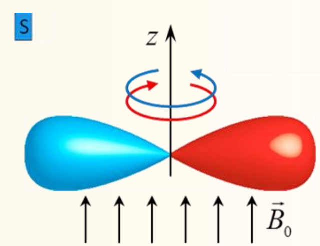
$$H_0 = \begin{pmatrix} 0 & c_0^2\vec{\nabla}\cdot\left(\rho_0\vec{\nabla}\right) \\ -1/\rho_0 & 0 \end{pmatrix}$$

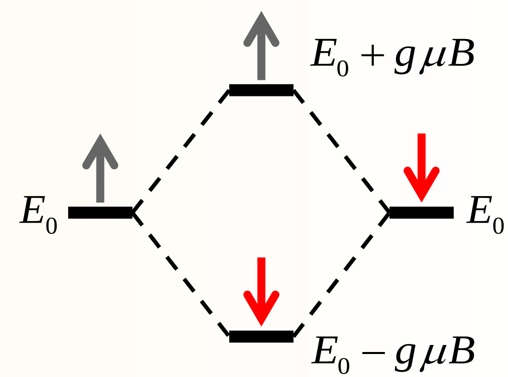
$$P = \begin{pmatrix} -i\left(\vec{v}\cdot\vec{\nabla}\right)-\vec{\nabla}\cdot(i\vec{v}) & 0 \\ 0 & -i\left(\vec{v}\cdot\vec{\nabla}\right) \end{pmatrix}$$

$$\omega^{\pm} = \omega_0 \pm \langle + | P | + \rangle$$

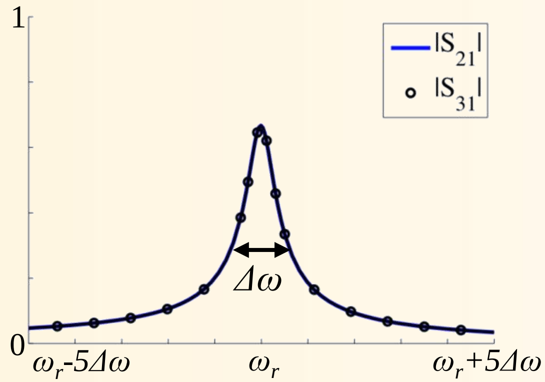


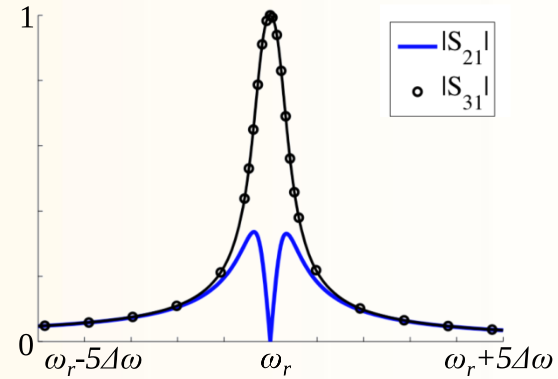
$$\omega_m^{\pm} = \omega_m \pm \frac{mv}{R_{av}}$$

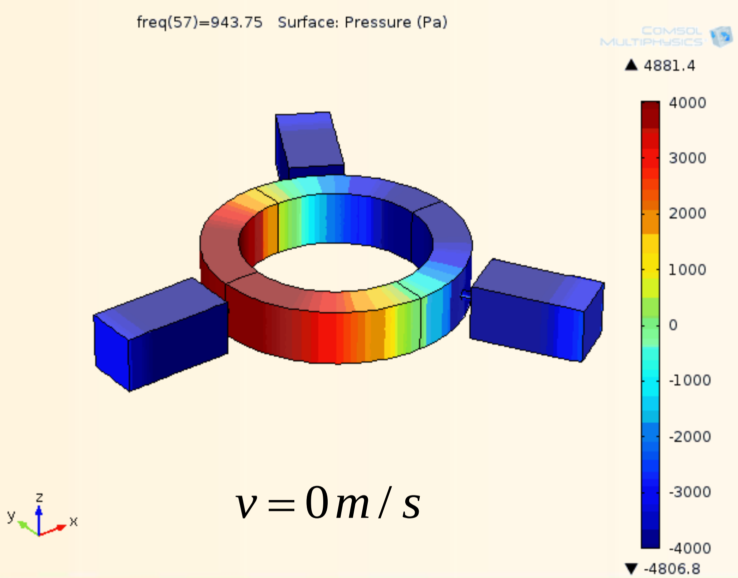




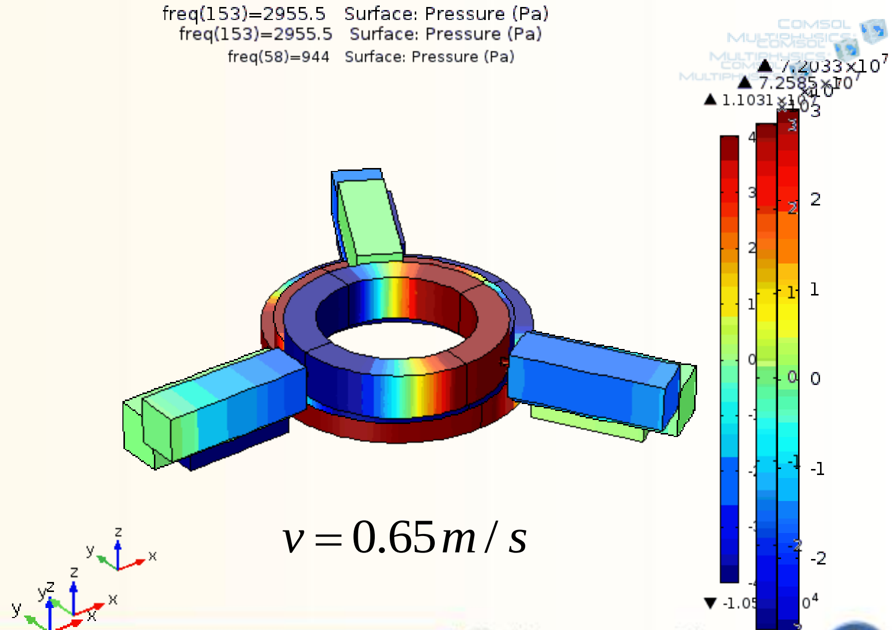
An Acoustic Circulator!





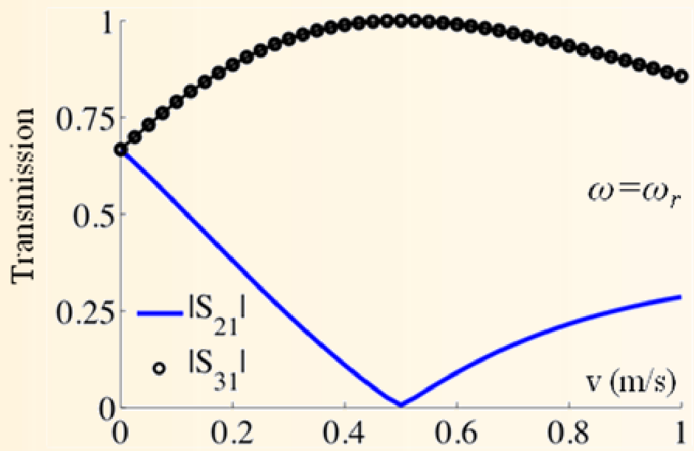


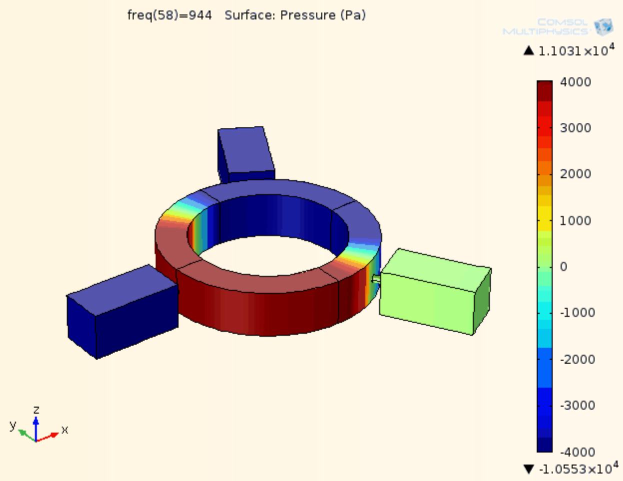
$v = 0\, m/s$



$v = 0.65\, m/s$

Mode Evolution





$$T_{1\to 2} = \left| \frac{2}{3} \left(\frac{e^{-i4\pi/3}}{1 - i(\omega - \omega^-)/\gamma^-} + \frac{e^{-i2\pi/3}}{1 - i(\omega - \omega^+)/\gamma^+} \right) \right|^2$$

$$T_{1\to 3} = \left| \frac{2}{3} \left(\frac{e^{-i2\pi/3}}{1 - i(\omega - \omega^-)/\gamma^-} + \frac{e^{-i4\pi/3}}{1 - i(\omega - \omega^+)/\gamma^+} \right) \right|^2$$

$$\downarrow$$

$$\omega^\pm = \omega_r \pm \gamma / \sqrt{3}$$

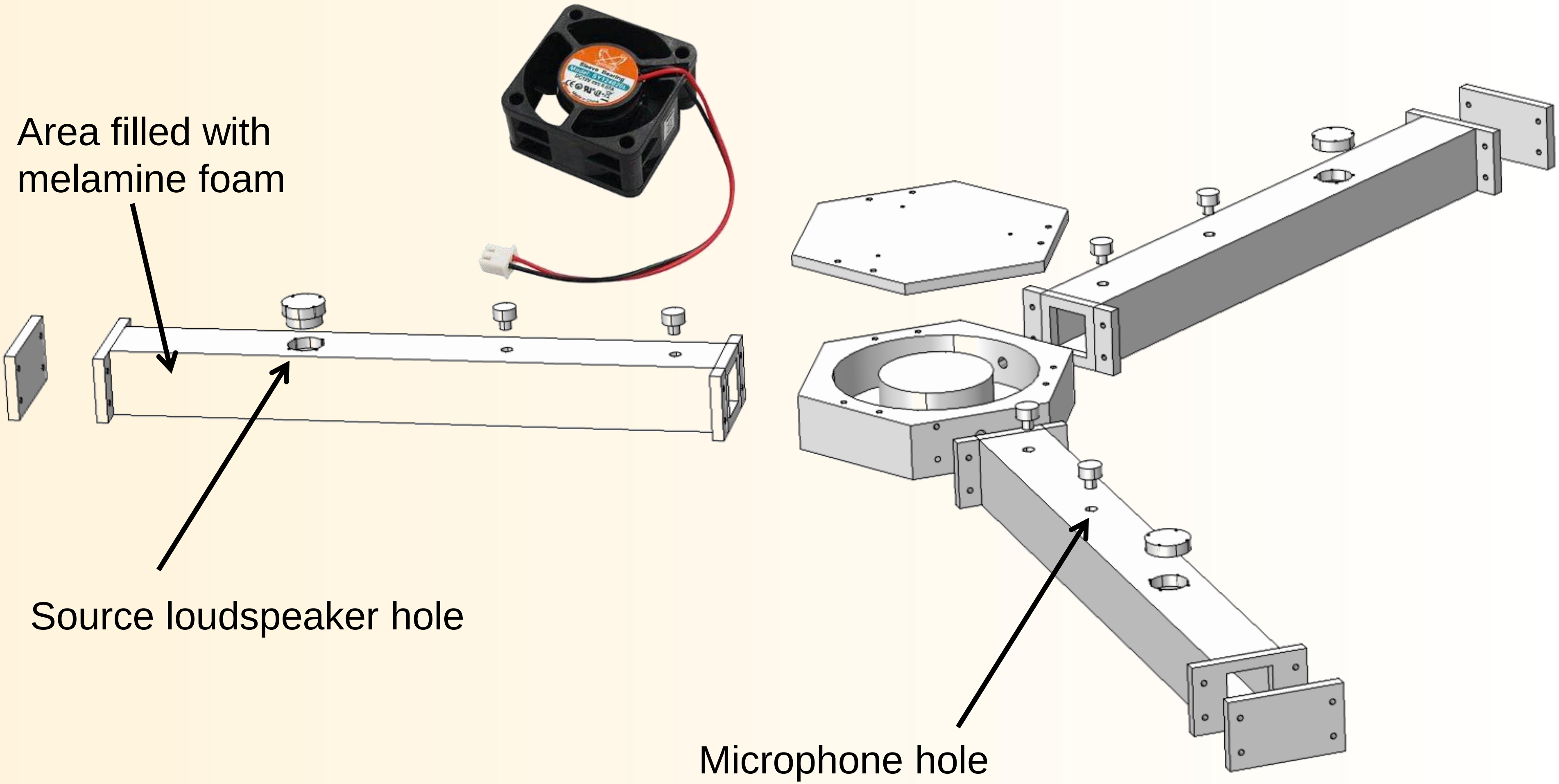
$$v_{opt} = \gamma R_{av} / \sqrt{3}$$

$$\omega_0 = c_0 / R_{av} \qquad Q = \omega_0 / (2\gamma)$$

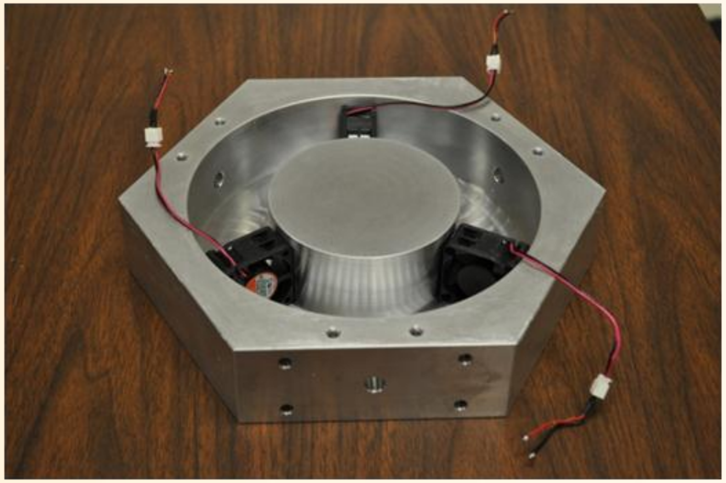
$$\boxed{v_{opt} / c_0 = 1 / \left(2Q\sqrt{3}\right)}$$

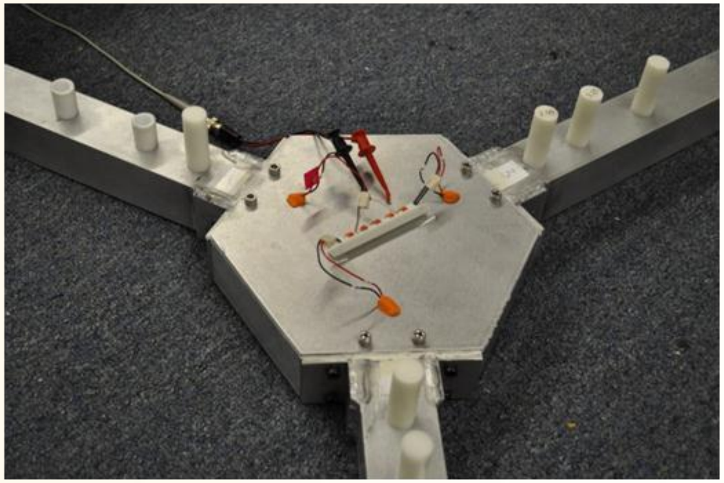
R. Fleury, D. L. Sounas, C. Sieck, M. Haberman, A. Alù, *Science* in press (2014)

An Acoustic Circulator



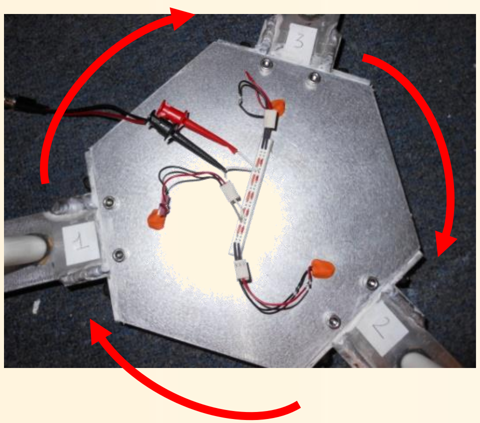
An Acoustic Circulator

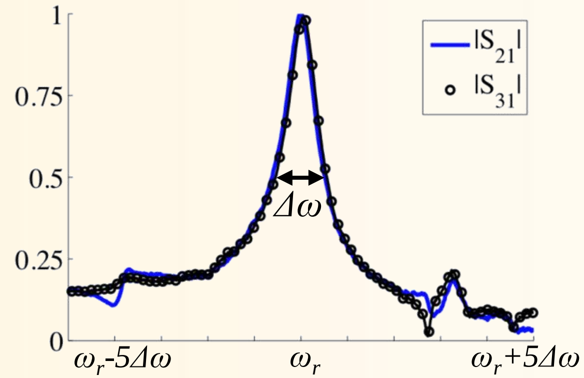


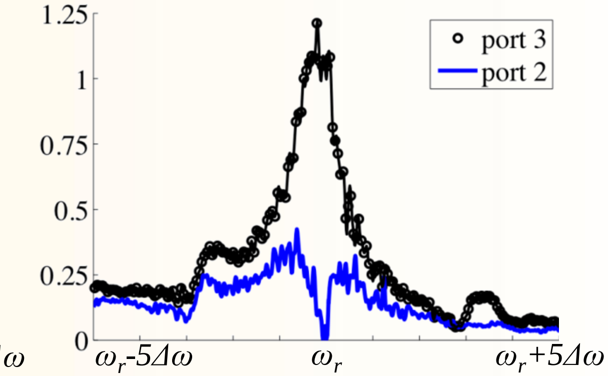


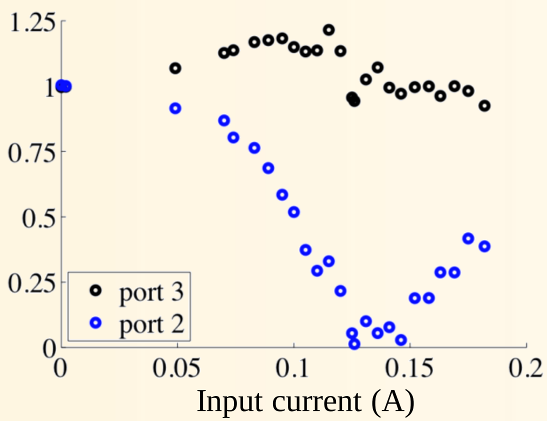
R. Fleury, D. L. Sounas, C. Sieck, M. Haberman, A. Alù, *Science* in press (2014)

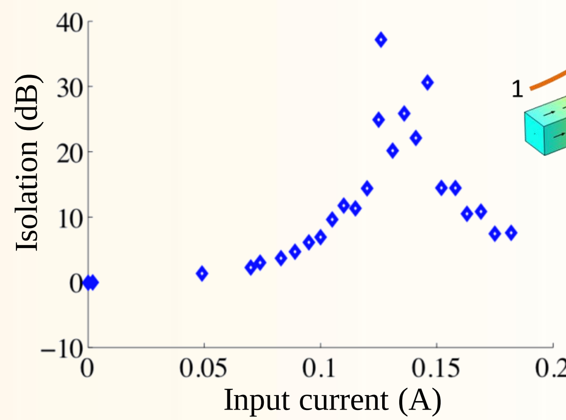
An Acoustic Circulator

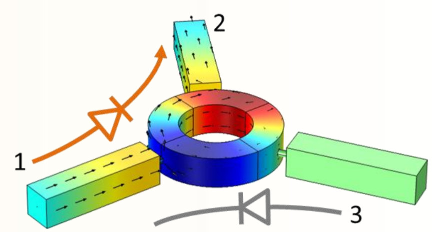








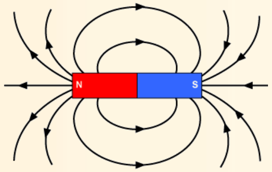




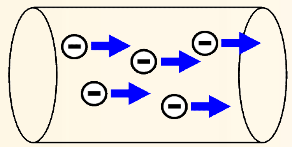
R. Fleury, D. L. Sounas, C. Sieck, M. Haberman, A. Alù, *Science* in press (2014)

How About EM Waves? Azimuthal Spatio-Temporal Modulation

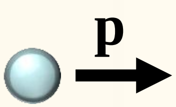
Magnetic Field



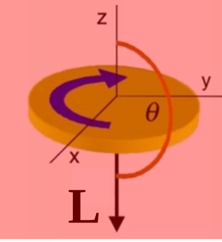
Direct current



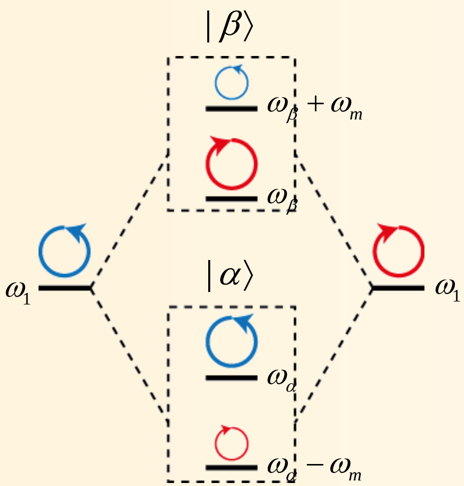
Linear Momentum

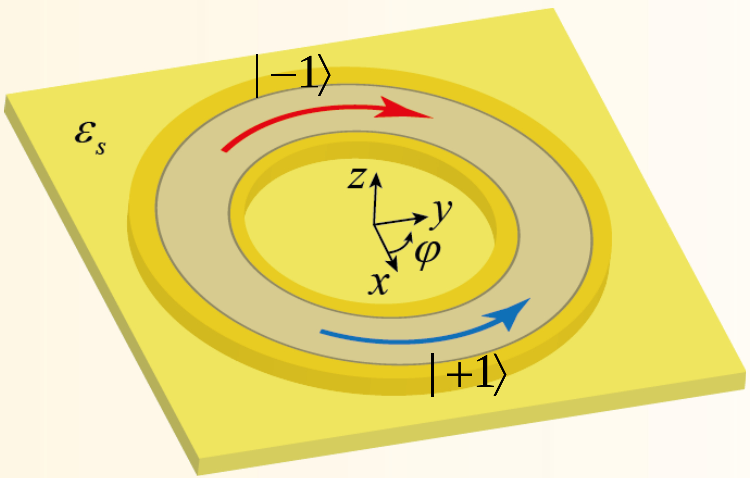


Angular Momentum

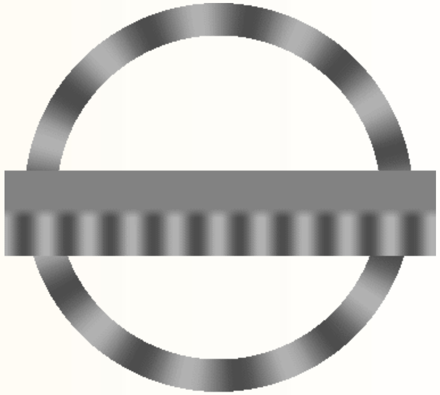


Effective Zeeman Effect





$$\Delta\varepsilon(\varphi,t)=\Delta\varepsilon_m\cos(\omega_m t - L_m\varphi)$$

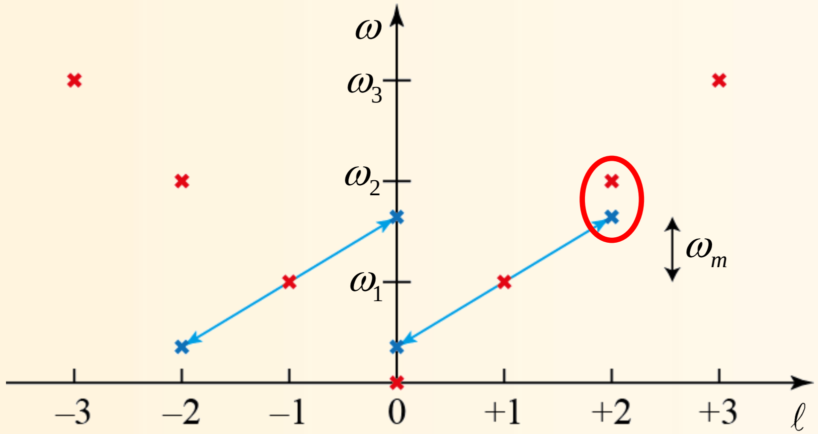


- ❑ Strong response & small footprint
- ❑ Microwaves to visible
- ❑ Weak modulation
- ❑ Moderate bandwidth

Sounas, D. L., Caloz, C. & Alù, A.
Nature Communications 4, No. 2407 (2013)

Optimal Choice of Azimuthal Spatio-Temporal Modulation

$$\Delta\varepsilon(\varphi,t)=\Delta\varepsilon_m\cos(\omega_m t-1\varphi)$$

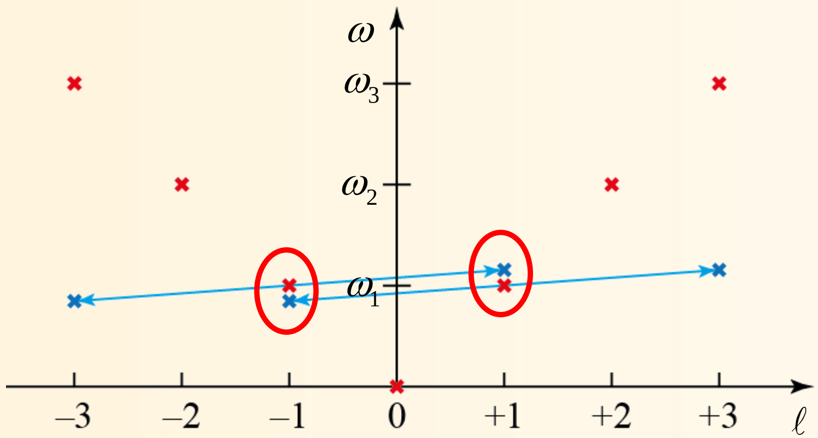


$$\omega_m=\omega_1$$

⇩

- ❑ Doable in microwaves
- ❑ **Impossible in optics!**

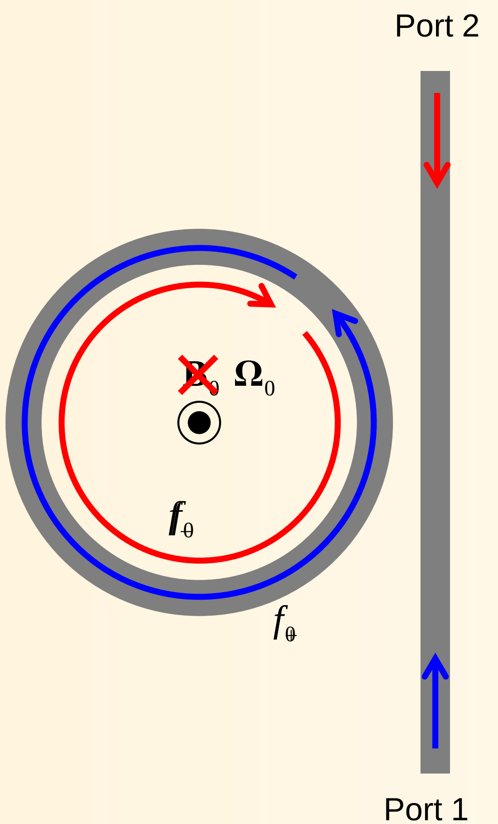
$$\Delta\varepsilon(\varphi,t)=\Delta\varepsilon_m\cos(\omega_m t-2\varphi)$$

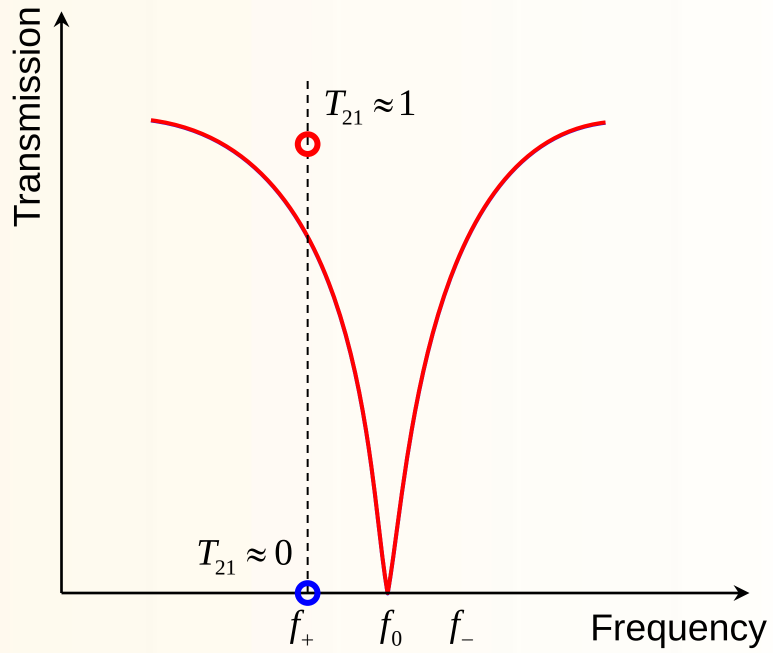


$$\omega_m \approx 0$$

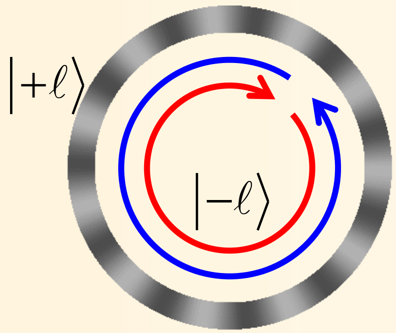
- ❑ Almost zero modulation can produce strong non-reciprocity!
- ❑ **Applicable over the entire EM spectrum**

Optical Isolator Principle





ST-Modulated Ring – Coupled-Mode Equations



$$\dot{a}_k = -i\omega_k a_k + i\omega_k \sum_\ell \kappa_{k\ell} a_\ell \qquad \kappa_{k\ell} = \int d\mathbf{r}^3 \Delta\varepsilon(\mathbf{r},t)\mathbf{E}_k^* \cdot \mathbf{E}_\ell$$

$$\int d\mathbf{r}^3 \left(\varepsilon \left|\mathbf{E}_k\right|^2 + \mu_0 \left|\mathbf{H}_k\right|^2\right) = 1 \quad \Rightarrow \quad \int d\mathbf{r}^3 \varepsilon \left|\mathbf{E}_k\right|^2 = \frac{1}{2}$$

$$\Delta\varepsilon(\rho,\varphi,z) = \Delta\varepsilon_{\mathrm{m}}(\rho,z)\cos(\omega_{\mathrm{m}} t - L_{\mathrm{m}}\varphi)$$

$$L_{\mathrm{m}} = 2\ell$$

$$\mathbf{E}_{\pm\ell}(\rho,\phi,z) = \mathbf{e}_\ell(\rho,z)e^{\pm i\ell\varphi} \quad \Rightarrow \quad 2\pi\int \varepsilon\left|\mathbf{e}_\ell\right|^2 \rho d\rho dz = \frac{1}{2}$$

$$\kappa_{+\ell,-\ell} = \kappa^*_{-\ell,+\ell} = \pi e^{-i\omega_{\mathrm{m}} t}\int \Delta\varepsilon_{\mathrm{m}}\left|\mathbf{e}_\ell\right|^2 \rho d\rho dz = \frac{1}{4}\frac{\int \Delta\varepsilon_{\mathrm{m}}\left|\mathbf{e}_\ell\right|^2 \rho d\rho dz}{\int \varepsilon\left|\mathbf{e}_\ell\right|^2 \rho d\rho dz} e^{-i\omega_{\mathrm{m}} t} \approx \frac{\Delta\varepsilon_{\mathrm{m}}}{4\varepsilon} e^{-i\omega_{\mathrm{m}} t}$$

$$\boxed{\begin{aligned} \dot{a}_{+\ell} &= -i\omega_\ell a_{+\ell} + i\frac{1}{2}\omega_\ell \kappa_\ell a_{-\ell} e^{-i\omega_{\mathrm{m}} t} \\ \dot{a}_{-\ell} &= -i\omega_\ell a_{-\ell} + i\frac{1}{2}\omega_\ell \kappa_\ell a_{+\ell} e^{i\omega_{\mathrm{m}} t} \\ \kappa_\ell &= 2\kappa_{+\ell,-\ell} e^{i\omega_{\mathrm{m}} t} \end{aligned}} \quad \Rightarrow$$

$$\left|\alpha\right\rangle = \left|+\ell\right\rangle e^{-i\omega_\alpha t} + \frac{\Delta\omega}{\omega_\ell \kappa_\ell}\left|-\ell\right\rangle e^{-i(\omega_\alpha - \omega_{\mathrm{m}})t}$$

$$\left|\beta\right\rangle = \left|-\ell\right\rangle e^{-i\omega_\beta t} - \frac{\Delta\omega}{\omega_\ell \kappa_\ell}\left|+\ell\right\rangle e^{-i(\omega_\beta + \omega_{\mathrm{m}})t}$$

$$\omega_\alpha = \omega_\ell - \Delta\omega/2$$

$$\omega_\beta = \omega_\ell + \Delta\omega/2$$

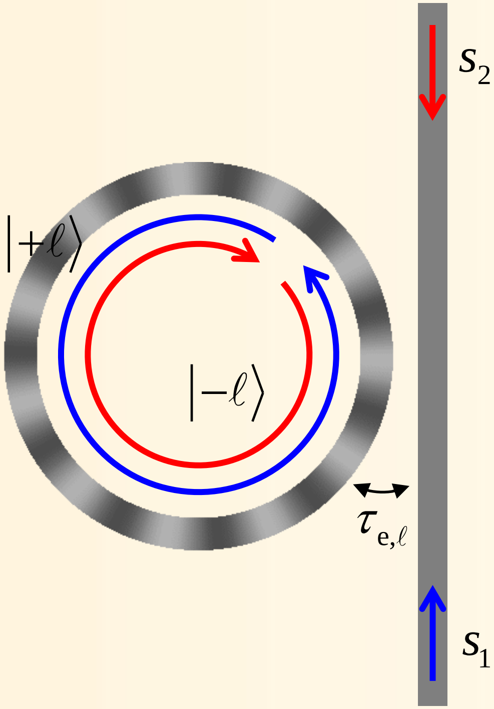
$$\Delta\omega = \sqrt{\omega_{\mathrm{m}}^2 + \omega_\ell^2\kappa_\ell^2} - \omega_{\mathrm{m}}$$

ST-Modulated Ring – Loss + Excitation

Loss $\Rightarrow$ $-i\omega_\ell \rightarrow -i\omega_\ell - \tau_\ell^{-1}$ $\Rightarrow$

$$\dot{a}_{+\ell} = (-i\omega_\ell - \tau_\ell^{-1})a_{+\ell} + i\frac{1}{2}\omega_\ell \kappa_\ell a_{-\ell} e^{-i\omega_{\mathrm{m}} t}$$

$$\dot{a}_{-\ell} = (-i\omega_\ell - \tau_\ell^{-1})a_{-\ell} + i\frac{1}{2}\omega_\ell \kappa_\ell a_{+\ell} e^{i\omega_{\mathrm{m}} t}$$

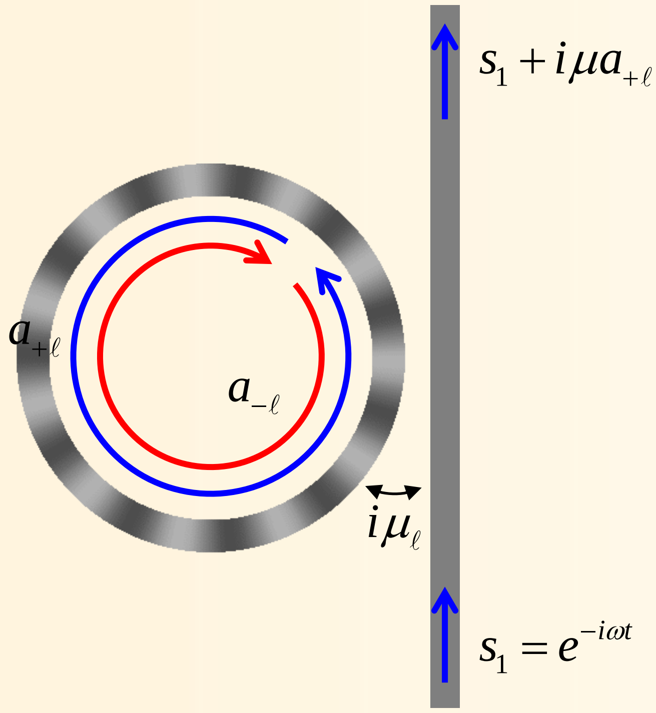


$$\dot{a}_{+\ell} = (-i\omega_\ell - \tau_\ell^{-1})a_{+\ell} + i\frac{1}{2}\omega_\ell \kappa_\ell a_{-\ell} e^{-i\omega_{\mathrm{m}} t} + i\mu_\ell s_1$$

$$\dot{a}_{-\ell} = (-i\omega_\ell - \tau_\ell^{-1})a_{-\ell} + i\frac{1}{2}\omega_\ell \kappa_\ell a_{+\ell} e^{i\omega_{\mathrm{m}} t} + i\mu_\ell s_2$$

$$\mu_\ell^2 = \frac{2}{\tau_{\mathrm{e},\ell}}$$

Transmission Coefficients



$$\dot{a}_{+\ell} = (-i\omega_\ell - \tau_\ell^{-1})a_{+\ell} + i\frac{1}{2}\omega_\ell\kappa_\ell a_{-\ell}e^{-i\omega_{\mathrm{m}}t} + i\mu_\ell s_1$$

$$\dot{a}_{-\ell} = (-i\omega_\ell - \tau_\ell^{-1})a_{-\ell} + i\frac{1}{2}\omega_\ell\kappa_\ell a_{+\ell}e^{i\omega_{\mathrm{m}}t}$$

$$a_{+\ell} = -\frac{\mu_\ell(\omega - \omega_{\mathrm{m}} - \omega_\ell + i\tau_\ell^{-1})}{(\omega - \omega_\ell + i\tau_\ell^{-1})(\omega - \omega_{\mathrm{m}} - \omega_\ell + i\tau_\ell^{-1}) - \omega_\ell^2\kappa_\ell^2/4}e^{i\omega t}$$

$$T_{21} = \frac{(\omega - \omega_\ell + i\tau_\ell^{-1} - i2\tau_{\mathrm{e},\ell}^{-1})(\omega - \omega_{\mathrm{m}} - \omega_\ell + i\tau_\ell^{-1}) - \omega_\ell^2\kappa_\ell^2/4}{(\omega - \omega_\ell + i\tau_\ell^{-1})(\omega - \omega_{\mathrm{m}} - \omega_\ell + i\tau_\ell^{-1}) - \omega_\ell^2\kappa_\ell^2/4}$$

Zero Transmission Condition

$$(\omega-\omega_\ell)(\omega-\omega_m-\omega_\ell)-\tau_\ell^{-1}(\tau_\ell^{-1}-2\tau_{e,\ell}^{-1})-\omega_\ell^2\kappa_\ell^2/4=0$$

$$(\omega-\omega_\ell)(\omega-\omega_m-\omega_\ell)-\omega_\ell^2\kappa_\ell^2/4\cong 0$$

$T_{21}=0$

$$\boxed{\omega=\omega_\alpha=\omega_\ell-\Delta\omega/2}$$

$$\omega=\omega_\beta+\omega_m=\omega_\ell+\Delta\omega/2+\omega_m$$

$$\tau_\ell^{-1}(\omega-\omega_\ell)+(\tau_\ell^{-1}-2\tau_{e,\ell}^{-1})(\omega-\omega_m-\omega_\ell)=0$$

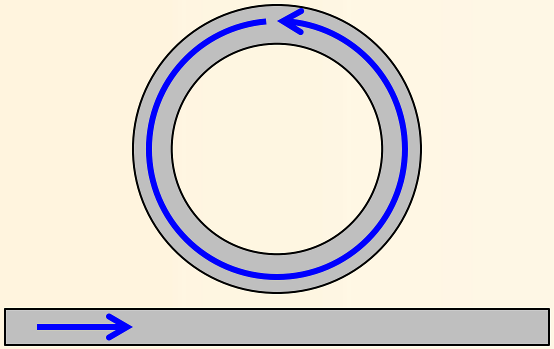
$\omega=\omega_\alpha$

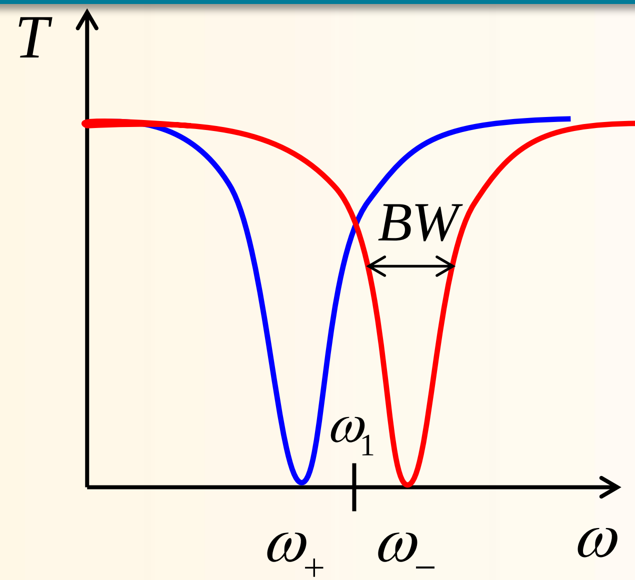
$$\boxed{\tau_\ell=\tau_{e,\ell}\frac{\Delta\omega+\omega_m}{\Delta\omega+2\omega_m}}$$

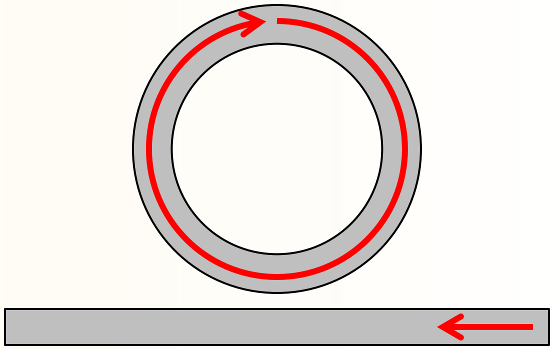
$\omega=\omega_\beta$

No solution

Bandwidth Requirements







$$\Delta\omega = \sqrt{\omega_m^2 + \omega_1^2 C_m^2} - \omega_m$$

$$\Delta\omega_{\max} = \frac{\omega_1 C_m}{\sqrt{3}}$$

$$C_m \sim \Delta\varepsilon_m$$

$$Q = \frac{\omega_1}{2BW}$$

$$\Delta\omega = \omega_- - \omega_+ = BW$$

$$\frac{\omega_1 C_m}{\sqrt{3}} = BW$$

$$\Delta\varepsilon_m \sim \frac{BW\sqrt{3}}{\omega_1} \sim \frac{1}{Q}$$

$$\boxed{Q\Delta\varepsilon_m \sim 1}$$

High Q allows weak modulation

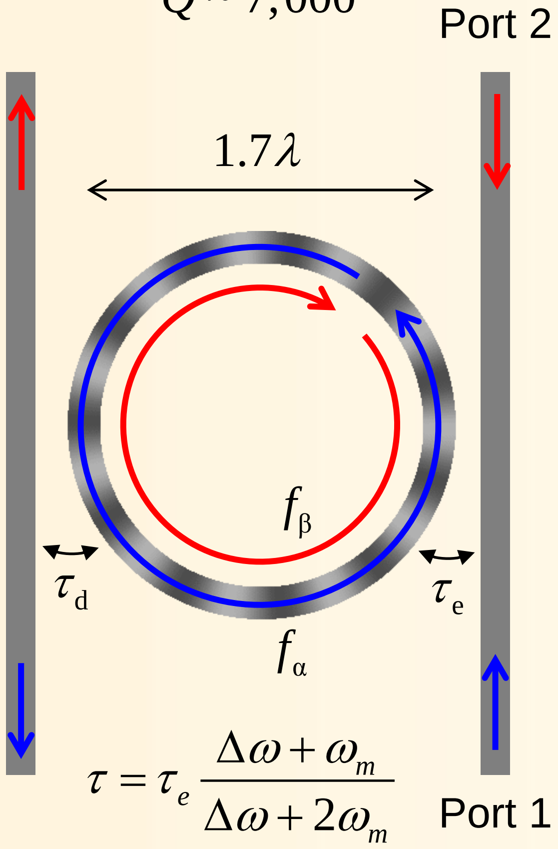
Ideal for RF as well as optics!

Angular-Momentum-Biased Optical Isolator

$$\Delta\varepsilon_m = 5\times10^{-4}\varepsilon \ !$$

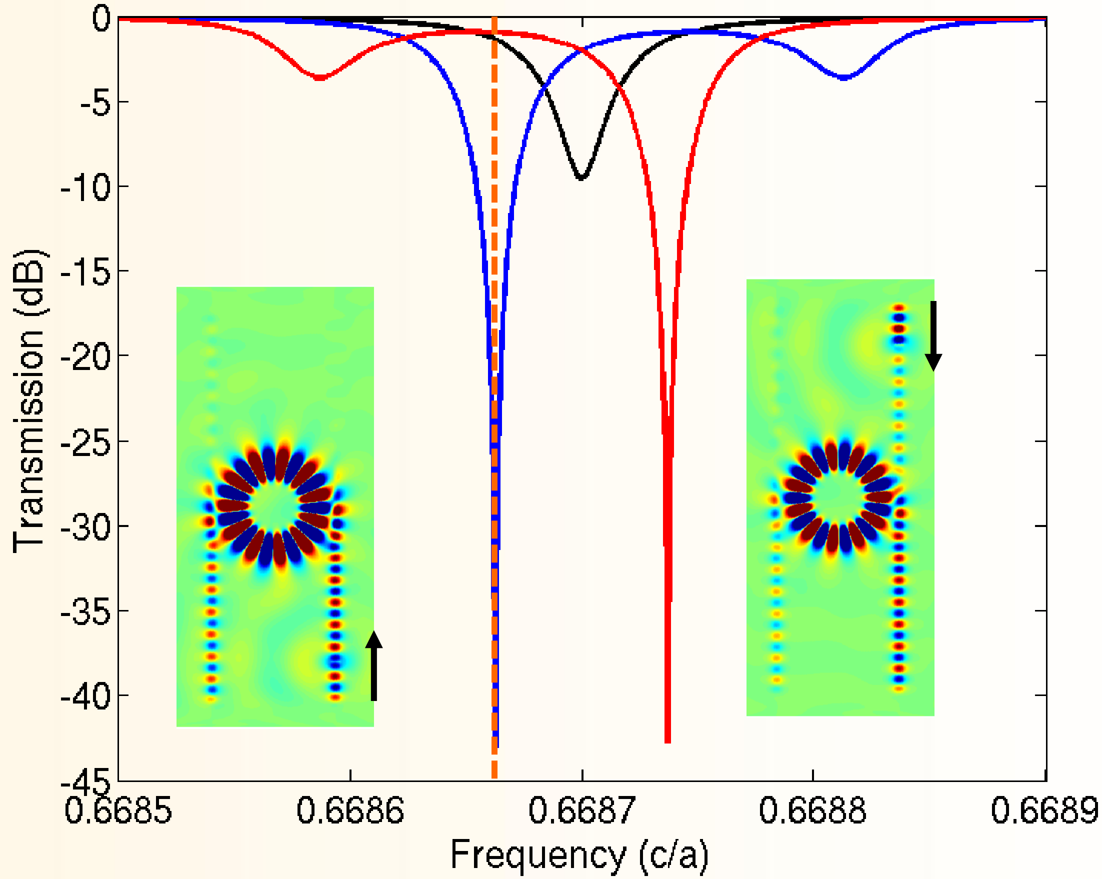
$$Q \sim 7,000$$

$$\boxed{Q\Delta\varepsilon_m \gg 1}$$

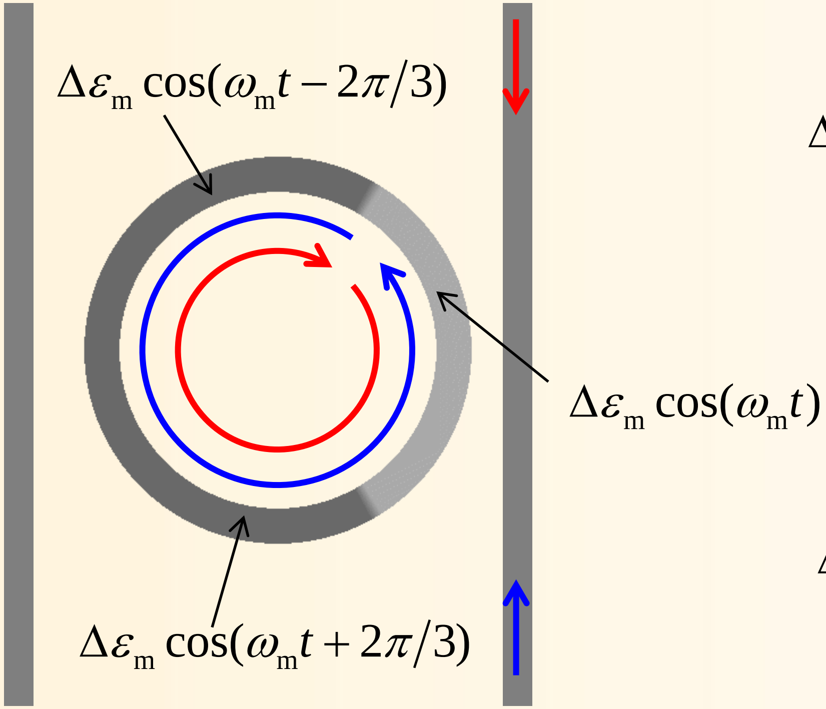


$$\tau = \tau_e \frac{\Delta\omega + \omega_m}{\Delta\omega + 2\omega_m}$$

$$\tau_d = \tau_e\left(1 + \frac{\Delta\omega}{\omega_m}\right)$$



ST Modulation Discretization: 3 Steps



$$\Delta\varepsilon=\Delta\varepsilon_{\mathrm{m}}\sum_{n=0}^{2}\cos[\omega_{\mathrm{m}}t-2(n-1)\pi/3]\mathrm{rect}\left(\frac{\varphi-\varphi_n}{\Delta\varphi}\right)$$

$$=\Delta\varepsilon_{\mathrm{m}}\sum_{k=-\infty}^{\infty}\mathrm{sinc}\left(\frac{3N+1}{3}\right)\cos\left[\omega_{\mathrm{m}}t-(3k+1)\varphi\right]$$

Poisson Summation Formula

$$\Delta\varepsilon=\Delta\varepsilon_{\mathrm{m}}\sum_{k=-\infty}^{\infty}\mathrm{sinc}\left(\frac{3N+1}{3}\right)\cos\left[\omega_{\mathrm{m}}t-(3k+1)\varphi\right]$$

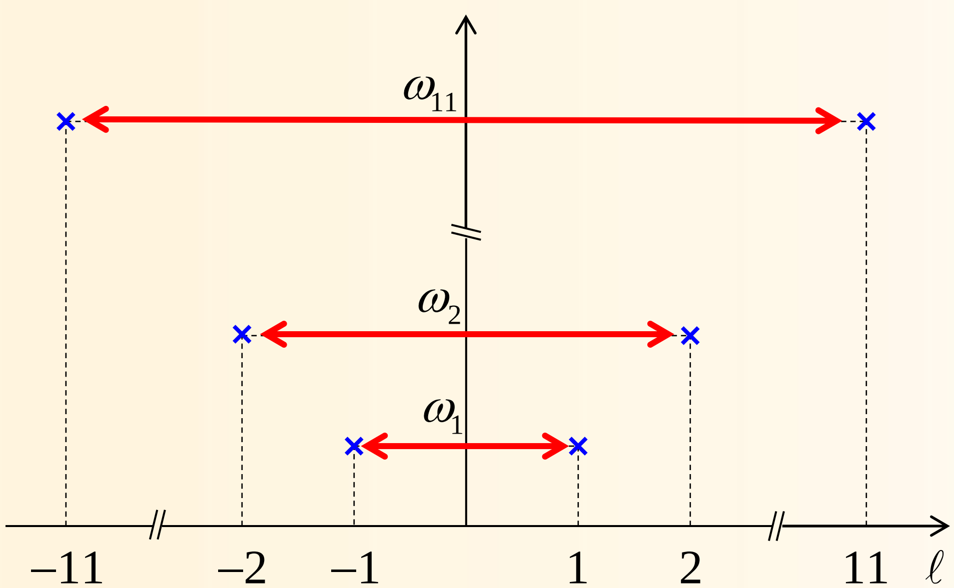
$$\Delta\varepsilon=\Delta\varepsilon_{\mathrm{m}}\,\mathrm{sinc}\left(\frac{1}{3}\right)\cos(\omega_{\mathrm{m}}t-\varphi)+\Delta\varepsilon_{\mathrm{m}}\,\mathrm{sinc}\left(\frac{4}{3}\right)\cos(\omega_{\mathrm{m}}t-4\varphi)+\ldots+\Delta\varepsilon_{\mathrm{m}}\,\mathrm{sinc}\left(\frac{22}{3}\right)\cos(\omega_{\mathrm{m}}t-22\varphi)+\ldots$$
$$+\Delta\varepsilon_{\mathrm{m}}\,\mathrm{sinc}\left(\frac{2}{3}\right)\cos(\omega_{\mathrm{m}}t+2\varphi)+\Delta\varepsilon_{\mathrm{m}}\,\mathrm{sinc}\left(\frac{5}{3}\right)\cos(\omega_{\mathrm{m}}t+5\varphi)+\ldots$$

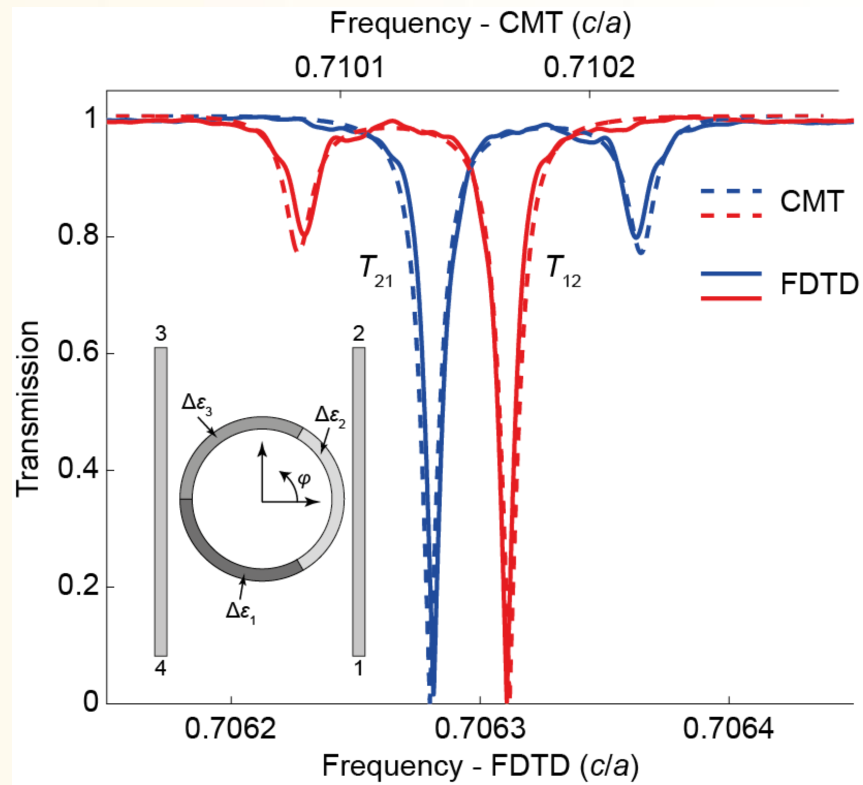
ST Modulation Discretization – 3 Steps

$$\Delta\varepsilon = \boxed{\Delta\varepsilon_{\mathrm{m}}\,\mathrm{sinc}\left(\frac{1}{3}\right)\cos(\omega_{\mathrm{m}}t-\varphi)} + \boxed{\Delta\varepsilon_{\mathrm{m}}\,\mathrm{sinc}\left(\frac{4}{3}\right)\cos(\omega_{\mathrm{m}}t-4\varphi)} + \ldots + \boxed{\Delta\varepsilon_{\mathrm{m}}\,\mathrm{sinc}\left(\frac{22}{3}\right)\cos(\omega_{\mathrm{m}}t-22\varphi)} + \ldots$$

$$+ \boxed{\Delta\varepsilon_{\mathrm{m}}\,\mathrm{sinc}\left(\frac{2}{3}\right)\cos(\omega_{\mathrm{m}}t+2\varphi)} + \boxed{\Delta\varepsilon_{\mathrm{m}}\,\mathrm{sinc}\left(\frac{5}{3}\right)\cos(\omega_{\mathrm{m}}t+5\varphi)} + \ldots$$

$\omega_{\mathrm{m}} \ll \omega_1$



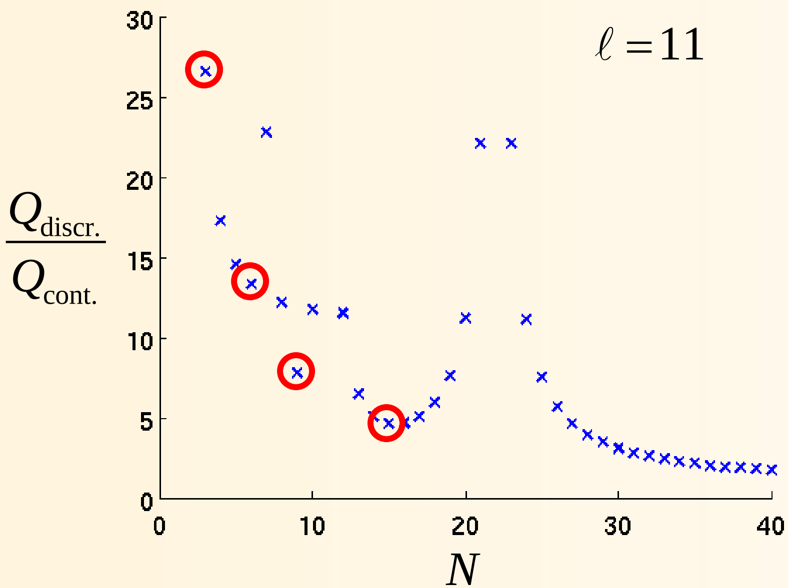


ST Modulation Discretization – N Steps

$$\Delta\varepsilon = \Delta\varepsilon_{\mathrm{m}} \sum_{n=0}^{N-1} \cos[\omega_{\mathrm{m}} t - L_{\mathrm{m}}\varphi_{\mathrm{n}}]\,\mathrm{rect}\left(\frac{\varphi-\varphi_n}{\Delta\varphi}\right)$$

$$= \Delta\varepsilon_{\mathrm{m}} \sum_{k=-\infty}^{\infty} (-1)^{k(N-1)}\,\mathrm{sinc}\left(\frac{L_{\mathrm{m}}+kN}{N}\right) \cos\left[\omega_{\mathrm{m}} t - \underbrace{(L_{\mathrm{m}}+kN)}_{L_{\mathrm{m,eff}}}\varphi\right]$$

ℓ-th pair $\Rightarrow$ $\left|L_{\mathrm{m,eff}}\right| = \left|L_{\mathrm{m}}+kN\right| = 2\ell$ $\Rightarrow$ $\Delta\varepsilon_{\mathrm{eff}} = \Delta\varepsilon_{\mathrm{m}}\,\mathrm{sinc}\left(\frac{2\ell}{N}\right)$ $\Rightarrow$ $Q_{\mathrm{discr.}} = \dfrac{Q_{\mathrm{cont.}}}{\mathrm{sinc}\left(\frac{2\ell}{N}\right)}$



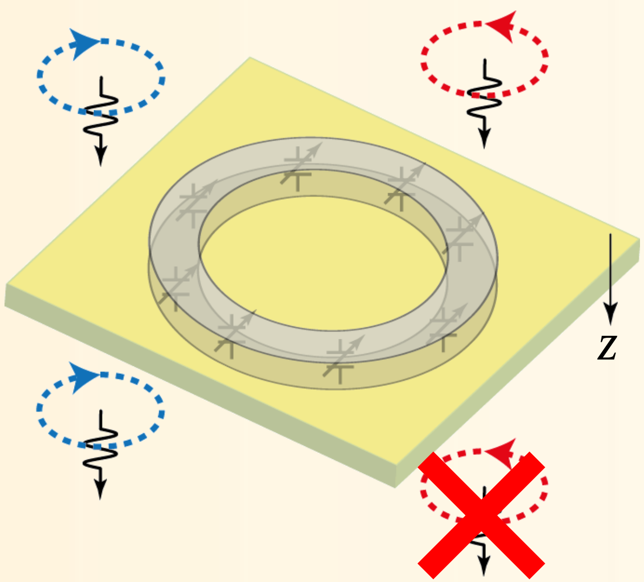
$N = 3$ $\quad Q_{\mathrm{discr.}} \approx 26 Q_{\mathrm{cont.}}$

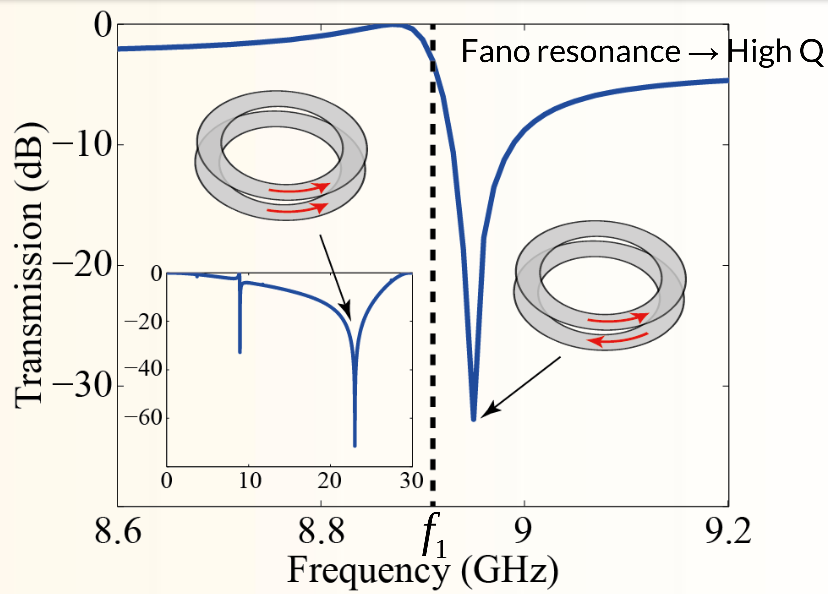
$N = 6$ $\quad Q_{\mathrm{discr.}} \approx 14 Q_{\mathrm{cont.}}$

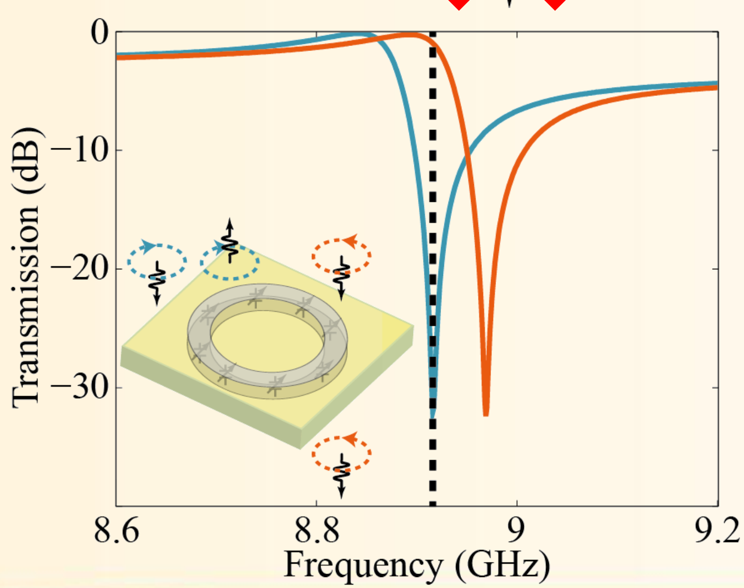
$N = 9$ $\quad Q_{\mathrm{discr.}} \approx 7 Q_{\mathrm{cont.}}$

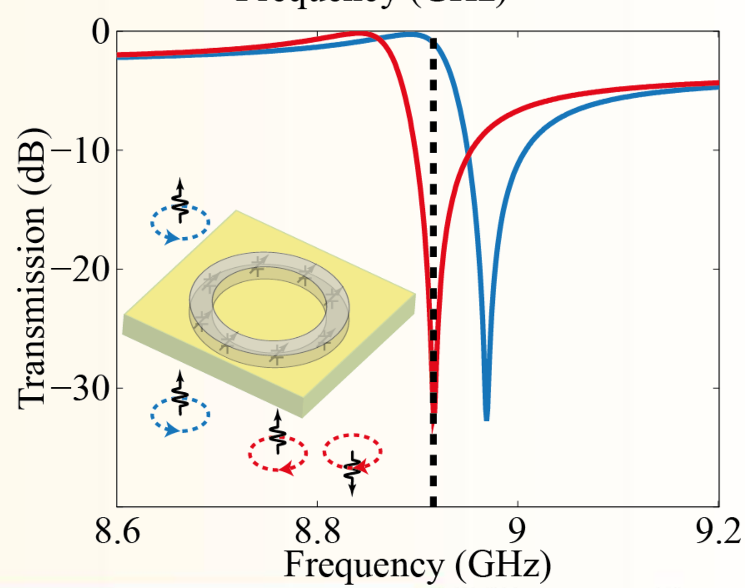
$N = 15$ $\quad Q_{\mathrm{discr.}} \approx 5 Q_{\mathrm{cont.}}$

RF Metasurface Isolator for Circularly Polarized Waves

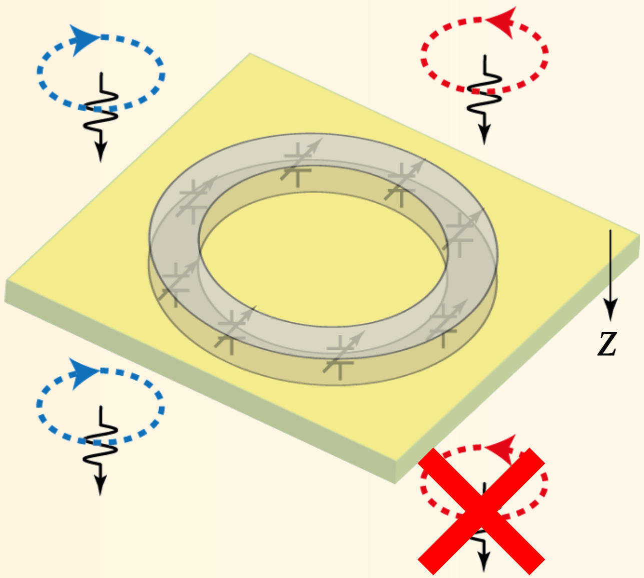




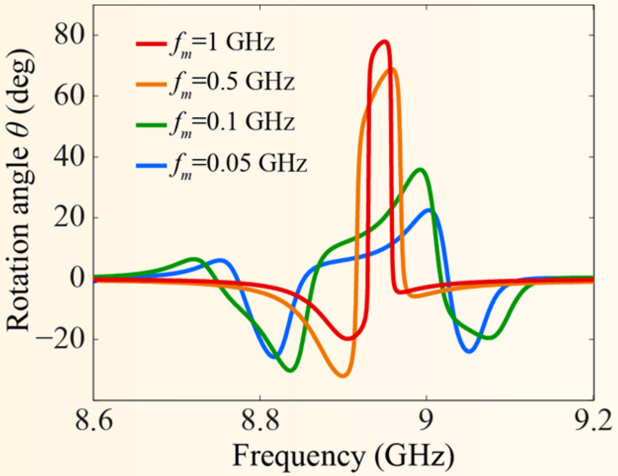




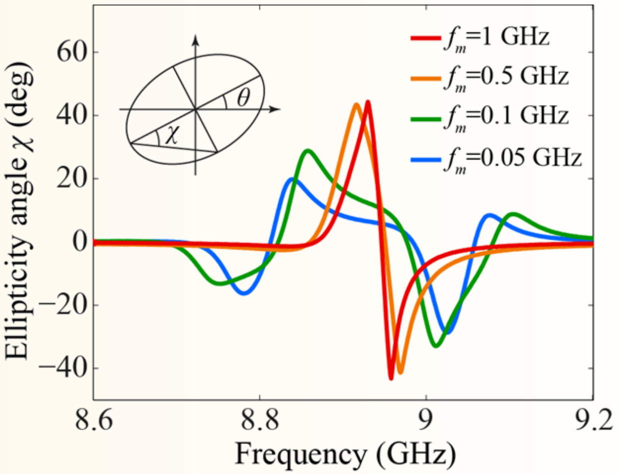
RF Isolator for Circularly Polarized Waves



6000º/wavelength !

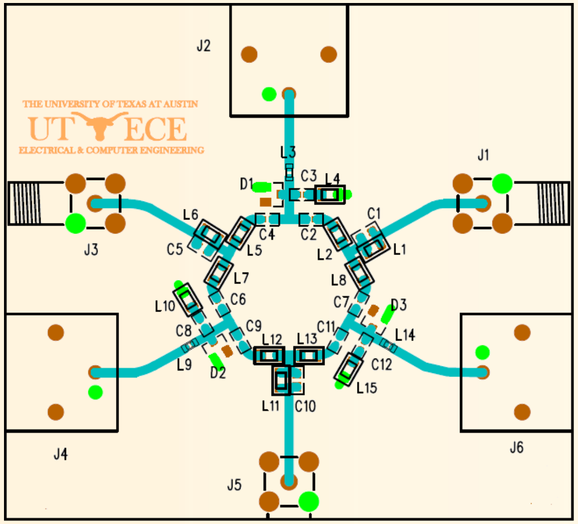


(a)

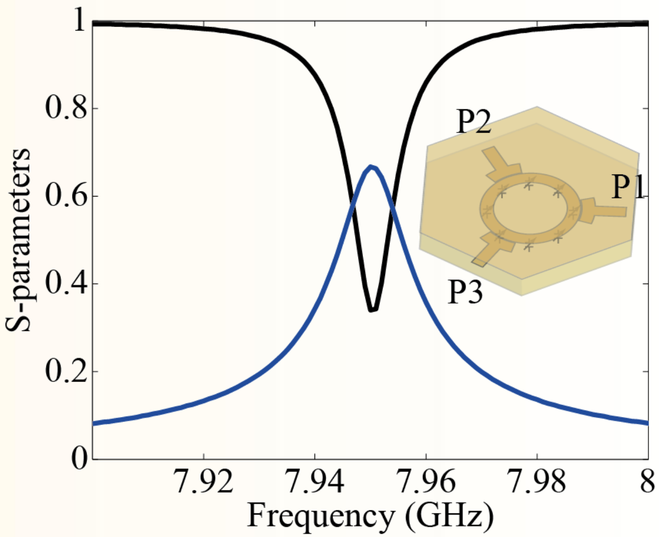


(b)

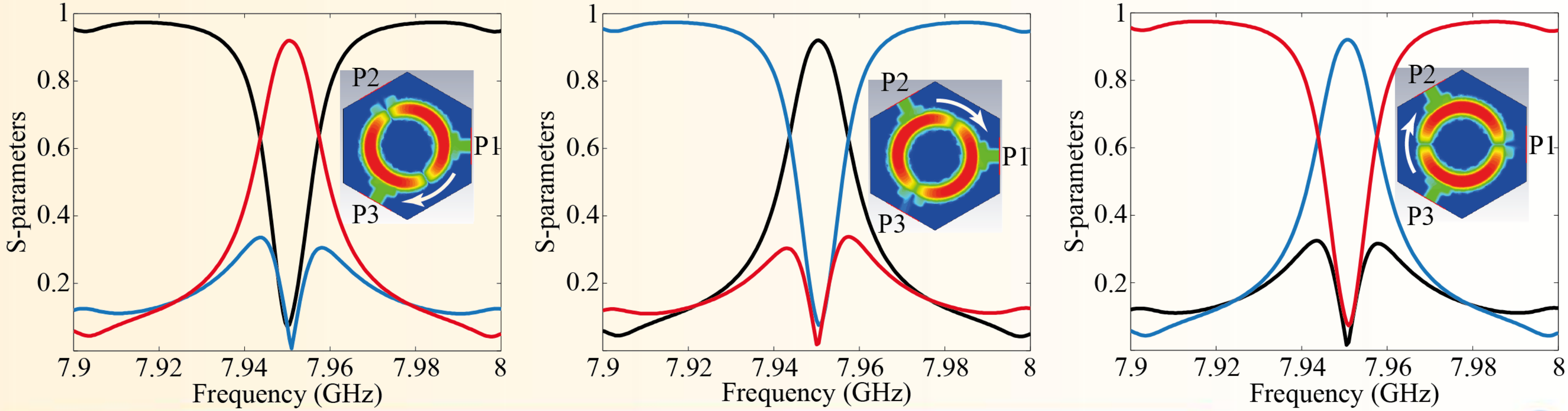
RF Magnet-Less Integrated Circulator



Without modulation

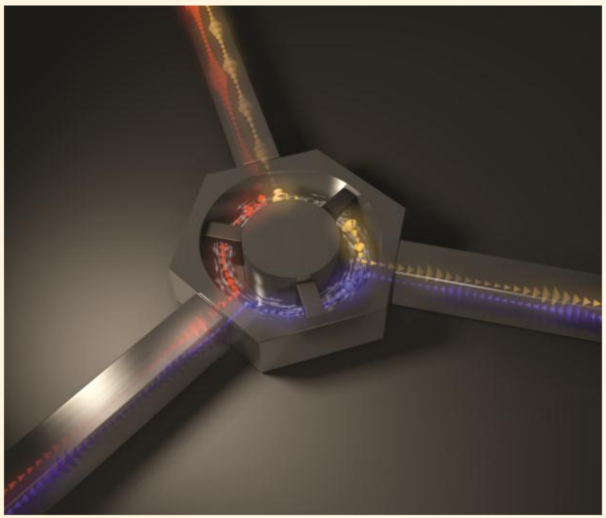


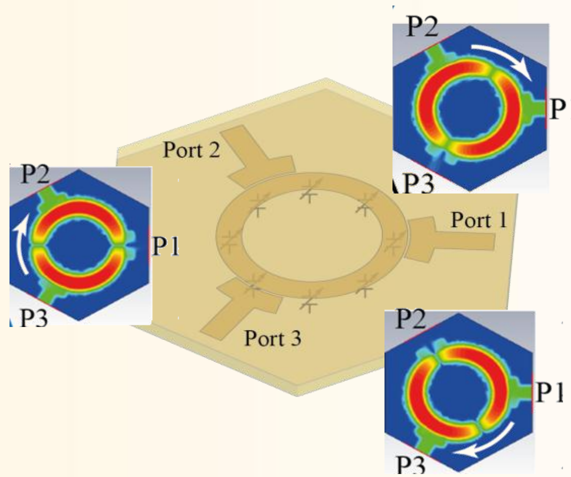
With modulation

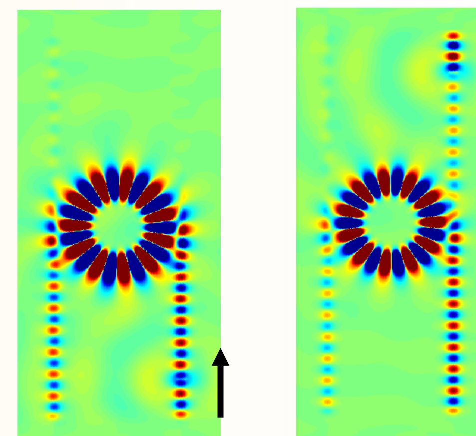


Non-Reciprocal Angular-Momentum Biased Metamaterials

Magnetic-free, linear, giant nonreciprocity at the subwavelength scale: angular-momentum biased meta-atoms, coupled to a circularly-symmetric high-Q resonance







D. L. Sounas, C. Caloz, and A. Alù, *Nature Communications* 4, No. 2407 (2013)
R. Fleury, D. L. Sounas, C. Sieck, M. Haberman, A. Alù, *Science* in press (2014)
D. L Sounas, A. Alù, *ACS Photonics* under review (2014)