Dynamic Mission Planning: Adversarial Conflicts and Optimal Dubins Path on a Sphere

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Outline

- ✓ Introduction
- ✓ Escorting of HVAA
- ✓ Virtual Target Selection for Multiple-Pursuer Multiple-Evader
- ✓ Optimal Dubins Path on a Sphere
- ✓ Recent Publications

Introduction

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Escorting of HVAA

Escorting of HVAA

- **Motivation.**
- Future mission planning will involve a high value aerial asset such as a bomber or AWACS being escorted by one or several wingmen.
- A similar concept includes a piloted, exquisite aircraft commanding several, relatively inexpensive, Unmanned Aerial Vehicles (UAVs) or Collaborative Combat Aircrafts (CCAs).

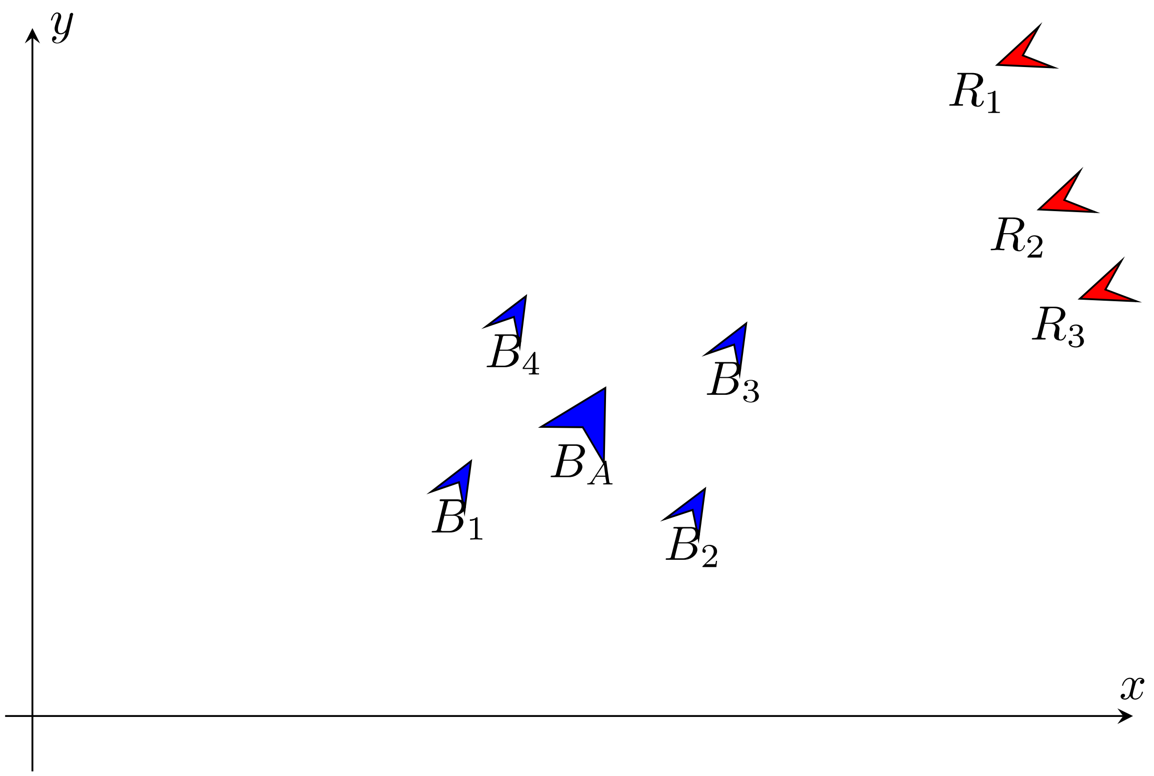
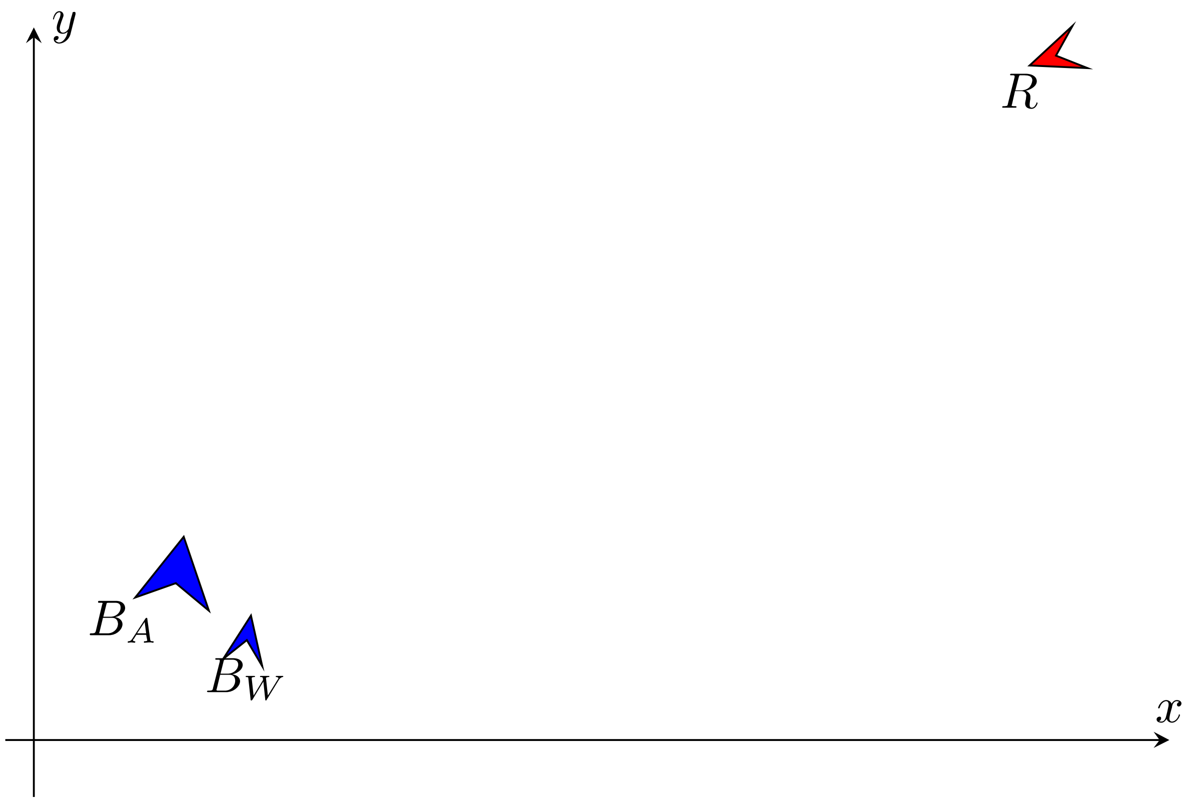


Fig. 1. Dynamic Mission Planning

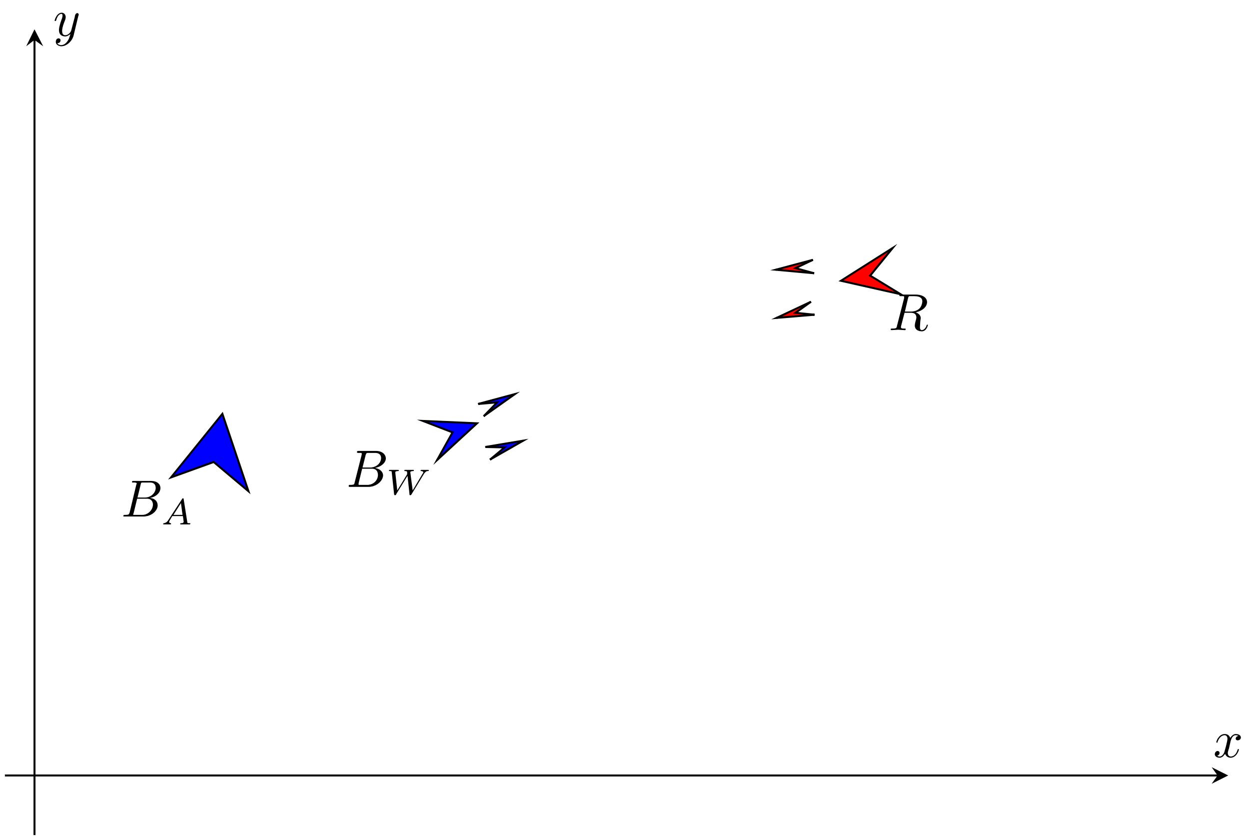
Escorting of HVAA

- **Problem.**
- Protect an HVAA which travels with constant heading.
- Wingman needs to block the red interceptor as far as possible from HVAA.
- Block: reach within fire range from interceptor.



Escorting of HVAA

- **Problem.**
- Red interceptor seeks to reach as close as possible to the HVAA and fire its weapons.
- Wingman aims at maximizing the terminal distance between the HVAA and interceptor at the moment it reaches blocking range.



Optimal Strategies

Optimal Strategies

- Reach-avoid differential game between B_W and R
- Both players know the speeds constant speed and heading of B_A
- R also knows the blocking parameter (range of weapon of B_W)

$$f = \frac{v_W}{v_R} < 1 \qquad \alpha = \frac{v_A}{v_R} < 1$$

$$\rho > 0$$

- Dominance regions explicitly considering both f and ρ are separated by the Cartesian Oval (CO)

$$r_W = fr + \rho$$

- r : distance R will travel to reach a given point in the CO
- r_W : distance between the point in the CO and the current position of B_W

Optimal Strategies

- Without loss of generality, assume that the initial position of the HVAA is $B_A = (0, 0)$ and $\theta_A = \frac{\pi}{2}$
- The positions of B_W and R in the relative frame are denoted by $B_W = (x_W, y_W)$ and $R = (x_R, y_R)$
- The LOS angle from R to B_W is

$$\lambda_W = \arctan\left(\frac{y_W - y_R}{x_W - x_R}\right)$$

- The current separation between B_W and R

$$d = \sqrt{(x_W - x_R)^2 + (y_W - y_R)^2}$$

Optimal Strategies

- **Theorem**: The optimal blocking point in the CO between R and B_W such that R minimizes and B_W maximizes the terminal distance between R and B_A, corresponds to the optimal distance r* which is the real solution of the polynomial equation

$$
\begin{aligned}
&n_4c_4r^8 + (n_4c_3 + n_3c_4)r^7 + (n_4c_2 + n_3c_3 + n_2c_4)r^6 + \big(n_4c_1 + n_3c_2 + n_2c_3 + n_1c_4 - 4\alpha^2 d^2(1-f^2)f\rho C_w^2\big)r^5 \\
&+ \big(n_4c_0 + n_3c_1 + n_2c_2 + n_1c_3 + n_0c_4 - 4\alpha d^2 C_w[\alpha C_w(\rho^2 + f^2K - f^2\rho^2) + 2(1-f^2)f\rho(x_RS_w - y_RC_w)]\big)r^4 \\
&+ \big(n_3c_0 + n_2c_1 + n_1c_2 + n_0c_3 - 4d^2[\alpha^2 f\rho C_w^2 + 2\alpha C_w(x_RS_w - y_RC_w)(\rho^2 + f^2K - f^2\rho^2) + (1-f^2)f\rho(x_RS_w - \\
&+ \big(n_2c_0 + n_1c_1 + n_0c_2 - 4d^2(x_RS_w - y_RC_w)[2\alpha f\rho C_wK + (x_RS_w - y_RC_w)(\rho^2 + f^2K - f^2\rho^2)]\big)r^2 \\
&+ \big(n_1c_0 + n_0c_1 - 4d^2 f\rho K(x_RS_w - y_RC_w)^2\big)r + n_0c_0 = 0
\end{aligned}
\tag{1}
$$

that minimizes the cost

$$
\begin{aligned}
J^2 &= x_R^2 + (\alpha r - y_R)^2 + r^2 - \frac{1}{d}\Big[(d^2 + r^2 - r_w^2)[(\alpha r - y_R)S_w - x_RC_w] \\
&\quad - \sqrt{4d^2r^2 - (d^2 + r^2 - r_w^2)^2}[(\alpha r - y_R)C_w + x_RS_w]\Big]
\end{aligned}
$$

Optimal Strategies

- Where

$$c_4 = \frac{9}{4}\alpha^2(1-f^2)^2$$

$$c_3 = -3\alpha(1-f^2)[2\alpha f\rho + y_R(1-f^2) + dS_w(1+\alpha^2)]$$

$$c_2 = d^2(1+\alpha^2(\alpha^2 - C_w^2 + 2S_w^2)) + 4\alpha^2 f^2\rho^2 + \frac{3}{2}\alpha^2(1-f^2)K + (1-f^2)^2(x_R^2+y_R^2) + 7\alpha y_R(1-f^2)f\rho$$
$$+ 2d(1+\alpha^2)((1-f^2)(y_R S_w + x_R C_w) + 2\alpha S_w f\rho) + 3d\alpha^2 y_R S_w(1-f^2)$$

$$c_1 = -\big[2\alpha d^2(y_R S_w^2 + x_R S_w C_w + \alpha^2 y_R) + 2\alpha^2 f\rho K + 2(1-f^2)f\rho(x_R^2+y_R^2) + \alpha y_R((1-f^2)K + 4f^2\rho^2)$$
$$+ d(1+\alpha^2)\alpha S_w K + 4d\alpha^2 y_R S_w f\rho + 2d(y_R S_w + x_R C_w)((1+\alpha^2)f\rho + \alpha(1-f^2)y_R)\big]$$

$$c_0 = \alpha^2 d^2 y_R^2 + \frac{\alpha^2}{4}K^2 + (x_R^2+y_R^2)f^2\rho^2 + \alpha y_R f\rho K + \alpha d y_R[\alpha S_w K + 2f\rho(y_R S_w + x_R C_w)]$$

Optimal Strategies

- Also

$$n_4 = -(1 - f^2)^2$$
$$n_3 = 4(1 - f^2) f \rho$$
$$n_2 = 2(d^2 + \rho^2 + d^2 f^2 - 3 f^2 \rho^2)$$
$$n_1 = 4 f \rho K$$
$$n_0 = -K^2$$
$$K = d^2 - \rho^2$$

Optimal Strategies

- *Proof*. Define

$$\lambda_1 = \arctan\left(\frac{\alpha r - y_R}{-x_R}\right)$$

We can also find

$$J_1^2 = x_R^2 + (\alpha r - y_R)^2$$

Also, solve for ϕ in the following

$$r_W^2 = d^2 + r^2 - 2dr\cos\phi$$

$$\cos\phi = \frac{d^2 + r^2 - r_w^2}{2dr}$$

$$\sin\phi = \frac{1}{2dr}\sqrt{4d^2r^2 - (d^2 + r^2 - r_w^2)^2}$$

Optimal Strategies

The cost to be minimized is obtained (in terms of J_1)

$$J^2 = J_1^2 + r^2 - 2J_1 r\cos(\phi - \theta)$$

where $\theta = \lambda_W - \lambda_1$

Expanding and substituting J_1

$$\begin{aligned} J^2 = {} & x_R^2 + (\alpha r - y_R)^2 + r^2 - 2rJ_1[\cos\phi\cos\theta + \sin\phi\sin\theta] \\ & = x_R^2 + (\alpha r - y_R)^2 + r^2 - \frac{1}{d}\Big[(d^2 + r^2 - r_w^2)[(\alpha r - y_R)S_w - x_R C_w] \\ & - \sqrt{4d^2r^2 - (d^2 + r^2 - r_w^2)^2}[(\alpha r - y_R)C_w + x_R S_w]\Big] \end{aligned}$$

where $C_w = \cos\lambda_W$, $S_w = \sin\lambda_W$

Optimal Strategies

Find the first derivative of J_2 with respect to r and set it equal to zero.

The resulting equation is as follows

$$
\begin{aligned}
& d[\alpha(\alpha r - y_R) + r] - \frac{\alpha S_w}{2}[d^2 + (1 - f^2)r^2 - 2f\rho r - \rho^2] \\
& - [(\alpha r - y_R)S_w - x_R C_w][(1 - f^2)r - f\rho] \\
& - \frac{\alpha C_w}{2}\sqrt{4d^2r^2 - [d^2 + (1 - f^2)r^2 - 2f\rho r - \rho^2]^2} \\
& - \frac{[(\alpha r - y_R)C_w + x_R S_w]\left[2d^2 r - [d^2 + (1 - f^2)r^2 - 2f\rho r - \rho^2][(1 - f^2)r - f\rho]\right]}{\sqrt{4d^2r^2 - [d^2 + (1 - f^2)r^2 - 2f\rho r - \rho^2]^2}} \\
& = 0
\end{aligned}
$$

After extensive simplification, eq. (1) is obtained.

Example

- Optimal strategies implemented by both Wingman and Red Interceptor for

$$\theta_A = 1.32\ rad$$

V(**x**) = 32.2388

- Minmax terminal distance between *H* and *R*.

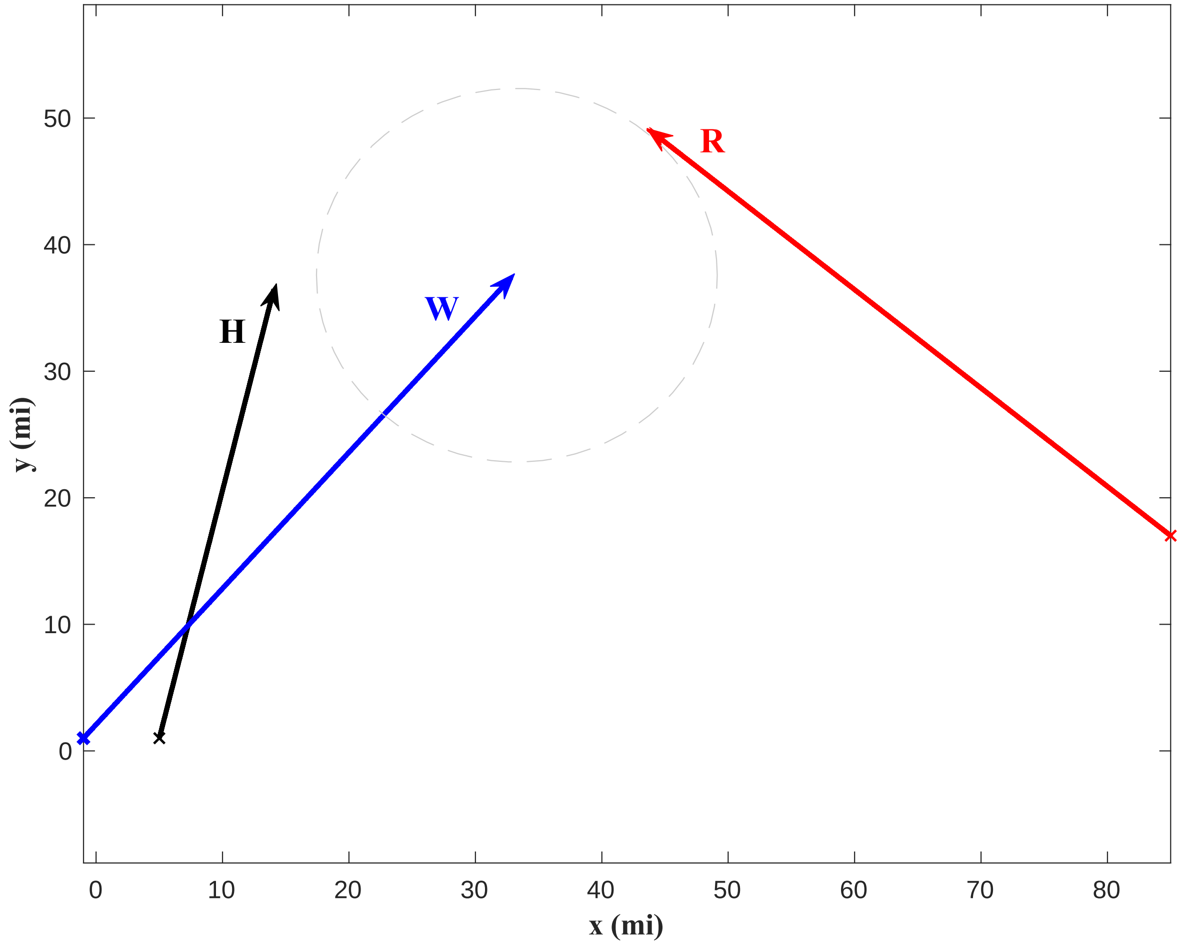


Fig. Optimal play.

Example

- Non-optimal strategies implemented by the Red Interceptor.

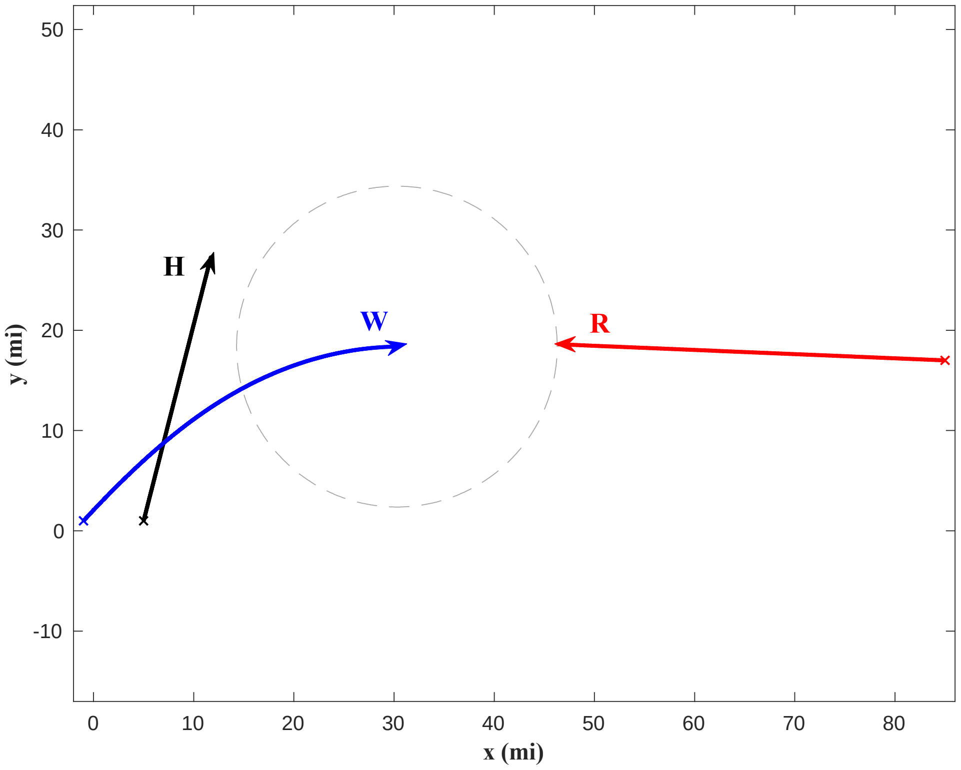


Fig. 'aggressive' maneuver by *R*.
$J = 35.5943 > V(\mathbf{x})$

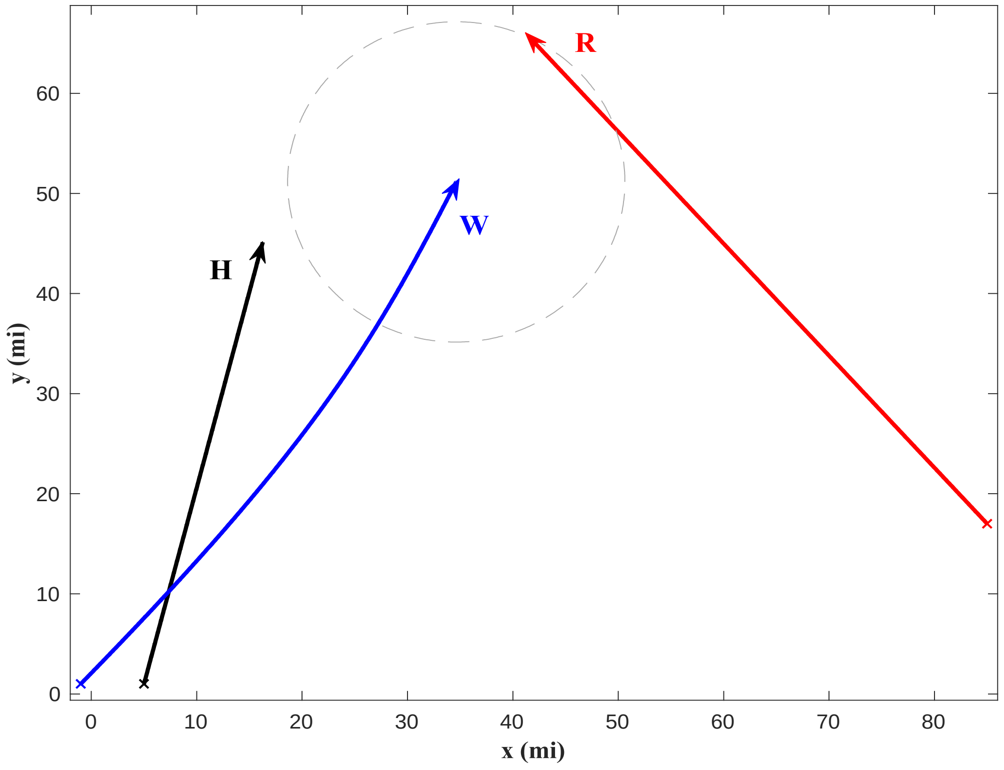


Fig. 'passive' maneuver by *R*.
$J = 32.5533 > V(\mathbf{x})$

Example

- Non-optimal strategies implemented by the Red Interceptor.

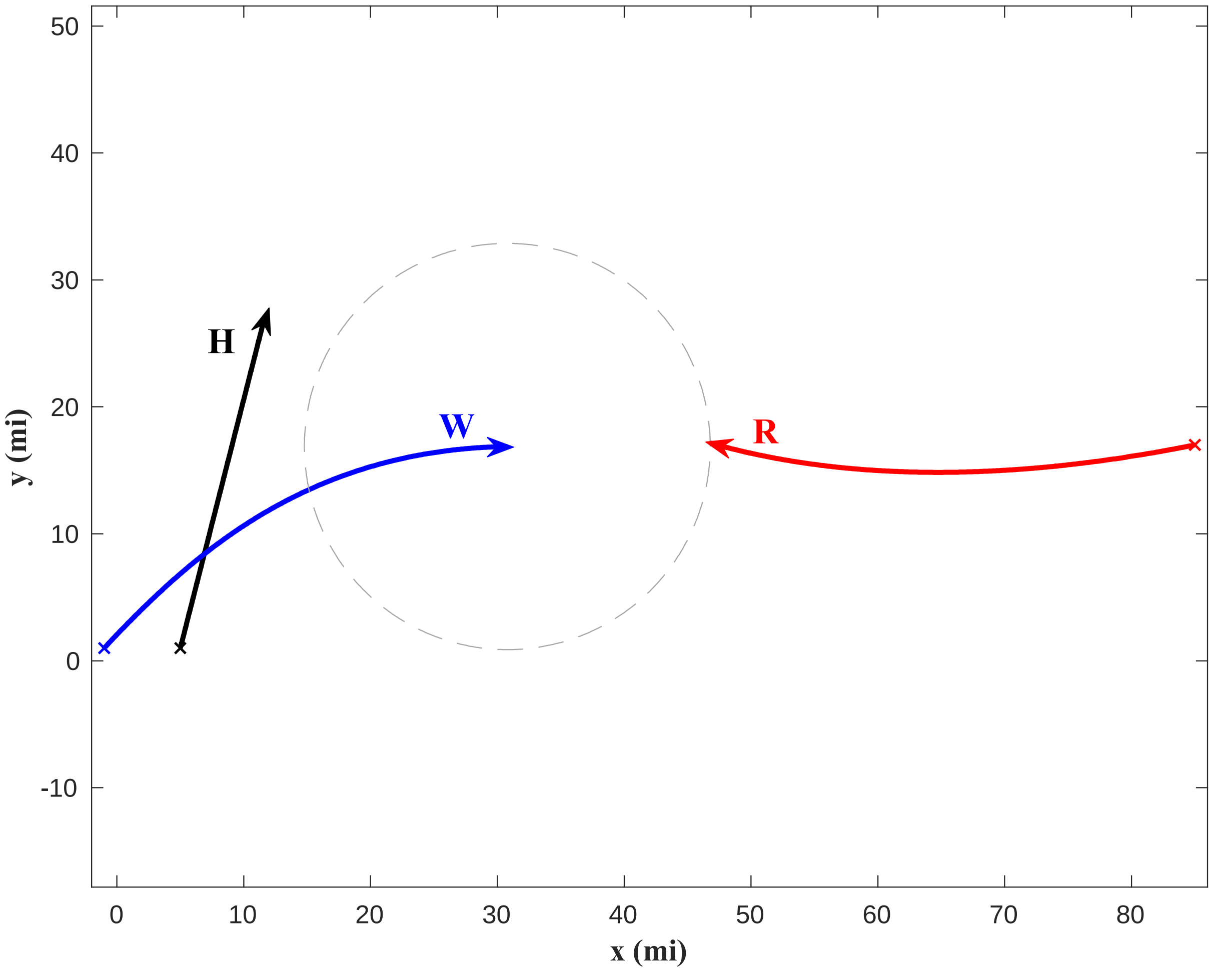


Fig. *R* implements Pure Pursuit on *H*.
$J = 36.4824 \; > \mathrm{V}(\mathbf{x})$

Escorting of HVAA

➢ Practical extensions.

- Low-end CCAs. Significantly slower than red interceptors, need to cooperate 2-on-1.
- High-end CCAs. Carry several weapons, could block sequential adversaries. Use terminal conditions of one stage for next stage.
- Include other tasks such as stationary enemies or tasks.
- Multiple CCAs, tasks and interceptors. Find optimal assignments.

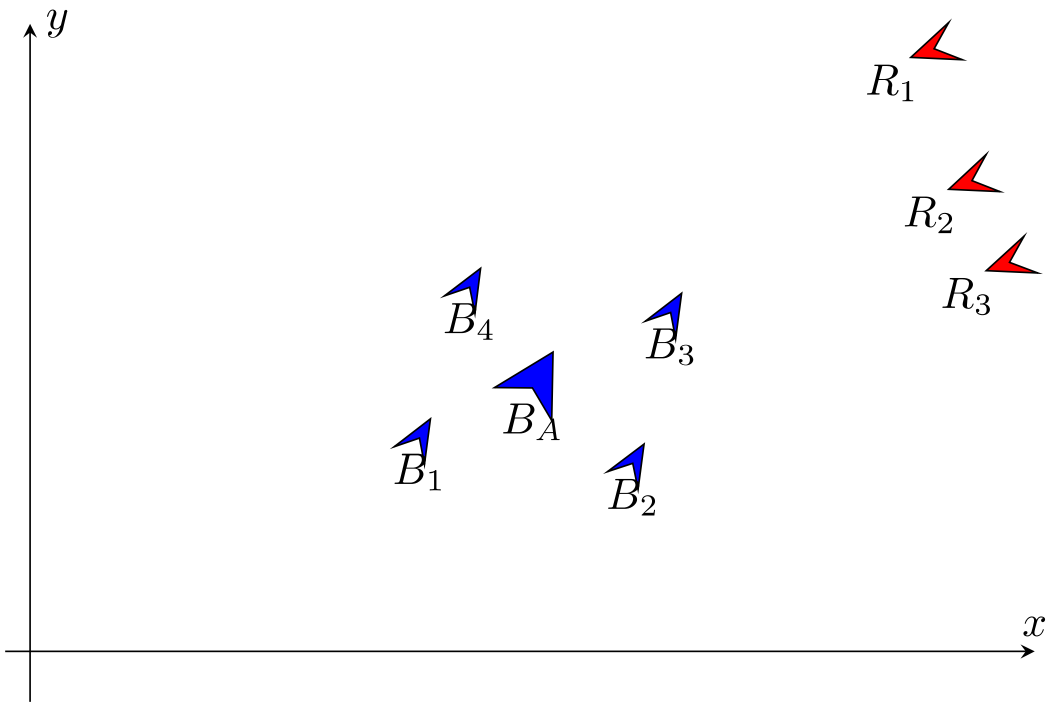


Fig. 1. Dynamic Mission Planning

Virtual Target Selection for Multiple-Pursuer Multiple-Evader

Virtual Target Selection for Multiple-Pursuer Multiple-Evader

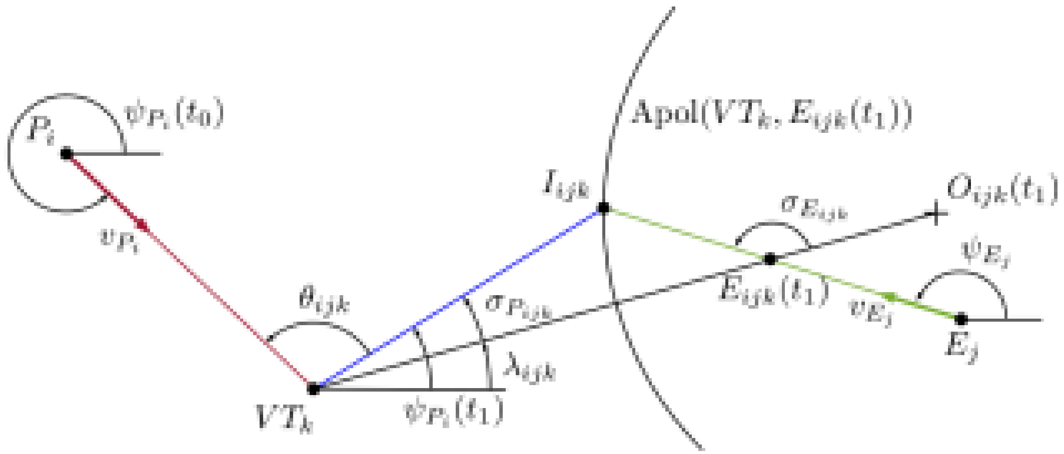
Background.

- Pursuit and weapon target assignment problems involving mobile agents represent a relevant class of problems for the aerospace and defense community
- Scalable methods for performing weapon target assignment are desired
- Perform the optimization for an overall fleet of vehicles rather than individually
- Delayed decisions made by the pursuers is of interest to provide agility and flexibility to operations.

Introduction

Problem Setup

- Consider many pursuers and evaders
- Faster pursuers aim to capture slower evaders
- Evaders stay on fixed-course
- Pursuers exhibit simple motion
- Pursuers navigate to a virtual target prior to engaging the evaders



Generalized Geometry

Objectives

- Pursuer intercept strategies to evaders by way of a virtual target
- Obtain the pursuer-evader assignments to minimize overall *energy* required by pursuer team.

Optimal Control Problem

State Space:

$$[x_{P_i}, y_{P_i}, x_{E_j}, y_{E_j}]^\top \forall i \in [1..N] \wedge j \in [1..M]$$

Controls: $\psi_P \in \Theta^N, \Theta \in [0, 2\pi] \subset \mathbb{R}$

Initial Conditions:

$$P_i = \left(x_{P_i}, y_{P_i}\right) \in \mathbb{R}^2, P \in \mathbb{R}^{N\times 2}, \quad P_0 = P(t_0)$$

$$E_i = \left(x_{E_i}, y_{E_i}\right) \in \mathbb{R}^2, E \in \mathbb{R}^{M\times 2}, \quad E_0 = E(t_0)$$

$$VT_i = \left(x_{VT_i}, y_{VT_i}\right) \in \mathbb{R}^2, P \in \mathbb{R}^{L\times 2}, VT_0 = VT(t_0)$$

Objective:

$$\min_{\psi_P} J = \min_{\psi_P}\left\{\pi - \theta_{ijk} + \int_{t_0}^{t_f} 1dt\right\}$$

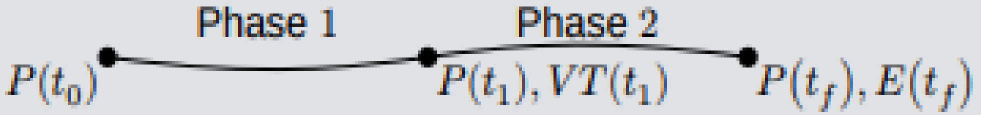
Dynamics:

$$\begin{aligned} \dot{x}_{P_i} &= v_{P_i}\cos\psi_{P_i} && \forall i \in [1...N] \\ \dot{y}_{P_i} &= v_{P_i}\sin\psi_{P_i} && \forall i \in [1...N] \\ \dot{x}_{E_j} &= v_{E_j}\cos\psi_{E_j} && \forall j \in [1...M] \\ \dot{y}_{E_j} &= v_{E_j}\sin\psi_{E_j} && \forall j \in [1...M] \end{aligned}$$

Equality Conditions:

$$P_f = P(t_f) = E_f = E(t_f)$$

$$P(t_1) = VT(t_1)$$



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Optimal Control Problem - Indirect Method

The optimal control problem is broken into two phases: Phase 1: $t \in [t_0, t_1)$ and Phase 2: $t \in [t_1, t_f]$.

Phase 1 Hamiltonian:

$$\mathcal{H}_I = p_{x_{P_i}}(t) v_{P_i} \cos\big(\psi_{P_i}(t)\big) + p_{y_{P_i}}(t) v_{P_i} \sin\big(\psi_{P_i}(t)\big)$$

Phase 1 Costate Dynamics:

$$\dot{p}_{x_{P_i}} = -\frac{\partial \mathcal{H}_I}{\partial x_{P_i}} = 0, \quad \dot{p}_{y_{P_i}} = -\frac{\partial \mathcal{H}_I}{\partial y_{P_i}} = 0$$

Costates are constant for Phase 1.

Phase 1 Stationarity Condition:

$$\frac{\partial \mathcal{H}_I}{\partial \psi_{P_i}} = 0 \Rightarrow -p_{x_{P_i}} v_{P_i} \sin\psi_{P_i} + p_{y_{P_i}} v_{P_i} \cos\psi_{P_i} = 0$$

The optimal control is constant for Phase 1 and is:

$$\psi^*_{P_i}(t) = \left\{ \operatorname{atan2}\left(y_{VT_k} - y_{P_i}, x_{VT_k} - x_{P_i}\right) \mid t \in [t_0, t_1) \right\}$$

Phase 2 Hamiltonian:

$$\mathcal{H}_{\mathrm{I}} = p_{x_{P_i}}(t) v_{P_i} \cos\big(\psi_{P_i}(t)\big) + p_{y_{P_i}}(t) v_{P_i} \sin\big(\psi_{P_i}(t)\big)$$

Phase 2 Costate Dynamics:

$$\dot{p}_{x_{P_i}} = -\frac{\partial \mathcal{H}_{\mathrm{I}}}{\partial x_{P_i}} = 0, \quad \dot{p}_{y_{P_i}} = -\frac{\partial \mathcal{H}_{\mathrm{I}}}{\partial y_{P_i}} = 0$$

Phase 2 Stationarity Condition:

$$\frac{\partial \mathcal{H}_{\mathrm{I}}}{\partial \psi_{P_i}} = 0 \Rightarrow -p_{x_{P_i}} v_{P_i} \sin\psi_{P_i} + p_{y_{P_i}} v_{P_i} \cos\psi_{P_i} = 0.$$

Therefore the optimal control is constant for Phase 2.

Phase 2 Solution Strategy:

- Pursuer takes a straight line course
- Intercept a slower evader
- Use Apollonius circle geometry

Apollonius Circle and the Interception Point

Time for P to reach VT:

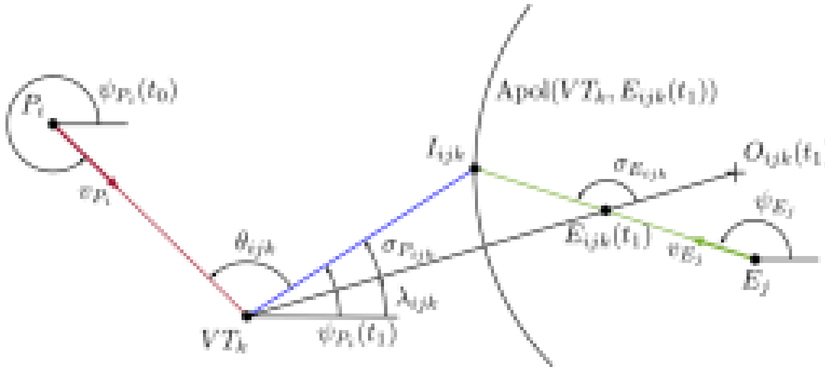
$$t_1 = \frac{1}{v_{P_i}}\sqrt{\left(x_{VT_k}-x_{P_i}(t_0)\right)^2+\left(y_{VT_k}-y_{P_i}(t_0)\right)^2}$$

Position of E when P is at VT :

$$E_{ijk}(t_1) = E_j(t_0) + t_1 v_{E_j}\hat{v}_{E_j}$$

Speed Ratio:

$$\mu_{ij} = \frac{v_{E_j}}{v_{P_i}}$$



Apollonius Circle

$$R_{ijk} = \frac{\mu_{ij}\overline{VT_k E_{ijk}(t_1)}}{1-\mu_{ij}^2}$$

$$O_{ijk}(t_1) = E_{ijk}(t_1) + \frac{\mu_{ij}\overline{VT_k E_{ijk}(t_1)}^2}{1-\mu_{ij}^2}\begin{pmatrix}\cos\lambda_{ijk}\\ \sin\lambda_{ijk}\end{pmatrix}$$

where

$$\overline{VT_k E_{ijk}} = \sqrt{\left(x_{E_{ijk}}(t_1)-x_{VT_k}\right)^2+\left(y_{E_{ijk}}(t_1)-y_{VT_k}\right)^2}$$

Interception point

$$I_{ijk} = \overline{VT_k I_{ijk}}\begin{pmatrix}\cos\left(\sigma_{P_{ijk}}+\lambda_{ijk}\right)\\ \sin\left(\sigma_{P_{ijk}}+\lambda_{ijk}\right)\end{pmatrix} + \begin{pmatrix}x_{VT_k}\\ y_{VT_k}\end{pmatrix}$$

where

$$\sigma_{E_{ijk}} = \psi_{E_j} - \lambda_{ijk}, \quad \sigma_{P_{ijk}} = \sin^{-1}\left(\mu_{ij}\sin\sigma_{E_{ijk}}\right)$$

$$\lambda_{ijk} = \mathrm{atan2}\left(y_{E_{ijk}}(t_1)-y_{VT_k}, x_{E_{ijk}}(t_1)-x_{VT_k}\right)$$

$$\overline{VT_k I_{ijk}} = \frac{\overline{VT_k E_{ijk}}}{1-\mu_{ij}^2}\left(\mu_{ij}\cos\sigma_{E_{ijk}} + \sqrt{1-\mu_{ij}^2\sin^2\sigma_{E_{ijk}}}\right)$$

Mixed Integer Linear Program Formulation

Solution Approach Revisited

Value Function

1. Use Optimal Control Theory to find the cost for the assignment of: $P_i \to VT_k \to E_j$
2. Optimal Control Theory allows for the utility of Apollonius circle geometry
3. Calculate the Length of $\overline{P_i VT_k}$ and $\overline{VT_k E_j}$
4. Calculate the maneuver θ_{ijk}
5. The cost of the assignment is

$$\overline{P_i VT_k} + \overline{VT_k E_j} + \pi - \theta_{ijk}$$

Assignment Problem

1. Obtain the cost for the assignment of: $P_i \to VT_k \to E_j$
2. Limit the number of VT_k cadidates to some maximum: M_V
3. Use a mixed integer linear program to find the optimal assignments.

Mixed Integer Linear Program (MILP) Formulation

- The cost for $P_i \rightarrow VT_k \rightarrow E_j$ is denoted as c_{ijk}

MILP Equation

$$\text{Objective:} \quad \min \sum_{i \in \mathcal{P}, j \in \mathcal{E}, k \in \mathcal{V}} c_{ijk} x_{ijk},$$

subject to

$$\text{Contraint 1:} \quad \sum_{i \in \mathcal{P}, k \in \mathcal{V}} x_{ijk} \geq 1, \forall j \in \mathcal{E},$$

$$\text{Contraint 2:} \quad \sum_{j \in \mathcal{E}, k \in \mathcal{V}} x_{ijk} = 1, \forall i \in \mathcal{P},$$

$$\text{Contraint 3:} \quad \sum_{i \in \mathcal{P}, j \in \mathcal{E}} x_{ijk} \leq y_k, \forall k \in \mathcal{V},$$

$$\text{Contraint 4:} \quad \sum_{k \in \mathcal{V}} y_k \leq M_V,$$

$$\text{Contraint 5:} \; x_{ijk}, y_k \in \{0, 1\}, \forall i \in \mathcal{P}, j \in \mathcal{E}, k \in \mathcal{V}.$$

Objective:
Find the assignment that minimizes the cost

Constraint 1:
Every E is assigned to at least one P at a VT

Constraint 2:
Every P is assigned to some VT and an E combination

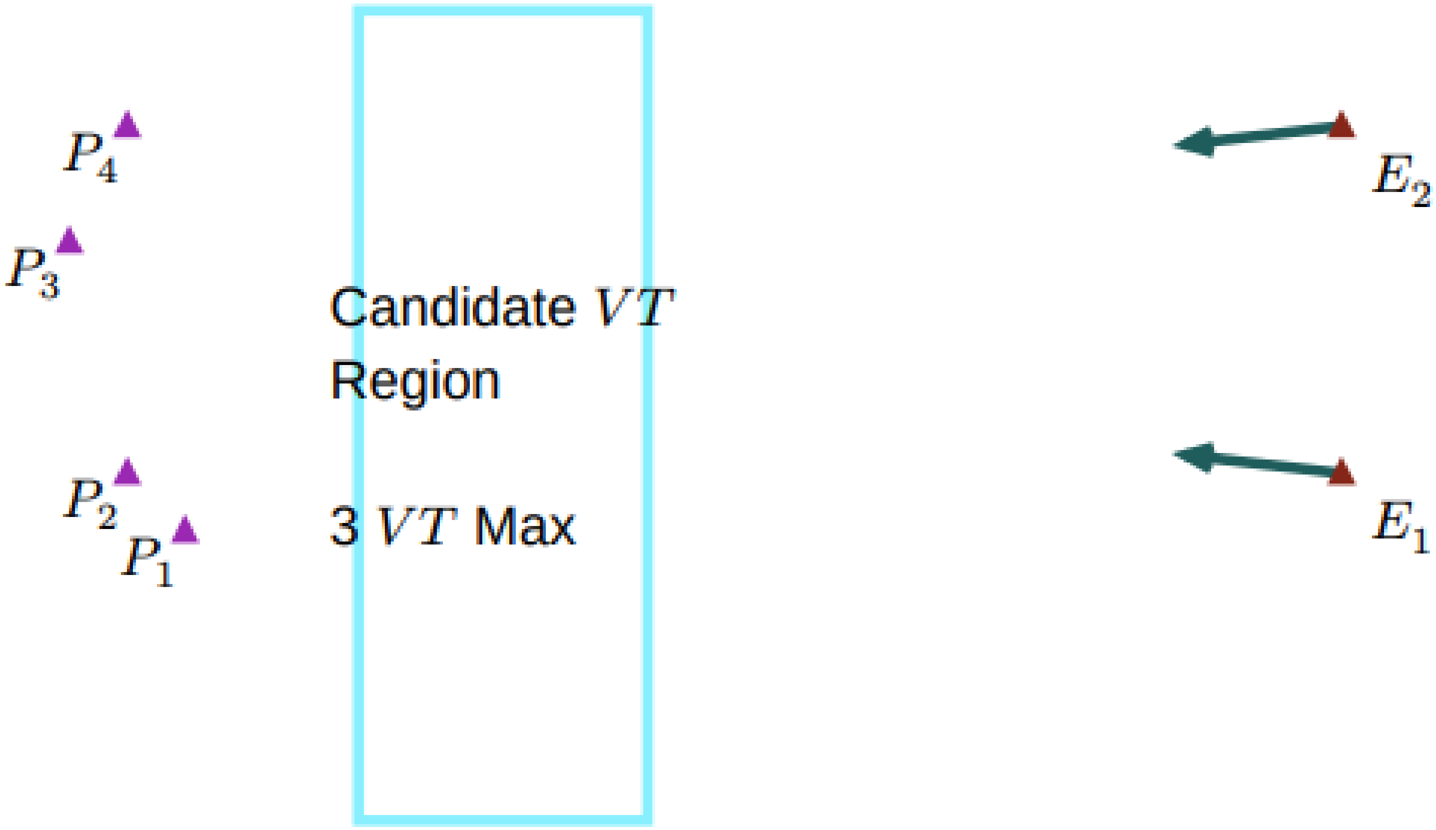
Constraint 3:
A VT_k, can only be used if the corresponding $y_k = 1$

Constraint 4:
Limit the maximum number of VT's.

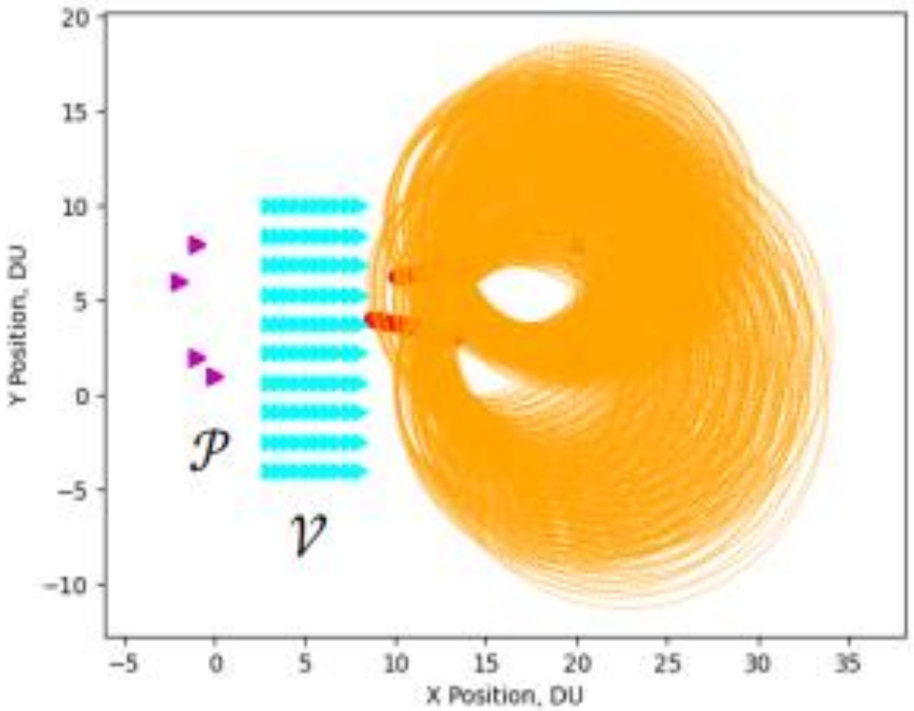
Constraint 5:
Binary constraints

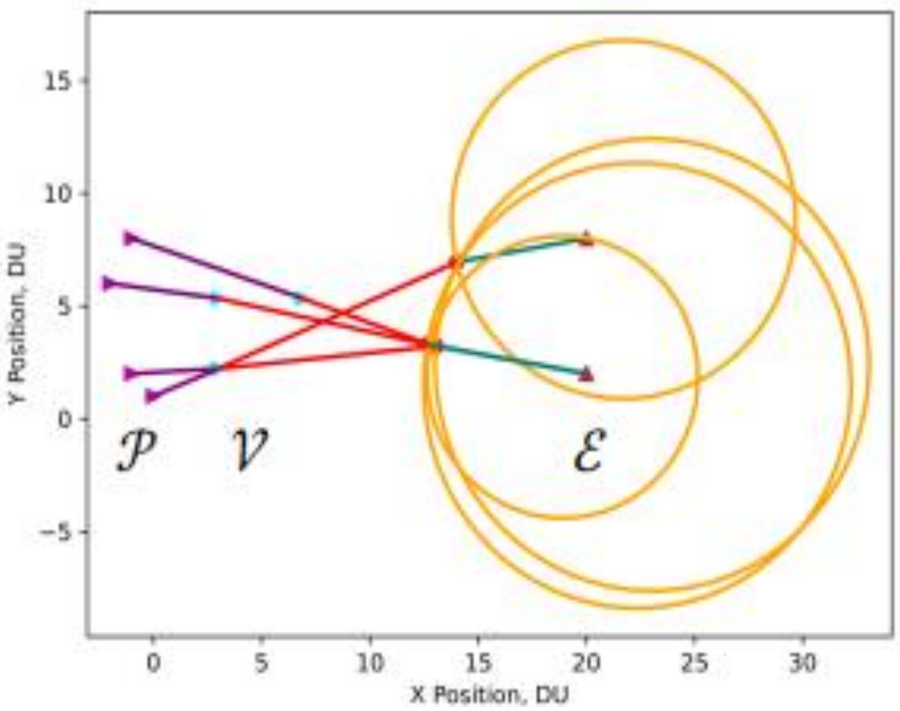
Example: Setup

Problem Setup: 4 Pursuers, 2 Evaders, 3 Virtual Targets



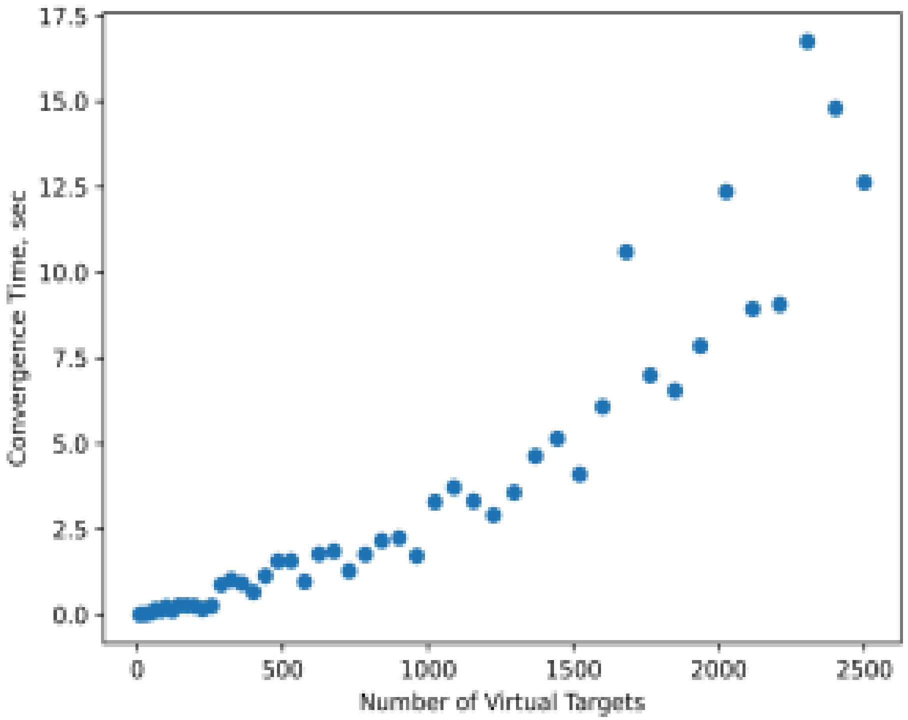
Example: Results - 100 Candidate Virtual Targets



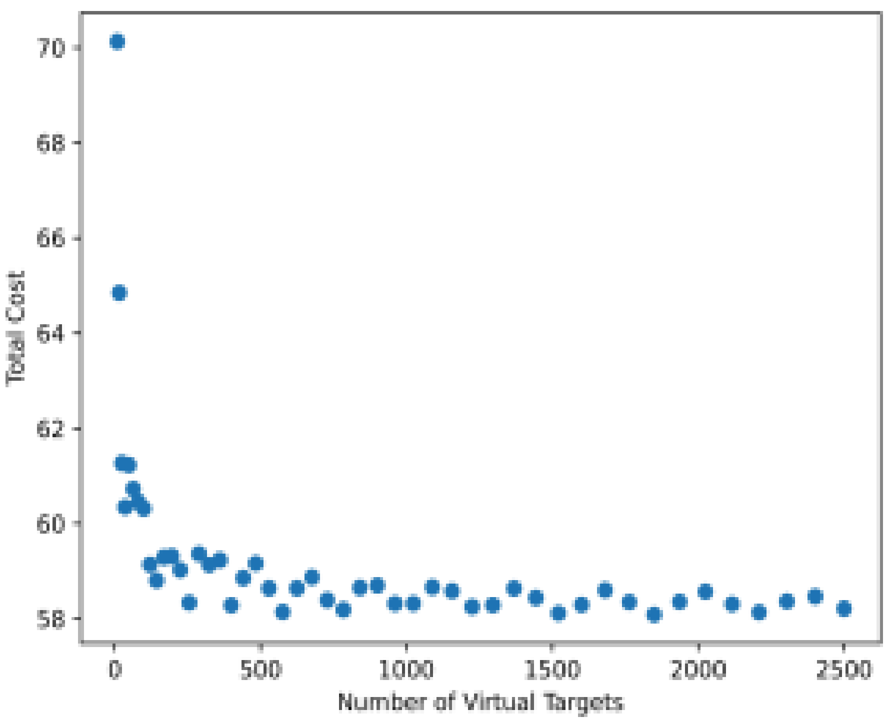


Covergence Time and Performance Tradeoff

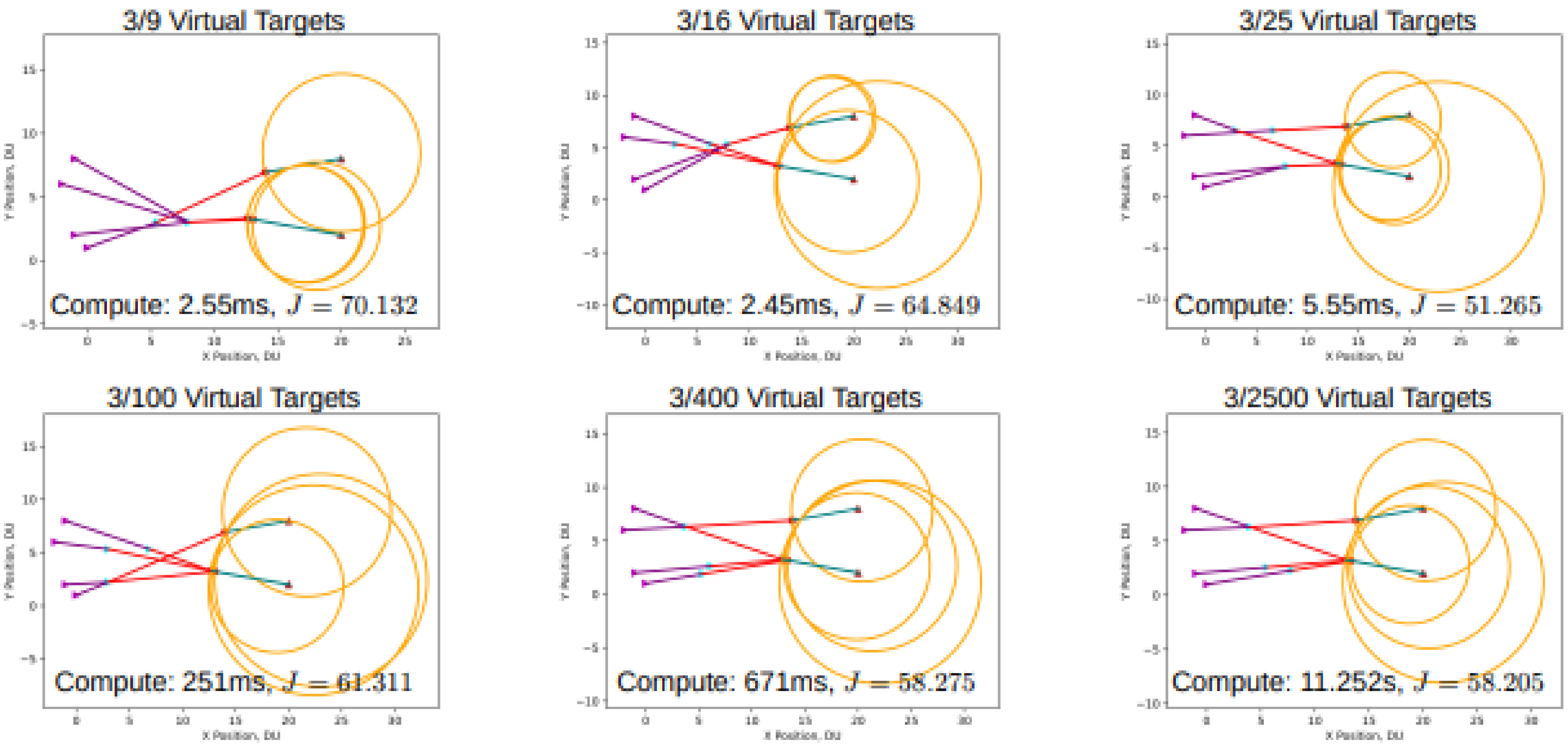
Convergence Time



Total Cost



Results: Various Number of Candidate VTs



Conclusions

Summary
- Multi-pursuer Multi-Evader Assignment Problem
- Pursuers navigate to virtual targets then to an assigned target
- Apollponius circle geomery and linear program solver leveraged
- Energy of the team is minimized

Observations
- A weighting factor on manuever or path travel changes the performance
- Solutions scale well for potential hardware applications

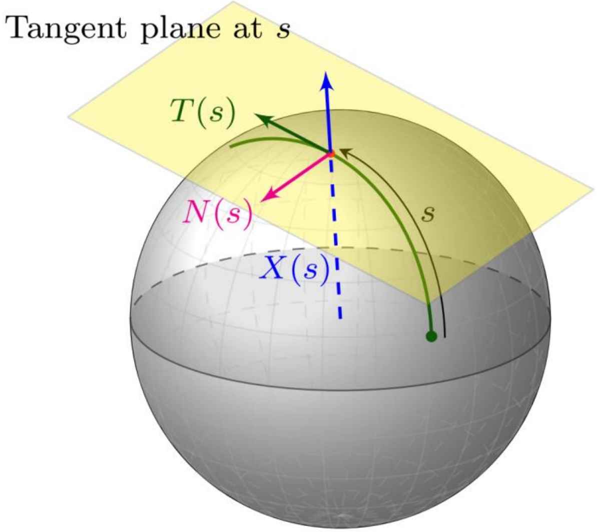
Future Work
- Ensuring path deconfliction of solutions
- Simulating higher-fidelity vehicles
- Performing software-in-the-loop and or hardware-in-the-loop tests
- Flight test where possible.

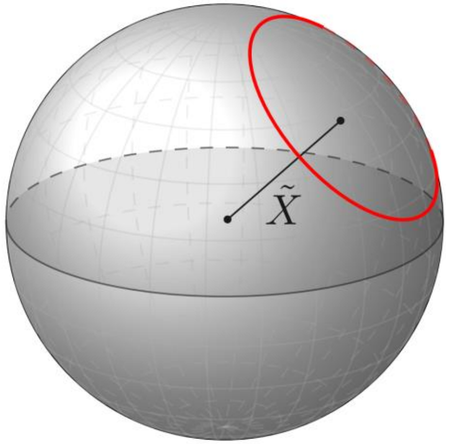
Optimal Dubins Path on a Sphere

Path Planning on a Sphere

- **Problem Statement:**
  - Find shortest geodesic curvature constrained path on surface of sphere.

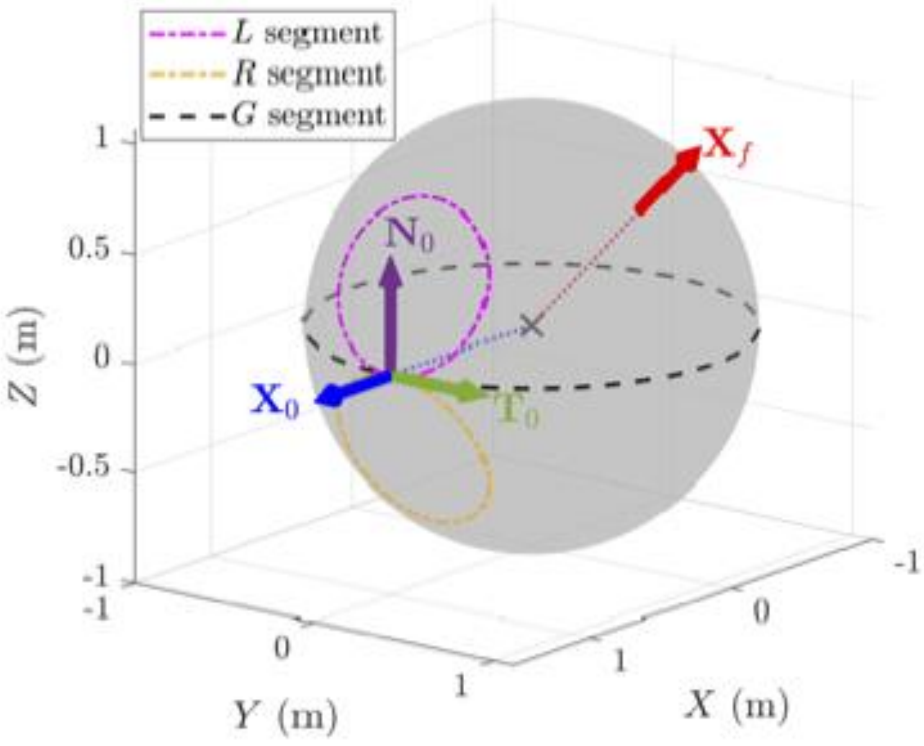
- **Results:**
  - Optimal path contained great circle arcs (G) and arcs corresponding to minimum turning radius (C).
  - Optimal paths are of type CCC, CGC, and degenerate paths for $r \leq \frac{1}{2}$.



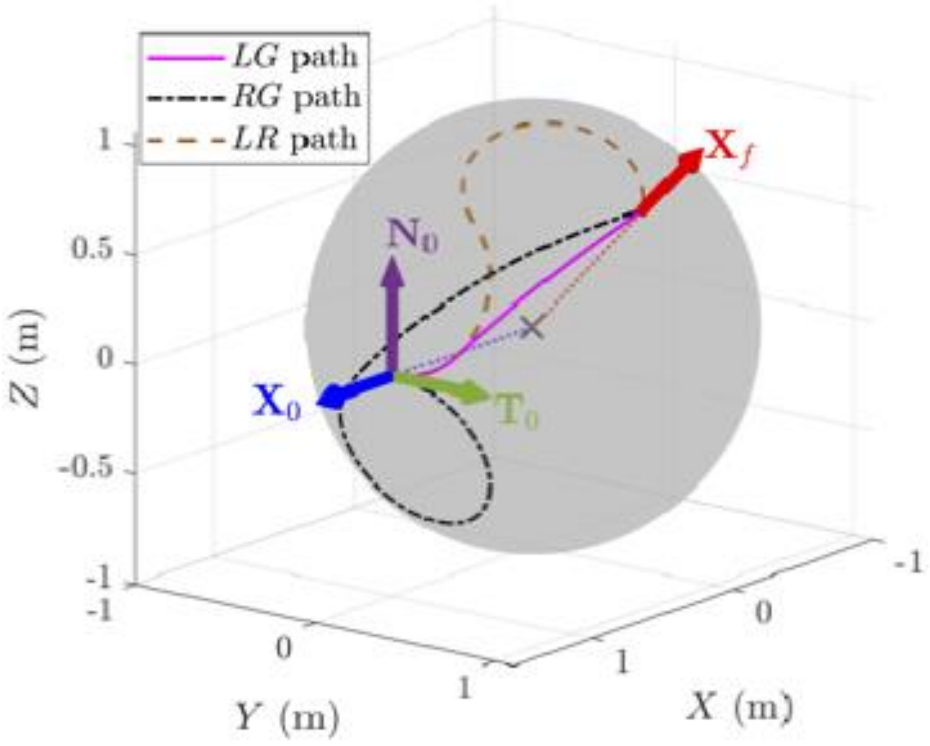


Free terminal orientation

- When the terminal orientation is free, the candidate solutions reduce to two segment paths, *CG* and *CC*.



Initial configuration and final position

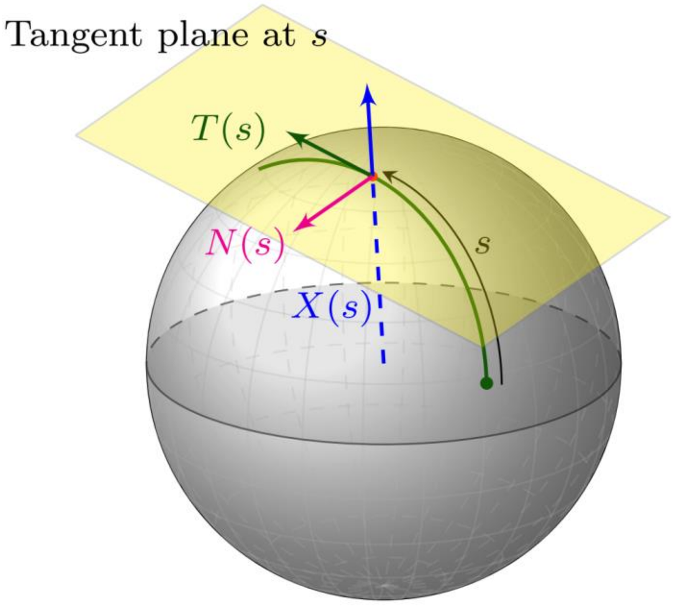


Candidate optimal paths

Overview of Modeling

- On a sphere, Sabban frame was considered.
  - s : Arc length
  - $X(s)$: Position vector
  - $T(s)$: Tangent vector
  - $N(s)$: $X(s)$ x $T(s)$.
- Sabban frame is given by

$$\frac{dX(s)}{ds} = T(s),$$
$$\frac{dT(s)}{ds} = -X(s) + u_g(s)N(s),$$
$$\frac{dN(s)}{ds} = -u_g(s)T(s).$$



s : Distance travelled by vehicle
$X(s)$: Position of vehicle on sphere
$T(s)$: Direction of vehicle's motion
$N(s)$: Lateral direction of vehicle

Overview of Modeling

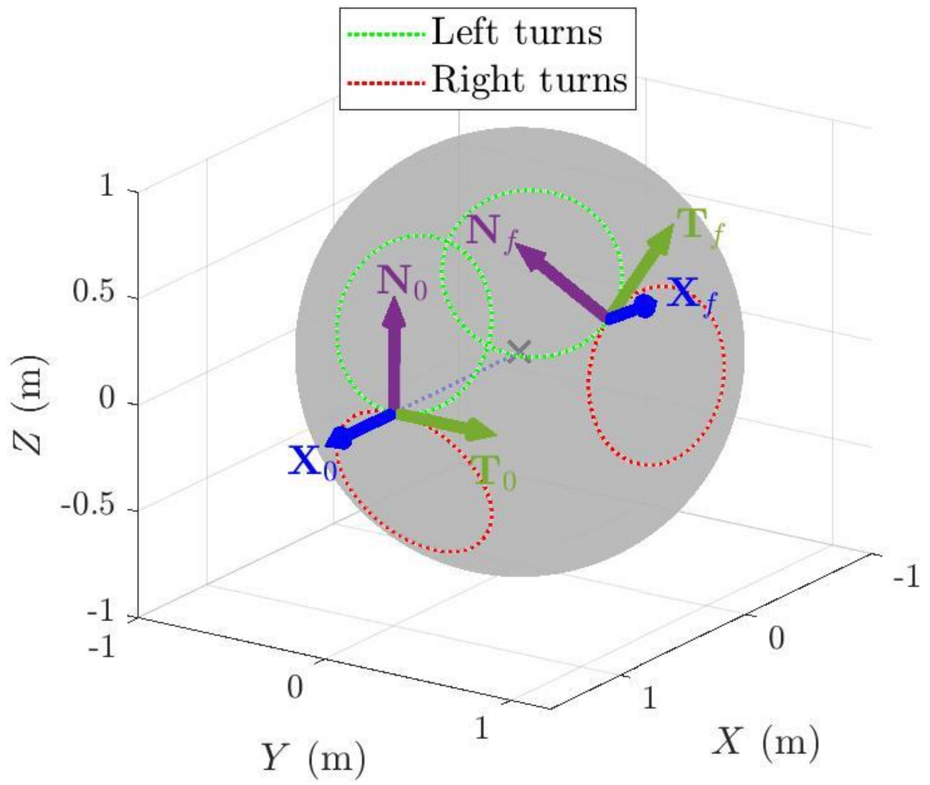
- **Goal:** Obtain shortest path connecting two configurations on a sphere.

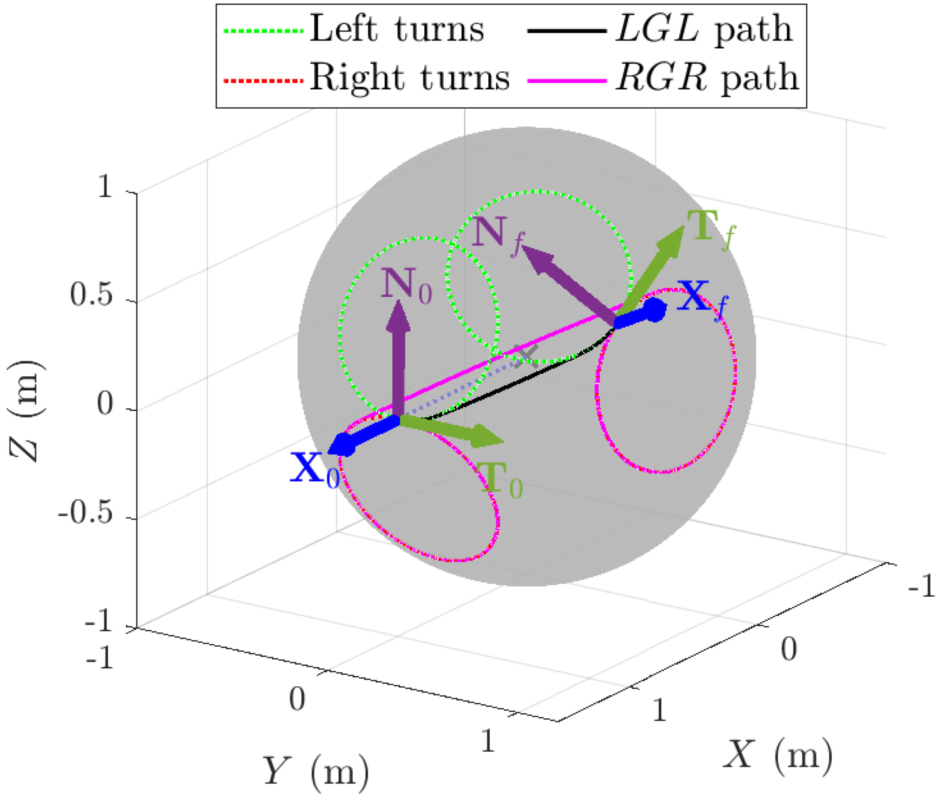
$$J = \min \int_0^L 1 ds \,,$$

subject to

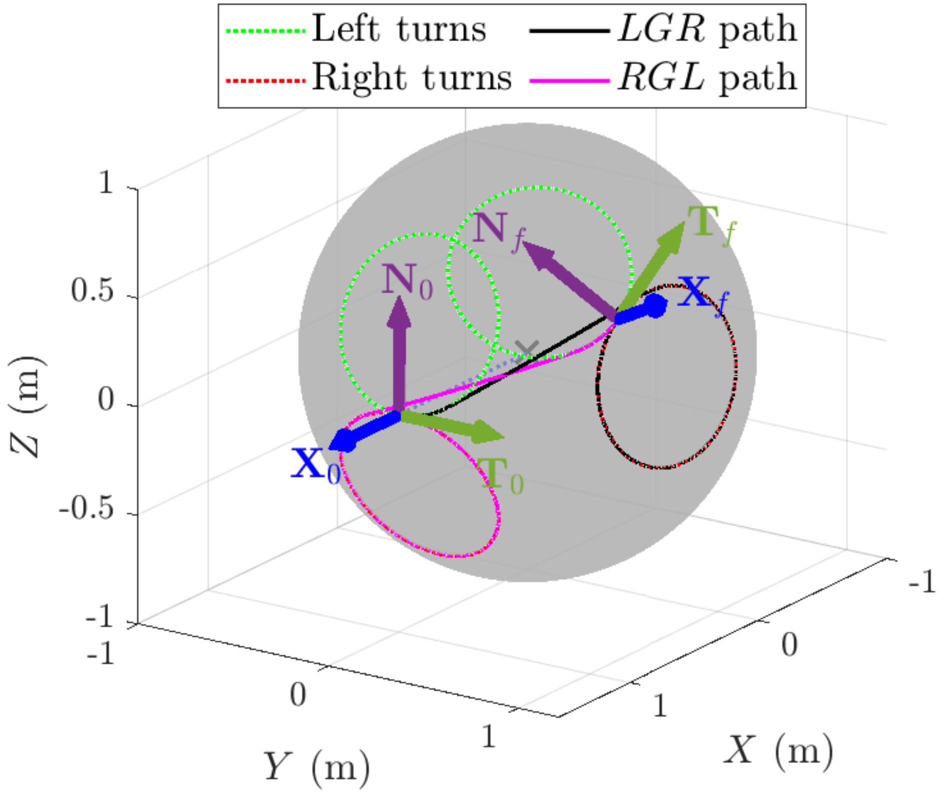
$$\frac{dX(s)}{ds} = T(s),$$
$$\frac{dT(s)}{ds} = -X(s) + u_g(s)N(s),$$
$$\frac{dN(s)}{ds} = -u_g(s)T(s),$$
$$R(0) = I_3,$$
$$R(L) = R_f.$$

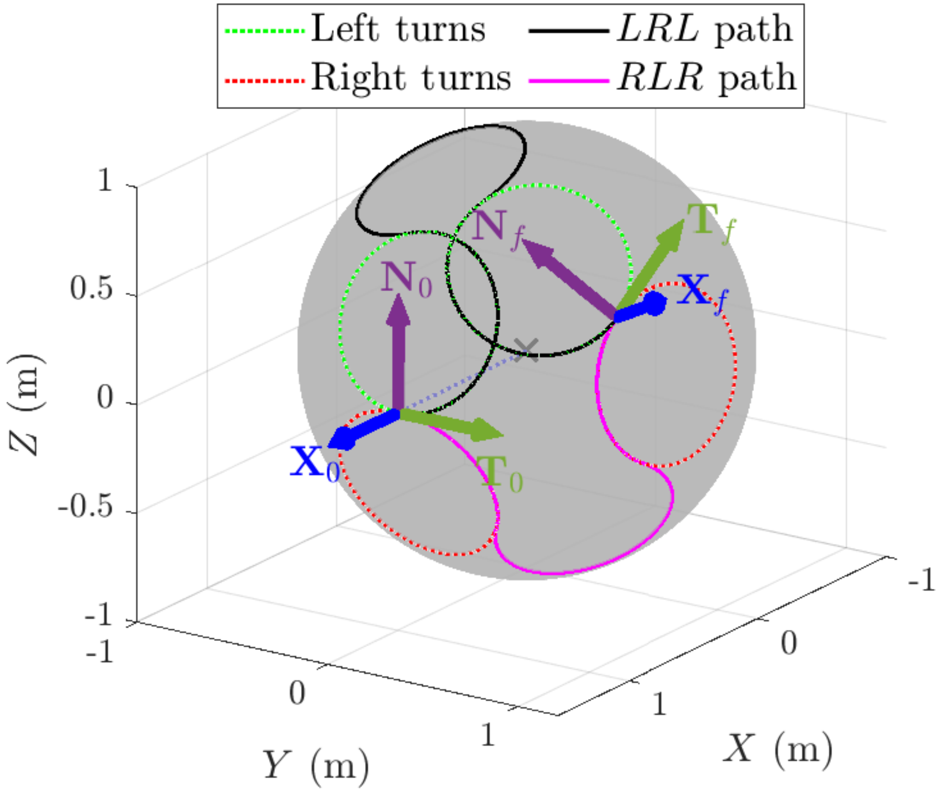
Paths connecting initial and final configuration





Paths connecting initial and final configuration

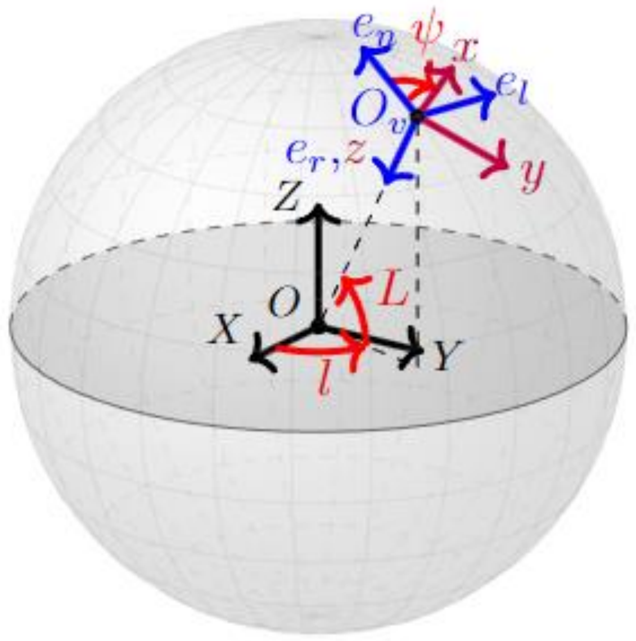




Model with spherical coordinates

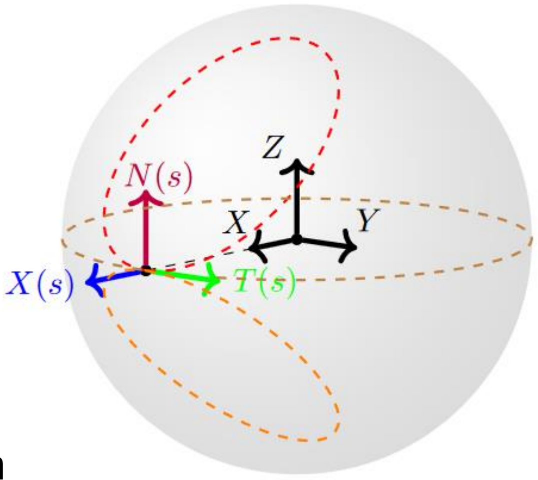
- Consider l: longitude, L: latitude, ψ: heading.
- Bounded force F can act along y-axis (vehicle's lateral direction).
- Model:

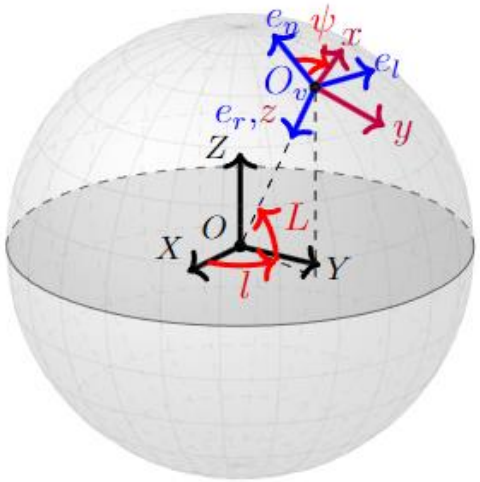
$$\frac{dL}{ds} = \cos\psi,$$
$$\frac{dl}{ds} = \frac{\sin\psi}{\cos L},$$
$$\frac{d\psi}{ds} = \tan L \sin\psi + \frac{1}{R}u,$$

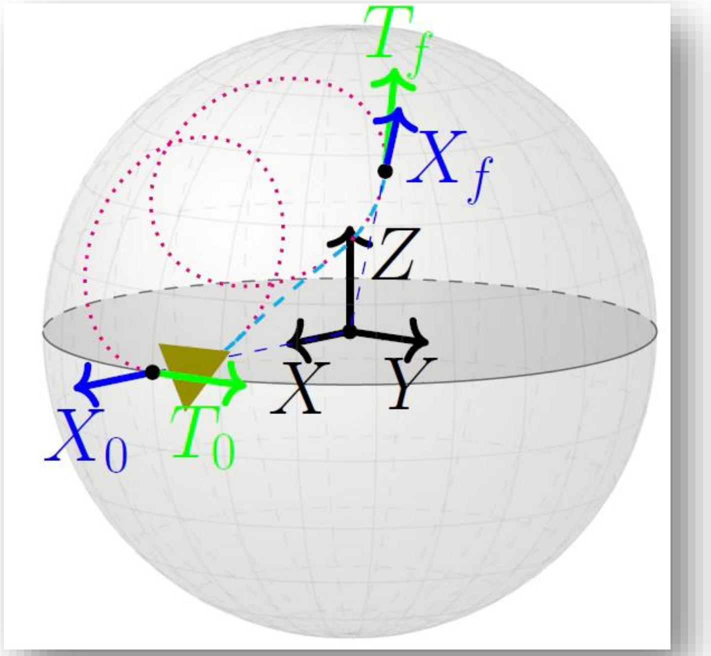


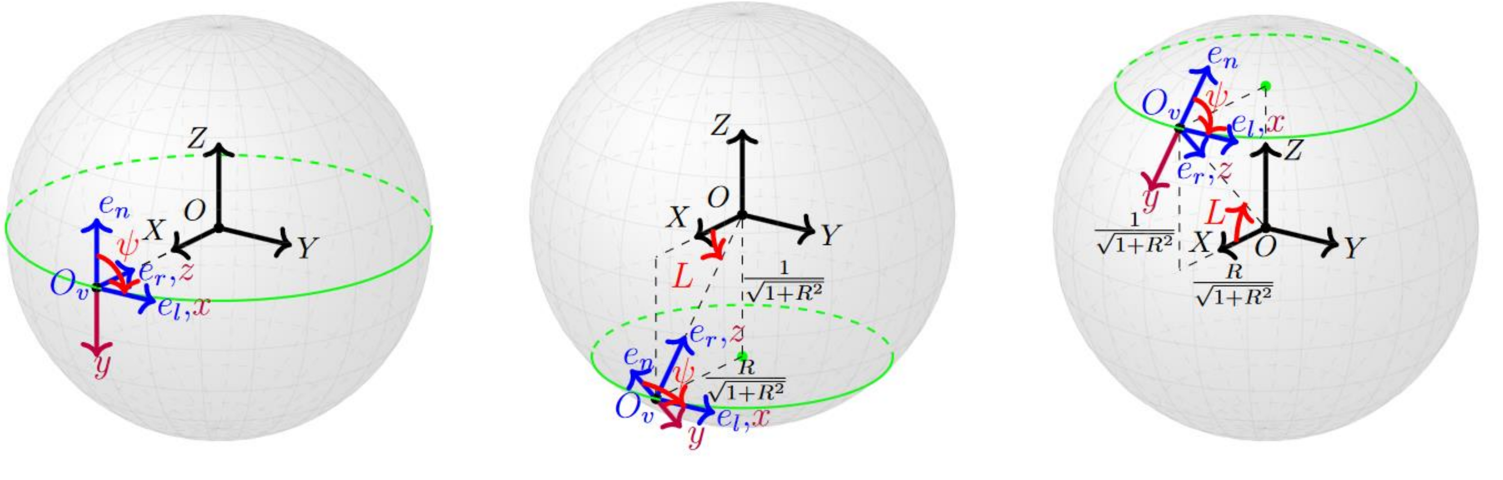
Equivalence of Model

- Two steps involved:
  - Show control inputs are the same, i.e., optimal segments are L, R, G
  - Show adjoint equation in both models evolve through same equation.
- Outcome:
  - Results from one approach transfer to other approach









(a) Great circular corresponding to $u = 0$ (b) Tight turn corresponding to $u = 1$ (c) Tight turn corresponding to $u = -1$

Fig. 4 Optimal segments for alternate sphere model

Recent Publications

- Von Moll, A., Fuchs, Z., Shishika, D., Maity, D., Dorothy, M. and Pachter, M., 2024. Turret escape differential game. *Journal of Dynamics and Games*, *11*(2), pp.100-114.– 10.3934/jdg.2023012
- Pachter, M. and Weintraub, I.E., 2024. The Synthesis of Optimal Control Laws Using Isaacs' Method for the Solution of Differential Games. *Journal of Optimization Theory and Applications*, vol. 202, pp.1137-1157. – 10.1007/s10957-024-02490-7
- Dillon, P.M., Zollars, M.D., Weintraub, I.E. and Von Moll, A., 2023. Optimal Trajectories for Aircraft Avoidance of Multiple Weapon Engagement Zones. *Journal of Aerospace Information Systems*, *20*(8), pp. 520-525. – 10.2514/1.1011224
- Weintraub, I.E., Von Moll, A. and Pachter, M., 2023, August. Range-Limited Pursuit-Evasion. In *NAECON 2023-IEEE National Aerospace and Electronics Conference* (pp. 28-35). IEEE. -- 10.1109/NAECON58068.2023.10365808
- Kurtoglu, D., Yucelen, T., Tran, D., Casbeer, D. and Garcia, E., 2024. Decentralized, norm-free, and adaptive event-triggered distributed control of nonholonomic mobile robots. *International Journal of Systems Science*, vol. 55, no. 14, pp. 2914–2932. -- 10.1080/00207721.2024.2364282
- Von Moll, A.L., Casbeer, D., Weintraub, I.E. and Pachter, M., 2024. Pure Pursuit of a Target on a Circular Trajectory. In *AIAA SCITECH 2024 Forum* (p. 0956). -- 10.2514/6.2024-0956
- S. Darbha, A. Pavan, K. Rajagopal, S. Rathinam, D W. Casbeer, and S. G. Manyam. "Optimal Geodesic Curvature Constrained Dubins' Paths on a Sphere." *Journal of Optimization Theory and Applications* 197, no. 3 (2023): 966-992. -- 10.1007/s10957-023-02206-3
- Kumar, D.P., Darbha, S., Manyam, S.G., Tran, D. and Casbeer, D.W., 2023, December. Optimal Geodesic Curvature Constrained Dubins' Path on Sphere with Free Terminal Orientation. In *2023 Ninth Indian Control Conference (ICC)* (pp. 377-382). IEEE. -- 10.1109/ICC61519.2023.10441900
- Weintraub, I.E., Von Moll, A.L., Casbeer, D. and Manyam, S.G., 2024. Virtual Target Selection for a Multiple-Pursuer Multiple-Evader Scenario. In *AIAA SCITECH 2024 Forum* (p. 0123). -- 10.2514/6.2024-0123
- Kurtoglu, D., Yucelen, T., Tran, D., Casbeer, D. and Garcia, E., 2024. Distributed Zone Allocation and Preservation in Multiagent

Recent Publications

- Kumar, D.P., Darbha, S., Manyam, S.G., Casbeer, D.W. and Pachter, M., 2024. Equivalence of Dubins Path on Sphere with Geographic Coordinates and Moving Frames. *arXiv preprint arXiv:2408.07206*.
- Wolek, A., Weintraub, I.E., Von Moll, A., Casbeer, D. and Manyam, S.G., 2024. Sampling-Based Risk-Aware Path Planning Around Dynamic Engagement Zones. *arXiv preprint arXiv:2403.05480*.
- Von Moll, A. and Weintraub, I.E., 2023. Basic Engagement Zones. Von Moll, A., & Weintraub, I. (2024). Basic engagement zones. Journal of Aerospace Information Systems, vol. 21, no. 10, pp. 885-891. -- 10.2514/1.I011394
- Mora, B., Von Moll, A., Weintraub, I., Casbeer, D. and Chakravarthy, A., 2023. Escape from an orbiting pursuer with a nonzero capture radius. In NAECON 2024-IEEE National Aerospace and Electronics Conference, pp. 200-205. -- 10.1109/NAECON61878.2024.10670675
- Weintraub, I.E., Von Moll, A. and Pachter, M., 2024. Minimum Time Escape from a Circular Region of a Dubins Car. *NAECON 2024 - IEEE National Aerospace and Electronics Conference, pp. 34-37*. -- 10.1109/NAECON61878.2024.10670348
- Von Moll, A. and Pachter, M., 2024. Complete Solution of the Lady in the Lake Scenario. *arXiv preprint arXiv:2401.14994*.
- D. P. Kumar, S. Darbha, S.G. Manyam, D. Casbeer, M. Pachter, “Equivalence of Dubins Path on Sphere with Geographic Coordinates and Moving Frames”, under review.
- Milutinovic, D., Von Moll, A., Manyam, S.G., Casbeer, D.W., Weintraub, I.E. and Pachter, M., 2024. Pursuit-Evasion on a Sphere and When It Can Be Considered Flat. *arXiv preprint arXiv:2403.15188*.

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Thank you!