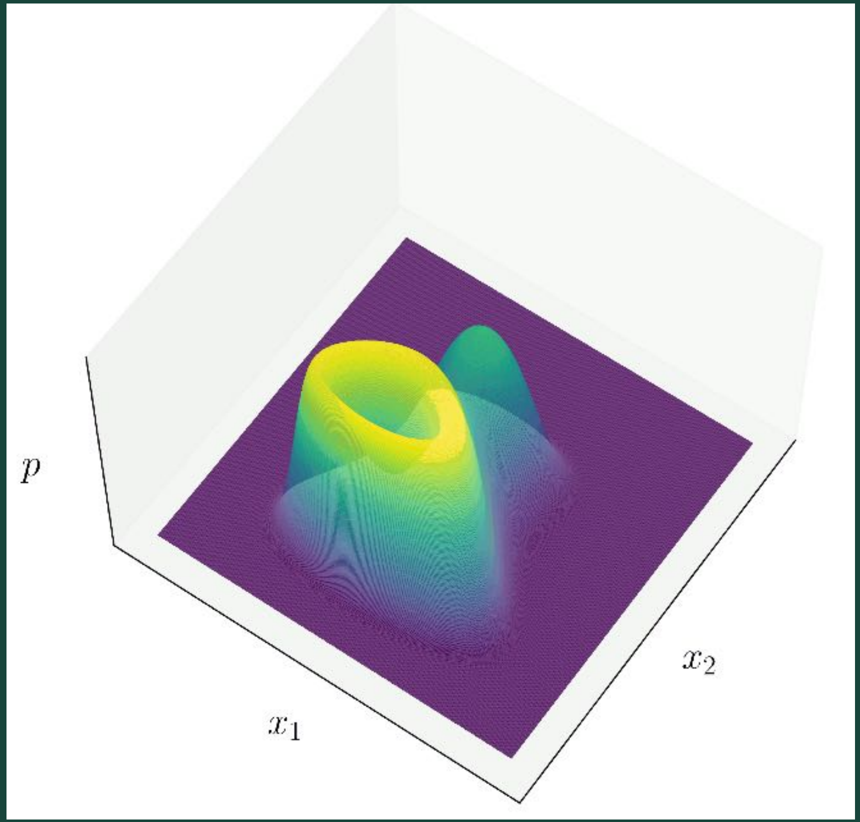
A Topological Approach for Detecting P-Bifurcations from Kernel Density Estimates



AFOSR Dynamics and Control Review, 8/26/2024

Firas A. Khasawneh, Ph.D.

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Computational Mathematics, Science, and Engineering

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Michigan State University

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Acknowledgements

Funding

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Collaborators


Elizabeth Munch


Sunia Tanweer


Max Chumley

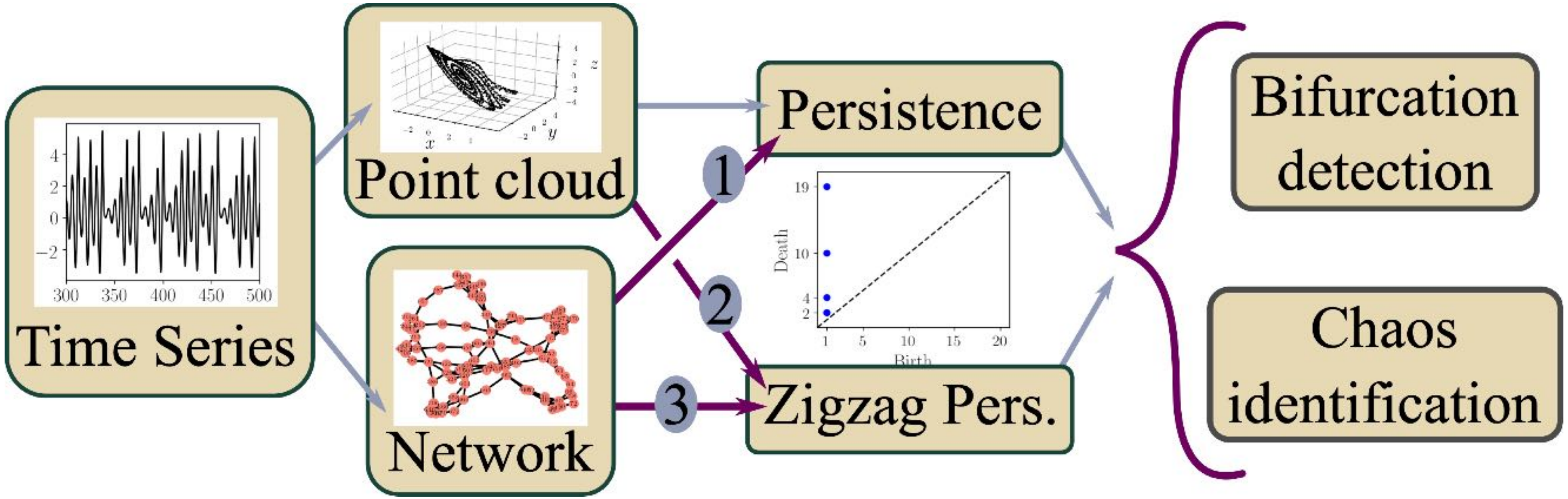

Audun Myers
(PNNL)


David Muñoz

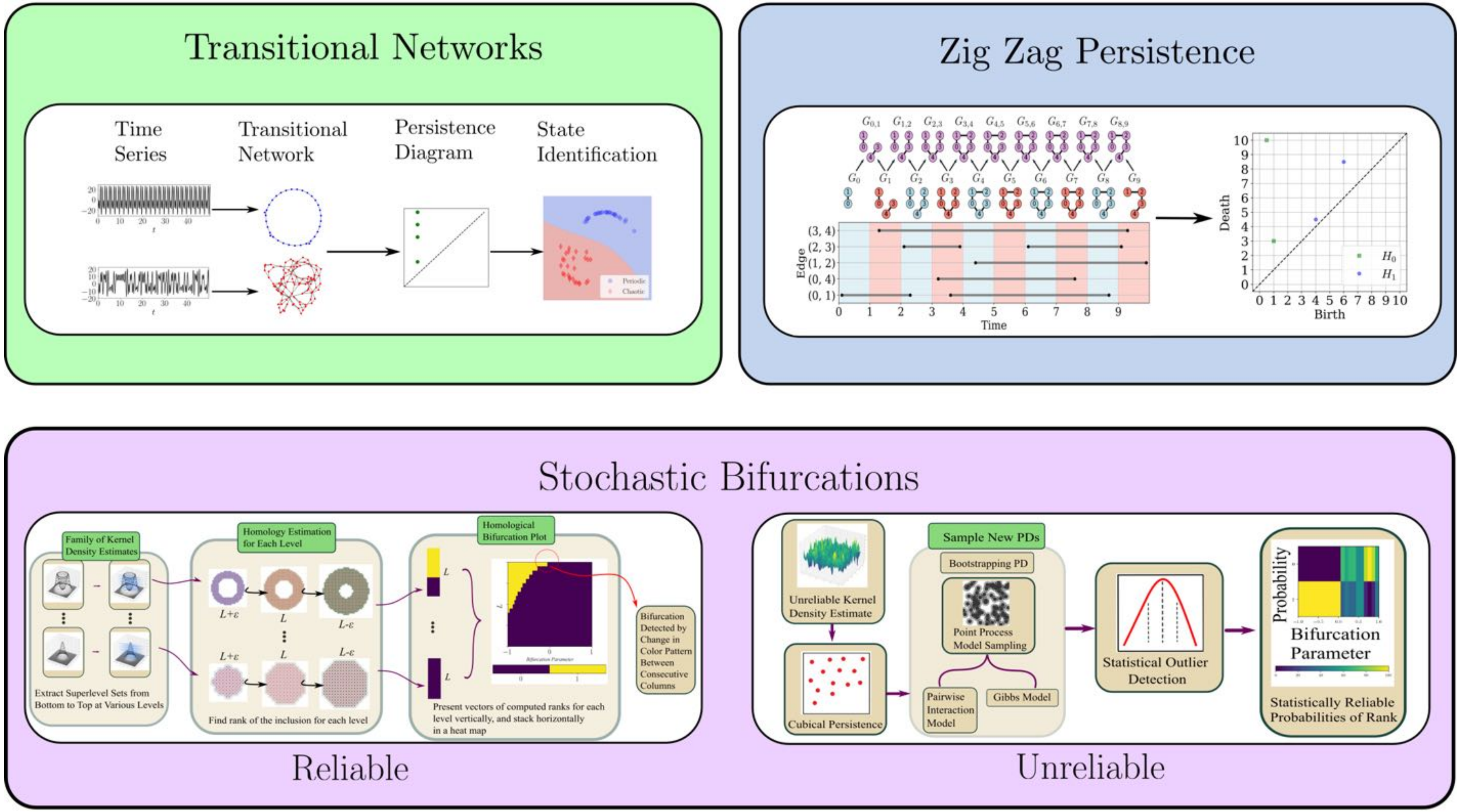

Sarah Tymochko
(UCLA-Math)

Time Series Analysis using Persistent Homology

1. Objective 1: Network representations of time series & their persistence
2. Objective 2: Zigzag persistence of point clouds from time series
3. Objective 3: Zigzag persistence of Network Representations of time series



Contents


Transitional Networks
Time Series
Transitional Network
Persistence Diagram
State Identification
Zig Zag Persistence
Edge
Time
Death
Birth
Stochastic Bifurcations
Family of Kernel Density Estimates
Extract Superlevel Sets from Bottom to Top at Various Levels
Homology Estimation for Each Level
Find rank of the inclusion for each level
Homological Bifurcation Plot
Present vectors of computed ranks for each level vertically, and stack horizontally in a heat map
Bifurcation Detected by Change in Color Pattern Between Consecutive Columns
Reliable
Unreliable Kernel Density Estimate
Cubical Persistence
Sample New PDs
Bootstrapping PD
Point Process Model Sampling
Pairwise Interaction Model
Gibbs Model
Statistical Outlier Detection
Probability
Bifurcation Parameter
Statistically Reliable Probabilities of Rank
Unreliable

Homology

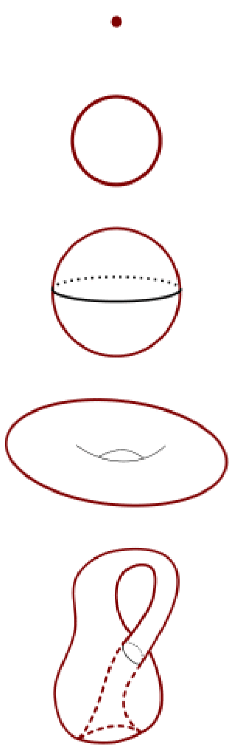
What is Homology?

A topological invariant which assigns a vector space, $H_k(X)$, to a given topological space X.

Dimension:

k is the dimension

- 0: Clusters
- 1: Holes
- 2: Voids



β_0	β_1	β_2	β_3
1	$\cdot$	$\cdot$	$\cdot$
1	1	$\cdot$	$\cdot$
1	$\cdot$	1	$\cdot$
1	2	1	$\cdot$
1	2	1	$\cdot$

Persistent Homology

A way to watch how the homology of a filtration (sequence) of topological spaces changes so that we can understand something about the space.

Given topological space K and filtration

$$K_0 \subseteq K_1 \subseteq K_2 \subseteq \cdots \subseteq K_n$$

gives a sequence of maps on homology

$$H_1(K_0) \to H_1(K_1) \to H_1(K_2) \to \cdots \to H_1(K_n)$$

Appearance $\to$ Birth (b)

Disppearance $\to$ Death (d)

Encoded on **Persistence Diagrams** as (b, d)

Point-Cloud Persistence (Vietoris Rips Complex)

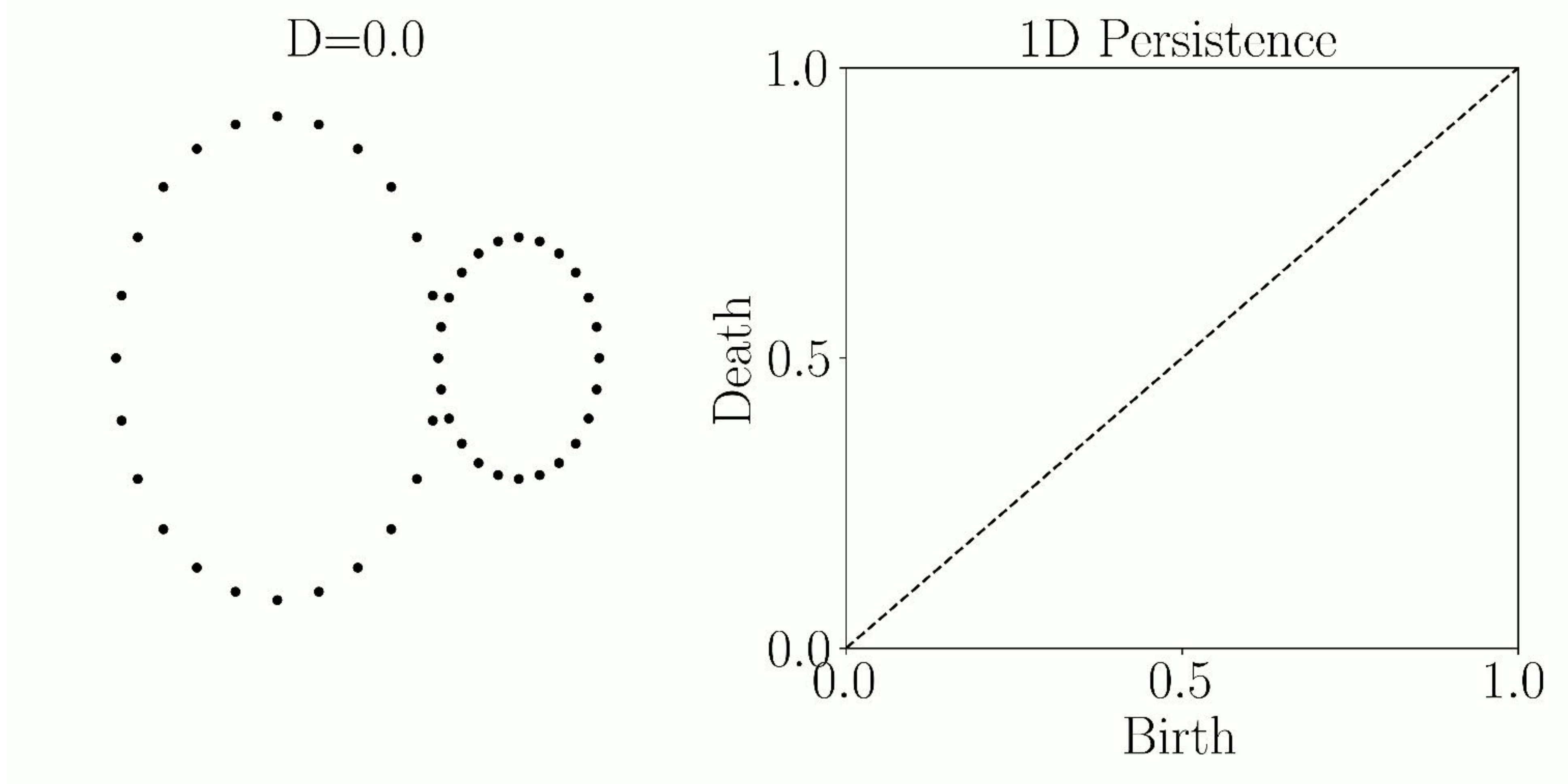
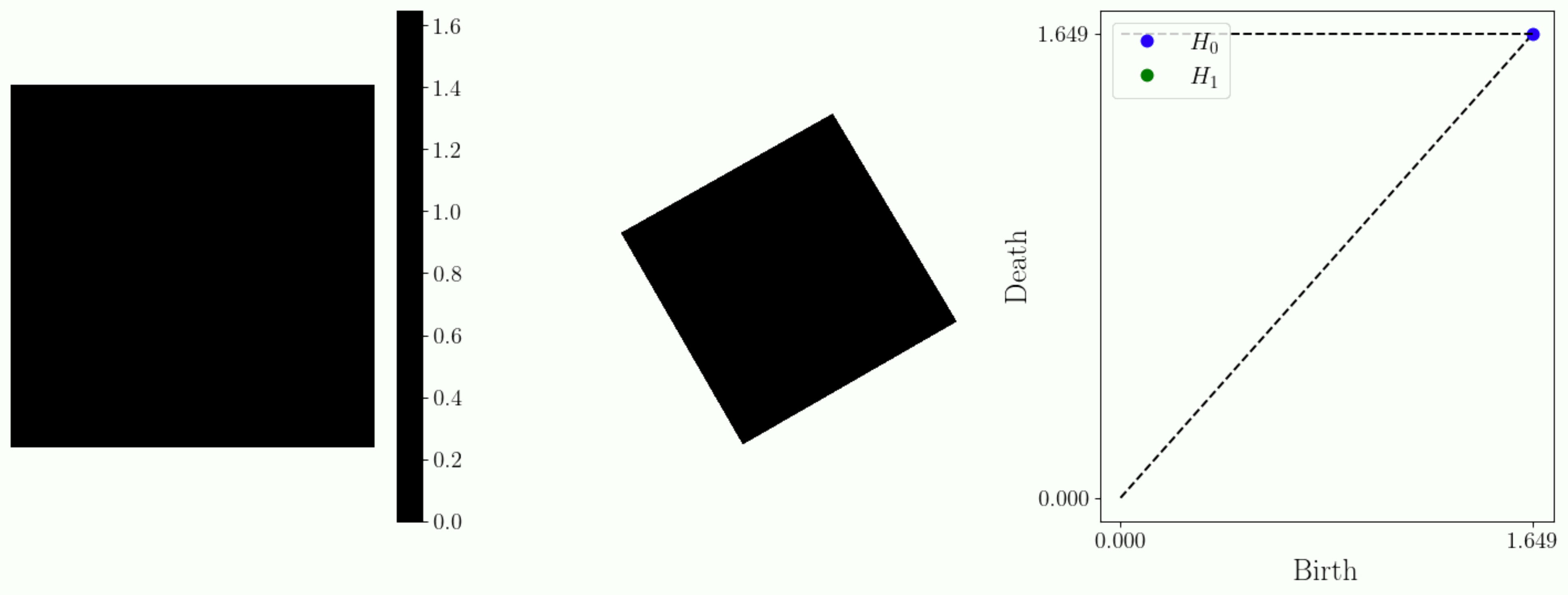
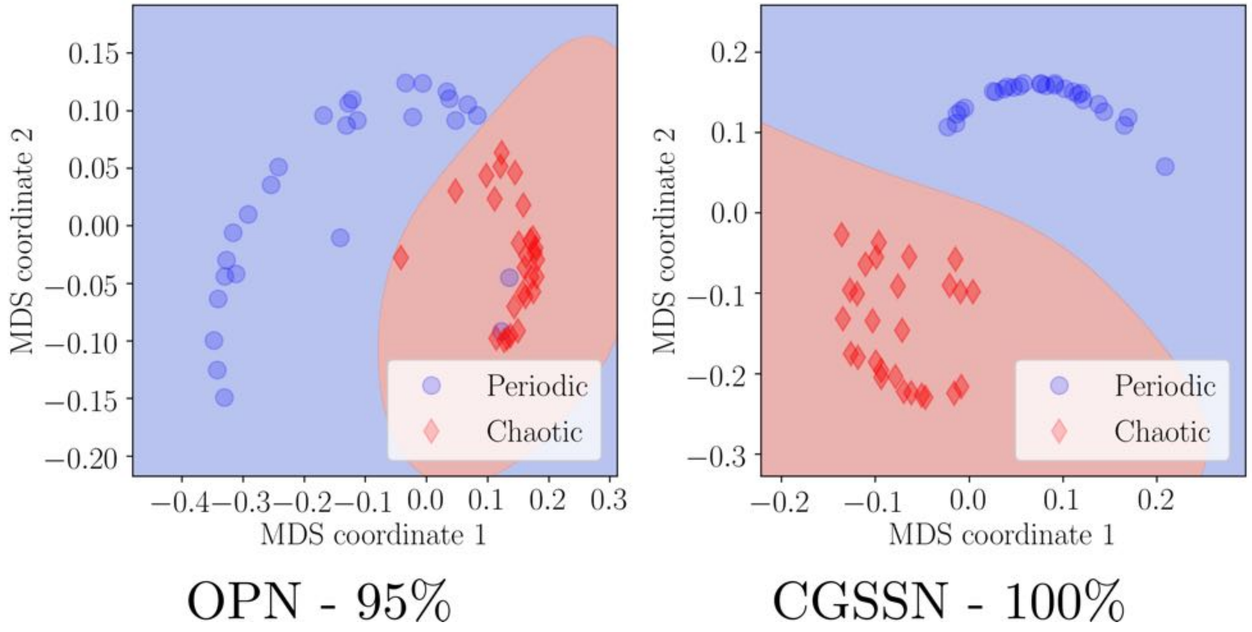
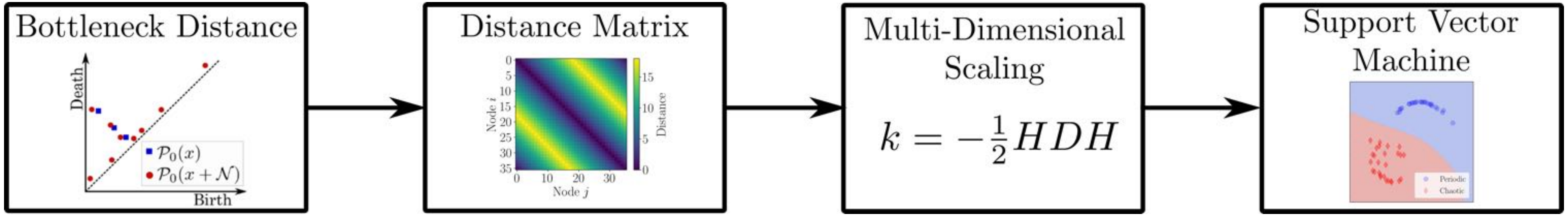


Image Cubical Persistence (Superlevel)



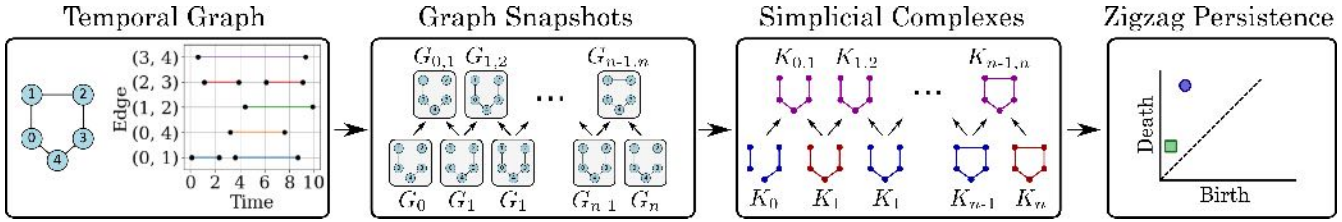
Chaos Detection with Persistent Homology of Networks



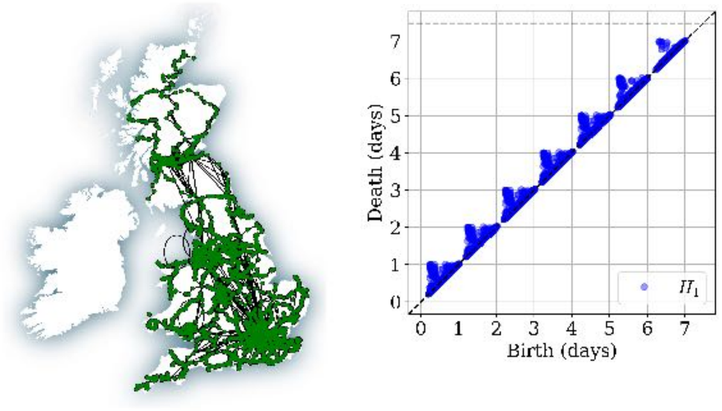


Myers, A. D., Chumley, M. M., Khasawneh, F. A., & Munch, E. (2023). "Persistent homology of coarse-grained state-space networks." Physical Review E, 107(3), 034303.

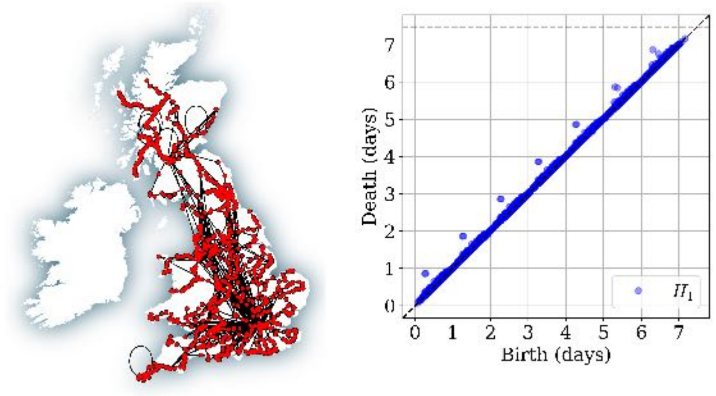
Temporal Networks and Zigzag Persistence

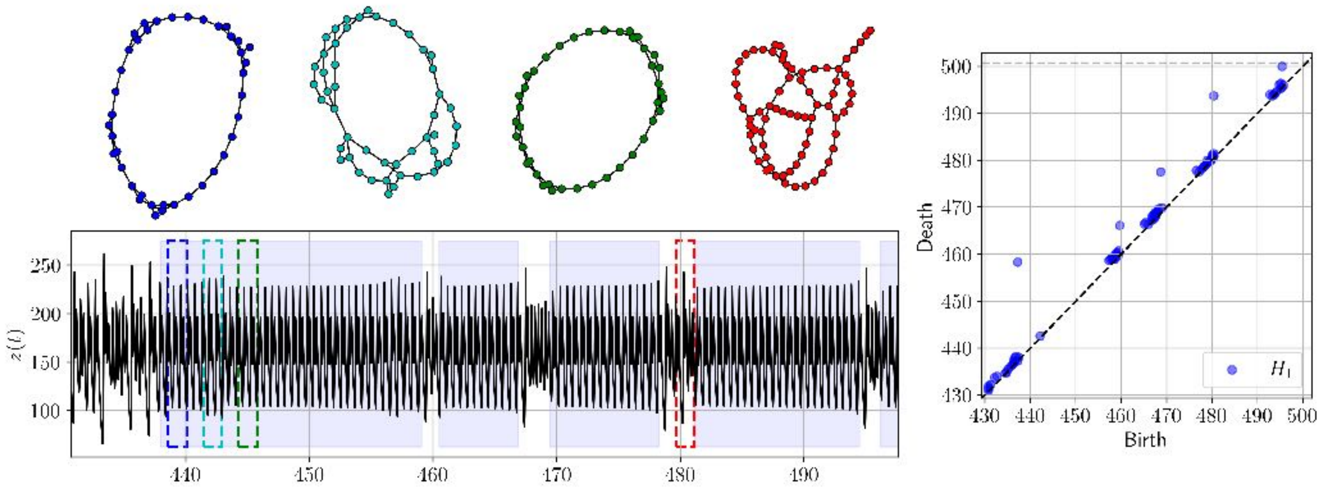


Rail



Coach



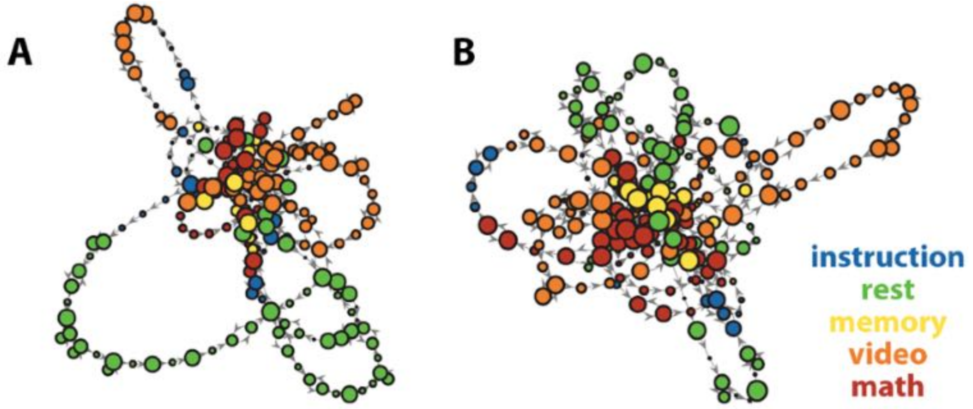


Great Britain Transportation Networks

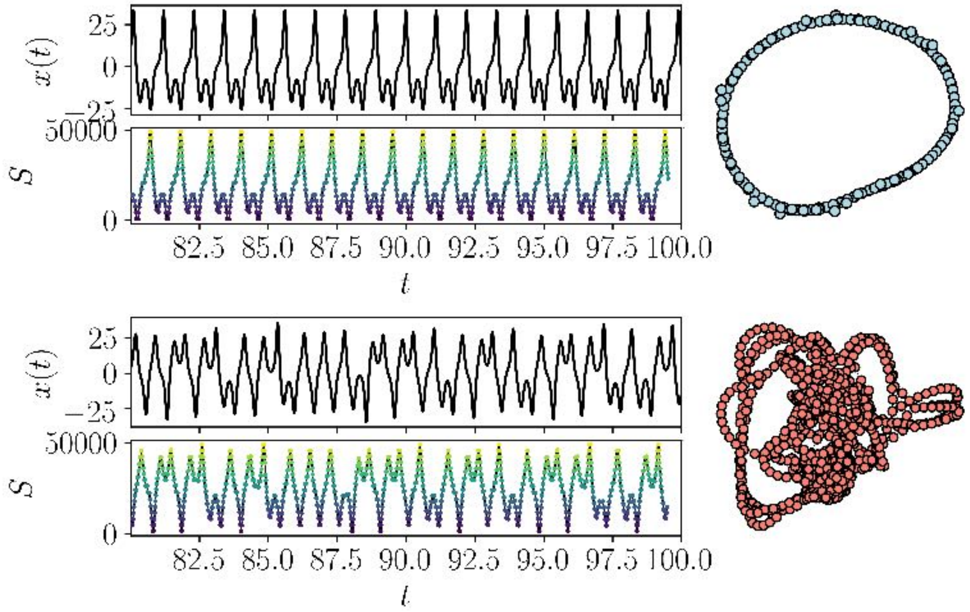
Intermittency Detection

Myers, A., Muñoz, D., Khasawneh, F. A., & Munch, E. (2023). Temporal network analysis using zigzag persistence. EPJ Data Science, 12(1), 6.

Study Structure of Directed Graphs



Temporal Mapper



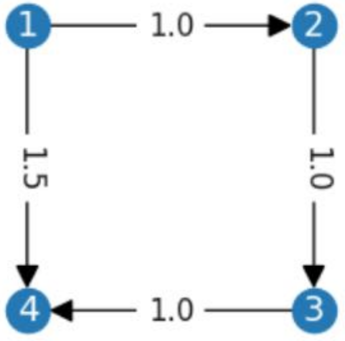
Network Representations of Timeseries

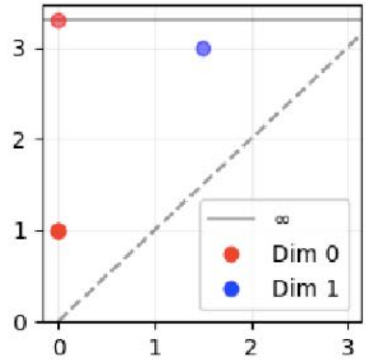
Walk-length Persistence

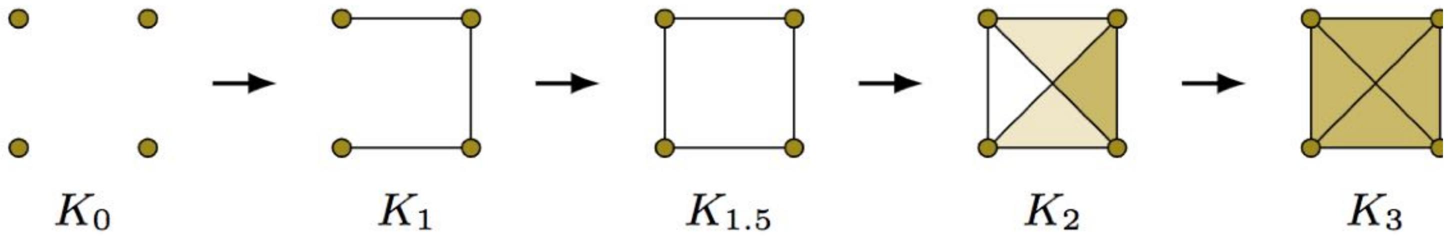
For a weighted digraph $D = (V, E, w)$ and $\sigma \subseteq V$, define

$$\mathfrak{f}(\sigma) = \inf \{w(\gamma) : \gamma \text{ is a walk in } D \text{ that contains all vertices in } \sigma\}$$

and the corresponding simplicial filtration $\{K_{\mathfrak{f} \leq \delta}\}_{\delta \in \mathbb{R}}$.

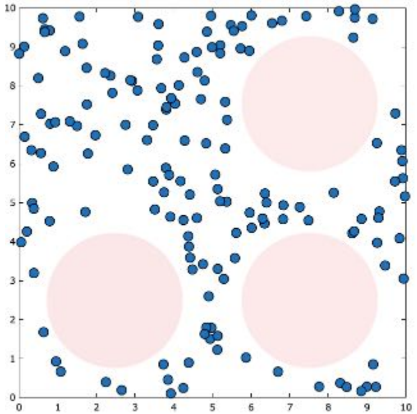


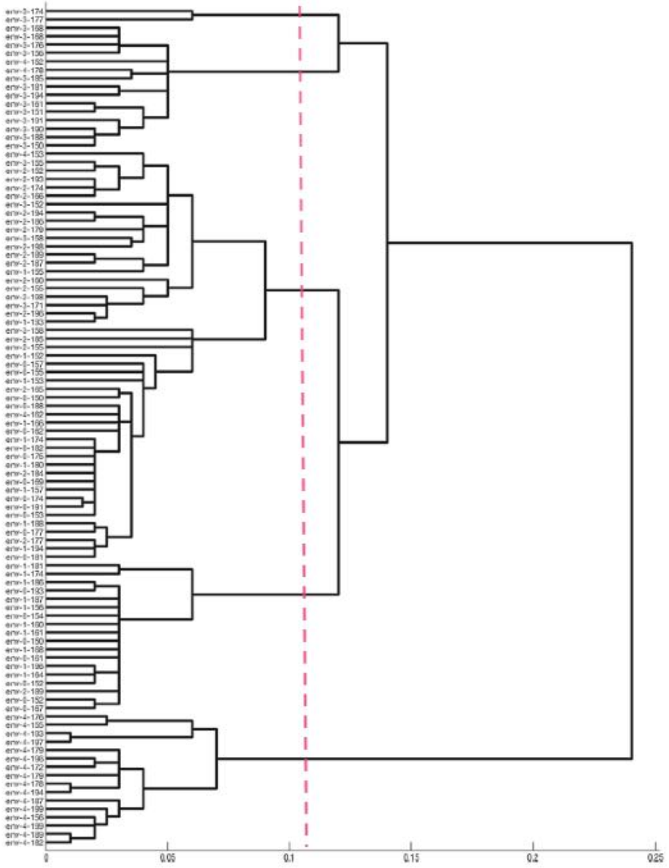




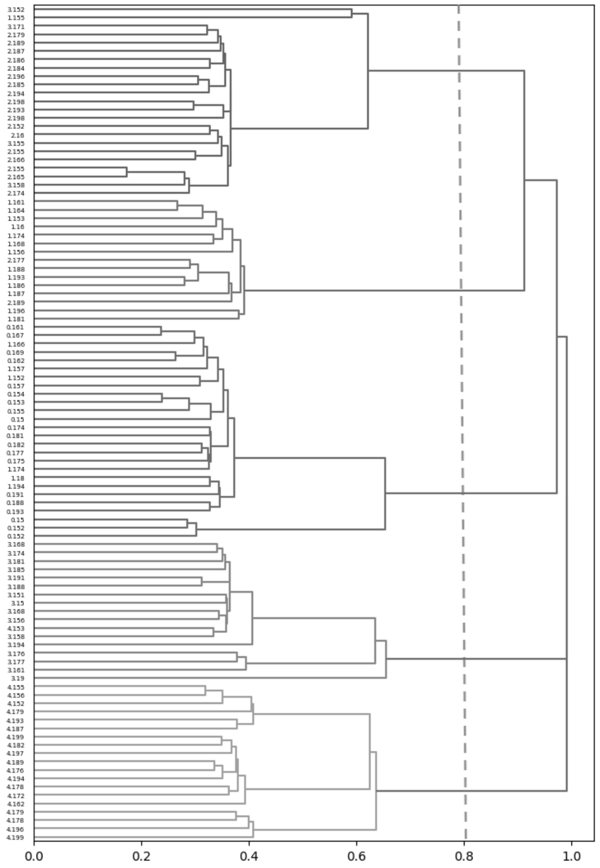
Detect Number of Holes in Hippocampal Networks

- Arenas with 0 to 4 holes.
- Place fields are matched with place cells in rodent's hippocampus.
- Time series spike data obtained from random trajectories.
- Network for each trajectory encode overall pairwise connections between place cells.





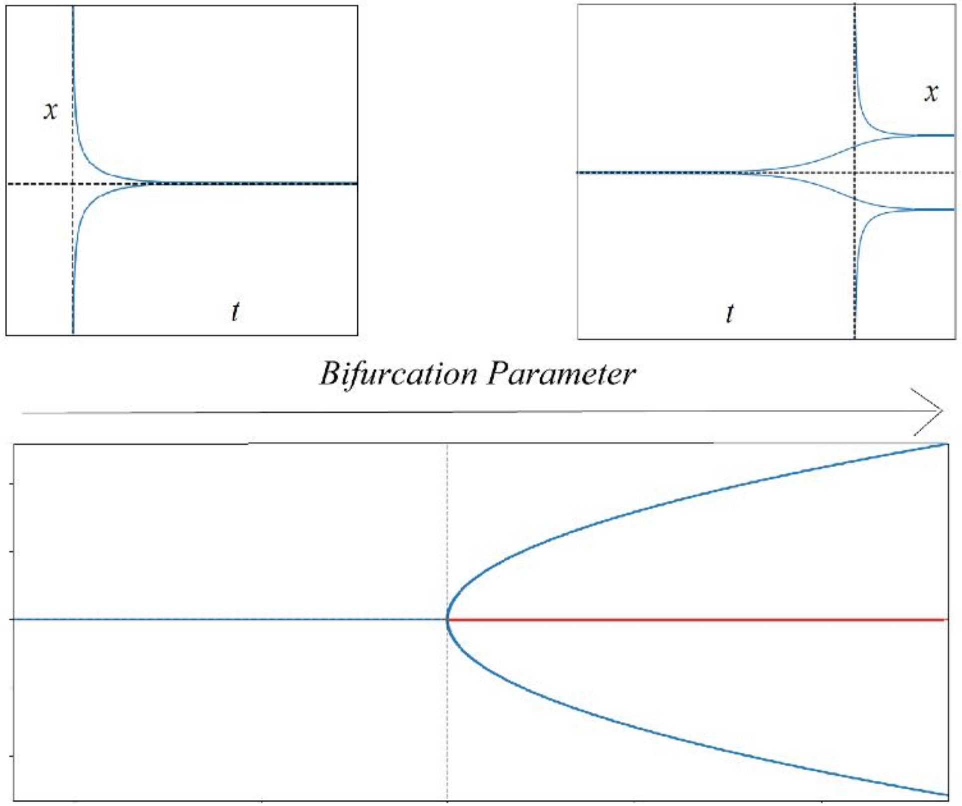
Dowker (Chowdhury, Mémoli, 2018)

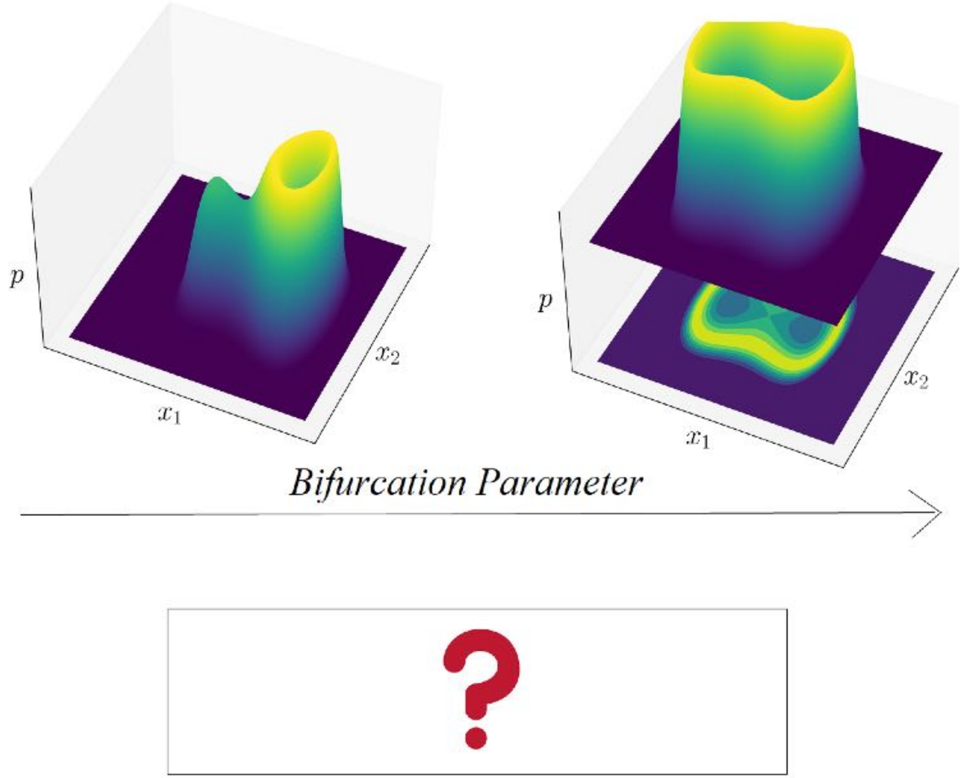


Walk-length

Walk-length shows more well-separated clusters and less missclassification on threshold (dotted line).

Stochastic Bifurcations: Objective

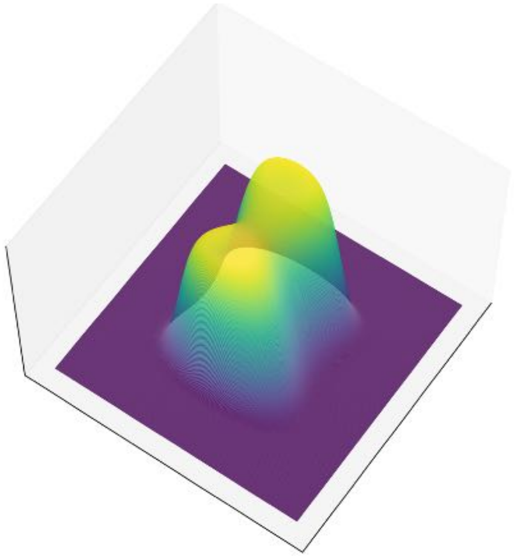




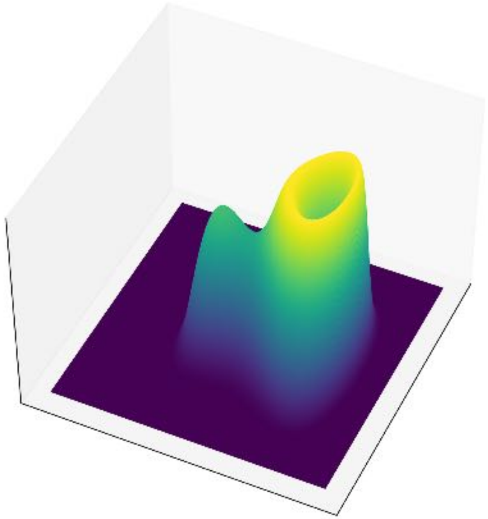
Objective: *Identifying bifurcations associated with topological changes in PDFs of stochastic dynamical systems*

Limitations

Visual Inspection

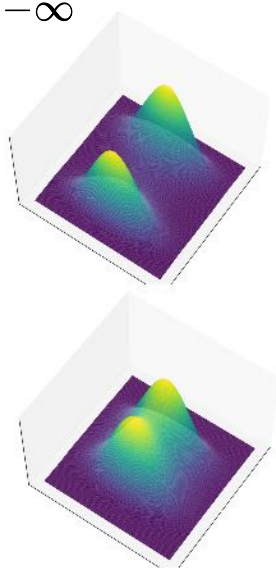


Quantifying Peaks

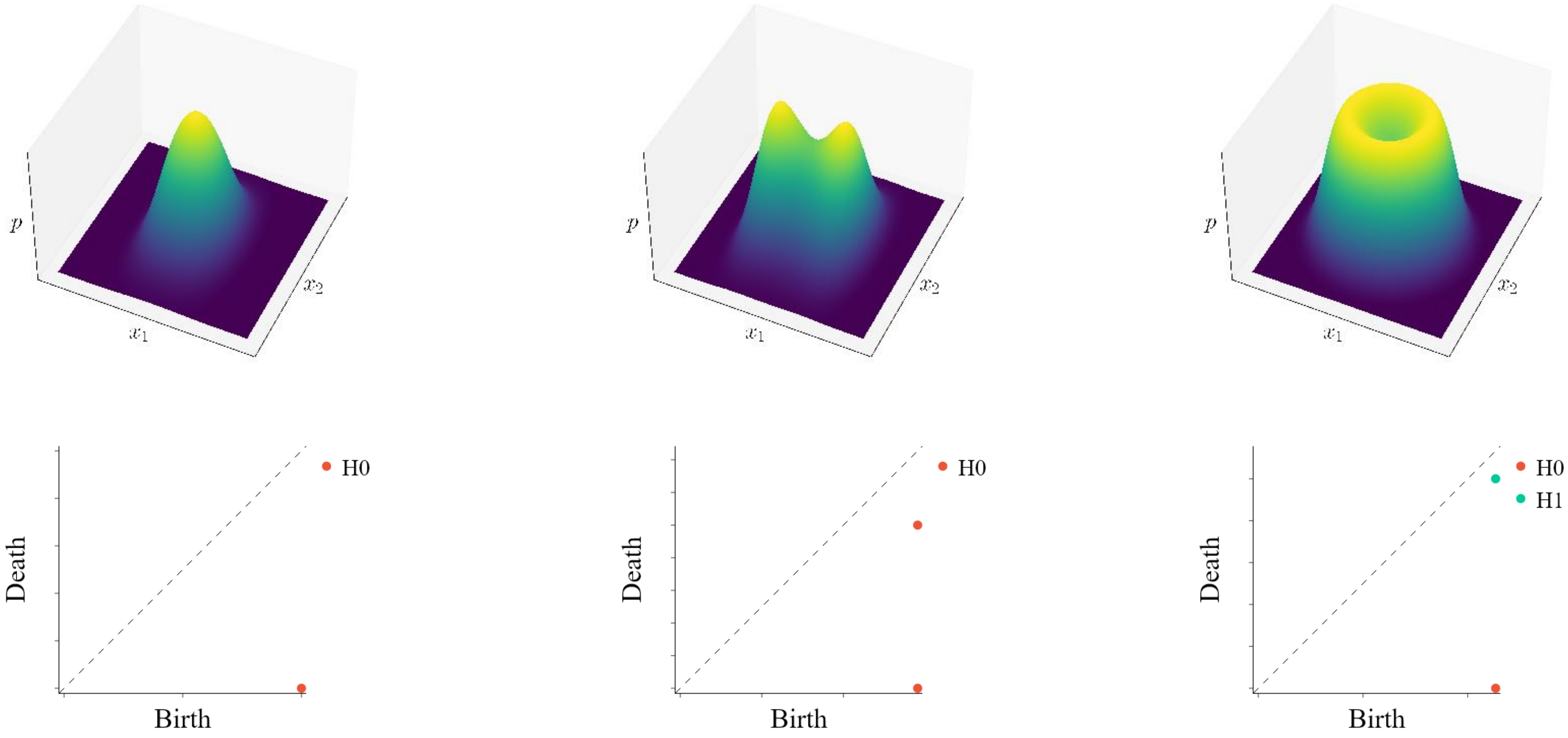


Shannon Entropy

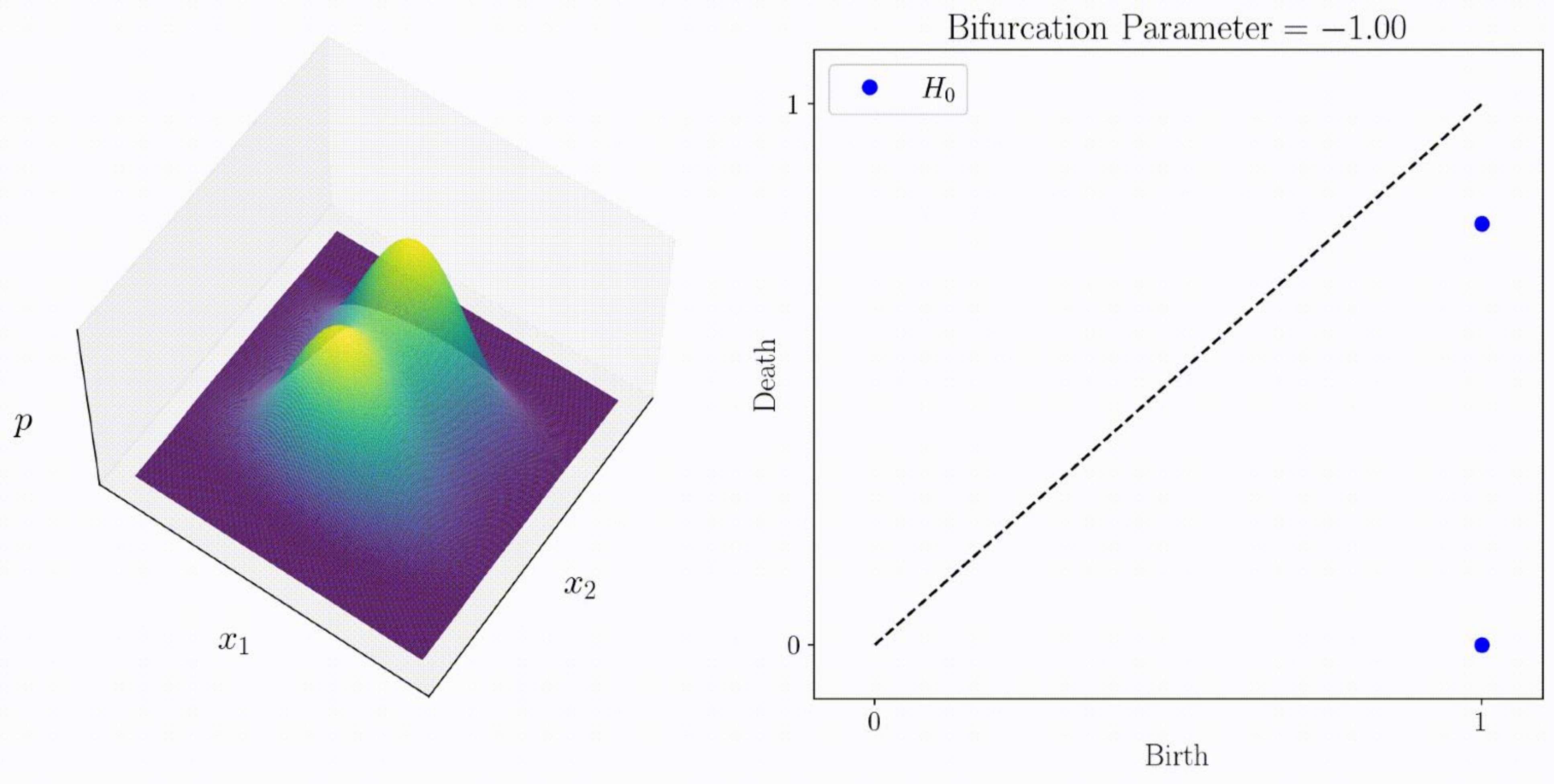
$$-\int_{-\infty}^{\infty} p \ln p dX$$



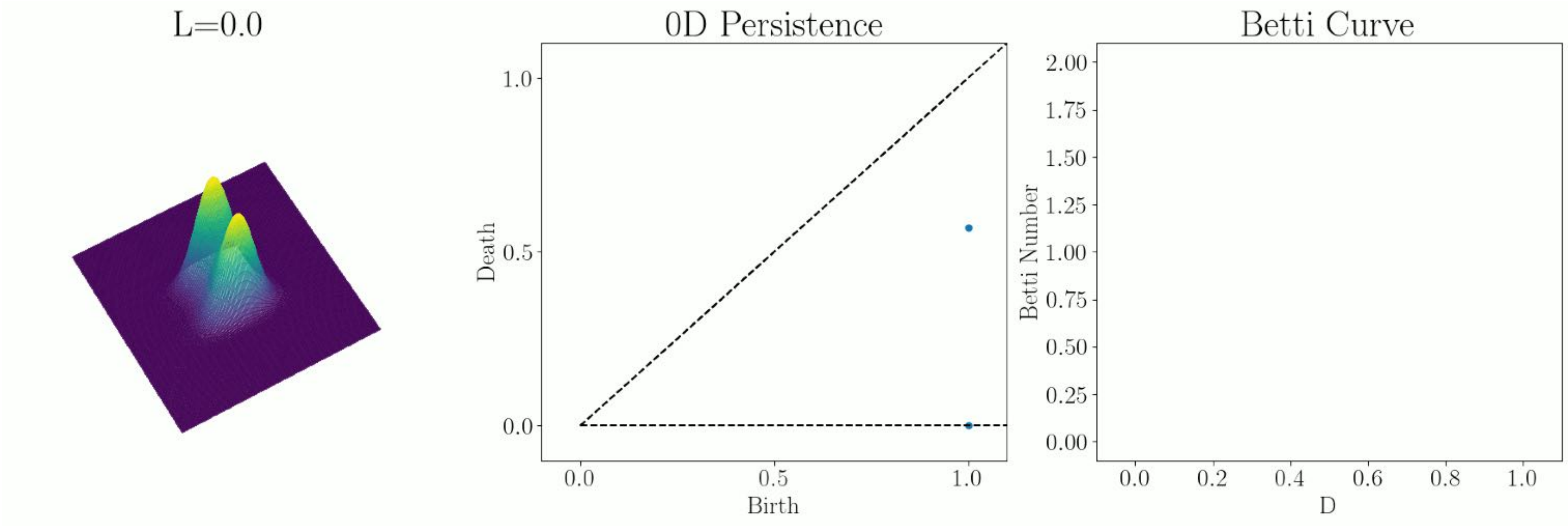
P-Bifurcation Topologies and Superlevel Persistences



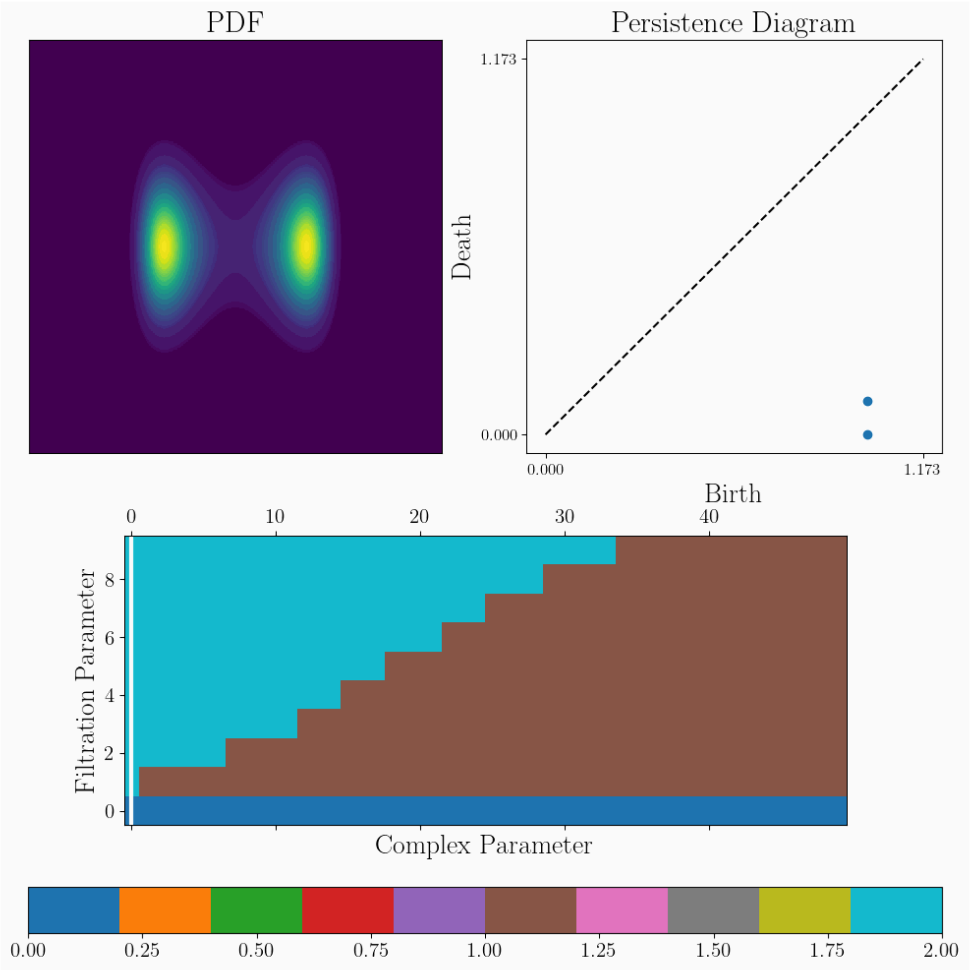
Stochastic Duffing Oscillator



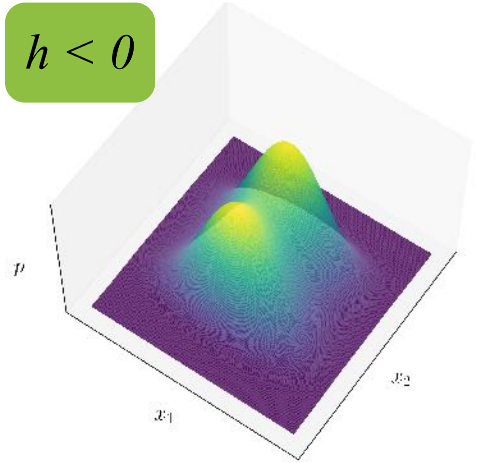
Betti Numbers/Vectors

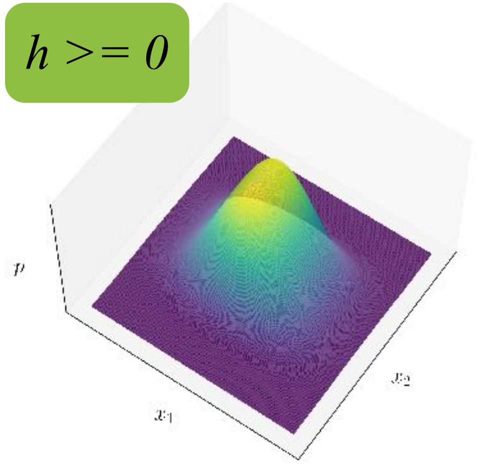


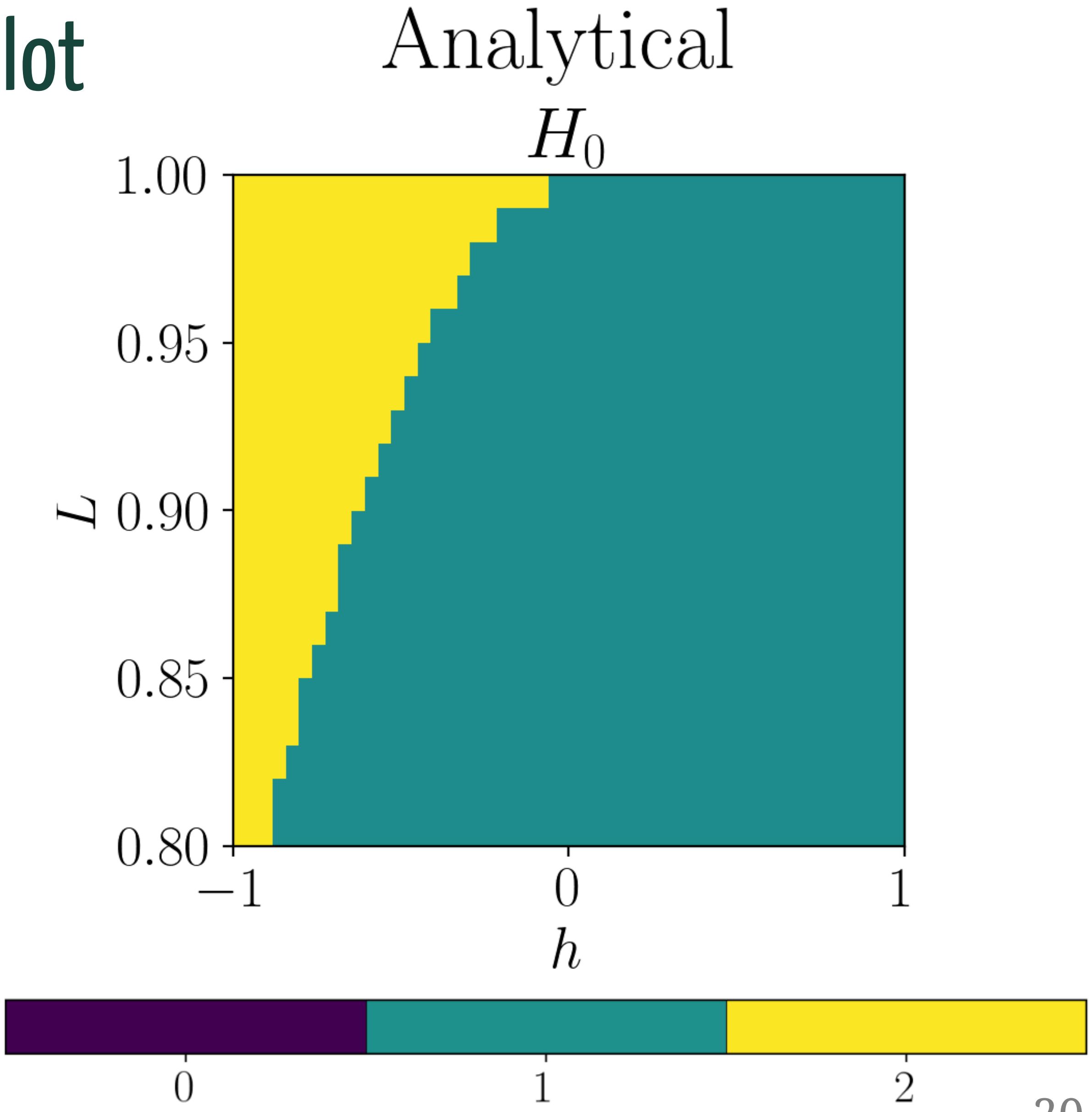
Homological Bifurcation Plot

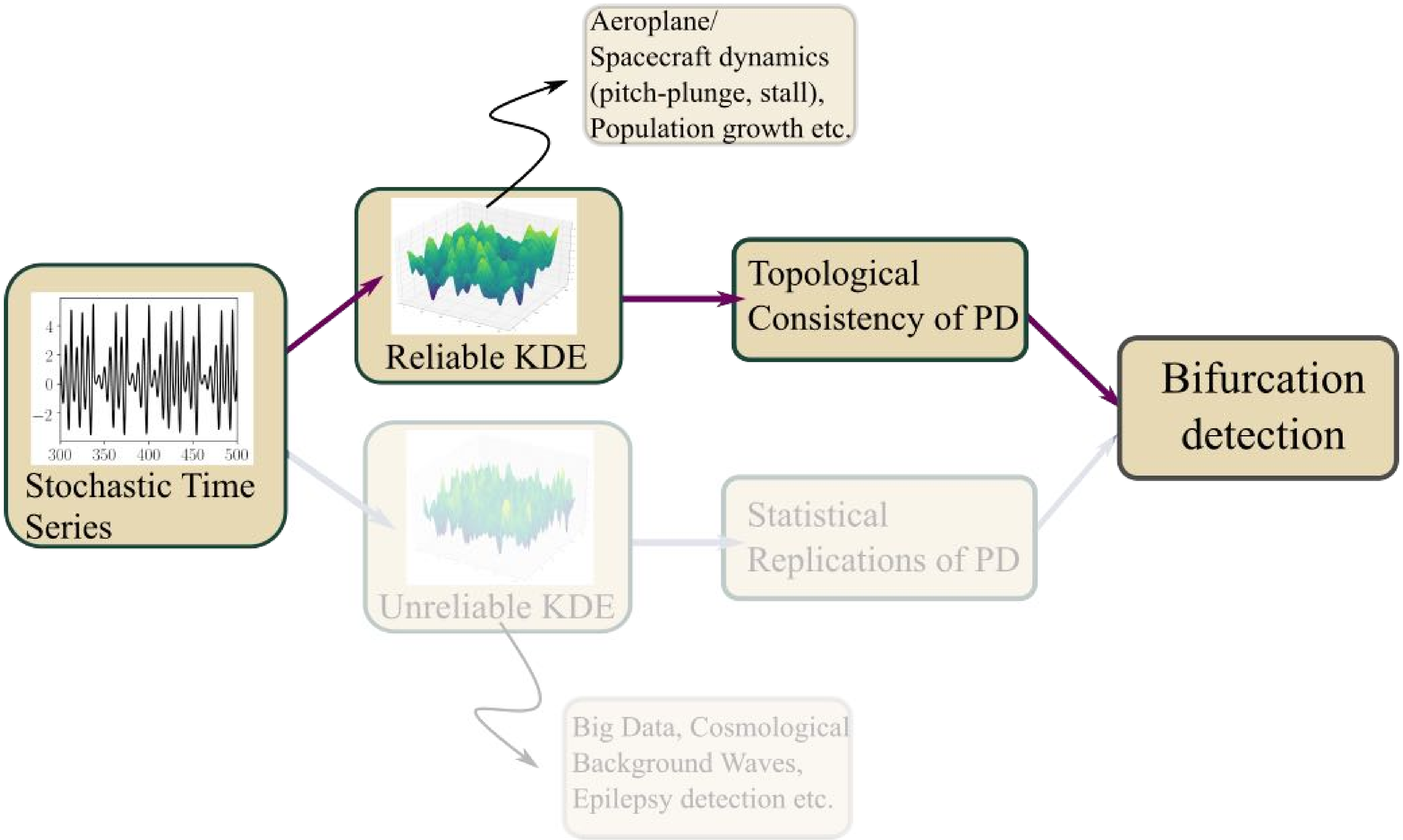


Homological Bifurcation Plot

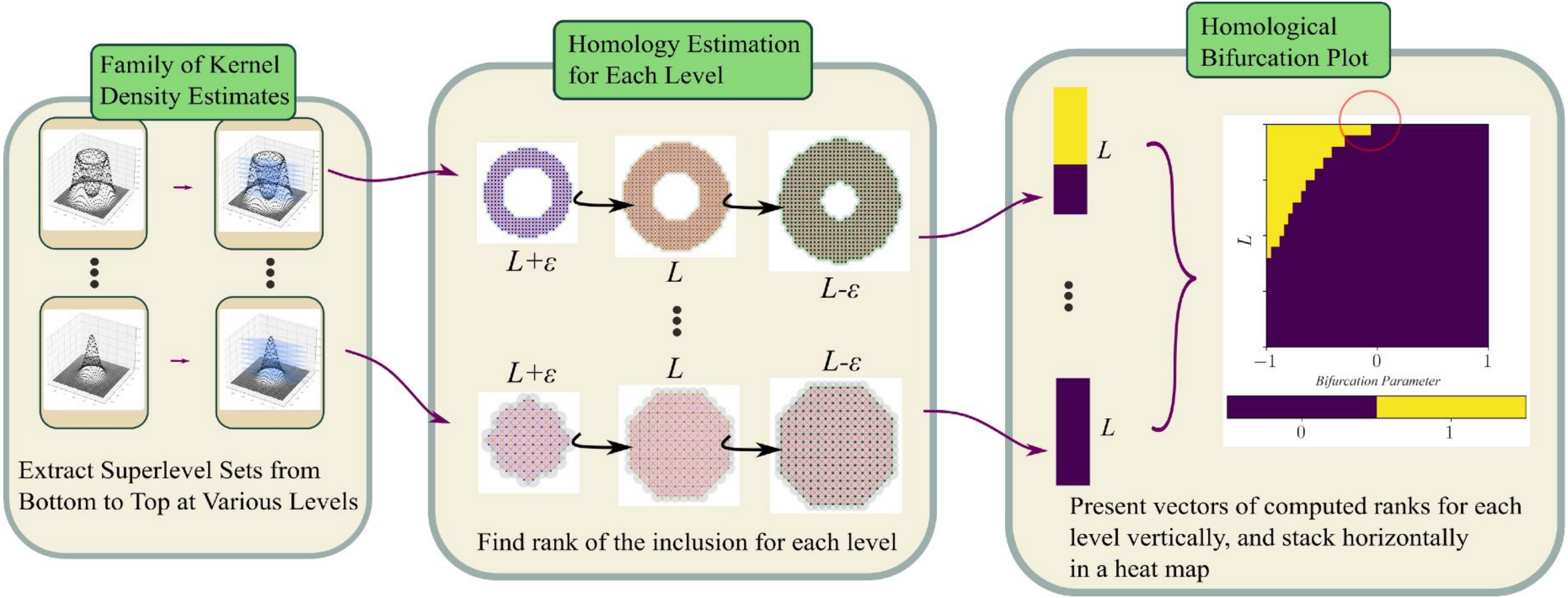







Aeroplane/
Spacecraft dynamics
(pitch-plunge, stall),
Population growth etc.
Reliable KDE
Topological
Consistency of PD
Bifurcation
detection
4
2
0
−2
300
350
400
450
500
Stochastic Time
Series
Unreliable KDE
Statistical
Replications of PD
Big Data, Cosmological
Background Waves,
Epilepsy detection etc.

Detection with Reliable KDEs: Method



Tanweer, S., A. Khasawneh, F., Munch, E., & R. Tempelman, J. (2024). A topological framework for identifying phenomenological bifurcations in stochastic dynamical systems. Nonlinear Dynamics, 112(6), 4687-4703.

Real-World Application: Aerofoil Flutter

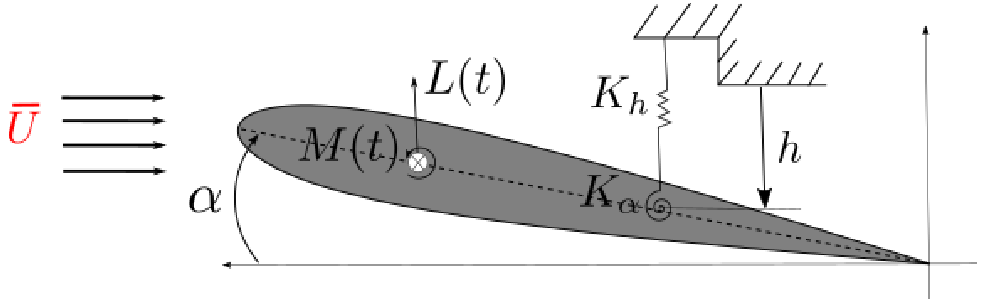
- Limit cycle flutter poses threat to flight safety
- Causes high amplitude oscillations
- Leads to fatigue damage and failure

$$M\ddot{h} + S\ddot{\alpha} + c_1\dot{h} + K_h h = L(t) = -0.1\alpha\bar{U}$$

$$S\ddot{h} + J_E\ddot{\alpha} + c_2\dot{\alpha} + K_\alpha \alpha = M(t) = 0.04\alpha\bar{U}$$

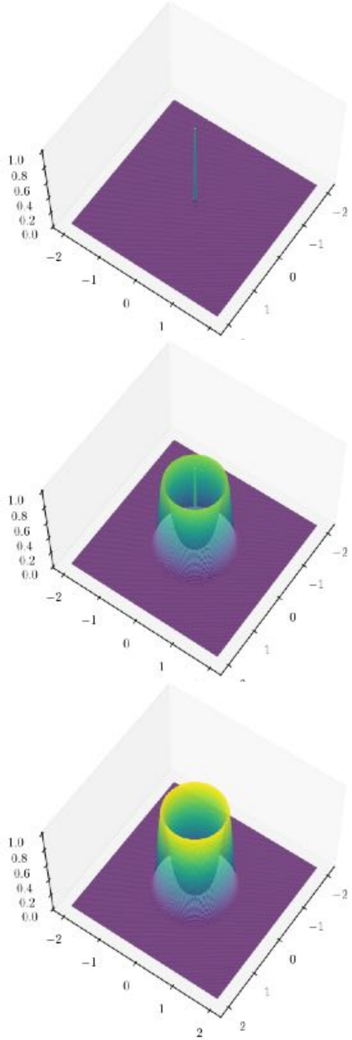
$$K_\alpha = k_1 + k_3\alpha^2 + k_5\alpha^4$$

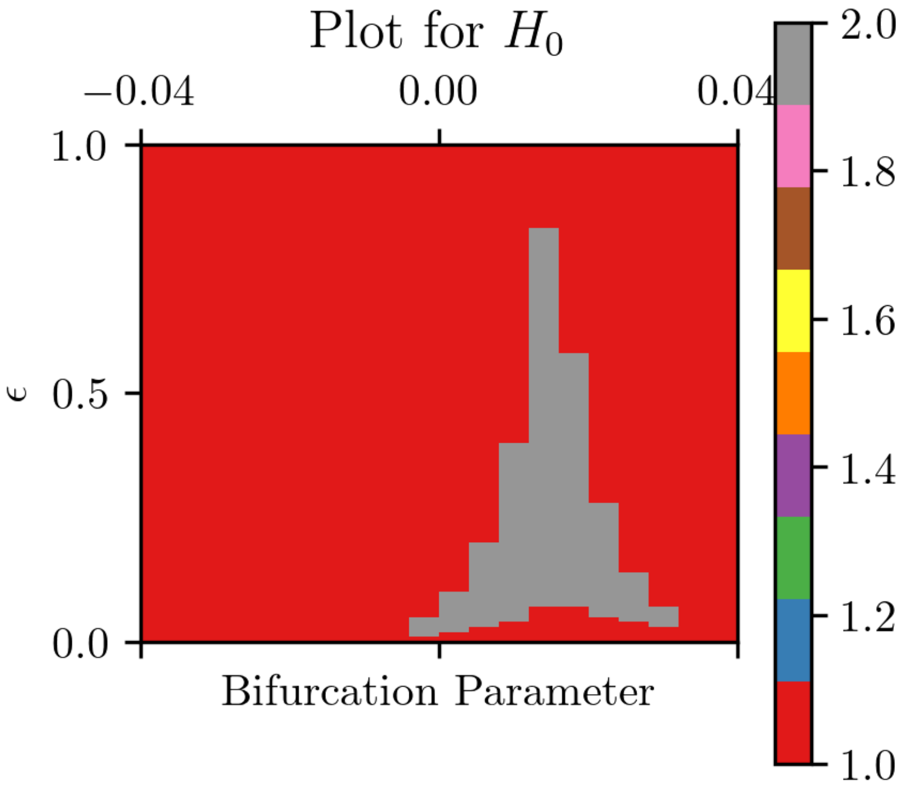
$$\bar{U} = U_f + \varepsilon + \xi(t)$$

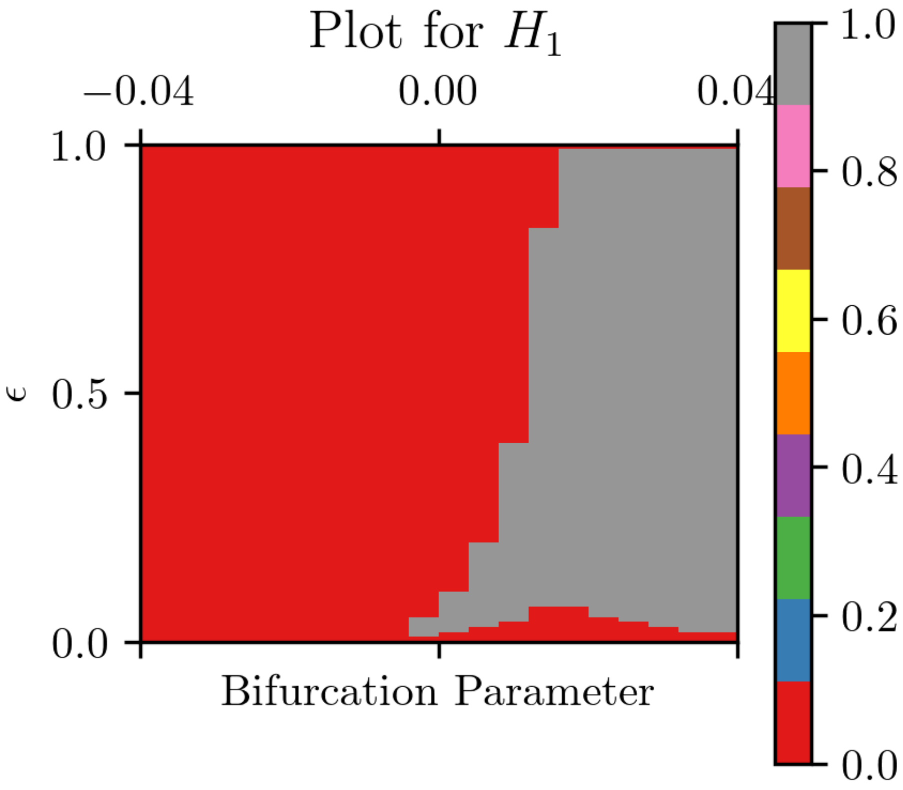


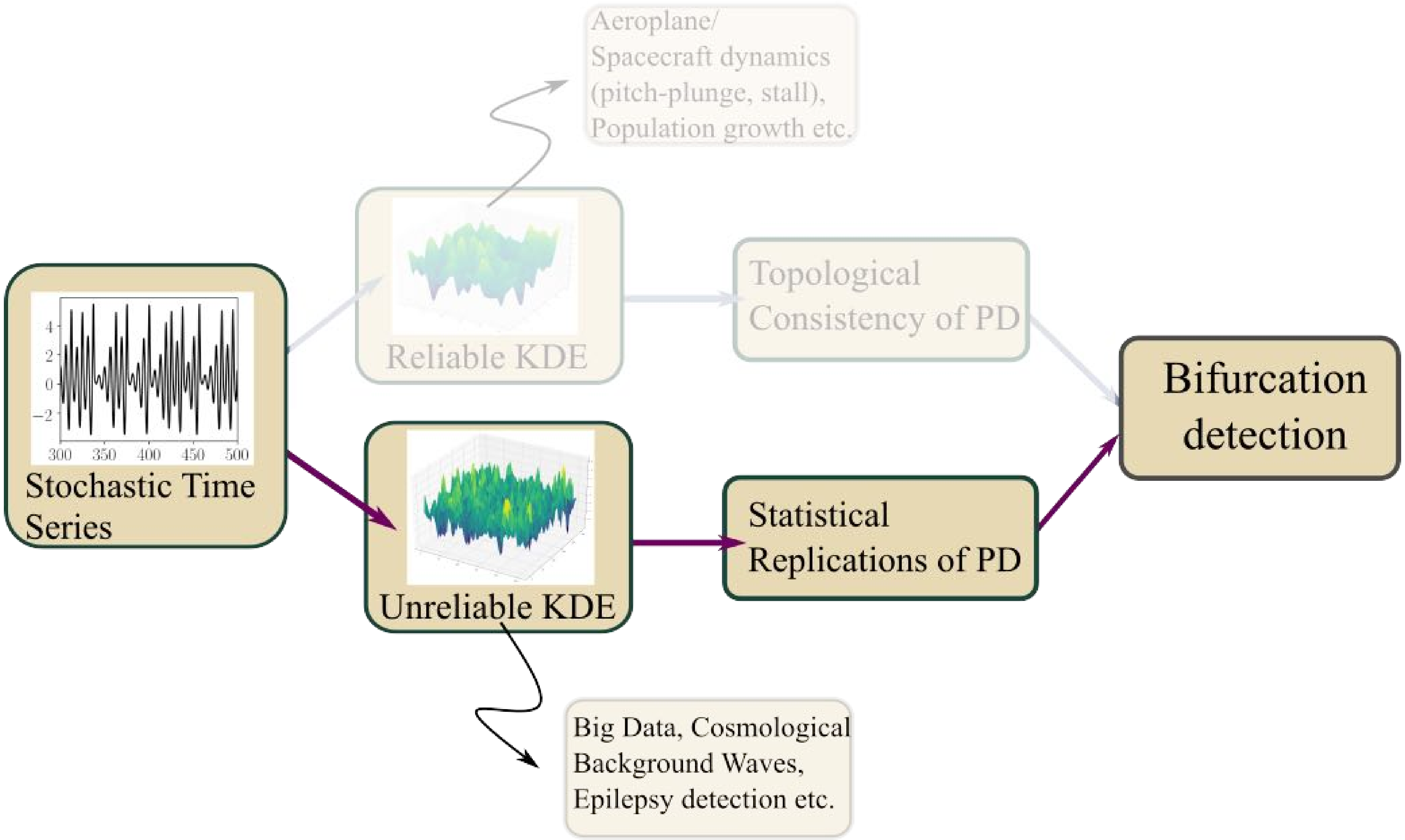
Y. Hao, Z. Q. Wu (2019). "RANDOM FLUTTER OF MULTI-STABLE AIRFOILS EXCITED PARAMETRICALLY IN STEADY FLOWS." Journal of Mechanics Vol. 35, No 3.

Homological Bifurcation Plot





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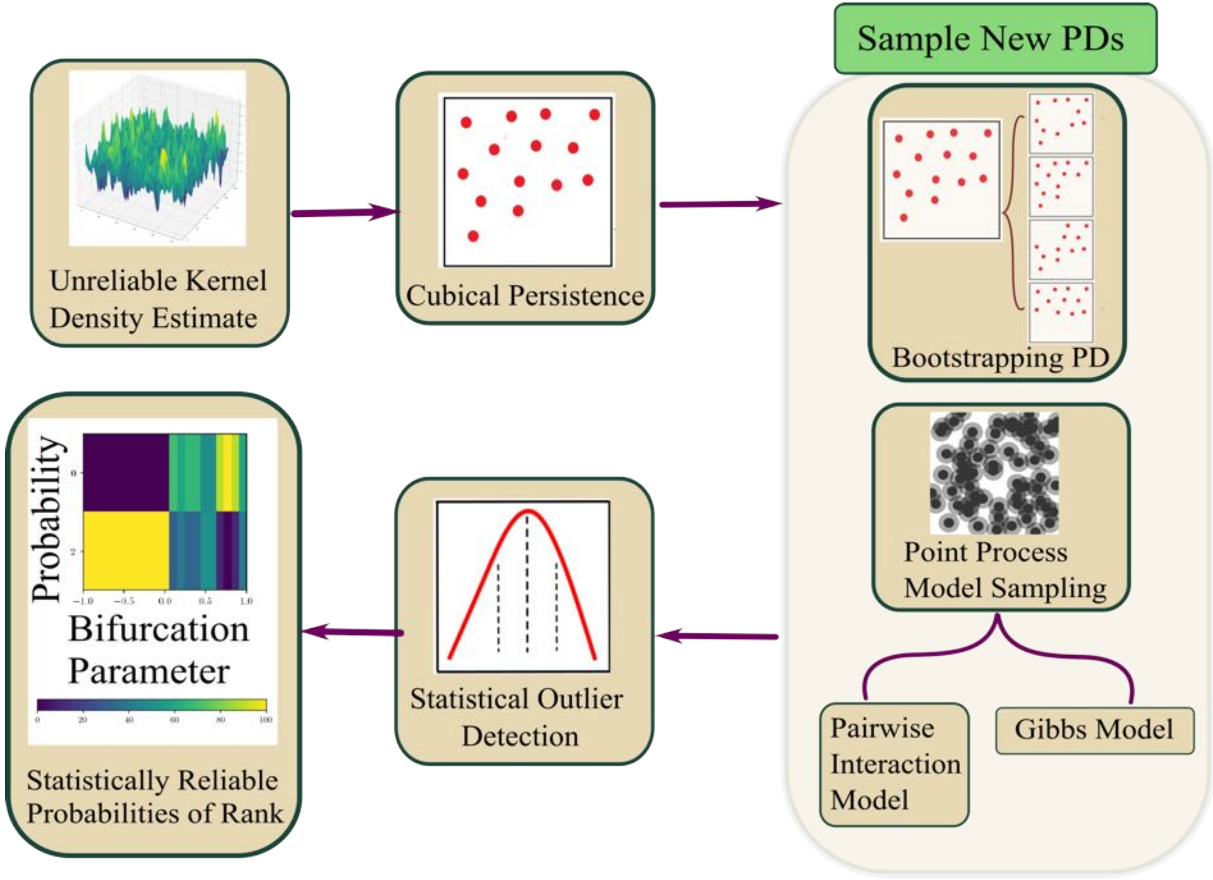
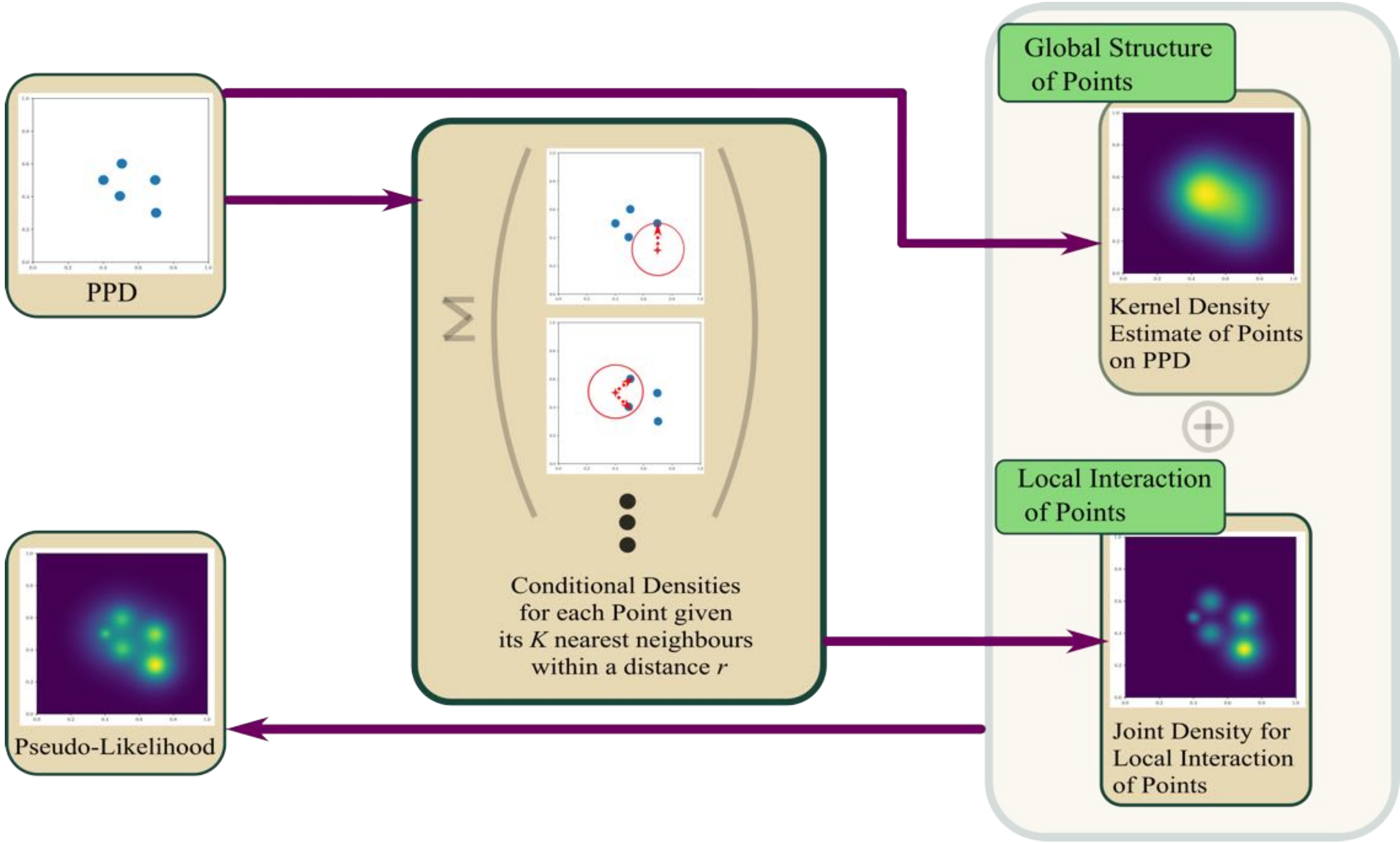
Replications of PD
Big Data, Cosmological
Background Waves,
Epilepsy detection etc.

Bifurcation Detection with Unreliable KDEs: Method

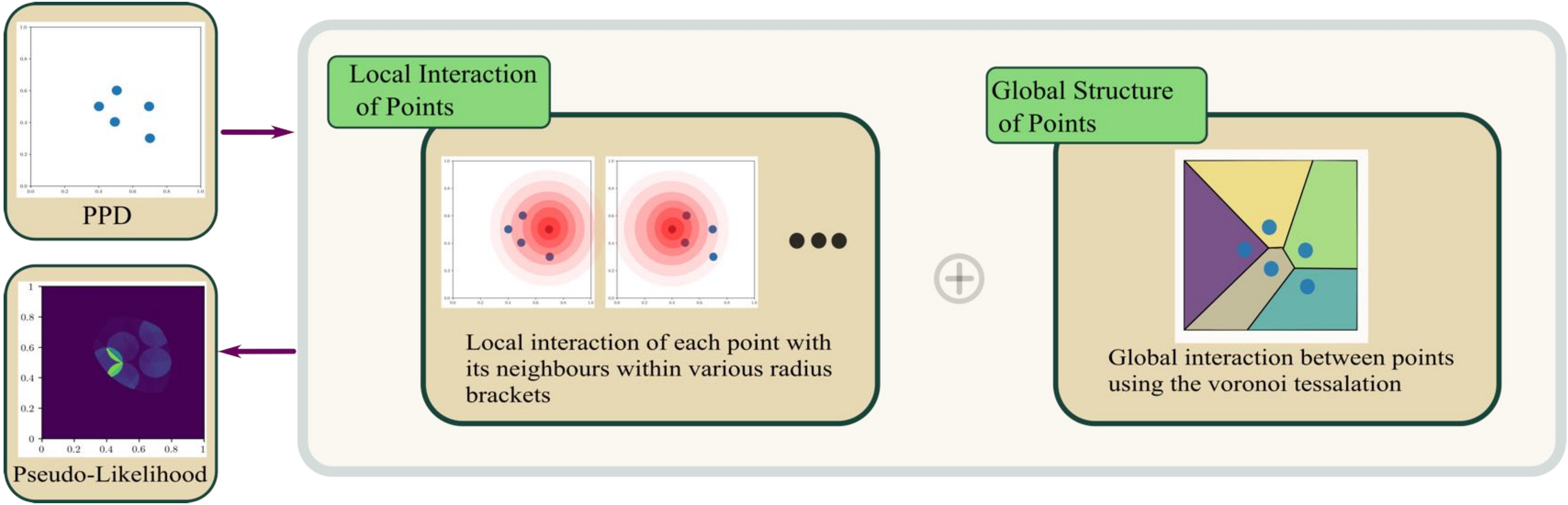


Tanweer, S., & Khasawneh, F. A. (2024). Topological detection of phenomenological bifurcations with unreliable kernel density estimates. Probabilistic Engineering Mechanics, 76, 103634.

Gibbs Modelling

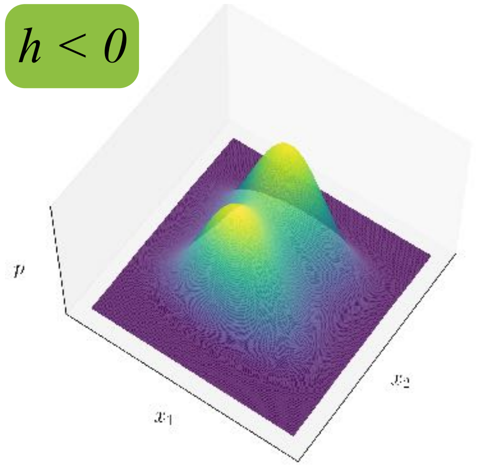


Pairwise Interaction Point Process Modelling

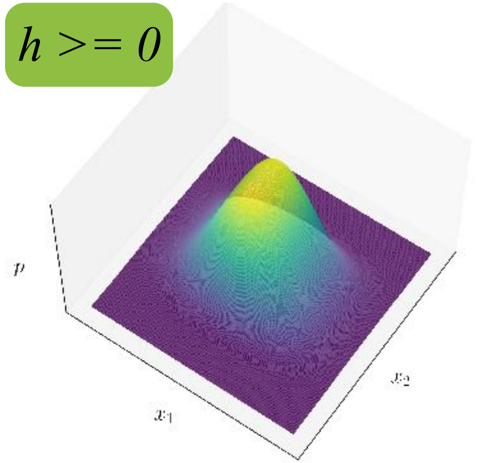


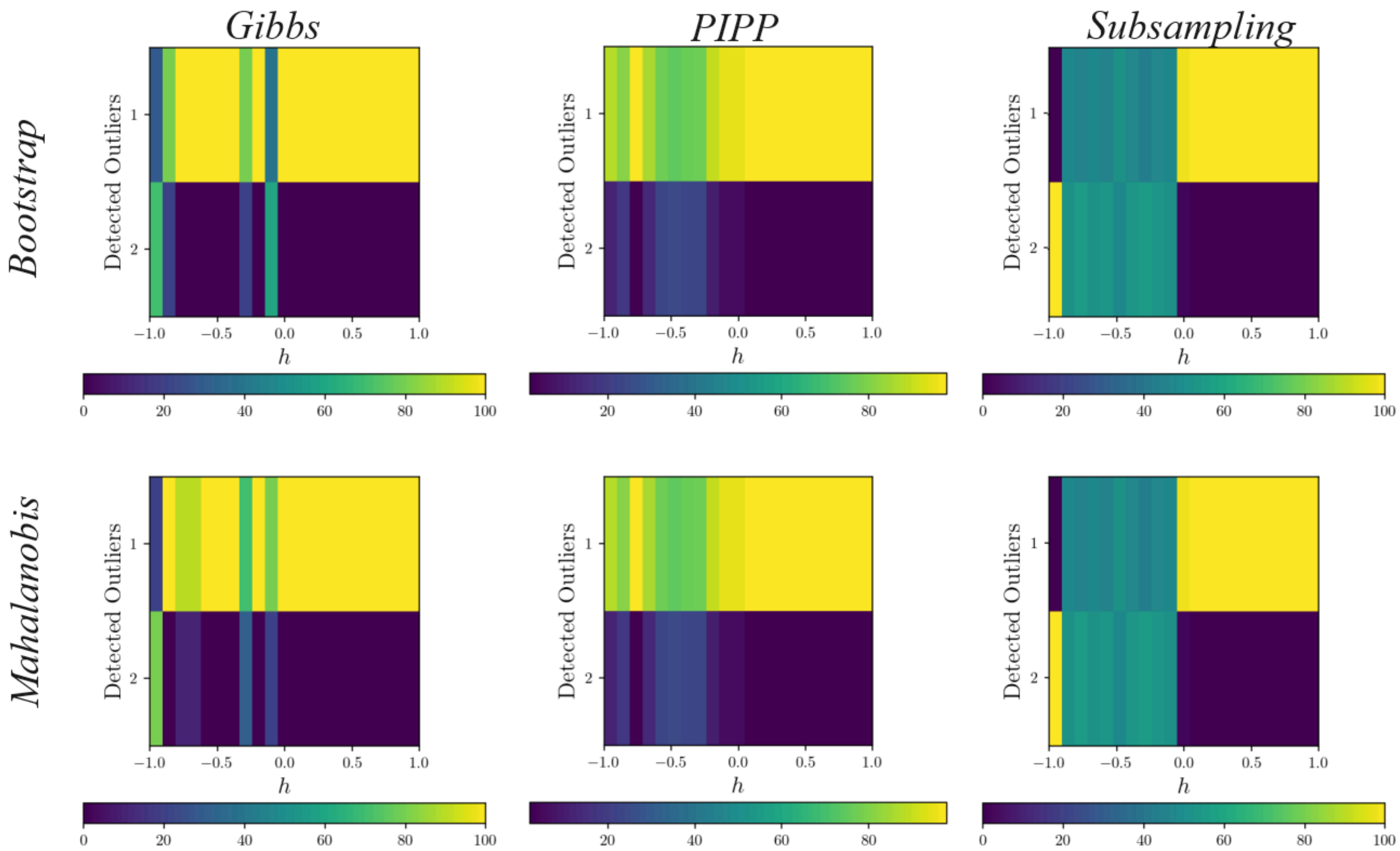
Example: Stochastic Duffing Oscilator

$h < 0$



$h >= 0$





Publications Over the Lifetime of the Project

Period 1 (10/15/2021-10/14/2022):

1. İsmail Güzel, Elizabeth Munch, and Firas A. Khasawneh. “Detecting bifurcations in dynamical systems with CROCKER plots”. In: Chaos: An Interdisciplinary Journal of Nonlinear Science 32.9 (Sept. 2022). Featured cover article., p. 093111. doi: 10.1063/5.0102421.

2. Audun D. Myers and Firas A. Khasawneh. “Damping parameter estimation using topological signal processing”. In: Mechanical Systems and Signal Processing 174 (2022), p. 109042. issn: 0888-3270. doi: 10.1016/j.ymssp.2022.109042.

3. Exploring Surface Texture Quantification in Piezo Vibration Striking Treatment (PVST) Using Topological Measures. Vol. Volume 2: Manufacturing Processes; Manufacturing Systems. International Manufacturing Science and Engineering Conference. V002T05A061. June 2022. doi: 10.1115/MSEC2022-86659.

Period 2 (10/15/2022-10/14/2023):

1. Myers, A., Khasawneh, F. A., & Munch, E. (2023). Persistence of weighted ordinal partition networks for dynamic state detection. SIAM Journal on Applied Dynamical Systems, 22(1), 65-89.

2. Chumley, M. M., Yesilli, M. C., Chen, J., Khasawneh, F. A., & Guo, Y. (2023). Pattern characterization using topological data analysis: Application to piezo vibration striking treatment. Precision Engineering, 83, 42-57.

3. Audun Myers, David Muñoz, Firas A. Khasawneh, and Elizabeth Munch. “Temporal network analysis using zigzag persistence”. In: EPJ Data Science 12.1 (Mar. 2023). doi: 10.1140/epjds/s13688-023-00379-5.

4. Audun D. Myers, Max M. Chumley, Firas A. Khasawneh, and Elizabeth Munch. “Persistent homology of coarse-grained state-space networks”. In: Phys. Rev. E 107 (3 Mar. 2023), p. 034303. doi: 10.1103/PhysRevE.107.034303.

Period 3 (10/15/2023-10/14/2024):

1. Tanweer, S., & Khasawneh, F. A. (2024). Topological detection of phenomenological bifurcations with unreliable kernel density estimates. Probabilistic Engineering Mechanics, 76, 103634.

2. Tanweer, S., Khasawneh, F. A., & Munch, E. (2024). Robust crossings detection in noisy signals using topological signal processing. Foundations of Data Science, 6(2), 154-171.

3. Tanweer, S., A. Khasawneh, F., Munch, E., & R. Tempelman, J. (2024). A topological framework for identifying phenomenological bifurcations in stochastic dynamical systems. Nonlinear Dynamics, 112(6), 4687-4703.

4. Myers, A. D., Chumley, M. M., & Khasawneh, F. A. (2024). Delay parameter selection in permutation entropy using topological data analysis. La Matematica, 1-34.

5. Chumley, M. M., Khasawneh, F. A., Otto, A., & Gedeon, T. (2023). A Nonlinear Delay Model for Metabolic Oscillations in Yeast Cells. Bulletin of Mathematical Biology, 85(12), 122.

Thank you!

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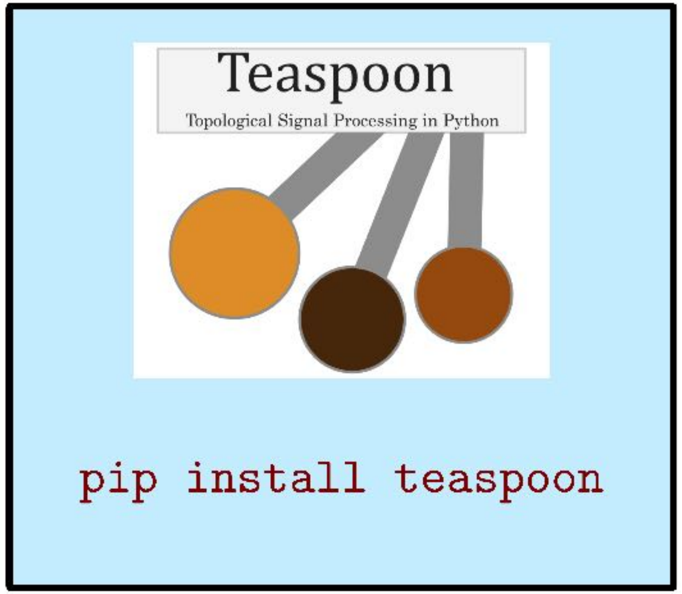
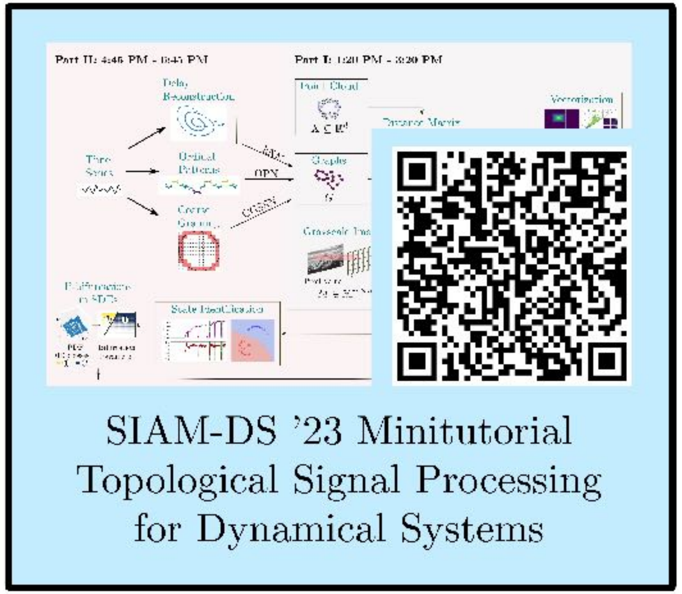
khasawn3@msu.edu





Period 3 (10/15/2023-10/14/2024):

1. Tanweer, S., & Khasawneh, F. A. (2024). **Topological detection of phenomenological bifurcations with unreliable kernel density estimates.** Probabilistic Engineering Mechanics, 76, 103634.

2. Tanweer, S., Khasawneh, F. A., & Munch, E. (2024). **Robust crossings detection in noisy signals using topological signal processing.** Foundations of Data Science, 6(2), 154-171.

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