Convexification of Motion Planning and Control through Liftings and Hypercomplex Numbers

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Abstract

The goal of the proposed research is the development of a rigorous mathematical and algorithmic framework to explore the role of parameterization in convexifying complex motion planning problems for controlled, autonomous, dynamical systems. Specifically, we will examine how novel parameterizations of control, configuration space, and logical constraints, can effectively be utilized to develop optimization-based solution methods and real-time executable algorithms for complex motion planning problems.

Over the last year, we have focused on several research directions in convexification and efficient solvability of nonconvex motion planning and control problems. Specifically, we provide an overview of our work on representation of temporal and logical specifications for autonomous trajectory optimization, followed by our analysis of the so-called funnel dynamics, characterizing the dispersion of controlled trajectories from those generated by guidance algorithms. We then turn our attention to duality-based motion-planning in presence of state constraints. Lastly, we present our work on direct first order optimization over a Riemannian quotient manifold for direct dynamic policy synthesis.

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*Partially supported by the Air Force Office of Scientific Research under the Award No: FA9550-20-1-0053, e-mail: `behcet@uw.edu, mesbahi@uw.edu`

Stability Margins of Neural Network Controllers

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Abstract

We will present a method to synthesize neural network controllers for a class of uncertain plants to guarantee closed-loop dissipativity specifications. Such specifications include Lyapunov stability or an L_2 gain bound with respect to disturbances. We employ an implicit neural network, which is a broad model that encompasses common neural network architectures, and we represent the controller as a Linear Time Invariant (LTI) dynamical system in feedback with activation functions. We then unify this representation with the dynamics of the plant to arrive at a Linear Fractional Transformation (LFT) representation of the closed-loop system. This LFT representation isolates the known part of the plant with the LTI part of the controller on the one hand, and the uncertain and nonlinear elements of the plant with the activation functions of the controller on the other. Next, we present a procedure to train the neural network controller to maximize a reward function while guaranteeing the prescribed closed-loop dissipativity property. Finally, we show that this methodology can be leveraged to establish disk margins for neural network controllers – an important step for their wider acceptance, e.g., in flight control where certification criteria include traditional stability margins.

*Partially supported by the Air Force Office of Scientific Research under the Award No: FA9550-21-1-0288, e-mail: `arcak@berkeley.edu`

New homotopy-based obstructions to global dynamic feedback stabilization

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Abstract

We investigate global dynamic feedback stabilization from a topological viewpoint. In particular, we consider the general case of dynamic feedback systems, whereby the total space (which includes the state space of the system and of the controller) is a fibre bundle, and derive conditions on the topology of the bundle that are necessary for various notions of global stabilization to hold. This point of view highlight the importance of distinguishing trivial bundles and twisted bundles in the study of global dynamic feedback stabilization, as we show that dynamic feedback defined on a twisted bundle can stabilize systems that dynamic feedback on trivial bundles cannot.

In particular, consider a control system $\dot{x} = f(x,u)$ with state-space M, a closed finite-dimensional smooth manifold, and control space U, a finite-dimensional smooth manifold. Then, we show that $x^* \in M$ is weakly/strongly 1-point globally stabilizable by dynamic feedback only if the closed-loop system has a total space E which is a *nontrivial* bundle.

*Partially supported by the Air Force Office of Scientific Research under the Award No: FA9550-20-1-0333, e-mail: `belabbas@illinois.edu`

Uncertainty-Aware Guidance for Multi-Agent Target Tracking subject to Intermittent Measurements using Motion Model Learning

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Abstract

This work presents a novel guidance law for target tracking applications where the target motion model is unknown and sensor measurements are intermittent due to unknown environmental conditions and low measurement update rate. In this work, the target motion model is represented by a neural network (NN) and trained by previous target position measurements. This NN-based motion model serves as the prediction step in a particle filter for target state estimation and uncertainty quantification. Then this estimation uncertainty is utilized in the information-driven guidance law to compute a path for the mobile agent to travel to a position with maximum expected entropy reduction (EER). The computation of EER is performed in real-time by computing the probability distribution by extending the particle representation from particle filter into information gain approximation. Simulation and hardware experiments are performed with a quadcopter agent and TurtleBot target to demonstrate that the presented guidance law outperforms two other baseline guidance methods.

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Biologically Plausible Optimization: Competitive Neural Circuits & Contraction Theory

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Abstract

This talk presents novel connections between biologically plausible continuous-time neural networks, optimization problems, and contraction theory. This work is motivated by the growing interest in modeling biologically plausible neural networks, ensuring their stability and robustness, and investigating their functional implications. Biologically-plausible networks may ultimately inform machine learning models.

We begin by presenting a normative top-down framework for analyzing and designing biologically plausible continuous-time recurrent neural networks for sparse reconstruction problems. Using contraction theory, we characterize the convergence properties of these dynamics, demonstrating that the convergence is linear-exponential. Furthermore, we show that a broader class of continuous-time contracting dynamics solving convex optimization problems also exhibits this convergence behavior. Finally, we discuss the potential of incorporating Hebbian learning into this framework and present related contractivity results for neural-synaptic networks.

*Partially supported by the Air Force Office of Scientific Research under the Award No: FA9550-22-1-0059, e-mail: `bullo@ucsb.edu`

Mean Field Games on Sparse and Dense Networks

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Abstract

For sequences of networks embedded in an m dimensional cube a generalization of the notion of graphons was introduced in [PEC, CDC 2022] in terms of weak measure limits of sequences of empirical measures of vertex densities (vertexons) on $[0,1]^m$ and the associated weak measure limits of sequences of empirical measures of edge densities (graphexons) on $[0,1]^{2m}$, both of which exist *regardless of the asymptotic sparsity or density* of the sequences. The current work presents an extension with Minyi Huang of the Graphon Mean Field Game (GMFG) theory of [PEC-Huang, SICON, 2021] to the Gaphexon Mean Field Game (GXMFG) set-up [ACC, CDC, 2024]. LQG GXMFG examples are presented for the ring and lattice sparse graphexon limits where the influence between agent populations on edges between neighbouring nodes is modeled by a second order PDE. Existence and uniqueness of solutions is established.

*Partially supported by the Air Force Office of Scientific Research under the Award No: FA9550-23-1-0015 e-mail: peterc@cim.mcgill.ca, peter.caines@mcgill.ca

Feedback Stabilization of Ensemble Systems

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Abstract

We consider in this talk discrete ensembles of linear, scalar control systems with single-inputs. Assuming that all the individual systems are unstable, we address the following problem: does there exist linear feedback control laws that can asymptotically stabilize the ensemble system? We provide necessary/sufficient conditions for feasibility of pole placement in the left half plane and for feedback stabilizability of the ensemble systems.

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*This work is supported by the Air Force Office of Scientific Research under the Award No: FA9550-20-1-0076. Email: `cxudong@wustl.edu`.

Certifying and training reachable sets for neural network controlled systems

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Abstract

We consider a nonlinear control system subject to disturbance with a state feedback controller parameterized as a feedforward neural network. For such systems, we propose a framework for training controllers with certified robust forward invariant polytopes, so that trajectories initialized within each such polytope always remain within the polytope. To train and certify forward invariant sets, we first construct a parameterized family of lifted control systems in a higher dimensional space, with the property that the neural controlled dynamical system is embedded in an invariant subspace of each lifted system. Next, we show that if the vector field of any lifted system satisfies a sign constraint at a single point, then a certain convex polytope of the original system is robustly forward invariant. Treating the neural network controller and the lifted system parameters as variables and backpropogating against this sufficient condition, we then propose an algorithm that trains neural network controllers to guarantee forward invariant polytopes of the closed-loop control system. The simplicity of the sufficient condition allows our approach to scale with system dimension, and we include an example with over 50 states. Moreover, we show that our algorithm outperforms state-of-the-art Lyapunov-based sampling approaches in runtime. In the last part of the talk, we turn our focus to computing overapproximated reachable sets for control systems evolving on Lie groups. For such systems, we combine our interval methods with results from geometric integration theory. In particular, we use interval bounds of the Baker-Campbell-Hausdorff formula to develop a Runge-Kutta-Munthe-Kaas reachability algorithm for estimating reachable sets on Lie groups at little computational cost.

*Partially supported by the Air Force Office of Scientific Research under the Award No: FA9550-23-1-0303, e-mail: `sam.coogan@gatech.edu`

Safe and Constrained Feedback Optimization of Dynamical Systems

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Abstract

Online feedback optimization (OFO) is a framework to design feedback controllers that regulate a dynamical system toward equilibria defined as optimal solutions of an optimization problem. By leveraging the interplay of OFO and control barrier functions (CBFs), we propose new strategies for the design of feedback controllers that: (i) drive the system to solutions of optimization problems with steady-state constraints on both inputs and states, and (ii) exhibit safety properties, in the sense that they render the set (or superset) of the feasible inputs and states forward invariant. In this talk, we first present a new state feedback controller based on CBF tools and a continuous approximation of the projected gradient flow; we show that the equilibria of the interconnection between the plant and the proposed controller correspond to critical points of the constrained (possibly nonconvex) optimization problem. Sufficient conditions to ensure that, for the closed-loop system, isolated locally optimal solutions of the optimization problem are locally exponentially stable are presented, and forward invariance of the set of feasible inputs is established. We then show how the proposed CBF-based OFO method can be augmented with safety filters to ensure forward invariance of a feasible set of states; we provide recent results on the emergence of spurious stable equilibria and saddle points when using a CBF-based filter, and provide remarks on open research questions that are the subject of our current research.

Supported by the Air Force Office of Scientific Research under the Award No: FA9550-23-1-0740,
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Real-time optimal distributed estimation and control of spatiotemporal processes using multi-domain methods and optimally-guided mobile sensors

Michael A. Demetriou and Nikolaos A. Gatsonis*

Abstract

The goal of this project is to develop an advanced theoretical and computational approach that enables the *real-time* estimation of spatiotemporal processes modelled by partial differential equations (PDEs) using mobile sensors. Under the proposed approach, the spatial domain of interest is decomposed into an inner domain surrounding the mobile sensor and an outer domain. The theoretical framework includes a state observer (Kalman filter or Luenberger observer) for the inner domain, a simplified naïve observer for the outer domain, and time-varying transmission conditions due to the mobile sensors.

The domain decomposition approach results in a hybrid estimator and reduces the computational cost needed for real-time implementation of a state observer by restricting the observer kernel to the inner domain. The hybrid estimator kernel in the inner domain is developed so that its support vanishes in the outer domain. Such a constraint ensures that no residual innovation terms appear in the outer domain filter thereby acting as an exogenous input. The proposed approach ensures that the optimality of the domain decomposition observer matches that of the observer designed for a single spatial domain.

The abstract framework for a hybrid domain decomposition filter employing a Kalman filter in the inner domain and a naïve observer in the outer domain is presented. Conditions for the sparsity of the filter gain kernel obtained from the solution to the operator differential Riccati equation are examined for both the case of static (immobile) and mobile sensors. The guidance of the mobile sensors is decoupled from the Kalman filter design leading to a trajectory-dependent operator differential Riccati equations generating optimal filter kernels.

Numerical results from a limiting case where the state observer in the inner domain is reduced to a Luenberger observer are presented. The hybrid estimator is implemented numerically with a parallelized, non-overlapping, domain-decomposition method using structured and uniform grids. The hybrid estimator is discretized in space with a finite-volume total variation diminishing method. The transmission conditions are implemented on the interfaces between adjacent subdomains for data communication. The time integration of

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the resulting semi-discrete equations is performed using a multi-step Runge-Kutta scheme with sub-cycling for the mobile sensor motion.

Dynamic Mission Planning: Adversarial Conflicts and Optimal Dubins Path on a Sphere

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July 23, 2024

Abstract

First, the problem of escorting a High Value Airborne Asset (HVAA) is considered. Low-cost Collaborative Combat Aircraft (CCA) are deployed to protect the HVAA from air interceptors which could be more lethal than the CCA. The planned trajectory of the low maneuverable HVAA is considered by the CCA and its adversary, the air interceptor, in order to design strategic approach and block maneuvers and solve the corresponding reach-avoid differential game. Then, the problem of virtual target selection considers M pursuers against N evaders. The virtual targets serve as an intermediary target for the pursuers, allowing the pursuers to delay their final assignment to the evaders. However, upon reaching the virtual target, the pursuers must decide which evader to capture. The objectives are to assign each pursuer to a virtual target and evader such that the pursuer team's energy is minimized, and to choose the virtual targets' locations for this minimization problem. Finally, the interesting problem of path planning in 3D to travel from one configuration (location and orientation) to another is considered. In particular, optimal Dubins paths on a (unit) sphere are analyzed for specific sets of minimum turning rates. Different models are presented and, by showing the equivalence of these models, the motion planning problem using the spherical coordinate description is also solved. The optimal path is obtained to be of type CGC, CCC, or a degenerate path of the same for $r \in (0, 1/2] \cup \{1/\sqrt{2}\}$, where $C = L,\ R$ denotes a tight left or right turn of radius r, and G denotes a great circular arc.

*Partially supported by the Air Force Office of Scientific Research under the Award No. 24RQ-COR002, e-mail: eloy.garcia.2@us.af.mil

The sub-Riemannian geometry of Optimal Mass Transport

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Abstract

The theory of Monge-Kantorovich Optimal Mass Transport (OMT) has spurred a fast developing phase of research across stochastic control, ensemble control, thermodynamics, data science, and several other fields in engineering and science. Specifically, OMT endowed the space of probability distributions with a rich Riemannian-like geometry and the Wasserstein W_2-metric, that proved fruitful in quantifying and regulating uncertainty in deterministic and stochastic systems, and the transport of ensembles of particles/agents in continuous and discrete spaces.

In the talk we will introduce a new type of transportation problems. The salient feature of these problems is that particles/agents to be transported are labeled and their relative position along their journey is of interest. Of particular importance in our program are closed orbits where particles/agents return to their original place after being transported along closed paths. Thereby, a control law is sought so that the distribution of the ensemble traverses a closed orbit in the Wasserstein manifold without mixing. This feature is in contrast with the classical theory of optimal transport where the primary object of study is the path of probability densities, without any concern about mixing along the flow which is expected and allowed. To this, in the theory we present, we explore a hitherto unstudied sub-Riemannian structure of the Monge-Kantorovich transport, where the relative position of particles along their journey is modeled by the holonomy of the transportation schedule.

This is joint work with Dr. Mahmoud Abdelgalil (maabdelg@uci.edu).

*Partially supported by the Air Force Office of Scientific Research under the Award No: FA9550-23-1-0096, e-mail: `tryphon@uci.edu`

Rigorous Optimal Uncertainty Quantification & Optimization

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Abstract

The success and survival of autonomous systems greatly depends on the systems' ability to quantify and reason about incomplete and uncertain information in actionable ways. Optimal Uncertainty Quantification (OUQ) provides a rigorous theoretical framework to this end by tractably bounding statistical measures of quantities of interest. Although OUQ is rigorous in theory, the rigor is lost in practice due to numerical solution approaches. This issue is further amplified when the quantities of interest are induced by dynamical systems. This works seeks to account for these issues and develop an *end-to-end rigorous OUQ* framework to support system, control, and mission design and optimization under uncertain and incomplete information by leveraging advancements in global optimization and validated numerics. In this talk we outline end-to-end rigorous OUQ, its core technology gaps, preliminary results, and potential applications.

Distribution Statement A: Approved for Public Release; Distribution is Unlimited. PA# AFRL-2024-3910, 7/18/2024

Mechanism Design for Control-theoretic Objectives

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Abstract

In this project, we consider incentive design in sequential decision making problems for control- oriented objectives. These problems can be expressed as Markov Stackelberg games. We start by providing the first algorithm that provably converges to an equilibrium in such games. To make these and other learning algorithms in multi-agent systems more practical, we then relax two assumptions – namely, each agent requiring information from all the other agents in such algorithms and the incentive designer possessing accurate information about the dynamics of the underlying Markov decision process.

*Partially supported by the Air Force Office of Scientific Research under the Award No: FA9550- 21-1-0231, e-mail: gupta869@purdue.edu

Topological Methods for Assured Transitions in Hybrid Systems

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August 29, 2024

Abstract

A critical and ubiquitous challenge for real-life implementation of high-level plans with low-level (hybrid) controllers is that of ensuring transitions from one domain of the state space to another despite significant state-estimation error resulting from model- and state-uncertainties. Such transitions need to be effected under two kinds of constraints: (a) transition into a target domain of (possibly) complex geometry must be guaranteed despite imprecise state feedback, and (b) invariance constraints reflecting the high-level plan must be maintained at the same time. Both problems are planning problems with deep topological roots.

In this talk, addressing (a) we will present our progress on developing and applying the theory of transition guarantees, together with new results on how to execute symbolic planning (e.g., LTL-based planning) in general settings where the atomic propositions form a good open cover (in the sense of satisfying the conditions of the nerve lemma) of the state space.

*Partially supported by the Air Force Office of Scientific Research under the Award No: FA9550-22-1-0429, e-mail: {danguralnik,wdixon}@ufl.edu

Fixed Time Stability, Uniform Strong Dissipativity, and Stability of Nonlinear Feedback Systems

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Abstract

In this work, we develop a new Lyapunov condition for fixed time stability that subsumes the classical fixed time stability results presented in the literature while providing an optimized estimate of the settling time bound that is less conservative than the existing results. Then, we introduce the notion of uniformly strongly dissipative dynamical systems and show that for a closed dynamical system (i.e., a system with the inputs and outputs set to zero) this notion implies fixed time stability. Specifically, we construct a stronger version of the dissipation inequality that implies system dissipativity and generalizes the notions of strict dissipativity and strong dissipativity while ensuring that the closed system is fixed time stable. The results are then used to derive new Kalman-Yakubovich-Popov conditions for characterizing necessary and sufficient conditions for uniform strong dissipativity in terms of the system drift, input, and output functions using continuously differentiable storage functions and quadratic supply rates. Furthermore, using uniform strong dissipativity concepts we present several stability results for nonlinear feedback systems that guarantee finite time and fixed time stability. For specific supply rates, these results provide generalizations of the feedback passivity and nonexpansivity theorems that additionally guarantee finite time and fixed time stability.

*Partially supported by the Air Force Office of Scientific Research under the Award No: FA9550-20-1-0038, e-mail: wm.haddad@aerospace.gatech.edu

A Compositional Framework for Non-convex Decision Problems

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Abstract

Multi-stage decision problems often require the computation of optimal decisions, such as in Markov processes, various classes of games, and many types of control systems. In this talk, we seek to understand how the effects of each decision propagate over time to influence the long-run behavior of a system. In their most general form, the computation of optimal decisions can require the solution of non-convex optimization problems. The solution to one such problem is a decision that, when applied to a system, affects its future states in ways that constrain future decisions. To formally model these dependencies, this talk will present a novel category – called AlgBifun – in which we can model the composition of non-convex optimization problems. We focus on the case of model-predictive control (MPC), and our main result shows that an MPC problem is a composition of morphisms in this category. We will also see how constraints on initial states, terminal states, and periodicity are readily formulated in this setting. This talk will also discuss how this categorical formalism automates the process of writing correct-by-construction software for solving MPC problems, as well as future work on connecting these categorical formalisms to other work in category theory on dynamical systems.

*Partially supported by the Air Force Office of Scientific Research under the Award No: FA9550-23-1-0120, E-mail: matthale@gatech.edu

Effective Whitney Stratification of Algebraic Maps and Applications

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Abstract

Motivated by the study of the singularities of kinematic systems, we consider the problem of developing effective computational methods to stratify algebraic maps between real algebraic varieties or full dimensional semi-algebraic sets. The goal of this stratification is to partition the codomain into regions where the fibers have locally constant topology, and to in turn study the geometry and topology of these fibers. A key step of the program is the computation of a Whitney stratification of a variety. For this later task we present several new algorithms which improve on the performance of our earlier methods. We also employ these stratifications to quantify several important geometric and topological properties of varieties.

*Partially supported by the Air Force Office of Scientific Research under the Award No: FA9550-22-1-0462, MH e-mail: mhelmer@ncsu.edu, VN email: nanda@maths.ox.ac.uk

Learning-Based Planning & Control with Persistent Safety for UAS

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Abstract

We discuss our recent results on $\mathcal{L}_1$ distributionally robust adaptive control ($\mathcal{L}_1$-DRAC) - A new class of robust adaptive controllers for nonlinear and uncertain continuous-time Markov processes. Nonlinear continuous-time Markov processes like Itô stochastic differential equations (SDEs) form a model class that can capture finite-dimensional dynamic phenomena with high-fidelity. However, robust control of such systems is a challenging prospect due to the presence of aleatoric and epistemic uncertainties. Moreover, standard robust control techniques for deterministic systems fail in enforcing even the conservative notions of robustness due to the nature of stochastic processes. Thus, we lift the notion of *robustness* to the space of *distributions over outcomes* for the control of nonlinear and uncertain stochastic systems. Using the architecture of $\mathcal{L}_1$ adaptive control, we design the $\mathcal{L}_1$-DRAC feedback operators to retain the standard implementation of feedback control, while simultaneously guaranteeing certificates of robustness in the space of distributions (probability measures).

*Partially supported by the Air Force Office of Scientific Research under the Award No: FA9550-21-1-0411, e-mail: `nhovakim@illinois.edu`

MPC based Feedback Control of Flow-Transport Systems: Analysis, Computation, and Machine Learning

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Abstract

We consider the nonlinear feedback control design for the problems of mass transport and fluid mixing via flow advection. The overall dynamics are governed by the transport equation coupled with the Stokes or the Navier-Stokes equations in an open bounded and connected domain. In the absence of molecular diffusion, transport and mixing occur due to pure advection. This essentially leads to a nonlinear control problem of a semi-dissipative system. The classic tools for treating parabolic systems, based on its intrinsic regularizing effects and model reduction, are no longer applicable. In this project, we employ the idea of Model Predictive Control (MPC) to construct a sub-optimal feedback law. We further analyze the mixing decay rate of the resulting closed-system and develop effective numerical schemes for implementing our control designs. On the other hand, we develop the algorithms by leveraging physics informed neural networks (PINNs) together with operator splitting and alternating direction method of multipliers. Various numerical experiments will be presented to demonstrate our ideas and designs.

*Partially supported by the Air Force Office of Scientific Research under the Award No: FA9550-23-1-0675, e-mail: Weiwei.Hu@uga.edu

Advanced Tools for Verification of Learning-based Control

Bin Hu, Peter Seiler, and Geir Dullerud*

Abstract

Deep learning models have tremendous potential for impacting the performance of autonomous aerospace and robotic systems. A barrier to the wide deployment of deep learning methods in safety-critical applications is the lack of tight closed-loop guarantees in terms of stability, safety, and robustness. In this talk, we will present advanced tools for certifying the control-theoretic guarantees of feedback systems with deep learning models in the loop. The first half of the talk focuses on the completeness of quadratic constraints (QCs) on ReLU/Leaky-ReLU/HouseHolder/Maxmin networks for tight closed-loop analysis of learning-based control systems. We start by presenting a complete set of QCs in the form of matrix copositivity conditions for the repeated ReLU mapping, and a theorem showing that our proposed QCs bound the repeated ReLU as tight as possible up to the sign invariance inherent in quadratic forms. Building on this result, we further obtain completeness results for different activation functions such as leaky ReLU, HouseHolder, and Maxmin, and develop related semidefinite programming conditions to certify learning-based control systems with such networks in the loop. In the second half of the talk, we present a "gainless" generalization of the classic small gain theorem, which can be potentially used for robustness analysis of control systems with self-attention networks in the loop. In this case, we consider the stability of a feedback interconnection in a generalized "bounded-input-bounded-output" (BIBO) sense by checking the worst-case output energy magnitude (rather than a ratio) subject to an input energy bound. Our theorem offers a stability condition which can be connected to the local sensitivity analysis of self-attention networks, making it more computationally friendly than the existing gain-based analysis.

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Distributional Control of Ensemble Systems

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Abstract

Applications involving the control of extensive ensembles of dynamical systems have emerged across pervasive areas of science and technology. This emergence has necessitated significant advancements in the theory of ensemble control in response to the arising nontraditional challenges. In this presentation, we will delve into the problems of distributional control for ensemble systems. The notion of distributional control arises from the utilization of distributional information inherent in the aggregated measurements to control ensemble systems. We will introduce the method of moment kernelization, which generates mappings of an ensemble system defined on a function space to a dual system defined on the corresponding moment space. We will demonstrate how this moment method bridges the concept and tools of optimal transport to distributional control and facilitates the design of ensemble control laws using computational neural networks. Additionally, we will formulate and discuss distributional control on fibered manifolds, exploring new frontiers in ensemble control methodologies.

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A new operator for dynamic mode decomposition in discrete time

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Abstract

This talk will focus on the consequences of the linearity of Koopman-like operators in their symbols. In continuous-time systems, the operators are linear in their symbols. We show that the said linearity can be exploited to incorporate partial knowledge into dynamic mode decomposition (DMD)-based system identification techniques. In addition, it will be shown that linearity facilitates norm-convergent finite-rank representations of Koopman generators of closed loop systems using open-loop data. Such representations will be shown to result in convergent system identification methods for control-affine systems.

In discrete time, the operators are not linear in their symbols, which renders DMD-based system identification difficult. In this talk, we show that if the system is affine in control, then DMD can nonetheless be used to model closed loops under different control laws using data collected under arbitrary open loop controllers. We show that spectral decomposition of a finite-rank representation of an operator that acts like the Koopman operator on kernels can be used to generate such models. Lack of compactness results in loss of convergence guarantees, but the methods are found to outperform existing approaches in numerical experiments.

*Partially supported by the Air Force Office of Scientific Research under the Award No: AFOSR FA9550-20-1-0127, e-mail: rkamalapurkar@ufl.edu

A Topological Approach for Detecting P-Bifurcations From Kernel Density Estimates

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We present a Topological Data Analysis (TDA) framework for the identification of qualitative shifts, namely P-type (phenomenological) bifurcations, within stochastic dynamical systems, defined by structural changes in the probability density functions (PDF) of the state variables. In contrast to traditional methods which require manual intervention and are suitable only for small state spaces, our TDA approach employs superlevel persistence to automatically construct a "homological bifurcation plot," which monitors changes in 0th and 1st homology group ranks via Betti vectors. The talk will explore the successful application of this method to stochastic oscillators, showcasing its effectiveness in algorithmically detecting P-bifurcations from both reliable and unreliable KDE inputs. We also address the challenge posed by real-world signals with sparse or even a single realization by evaluating different approaches for generating an ensemble of persistence diagrams starting with only one diagram.

*This material is based upon work supported by the Air Force Office of Scientific Research under award number FA9550-22-1-0007, e-mail: `khasawn3@msu.edu, muncheli@msu.edu`

Developments in Suboptimal Model Predictive and Constrained Control

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Abstract

When implemented sub-optimally using time-distributed optimization, the closed-loop properties of receding horizon model predictive controllers (RH-MPC) become dependent on the optimization algorithm, the number of iterations per time step, on the warm-starting strategy and the model used in the optimization. In the shrinking horizon MPC (SH-MPC) setting (when the end time of the maneuver is fixed and the prediction horizon over which the underlying optimal control problem is solved decreases with time) this leads to questions on the extent of inexactness, the number of iterations that need to be performed and the properties of the cost function which ensure that the trajectory enters a specified terminal set or cumulative sub-optimality of the maneuver is acceptable. Supervisory schemes can be augmented to MPC controllers to ensure desirable properties with the inexact implementation. In particular, we will describe a control strategy which combines an input sequence generator, based on solving an optimal control problem for a lower-fidelity disturbance-free linear model, with a prediction-based supervisory scheme that adjusts the reference command to ensure constraints satisfaction for the original system which could be nonlinear and have disturbances/uncertainties. We will also describe an iteration governor for RH-MPC that augments a suboptimal input-constrained RH-MPC by performing online selection of the reference command and the number of optimization iterations used to generate a control input. The reference selection procedure allows for stabilization of the system with shorter MPC prediction horizons, while the online iteration scheduling reduces the computational effort used to generate control inputs. We will also discuss the regret analysis for SH-MPC when a fixed number of solver iterations and a warm-start are utilized at each time step to solve the underlying Optimal Control Problem (OCP), leading to bounds on the loss of performance (regret) and on the difference between suboptimal SHMPC and optimal solutions. Finally, in the setting of bilevel optimization we will highlight input-to-state stability properties with respect to disturbances such as due to inexact computations.

*Partially supported by the Air Force Office of Scientific Research under the Award No: FA9550-20-1-0385, e-mail: `ilya@umich.edu`

Boundary Stabilization of a Bending and Twisting Beam

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Abstract

We consider the stabilization of the bending and twisting of a rectangular cantilever beam of moderate to high aspect ratio using boundary control. This is a first step to stabilizing the bending and twisting of a wing, but in this study we ignore aerodynamic forces that act on a wing. It has been stated that "even for relatively low aspect ratios ($4 < AR < 5$) of thin rectangular cantilever plates, the model based on the beam theory is closer to the experimental model compared to the plate theory."

We start with the model presented in Section 3.6 of classical treatise *Aeroelasticity* by Bisplinghoff, Ashley and Halfman. This is a linear model where the bending modes are described by the beam equation and the twisting modes are described by the wave equation. But the bending and twisting modes are inertially coupled. and this introduces considrable complexity in the model. We also consider a nonlinear extension of this linear model where the bending of the beam increases its torsional rigidity and the twisting of the beam increases its bending rigidity.

The linear model is neutrally stable so the bending and the twisting oscillations do not decay. We seek a feedback to asymptotically stabilize these oscillations. We add two actuators located where the cantilever beam is joined to its support. One actuator affects the bending of the beam at its support and other affects the twisting of the beam at its support. We find a state feedback control law which drives the oscillations to zero. To find such a feedback we set up and solve a Linear Quadratic Regulator (LQR). Then we use nonLinear nonQuadratic Regulation (nLnQR) to stabilize the nonlinear extension.

*This work was funded by AFOSR under grant FA9550-20-1-0318, e-mail: `ajkrener@ucdavis.edu`

Offline Learning of Gain Maps Enables Online-Adaptive PDE Control

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Abstract

Adaptive backstepping control of unstable PDEs, emerged in 2005, extended to PDEs a collection of methodologies developed for nonlinear ODEs in 1991-1994. It translated differential geometric design and analysis techniques invented for adaptive stabilization of ODEs to the functional analytic setting, in which such techniques are deployed to guarantee stabilization of PDEs with unknown spatially-dependent parameter functions.

However, in these adaptive control methods for PDEs, which have, since 2005, spawned advances captured in quite a few papers and three books (one book for parabolic PDEs, one for hyperbolic PDEs, and one for delay systems), one challenge was unresolved and left to the practitioners and heuristics. How to solve, *at each time step*, the kernel PDEs in spatial time variables, which govern the controller gains, and whose inputs are the real-time estimates of the unknown functional parameters of the PDEs? This is a considerably greater numerical challenge than, say, solving a new Riccati equation at each time step of implementation of adaptive optimal control.

In this briefing we report on the development of neural operators for gain kernel PDEs, where such nonlinear operators replace the online solution of the PDEs by approximating their solutions, typically with a thousandfold increase in speed.

The principal results are the guarantees of stability and convergence of the PDE plant state under such adaptive controllers with neurally-approximated gain functions. Designs and theorems are provided for both hyperbolic and parabolic PDEs. The theory comes with two mild conditions: that the neural operator be trained with a sufficient given accuracy, and that the adaptation gain be below a given value, both of which are explicitly quantified based on the set of possible unknown parameter functions being estimated.

The methodology presented combines offline and online learning for control. The plant parameter estimates, learned online, are fed into an approximation of the mapping from the plant parameters into control gains, learned offline. Almost the entire computational cost is shifted from online to offline, while stability and convergence remain guaranteed.

*Supported by the Air Force Office of Scientific Research under the Award No: FA9550-22-1-0265, e-mail: `krstic@ucsd.edu`

The stability and convergence are guarantees in the face of three classes of perturbations that arise in neurally-implemented adaptive control: operator approximation error, parameter estimation error, and parameter update rate perturbation. This trio of perturbations are dominated with a variety of techniques in Lyapunov theory, functional analysis, and operator approximation theory.

Obstructions to feedback stabilization

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Abstract

This talk concerns the problem of asymptotically stabilizing a compact subset of a manifold using a smooth, time-independent, feedback control law. One obstruction to doing so arises from a coordinate-free generalization of Brockett's necessary condition (joint with D. E. Koditschek). Another obstruction is based on a coordinate-free generalization of Coron's and Mansouri's homological necessary conditions. The latter obstruction follows from a "homotopy theorem" for a pair of smooth vector fields sharing a compact attractor: one can be continuously deformed into the other through nonzero vector fields over sufficiently small annular regions surrounding the attractor. The mentioned obstructions are independent in the "fiber bundle picture" of control, but the latter is stronger in the "vector bundle picture", extending the classical relationship between Brockett's and Coron's conditions. Curiously, none of these obstructions can detect non-stabilizability of periodic orbits, leading to ongoing work with A. M. Bloch with applications to nonholonomic systems.

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Non-convexity analysis: Online optimization and online learning

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Abstract

In this talk, we study two non-convex problems of online optimization and online learning. In online optimization, we consider a sequence of unknown nonlinear functions that evolve over time, and the goal is to find a global minimum for each function. This problem is well studied in the convex case but the non-convex case is at its infancy. Using tools for the analysis of nonlinear dynamical systems, we develop an efficient algorithm to solve the problem, and study how the resulting regret bound (due to lack of knowledge about the function or its complex optimization landscape) can be quantified. For online learning, we consider the problem of learning a nonlinear dynamical system whose input is subject to adversarial attacks. Although the resulting optimization problem is highly non-convex, we design an efficient estimator and show that the correct dynamics of the system can be learned without any errors even when the states of the system are under attack more than half of the time. This work offers the first positive result in the literature on learning nonlinear systems under attacks with a zero error.

*Partially supported by the Air Force Office of Scientific Research under the Award No: FA9550-21-1-0250, e-mail: lavaei@berkeley.edu

Geometric Adjoint Sensitivity Analysis for Lie Groups and PDEs

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Abstract

Adjoint systems are widely used to inform control, optimization, and design in systems described by ordinary differential equations or differential-algebraic equations. We explore the geometric properties and develop geometric discretization methods for such adjoint systems. In particular, we utilize symplectic and presymplectic geometry to investigate the properties of adjoint systems associated with ordinary differential equations and differential-algebraic equations, respectively. We show that the adjoint variational quadratic conservation laws, which are key to adjoint sensitivity analysis, arise from (pre)symplecticity of such adjoint systems. We discuss various additional geometric properties of adjoint systems, such as symmetries and variational characterizations. As an application of this geometric framework, we discuss how the adjoint variational quadratic conservation laws can be used to compute sensitivities of terminal or running cost functions. Furthermore, we develop structure-preserving numerical methods for such systems using Galerkin Hamiltonian variational integrators which admit discrete analogues of these quadratic conservation laws.

We also consider the generalization to flows on Lie groups by studying Lie group variational integrators with a novel Type II variational principle on the cotangent bundle of a Lie group which allows for Type II boundary conditions, i.e., fixed initial position and final momenta. In general, such Type II variational principles are only globally defined on vector spaces or locally defined on general manifolds; however, by left translation, we are able to define this variational principle globally on cotangent bundles of Lie groups.

Finally, we investigate the geometric structure of adjoint systems associated with evolutionary partial differential equations at the fully continuous, semi-discrete, and fully discrete levels and the relations between these levels. We show that the adjoint system associated with an evolutionary partial differential equation has an infinite-dimensional Hamiltonian structure, which is useful for connecting the fully continuous, semi-discrete, and fully discrete levels.

*Partially supported by the Air Force Office of Scientific Research under the Award No: FA9550-23-1-0279, e-mail: mleok@ucsd.edu

System Theoretic Dissipativity For Nash Equilibria Learning In Large Agent Populations

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Abstract

In this talk, we discuss new methods based on system-theoretic dissipativity to understand how large populations of agents engaged in strategic interactions learn and adapt their strategies over time. Our goal is to provide guarantees that these populations will reach Nash equilibria, even without detailed knowledge of the specific learning rules used by the agents. We begin by presenting new results that, for the first time, allow for hybrid learning rules combining elements from different established classes, which were previously considered only in isolation.

The main focus will be on a new approach using counterclockwise (CCW) dissipativity, originally proposed to study the stability and robustness of nonlinear feedback systems. We demonstrate that a memoryless payoff mechanism is CCW if and only if it is a potential game. Our work shows that payoff mechanisms satisfying CCW, whether memoryless or dynamic, are guaranteed to steer the population towards Nash equilibria for the most extensive class of hybrid learning rules considered to date. This means that Nash equilibria can be used to predict the long-term strategic behavior of the population.

Our findings have significant applications for the modeling and distributed coordination of large multi-agent systems, providing a robust framework for predicting and guiding the behavior of such systems in various contexts.

*Partially supported by the Air Force Office of Scientific Research under the Award No: FA9550-23-1-0467, e-mail: nmartins@umd.edu

Active Learning for Control-Oriented Identification of Nonlinear Systems

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Abstract

Model-based reinforcement learning is an effective approach for controlling an unknown system. It is based on a longstanding pipeline familiar to the control community in which one performs experiments on the environment to collect a dataset, uses the resulting dataset to identify a model of the system, and finally performs control synthesis using the identified model. As interacting with the system may be costly and time consuming, targeted exploration is crucial for developing an effective control-oriented model with minimal experimentation. Motivated by this challenge, recent work has begun to study finite sample data requirements and sample efficient algorithms for the problem of optimal exploration in model-based reinforcement learning. However, existing theory and algorithms are limited to model classes which are linear in the parameters. Our work instead focuses on models with nonlinear parameter dependencies, and presents the first finite sample analysis of an active learning algorithm suitable for a general class of nonlinear dynamics. In certain settings, the excess control cost of our algorithm achieves the optimal rate, up to logarithmic factors. We validate our approach in simulation, showcasing the advantage of active, control-oriented exploration for controlling nonlinear systems.

*Partially supported by the Air Force Office of Scientific Research under the Award No: FA9550-24-1-0102, e-mail: nikolai.matni@seas.upenn.edu

Stat-duality, HJ PDEs with low-dimensional nonlinearities and the Schrödinger equation

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Abstract

As is well-known, classical methods for solution of nonlinear HJ PDEs (Hamilton-Jacobi partial differential equations) suffer from the curse-of-dimensionality. A single nonlinearity in either the dynamics or the cost of an otherwise linear-quadratic problem typically affects the behavior of the solution over all dimensions yielding non-quadratic behavior in all of the arguments. Stat-duality is employed to move all nonlinearities into a single function within the running cost. This is a function only of newly introduced control processes that are introduced via the stat-duality. These new control processes have dimension equal to that of the low-dimensional nonlinearity, say dimension m. All remaining elements of the problem are linear-quadratic. Solution of the problem is reduced to that of a nonlinear staticization control problem of dimension m, essentially independent of the original space dimension. Previously, we had developed the algorithm to completion only for the case of nonlinearities that were in the running cost of the original problem. Although the main result handled nonlinearities in both the running cost and the dynamics, the case of nonlinearities in the dynamics presented unexpected difficulties due to the structure of the Hamiltonian, where in particular, the dynamics nonlinearity appears within an inner product with the co-state. We briefly discuss the mathematical and algorithmic extensions. In contrast to the case of a running-cost nonlinearity, the dimension of a nonlinearity in the dynamics is described in terms of both the dimension of the domain of the nonlinearity and the dimension of the directly-affected range components. Examples with nonlinearities in both the running cost and the dynamics will be presented; the algorithms remain exceptionally fast.

The Schrödinger equation is an emblematic (complex-valued) second-order PDE. The corresponding initial value problem is considered, and the dequantized form of the equation is used for propagation. One should note that this form is that of an HJ PDE with formal stochastic-control interpretations, and hence this component of our effort has wider control-theory ramifications. New tools from ongoing staticization research are brought to bear. Similar to the max-plus curse-of-dimensionality-free algorithms for deterministic nonlinear control problems, one can propagate solutions in a dual-space, where that space

*Partially supported by the Air Force Office of Scientific Research under the Award No: FA9550-18-1-0051, e-mail: wmceneaney@ucsd.edu

is now obtained through stat-duality. The concept of discrete-staticization is generated, and convergence to standard staticization is shown, yielding convergence of certain stat-based approximations. The original-space state is described through complex-valued matrices describing quadratic functions, and the stat-dual of a space of such is employed to obtain the dual-space propagation. One should note that the success of previous efforts at application of max-plus curse-of-dimensionality-free methods to second-order HJ PDEs had been somewhat attenuated due to the necessity of a distributive-property step.

The included material reflects also the efforts of Prof. P.M. Dower (Univ. of Melbourne), Prof. H. Kaise (Univ. of Kumamoto) and Y. Zheng (UCSD).

Backward Map for Filter Stability Analysis

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Abstract

Dissipation is at the heart of any stability theory for dynamical systems. For Markov processes, dissipation is referred to as the variance decay. This talk is concerned with extension of variance decay to the study of conditioned Markov processes (nonlinear filter). Specifically, the following questions are of interest:

Q1. What is the appropriate notion of variance decay for a nonlinear filter? And how is it related to filter stability?

Q2. What is the appropriate generalization of the dissipation equation for the nonlinear filter?

Q3. How is it related to the hidden Markov model (HMM) properties such as ergodicity, observability, and detectability?

In this talk, we provide an answer to each of these questions based on a newly discovered duality between nonlinear filtering and optimal control.

*Partially supported by the Air Force Office of Scientific Research under the Award No: FA9550-23-1-0060, e-mail: `mehtapg@illinois.edu`

Mathematically Justified Computational Platform for Nonlinear Dynamics

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Abstract

We present current progress on the development of a *combinatorial topological, computationally efficient, theory of dynamics.* The broader goal consists of at least three overlapping parts: theory, applicability, and applications. However, we focus on applicability, that is, solving the mathematical, algorithmic, and computational challenges that allow the theory to be used to solve problems of practical interest.

Here we present developments related to the identification of bifurcations. Algebraic structures such as the lattices of attractors, repellers, and Morse representations provide a computable description of global dynamics. For parameterized families of dynamical systems, these structures form a sheaf over the parameter space. Sheaf cohomology provides information about the global sections of the attractor sheaf and changes in this structure that detect bifurcations.

Advancements in the theory and implementation of the Dynamic Signatures Generated by Regulatory Networks (DSGRN) framework have facilitated the computation of cellular sheaf cohomology across the parameter space. In particular, we have used attractor sheaves to discover paths in parameter space that exhibit hysteresis, as well as higher-codimension bifurcations.

*Partially supported by the Air Force Office of Scientific Research under the Award No: FA9550-23-1-0011, e-mail: mischaik@math.rutgers.edu

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Hybrid Geometric Control and Morse Theory

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Abstract

We use Morse theory to construct a hybrid feedback control law to robustly and globally asymptotically stabilize a given desired point in a compact connected manifold. The method is inspired by a trick performed for the stabilization of a point on the unit circle. The proposed hybrid geometric control law exploits a logic variable to implement hysteresis-based switching between two smooth vector fields on the manifold, one of which is the gradient of a Morse function having the desired point as its global minimum.

*Partially supported by the Air Force Office of Scientific Research under the Award No: FA9550-23-1-0313, e-mail: `rmont@ucsc.edu, ricardo@ucsc.edu`

Glimmers of autonomy: Structure-aware reachability analysis and control synthesis for unknown nonlinear systems

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Abstract

Following a sudden adverse event, a system may become incapable of pursuing its original task regardless of its control policy. Our previous work focused on real-time task reassignment: recognizing the task as potentially impossible to complete given the current information about the system, and proposing a certifiably completable alternative. Naturally, this method is meaningful only if the identified set of alternative tasks is not prohibitively conservative, and the whole framework only makes sense if the planner can subsequently synthesize in real time a control policy that drives the system to complete the chosen task. In this year's talk, I will describe three directions of efforts in service of this high-level capability: (i) more realistic description of completable tasks which considers the overall energy that may need to be expended to complete a task, (ii) reduction of conservatism of the identified set through side information about the system's operating manifold, and (iii) synthesis of a provably correct control law – relying on continual online learning – which enables the system to complete the desired task.

*Partially supported by the Air Force Office of Scientific Research under the Award No: FA9550-23-1-0131, e-mail: mornik@illinois.edu

Differential games relevant to the warfighter

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Abstract

A classical differential game and its connection to some new differential games which are relevant to the warfighter are discussed. First the complete solution of the classical Lady in the Lake differential game is reported. A Focal Line (FL) and a Universal Line (UL), and their respective tributaries, are introduced. The UL is akin to a singular arc in optimal control, but the existence of a FL is a unique feature of differential games. The two tributary regions are separated by a spiral of Archimedes curve. In addition, in this differential game a new feature is introduced - an *internal* semipermeable "surface" is present. The connection of the Lady in the Lake game to differential games where a directed energy weapon is employed to protect a high value target is emphasized. In addition, new, off the beaten path, and currently relevant, pursuit-evasion scenarios are addressed where a *fast* Evader is engaged by a Pursuit team. In these games a novel type of optimal/equilibrium trajectories is discerned, where the Evader is temporarily in contact with one of the Pursuers before capture is finally effected. The closed form optimal/equilibrium strategies of the agents and the Value functions of the differential games are obtained. The optimal/equilibrium strategies are in state feedback form, are realizable in real time, and thus are actionable and useful to the warfighter.

*Partially supported by the Air Force Office of Scientific Research under Project Task: 3004/FY19, e-mail: `meir.Pachter@afit.edu`

Dynamic shape state space modeling with change points

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Abstract

The primary objective of this project is to enable data-driven adaptive control of dynamically evolving object shapes, where the control input-output relationship undergoes abrupt regime changes. To achieve this, we explore dynamic state space models for shape evolution and develop control models capable of handling these regime transitions. In this talk, we introduce our singly-transported vector field (SVF) approach, which simplifies the representation of complex shape evolution on a Riemannian manifold into a vector field, allowing the shape evolution to be modeled as a multivariate time series. We also present our change point detection methods to identify regime changes in shape evolution, facilitating the adaptation of control logic. Our new approaches are tested using simulated datasets and 3D print datasets provided by our collaborator at the Air Force Research Laboratory.

*Partially supported by the Air Force Office of Scientific Research under the Award No: FA9550-23-1-0673, e-mail: chiwpark@uw.edu

Hybrid Lie-Bracket Averaging for Model-Free Optimization and Control

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Abstract

Dynamical systems characterized by oscillatory behaviors and well-defined average vector fields have traditionally been subjects of stability analysis through methodologies rooted in averaging theory. Such tools have also found application in the stability analysis of systems that combine continuous-time dynamics and discrete-time dynamics, referred to as hybrid dynamical systems. However, in contrast to the existing results available in the literature for continuous-time systems, averaging results for hybrid systems have mostly been limited to standard, or *first-order*, averaging methods. This limitation prevents their direct application in the analysis and design of systems and algorithms that require high-order averaging techniques. Such techniques are typically necessary for solving model-free control and optimization problems with geometric constraints. To address this gap, we introduce new high-order averaging results for the stability analysis of hybrid dynamical systems with high-frequency periodic flow maps. The considered systems allow for the incorporation of set-valued flow maps and jump maps, effectively modeling well-posed differential and difference inclusions. By imposing appropriate regularity conditions on the hybrid system's data, results on closeness of solutions and semi-global practical asymptotic stability for sets are established. In this way, our findings yield hybrid Lie-bracket averaging tools that extend those found in the literature on ordinary differential equations. These theoretical results are then applied to the study of different applications in the context of model-free stabilization of systems with unknown control directions and model-free optimization. In both scenarios, we illustrate the limitations of existing hybrid and non-hybrid techniques, and how the proposed tools can overcome such limitations. We also discuss how some of the proposed algorithms can achieve *global* stability results in $\mathbb{R}^n$ by relying on a novel second-order averaging tool for systems with bounded vector fields.

*Partially supported by the Air Force Office of Scientific Research under the Award No: AFOSR YIP: FA9550-22-1-0211, e-mail: `poveda@ucsd.edu`

Data-guided Learning and Control of Higher Order Structures

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Abstract

The objective of this project is to develop a data-guided framework for learning, estimation and control of higher order structure, function, and dynamics. Underlying our framework are the concepts of tensor algebra, hypergraphs, and nonlinear dynamics and control. In this talk we present a framework for biomarker identification based on observability guided sensor selection. Our methods, Dynamic Sensor Selection (DSS) and Structure-Guided Sensor Selection (SGSS), utilize temporal models and experimental data, offering a template for applying observability theory to high dimensional and sparsely time sampled data obtained from biological systems. Unlike conventional methods that assume well-known, fixed dynamics, DSS adaptively selects biomarkers or sensors that maximize observability while accounting for the time-varying nature of biological systems. Additionally, SGSS incorporates structural information and diverse data sources to identify sensors which are resilient to inaccuracies of the identified model of the underlying system.

Partially supported by the Air Force Office of Scientific Research under the Award No: FA9550-22-1-0215, e-mail: `indikar@umich.edu`

A New Operator for Learning Nonlinear Dynamics

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Abstract

In this talk we investigate a different approach to learning dynamics through certain operators. Traditionally, operators associated with dynamical systems are the Koopman operators and their generators. Koopman operators naturally lead to linear models of the system state, which have performed well in many situations, including in data driven modeling of fluid dynamics.

However, in the creation of the linear model, accurate nonlinear approximations of the dynamics are discarded. This nonlinear approximation is obtained through the application of an approximation of the Koopman generator to the full state observable. This perspective on operator learning opens the door for different operators and different strategies for learning dynamical systems. In this talk, we will demonstrate that a similar approach to learning nonlinear dynamics by examining weighted composition operators.

*Partially supported by the Air Force Office of Scientific Research under the Award No: FA9550-21-1-0134, e-mail: rosenfeldj@usf.edu

Variational closures for composite homogenised fluid flows

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Abstract

The Stochastic Advection by Lie Transport (SALT) is a variational formulation of stochastic fluid dynamics introduced to model the effects of unresolved scales, whilst preserving the geometric structure of ideal fluid flows. In this report, we show that the SALT equations can arise from the decomposition of the fluid flow map into its mean and fluctuating components. The fluctuating component is realised as a prescribed stochastic diffeomorphism that introduces stochastic transport into the system and we construct it using homogenisation theory. The dynamics of the mean component are derived from a variational principle utilising particular forms of variations that preserve the composite structure of the flow. Using the new variational principle, we show that SALT equations can arise from random Lagrangians and are equivalent to random PDEs. We also demonstrate how to modify the composite flow and the associated variational principle to derive related theories such as Generalised Lagrangian Mean.

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†Partially supported by the Air Force Office of Scientific Research under the Award No: FA8655-21-1-7034

Some topics at the intersection of control, dynamics, and learning from data

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Abstract

Data-driven modeling typically involves simplifications of systems through dimensionality reduction (less variables) or through dimensionality enlargement (more variables, but simpler, perhaps linear, dynamics). Autoencoders with narrow bottleneck layers are a typical approach to the former (allowing the discovery of dynamics taking place in a lower-dimensional manifold), and autoencoders with wide layers provide an approach to the later, with "neurons" in these layers thought of as "observables" in Koopman representations. In this talk, I'll briefly discuss some theoretical results about each of these. (Joint work with M.D. Kvalheim on dimension reduction and with Z. Liu and N. Ozay on Koopman representations.)

Training of such autoencoders (or more general vector functions) is typically performed by gradient descent. Thus, it is natural to ask about the dynamics of such training, especially in the presence of errors in the estimation of the gradient. Inputs that represent perturbations from the true gradient arising from adversarial attacks, wrong evaluation by an oracle, early stopping of a simulation, inaccurate and very approximate digital twins, stochastic computations (algorithm "reproducibility"), or learning by sampling from limited data, can be formulated in terms of input to state stability (ISS), which quantifies the graceful degradation of performance of transient and asymptotic behavior. We present results for such disturbed gradient systems in the context of regression learning (joint work with A.C.B. de Oliveira and M. Siami) as well as in direct LQR algorithms (joint work with L. Cui and Z.P. Jiang).

*Partially supported by the Air Force Office of Scientific Research under the Award No: FA9550-21-1-0289, e-mail: e.sontag@northestern.edu, eduardo.sontag@gmail.com

Robust Optimal Estimation and Control for Dynamic Systems with Additive Heavy-Tailed Uncertainty for Aerospace Applications

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Abstract

The multivariate Cauchy estimator (MCE) is an analytic and recursive state estimation algorithm derived by using Bayes' rule and applied to discrete-time linear dynamic systems. Unlike the Kalman filter, the MCE models its additive measurement and process noises as Cauchy distributed rather than Gaussian. This structural change allows the MCE to analytically produce rich, sometimes non-symmetric, and even multiple peaked conditional probability density functions of a state space given its measurement history. Several contributions to the MCE algorithm have recently been proposed and implemented, that when used collectively, yield a fast and robust multivariate estimation algorithm for linear or nonlinear dynamic systems. The robust and real-time performance of the MCE is compared to the EKF/UKF in three motivating examples, which include a nonlinear pendulum simulation with a misidentified system parameter, a three-state homing missile with simulated impulsive radar noise, and a nonlinear seven-state low Earth orbit (LEO) state estimator with a realistic impulsive atmospheric uncertainty. Understanding the fundamental structure of this estimator, which propagates the characteristic function of the unnormalized conditional probability density function, and how to embed its Bayesian construction into new stochastic control formulations, including measures of robustness, convergence, and stability, is our current focus.

*Partially supported by the Air Force Office of Scientific Research under the Award No: FA9550-24-1-0161, e-mail: speyer@g.ucla.edu

Structure and dynamics of working language

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Abstract

This grant is funded by three AFOSR programs: Dynamics and Control, Computation and Learning, and Information Assurance. Its goal is to account for the structure and dynamics of working language using the compositional and relational mathematics of category theory.

Language *works* in the sense of basic physics: when I say "pass the salt," a coordinated symphony of feedback-driven activity moves 10^{20} atoms of salt-shaker through space, until some collected crystals of NaCl land on my soup. How can such working language come to exist, and what is the barest mathematical setting in which to account for it?

This being the first year that I'm reporting on this grant, I'll begin by explaining the problem. I'll then run through my group's current way of thinking about it, which involves very basic but fancy-sounding notions from category theory: the free monad monad and the cofree comonad comonad, and the fact that the former is a module over the latter. I will unpack this as best I can—keeping in mind the background of the audience—and present it in basic terms, e.g. flow charts acting on open discrete dynamical systems.

*Partially supported by the Air Force Office of Scientific Research (AFOSR) under the Award Number: FA9550-23-1-0376, e-mail: david@topos.institute, dspivak@gmail.com

Information-Geometric Path Planning: Roles of Information Theory in Motion Planning and Future Research Opportunities

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Abstract

This YIP project has been concerned with the motion planning problem in Gaussian belief spaces for minimum sensing navigation. In our approach, we formulate a shortest path problem on a Gaussian belief space equipped with a quasi-pseudometric representing the minimum information gain required to steer the belief state. Over the past years, sampling-based algorithms (namely RRT*- and PRM*-based schemes) have been developed and their effectiveness has been demonstrated experimentally.

In this presentation, I will summarize the overall achievements of the project. New research directions identified through the project, such as deep-learning-based path planning and information-theoretic motion planning via the path integral methods, will also be discussed. After describing various roles that information theory can play in motion planning, I will introduce a new research project titled "Motion Planning, Partial Observability, and Quantum Mechanics: Advancing the Frontiers of Path Integral Control Theory."

*Partially supported by the Air Force Office of Scientific Research under the Award No: FOA-AFRL-AFOSR-2019-0003, e-mail: ttanaka@utexas.edu

Converse theorems for strong forward invariance with applications to interconnections

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Abstract

We consider Lyapunov-like characterizations of strong forward invariance of a compact set for a constrained differential inclusion. In particular, we present new converse theorems for strong forward invariance. We show how the converse theorem for locally Lipschitz differential inclusions allows us to recover a recent result, in the setting of assume/guarantee contracts, on strong forward invariance for interconnected locally Lipschitz differential inclusions. The necessary Lyapunov-like conditions in this special case motivate sufficient Lyapunov-like conditions for strong forward invariance for interconnections of more general constrained differential inclusions and hybrid inclusions.

*Partially supported by the Air Force Office of Scientific Research under the Award No: FA9550-21-1-0452, e-mail: teel@ucsb.edu

Opportunistic Stochasticity in Shortest Path Problems: from causal PDE-discretizations to efficient routing of autonomous vehicles

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Abstract

Stochastic Shortest Path (SPP) problems are Markov Decision Processes that have many applications in discrete settings, but can also be used to produce discretizations of stationary Hamilton-Jacobi-Bellman PDEs describing the cost of optimal controls in indefinite-horizon problems (where the process terminates upon reaching a prescribed target). For *deterministic* shortest path problems on graphs, the solution is easy to find via non-iterative label-setting algorithms such as Dijkstra's classical method. Unfortunately, label-setting algorithms are inapplicable to general SSPs, but there are well-known non-trivial examples of SSPs for which they produce the right answer. This observation led to development of (non-iterative) Dijkstra-like methods for Eikonal PDEs describing isotropic optimal control problems. Much subsequent work has focused on extending such non-iterative techniques to more general (anisotropic) problems.

We will present a broad subclass of SSPs called *Opportunistically-Stochastic Shortest Path (OSSP)* problems, for which the applicability of label-setting algorithms is easy to guarantee a priori. We use OSSPs in two very different applications: (1) finding a simple geometric characterization of anisotropic min-time control problems in 2D and 3D, for which label-setting methods are applicable; and (2) introducing a new stochastic model of optimal lane-switching by autonomous vehicles and showing that it can be solved by a Dijkstra-like method. Both of these are joint work with a Ph.D. student Mallory Gaspard.

*Partially supported by the Air Force Office of Scientific Research under the Award No: FA9550-22-1-0528, e-mail: vladimirsky@cornell.edu

Understanding and Leveraging Synchronization in Reservoir Computing

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Abstract

Reservoir Computing (RC) has garnered much interest from both the machine learning, dynamical systems, and mathematics communities. This is due to the small memory footprint, fast training times, and unusual architecture, yet able to produce good predictions of noisy and chaotic measurements. Recent publications have linked the concept of Generalized Synchronization with RC, which partially explains the unusual architecture and predictive capabilities of RC. In this work, we present an intuitive manner of understanding RC from the perspective of established dynamical systems theory. We then leverage synchronization of nearly-identical systems to create observers of experimental thruster and facility measurements. These inferred measurements are synchronized with the original system, so long as a subset of measurements is continually presented. So far, all the results have been empirical and future effort will be focused on establishing more theoretical guarantees.

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