

# Thermalization of radiation-induced carriers in insulators and wide bandgap semiconductors **and semiconductor devices (HEMTs)** (GaN, diamond, $\beta$ -Ga<sub>2</sub>O<sub>3</sub>, **Si, SiC**,... for now)

Massimo V. Fischetti and Dallin O. Nielsen  
The University of Texas at Dallas, Dept. of Materials Science and Engineering  
Emails: [max.fischetti@utdallas.edu](mailto:max.fischetti@utdallas.edu), [dallin.nielsen@utdallas.edu](mailto:dallin.nielsen@utdallas.edu)

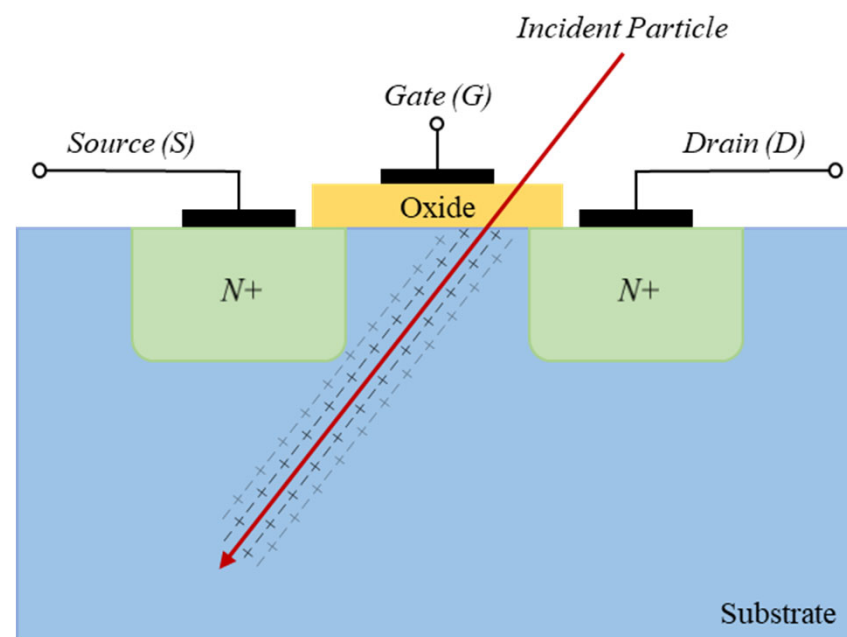
in collaboration with C. Van de Walle (UCSB),  
Laura Nichols, S. Pantelides, R. Schrimpf, and D. Fleetwood (Vanderbilt)

# The problem



## Simulate single-event effects/upsets (SEE/SEUs) in electron devices

- Basic ingredients
  1. Spatial and energetic distribution of the electron-hole (e-h) pairs after the (proton) strike
  2. Thermalization and diffusion of the e-h pairs
  3. Properties of possible precursor defects at interfaces/bulk (permanent damage)
- Present state of the art to obtain these ingredients
  1. High-energy binary collision Monte Carlo code (e.g., Geant-4, MRED)
  2. Assume instantaneous complete thermalization + drift-diffusion
  3. Empirical cross sections and threshold/activation energies
- Our approach
  1. Use DTF below 100 eV (close the '10-100 eV gap')
  2. Consider e-h pair distributions from step 1 and use full-band Monte Carlo
  3. Cross sections and activation energies also from *ab initio* calculations



- Review and **new results**:

1. Calculated energy-loss function (ELF) from DTF up to 100 eV  
Calculated rates for carrier-plasmon, carrier-phonon interactions and impact ionization
  - GaN,  $\beta$ -Ga<sub>2</sub>O<sub>3</sub>, C (diamond), **Si, and SiC (fcc)**
2. Used full-band MC to study carrier relaxation (bulk)  
**Used distributions after fast relaxation (~100 fs) as initial configuration in Monte Carlo device-simulation code: A GaN/AlGaN HEMT**
  - Major coding work on Anduril
  - Test code to calculate hot-electron damage generation in a high- $V_{DS}$ -stressed GaN/AlGaN HEMT (empirical cross section and threshold energy for defect creation)
  - Test code to calculate radiation-induced damage and current transient (upset) during an SEE
3. Cross sections and activation energies from *ab initio* calculations; with Laura Nichols and Sok Pantelides (VU), and Xiaoguang Zhang (UF) – just starting
  - De-hydrogenation of  $V_{Ga}$ -H<sub>3</sub> complexes at the GaN-AlGaN interface of a p-channel HEMT

# Summary of new results



1. Thermalization of 100 eV electrons in Si and SiC
2. Damage generated by hot electrons at the GaN-AlGaN interface of a HEMT stressed at high drain bias
3. Damage induced by 100 eV electrons in a GaN/AlGaN HEMT
4. Dehydrogenation of  $V_{\text{Ga}}\text{-H}_3$  complexes at the GaN-AlGaN interface

Only “proof-of-concept” (or ‘work in progress’) for now.

Missing: 1. GaN hole transport in MC device code

2. Reliable information about defect precursors

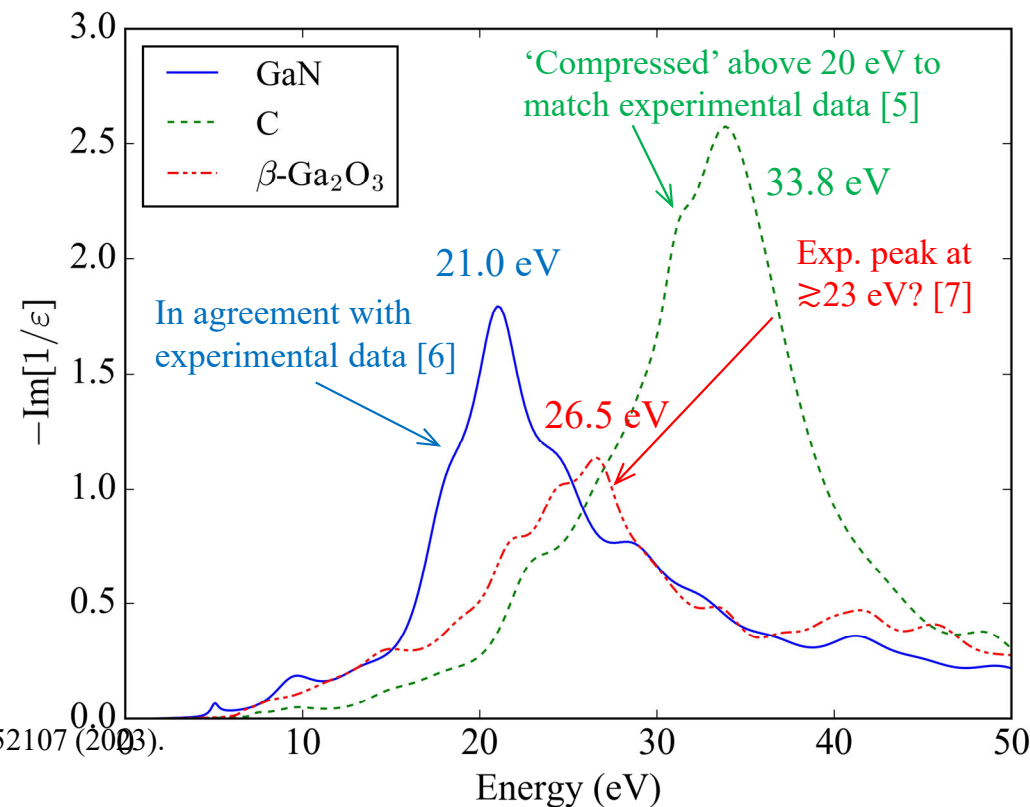
3. Pair momentum distribution in the ‘streak’

(... but probably already isotropic in  $\sim 100$  fs)

# First Principles Calculations [1]



- DFT and DFPT (*Quantum ESPRESSO* [2])
  - Band structure
    - “Scissor operator” to fit gap including also excitonic effects when significant (diamond)
  - Phonon dispersion
  - Dielectric function (TDDFT, *turboEELS* [3])
    - Plasmon emission and impact ionization from energy-loss (EL) function  $\text{Im}\left(\frac{-1}{\epsilon(\mathbf{q},\omega)}\right)$
- EPW [4]: Carrier-phonon matrix elements
- Loss function:
  - Stronger peaks  $\rightarrow$  higher plasmon emission rate
  - GaN and  $\beta\text{-Ga}_2\text{O}_3$  plasmon energies lower than C  $\rightarrow$  slower carrier thermalization



Calculated energy-loss functions for C (diamond), GaN (wurtzite) [1], and  $\beta\text{-Ga}_2\text{O}_3$

[1] D. O. Nielsen *et al.*, Phys. Rev. B **108**, 155203 (2023); Appl. Phys. Lett. **123**, 252107 (2023).

[2] P. Giannozzi, *et al.*, J. Phys. Cond. Matt. **21**, 395502 (2009).

[3] I. Timrov, *et al.*, Comput. Phys. Commun. **196**, 460 (2015).

[4] S. Ponc , *et al.*, Comput. Phys. Commun. **209**, 116 (2016).

[5] J. Ca as *et al.*, Appl. Surf. Sci. **461**, 93 (2018).

[6] R. Dhall, *et al.*, Appl. Phys. Lett. **112**, 061102 (2018).

[7] C. Fares *et al.*, ECS J. Solid State Sci. Technol. **8**, Q3154 (2019) (Surface losses from REELS. Data only up to 25 eV).

# Scattering Rates: Calculation [1]

- Carrier-phonon scattering (Fermi's Golden Rule, FGR):

$$\frac{1}{\tau_n^{(\eta)}(\mathbf{k})} = \frac{2\pi}{\hbar} \sum_{n', \mathbf{q}} |g_{nn'}^{\eta}(\mathbf{k}, \mathbf{q})|^2 \left( N_{\mathbf{q}} + \frac{1}{2} \mp \frac{1}{2} \right) \delta [E_n(\mathbf{k}) - E_{n'}(\mathbf{k} \pm \mathbf{q}) \pm \hbar\omega_{\mathbf{q}}^{(\eta)}]$$

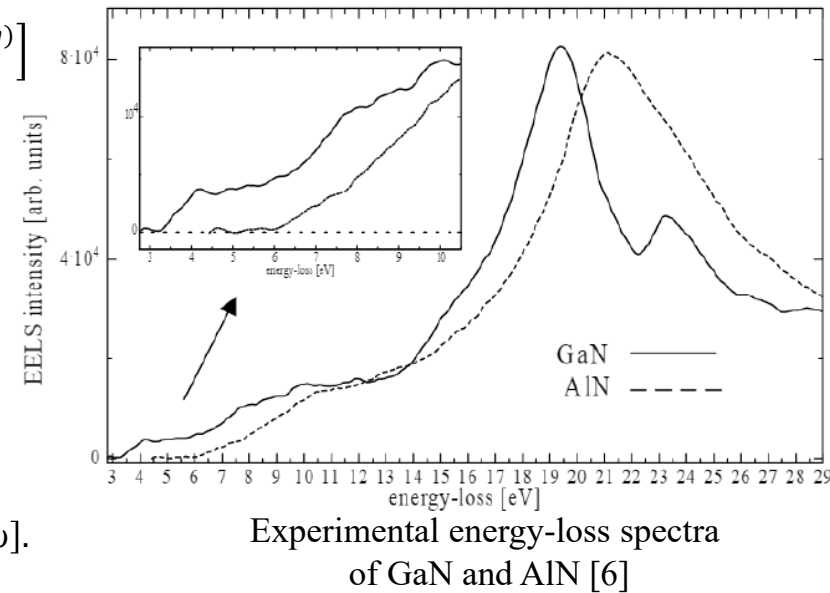
- $N_{\mathbf{q}} = 1/(e^{\hbar\omega_{\mathbf{q}}/k_B T} - 1)$  (assuming  $T=300$  K)
- Evaluation of delta function done by Blöchl's tetrahedron method [2].

- Carrier energy-loss rate (ELR)

- Electron energy loss spectroscopy (STM-EELS)
  - Peaks = impact ionization and plasma excitations (see figure)
- Total rate using dielectric function (directly related to EEL cross section), FGR and dissipation fluctuation theorem [3-5]:

$$\frac{1}{\tau_n^{\text{ELR}}(\mathbf{k})} = \frac{2\pi}{\hbar} \sum_{n'} \int \frac{d\mathbf{q}}{(2\pi)^3} \frac{e^2 \hbar}{q^2} \int \frac{d\omega}{2\pi} \text{Im} \left[ \frac{-1}{\epsilon(\mathbf{q}, \omega)} \right] \delta [E_n(\mathbf{k}) - E_{n'}(\mathbf{k} + \mathbf{q}) \pm \hbar\omega].$$

- Assume a low radiation-dose rate: Small density, no conduction-band el-el scattering, no heating



[1] D. O. Nielsen *et al.*, Phys. Rev. B **108**, 155203 (2023); Appl. Phys. Lett. **123**, 252107 (2023)..

[2] P. E. Blöchl, O. Jepsen, and O. K. Andersen, Phys. Rev. B **49**, 16223 (1994).

[3] P. Nozières and D. Pines, *Theory of Quantum Liquids*, Advanced Book Classics Series (Westview Press, Boulder, CO, 1999).

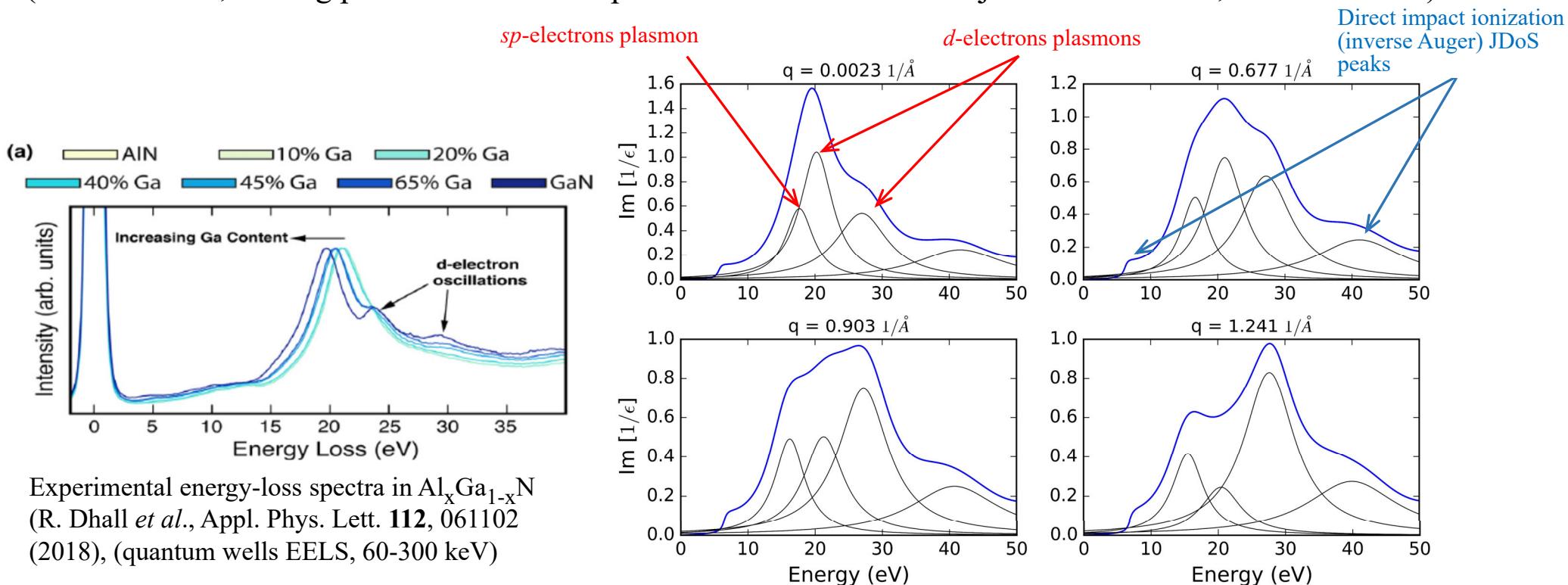
[4] H. Nyquist, Phys. Rev. **32**, 110 (1928)/

[5] H. B. Callen and T. A. Welton, Phys. Rev. **83**, 34 (1951).

[6] G. Brockt and H. Lakner, Micron **31**, 435 (2000).

# The energy-loss function $\text{Im} \left[ -\frac{1}{\epsilon(\mathbf{q}, \omega)} \right] (\text{GaN})$

(from the RPA, finding plasmons and JDoS peaks via Lorentzian fits... just to understand, but academic\*)



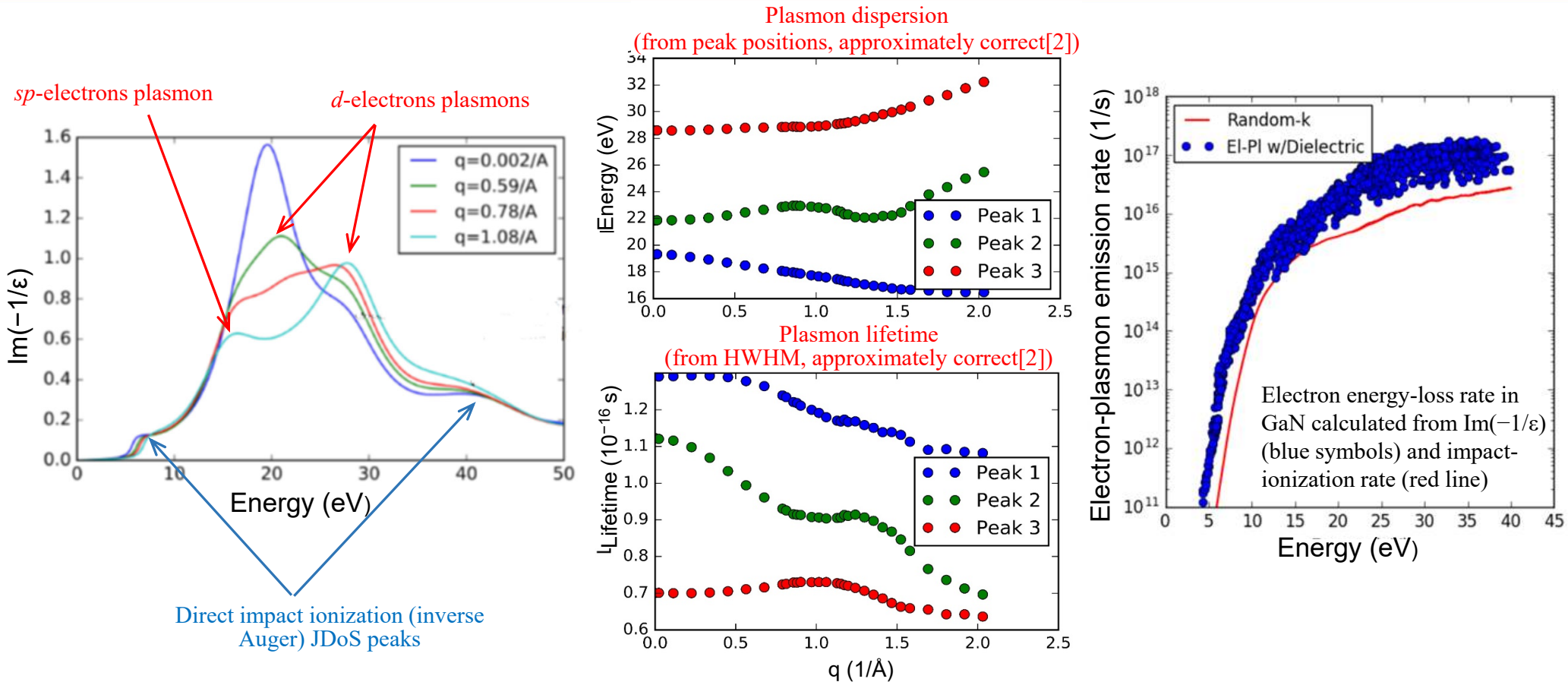
Experimental energy-loss spectra in  $\text{Al}_x\text{Ga}_{1-x}\text{N}$   
(R. Dhall *et al.*, Appl. Phys. Lett. **112**, 061102  
(2018), (quantum wells EELS, 60-300 keV)

(\*Except for 'vertex corrections')

$$\epsilon^{(r)}(\mathbf{q}, \omega) = 1 - \frac{e^2}{q^2 \epsilon_0} \sum_{\mathbf{G}, \mathbf{k}, \mu, \mu'} \frac{[p(\mathbf{k}, \mu) - p(\mathbf{k} + \mathbf{q}, \mu')][\langle \mathbf{k} + \mathbf{q} + \mathbf{G}, \mu' | e^{i\mathbf{q} \cdot \mathbf{r}} | \mathbf{k}, \mu \rangle]^2}{E_\mu(\mathbf{k}) - E_{\mu'}(\mathbf{k} + \mathbf{q}) + \hbar\omega + i\hbar s}$$



# $sp$ - and $d$ -electrons plasmon: Dispersion and lifetime (GaN) [1]



[1] D. O. Nielsen *et al.*, Phys. Rev. B **108**, 155203 (2023).

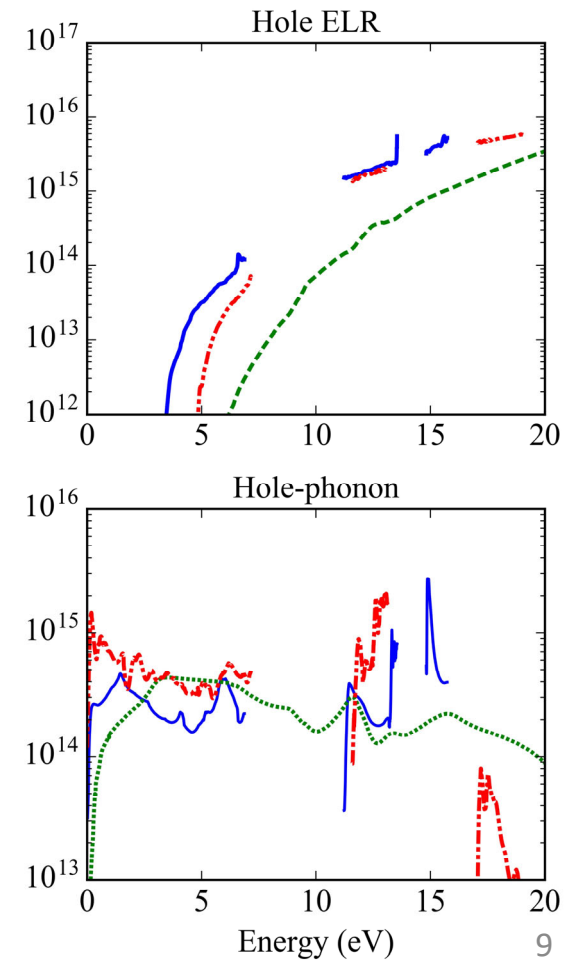
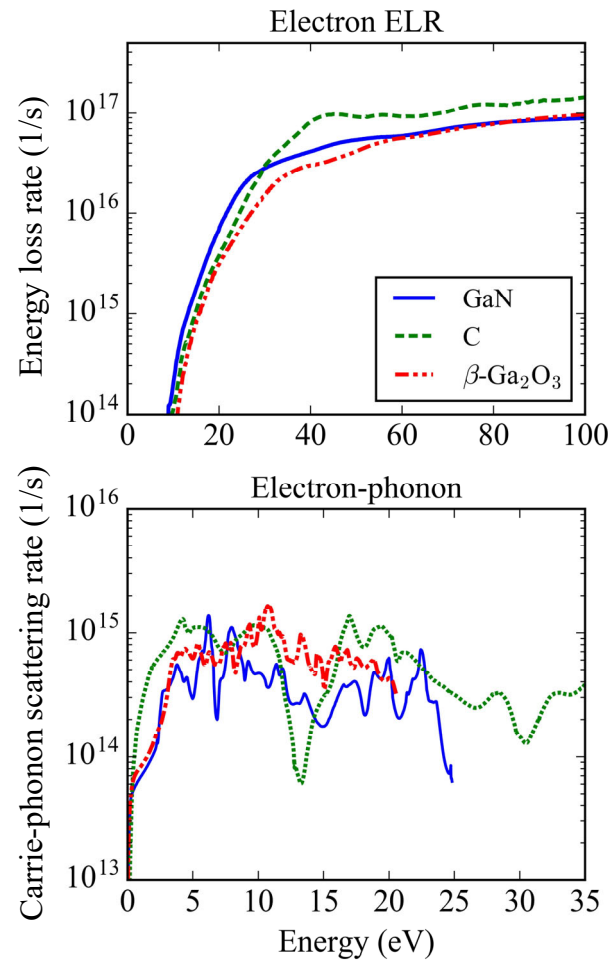
[2] P. Hamann *et al.*, Contrib. Plasma Phys. **10**, e202000147 (2020)



# Scattering/Energy-Loss Rates



- Phonon scattering and ELR
- Electrons:
  - Phonon scattering dominant for low energies ( $\lesssim 10$ -15 eV)
  - ELRs flatten as energy increases (reaches  $\sim 10^{17}$  1/s; broadening still acceptable)
  - Quinn, Ferrell, Pines, Penn calculated rates of similar magnitude [2-5]
  - Ga-based materials driven to high magnitudes by dense  $d$  bands; diamond by large loss function
- Holes:
  - Phonon scattering dominant for low energies
  - GaN spikes at  $\sim 13$  and 15 eV from  $d$  bands



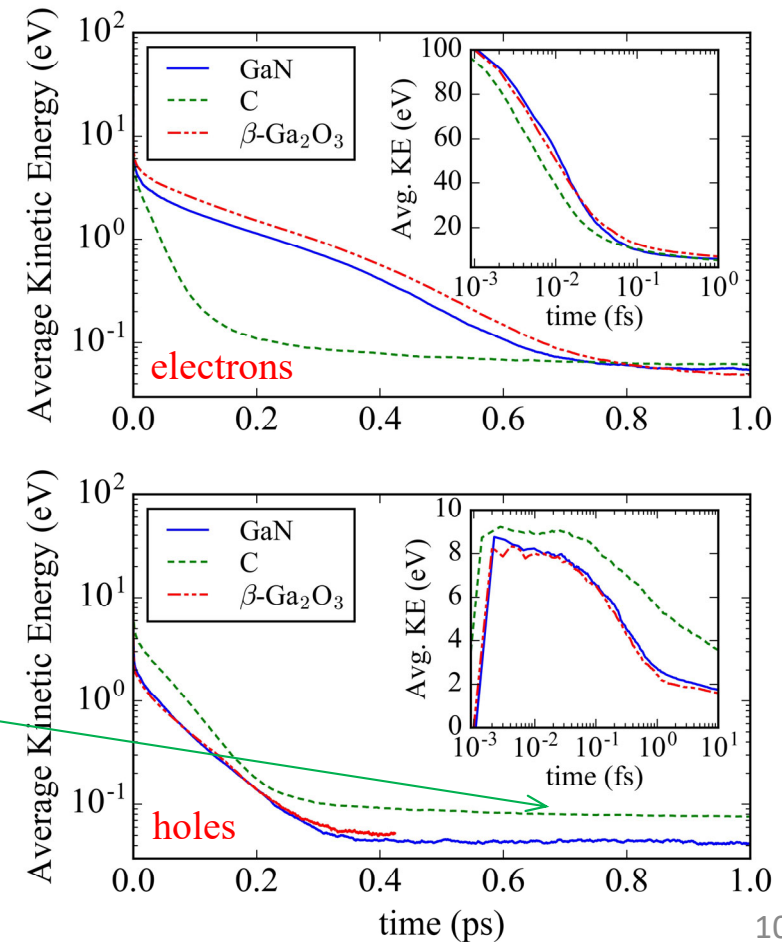
- [1] D. O. Nielsen *et al.*, Phys. Rev. B **108**, 155203 (2023) (GaN only).  
[2] D. Pines, Rev. Mod. Phys. **28**, 184 (1956).  
[3] J. J. Quinn and R. A. Ferrell, Phys. Rev. **112**, 812 (1958).  
[4] J. J. Quinn, Phys. Rev. **126**, 1453 (1962).  
[5] D. R. Penn, Phys. Rev. **35**, 482 (1987).

# Hot Electron Thermalization



- Solved semi-classical BTE
  - Full bands [1,2]
  - Zero field, 300 K
  - 1000 electrons ( $E \sim 100$  eV)
- Electrons
  - Rapid energy loss ( $t < 10^{-16}$  s)
  - Plasmon emission/impact ionization
  - Thermalization rate decreases ( $t > 10^{-16}$  s)
  - Phonon scattering to thermal when energy falls below impact-ionization threshold
- Holes
  - Initial hole buildup
  - No rapid initial decrease (no plasma excitations)
  - Steeper slope (larger frame)
- GaN
  - Electrons thermalize in  $\sim 1$  ps; holes in  $\sim 0.5$  ps
  - Average energy per pair  $\sim 8.9$  eV/pair [3,4]
- C (diamond)
  - Electrons thermalize in  $\sim 0.2$  ps; holes in  $\sim 0.3$  ps (large plasmons energy)
  - Average energy per pair  $\sim 12.87$  eV/pair [5]
- $\beta$ -Ga<sub>2</sub>O<sub>3</sub>
  - Electrons thermalize in  $\sim 1$  ps; holes in  $\sim 0.5$  ps
  - Average energy per pair  $\sim 11.24$  eV/pair [4]

Slow hole thermalization in C due to small hole-phonon scattering rates below  $\sim 0.5$  eV

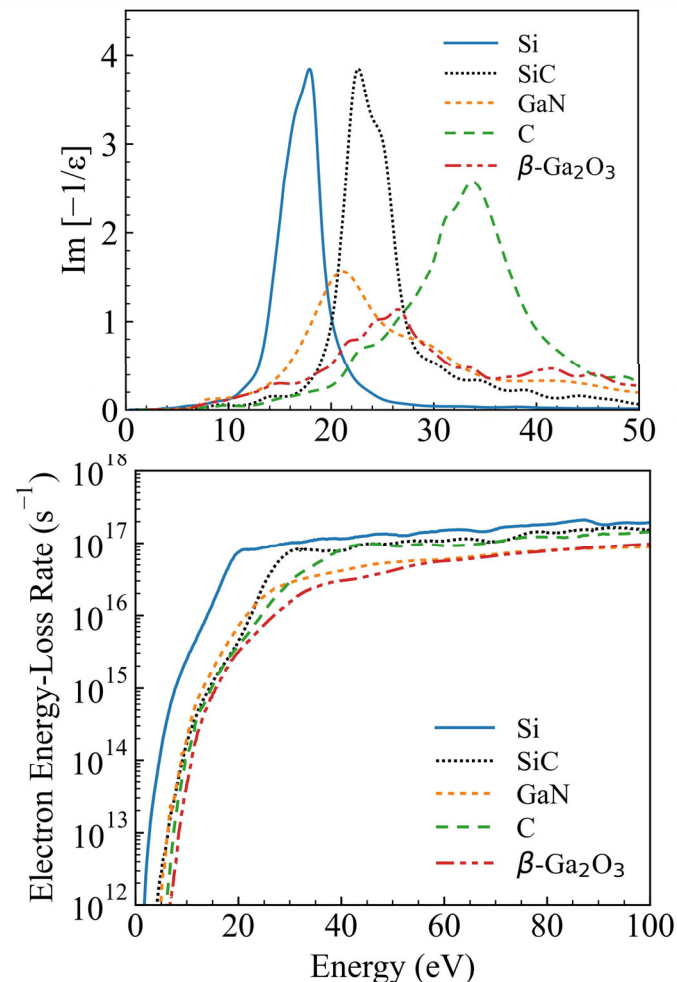


- [1] H. Shichijo and K. Hess, Phys. Rev. B **23**, 4197 (1981).
- [2] M. V. Fischetti and S. E. Laux, Phys. Rev. B **38**, 14 (1988).
- [3] P. J. Sellin and J. Vaitkas, Nucl. Instrum. Methods Phys. Res. A **557**, 479 (2006).
- [4] E. B. Yakimov, *et al.*, Appl. Phys. Lett. **118**, 202106 (2021).
- [5] J. W. Keister, *et al.*, J. Synchrotron Rad. **25**, 407 (2018).

1. Thermalization of 100 eV electrons in Si and SiC
  - Small gap and Lorentzian broadening used by TurboEELs for ELF caused issues. Used “epsilon.x” instead.
2. Damage generated by hot electrons at the GaN-AlGaN interface of a HEMT stressed at high drain bias
  - Two defect precursors with small/slow observed  $V_{th}$ -shift ( $\sim 10$  mV/hour) [1]:  
$$N_1^\infty = 1.46 \times 10^{11} \text{ cm}^{-2} \quad \sigma_1 = 8.2 \times 10^{-16} \text{ cm}^2 \quad E_1 = 0.6 \text{ eV}$$
$$N_2^\infty = 3.6 \times 10^{10} \text{ cm}^{-2} \quad \sigma_2 = 2.3 \times 10^{-16} \text{ cm}^2 \quad E_2 = 1.4 \text{ eV}$$
  - No GaN holes in Anduril (yet): Possible  $V_{th}$ -shift due to floating-body effects; faster/larger predicted  $V_{th}$ -shift (6 orders-of-magnitude): Increase  $E_1, E_2$ , decrease  $\sigma_1, \sigma_2$
3. Damage induced by 100 eV electrons in a GaN/AlGaN HEMT
  - No GaN holes in Anduril (yet): Only qualitative estimates for small density
4. Dehydrogenation of  $V_{Ga}$ -H<sub>3</sub> complexes at the GaN-AlGaN interface (p-channel HEMT)
  - Provided Laura and Sok hot-hole distributions to test their codes

# 1. Thermalization in Si and SiC

- Loss function of Si
  - Peak near predicted value (el gas: 16.4 eV)
  - Largest peak magnitude
- Loss function of SiC
  - Peak magnitude and area similar to Si
- Energy-loss rates
  - Si: reaches  $10^{17} \text{ s}^{-1}$  at  $\sim 20 \text{ eV}$ ; SiC: at  $\sim 30 \text{ eV}$ 
    - collisional broadening ( $\sim 33 \text{ eV}$ )
  - Caused by large loss function (peak magnitude and area)
  - Si: Small gap  $\rightarrow$  large DOS for generated pairs
  - SiC: Large hole DOS  $\rightarrow$  relatively large JDOS

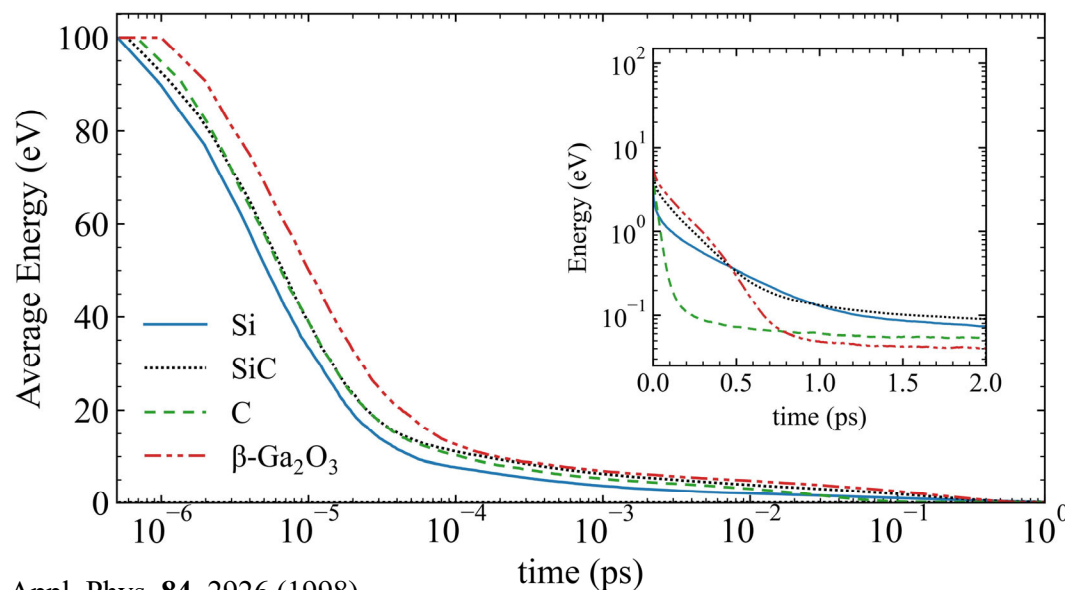


[1] D. O. Nielsen *et al.*, Phys. Rev. B **108**, 155203 (2023) (GaN only).

[2] D. O. Nielsen and M. V. Fischetti, Appl. Phys. Lett. **123**, 252107 (2023) ( $\text{Ga}_2\text{O}_3$  and C)

# 1. Thermalization in Si and SiC

- Initially, energy drops fastest in Si → high ELR
  - SiC and C nearly overlap → similar ELR
  - $\beta$ -Ga<sub>2</sub>O<sub>3</sub> lowest ELR (GaN very similar)
- Inset figure:
  - Final stages depend on phonon scattering rates
  - Low phonon scattering rates in Si and SiC cause slow thermalization.
- Pair creation energy:
  - Si: 3.4 eV/pair (Exp: 3.6 eV/pair [1])
  - SiC: 7.2 eV/pair (Klein's expression: 7.21 eV/pair [2])
  - GaN: 8.9 eV/pair
  - $\beta$ -Ga<sub>2</sub>O<sub>3</sub>: 11.2 eV/pair
  - C: 12.9 eV/pair } (slide 10)



[1] F. Scholze, H. Rabus, and G. Ulm, J. Appl. Phys. **84**, 2926 (1998).

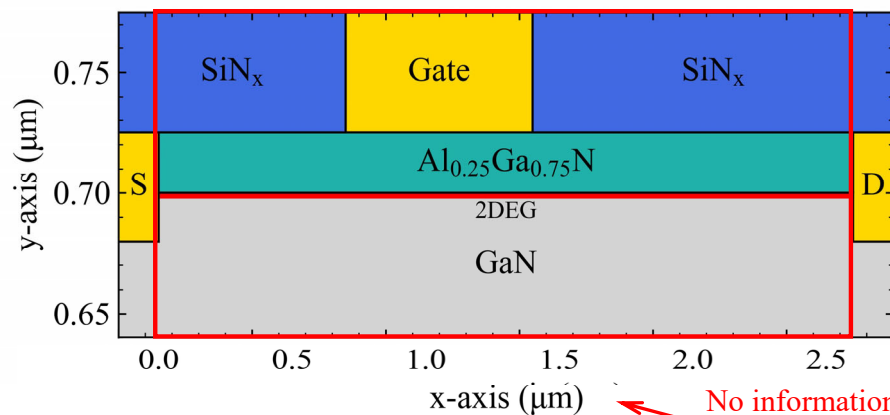
[2] C. A. Klein, J. Appl. Phys. **39**, 2029 (1968).



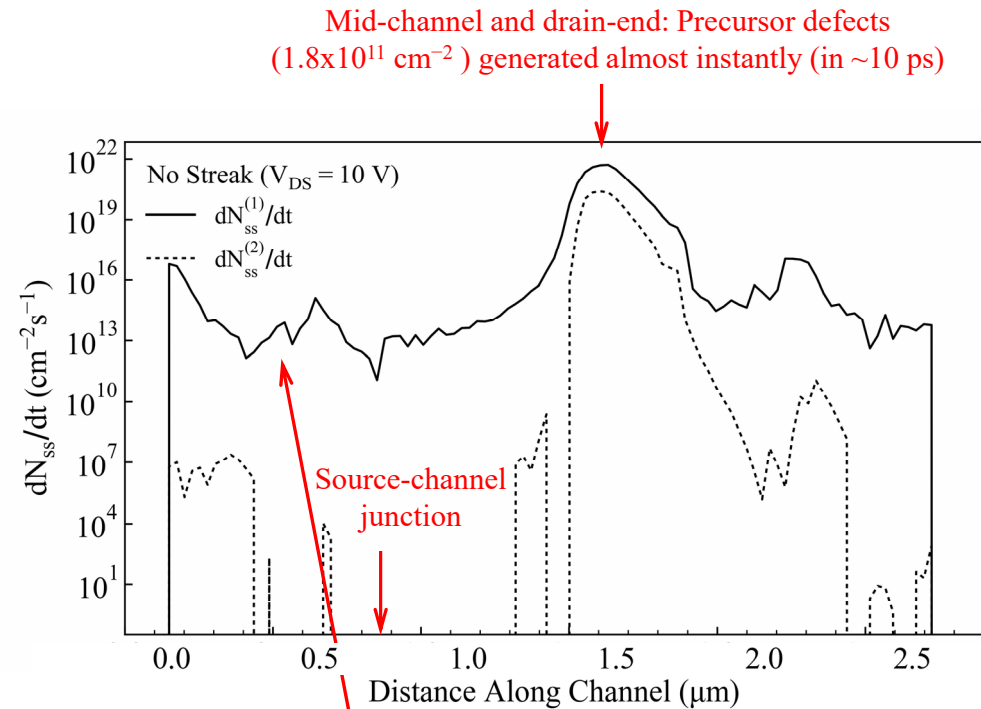


## 2. Damage generated at the GaN-AlGaN interface of a HEMT

- Hot-electron induced damage at the GaN-AlGaN interface of n-channel HEMTs [1]
- With published parameters ( $N_{\infty}$ ,  $\sigma_i$ ), found very high defect-generation rate near the source,  $\sim 10^{13} \text{ cm}^{-2}\text{s}^{-1}$  @  $V_{DS}=10 \text{ V}$ ; **but**: very slow  $V_{th}$  shift ( $\sim 10 \text{ mV/hours}$ ), seen in experiments,  $\sim 10^6 \text{ cm}^{-2}\text{s}^{-1}$  @  $V_{DS}=20 \text{ V}$  [1]



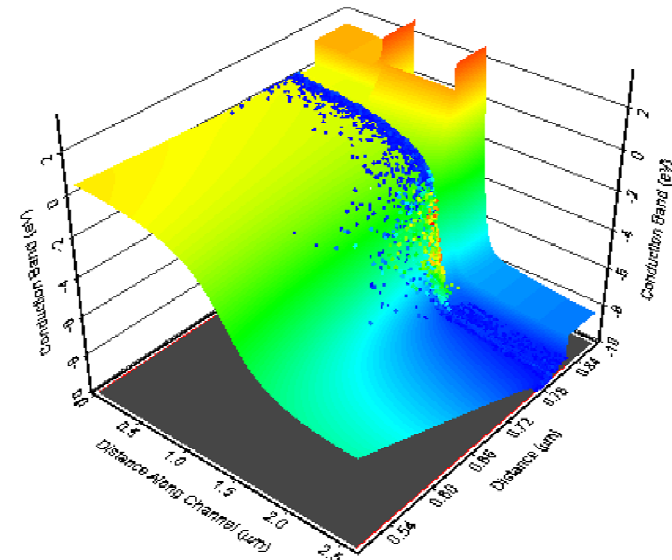
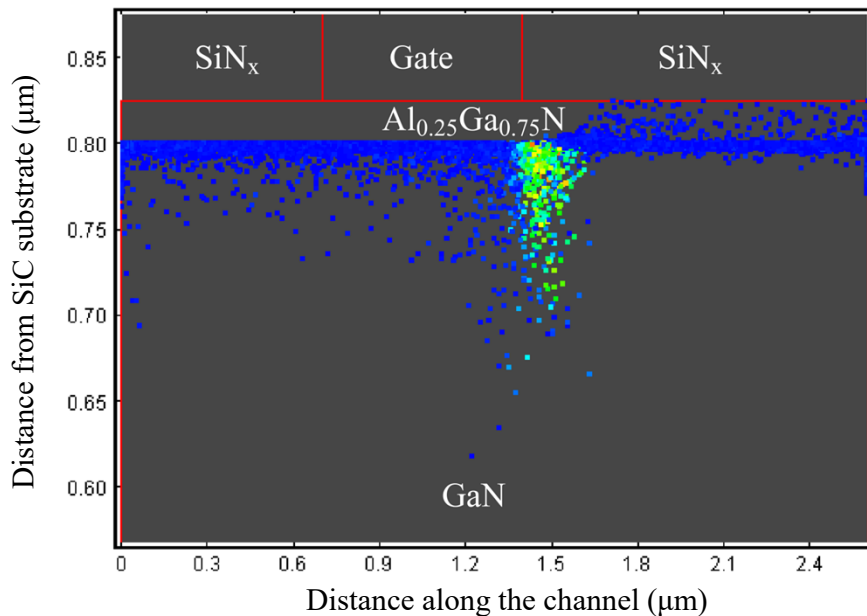
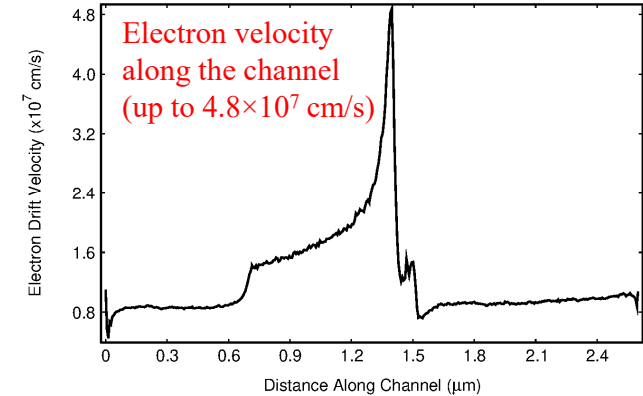
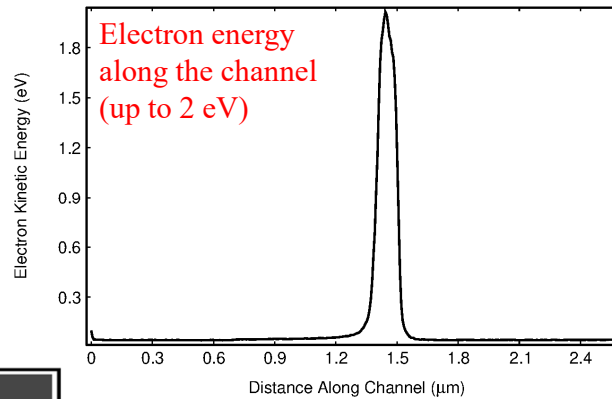
No information about SiC body contact. Crucial to assess role of holes (floating body?, parasitic bipolar?)





## 2. Damage generated at the GaN-AlGaN interface of a HEMT

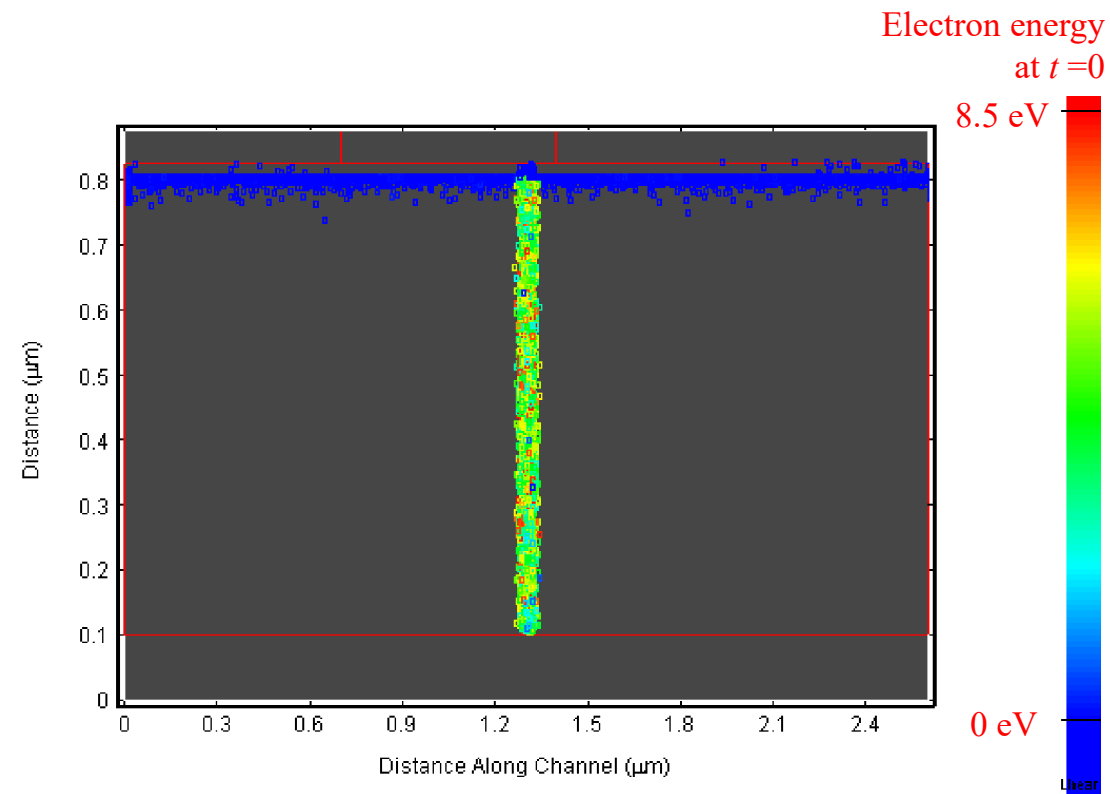
- GaN-AlGaN n-channel HEMT;  $V_{DS} = 10$  V,  $V_{GS} = 0$  V
- Hot electrons near drain-edge of gate → Large defect generation rate near drain
- Mostly thermal electrons near source → Small defect generation rates



### 3. Damage induced by 100 eV electrons in a GaN/AlGaN HEMT

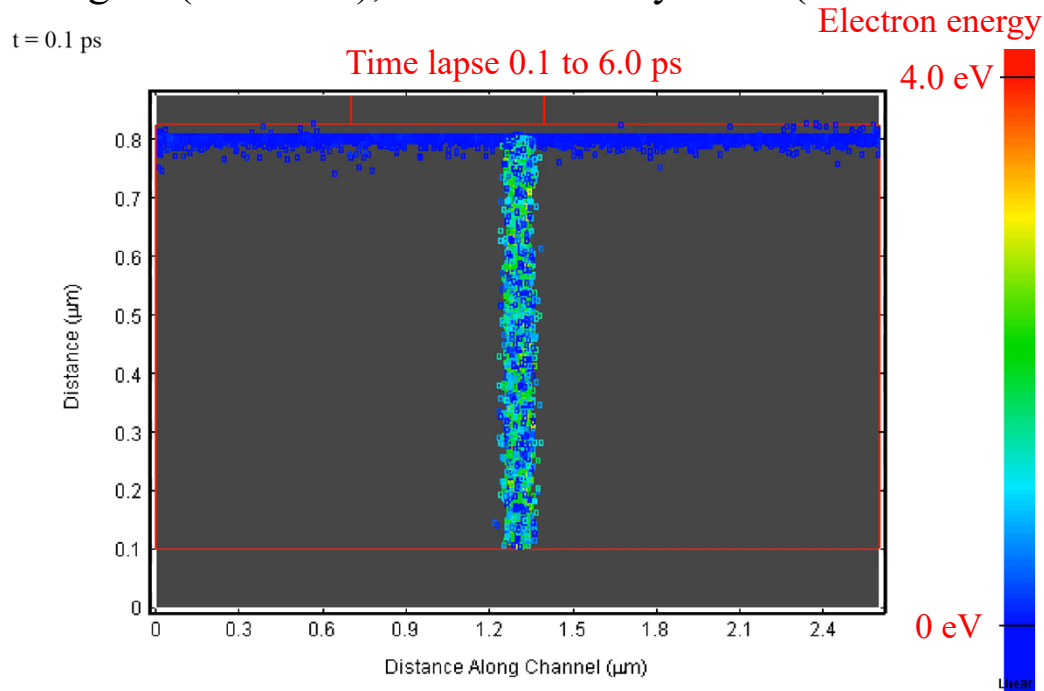


- Mimic with optically-generated pairs ( $h\nu = 12.1$  eV) the spatial and energetic pair distribution from 100 eV relaxation after 120 fs:
  - Average electron energy  $\sim 8.7$  eV
  - Gaussian width  $\sim 20$  nm
  - Initial peak density  $\sim 10^{18} \text{ cm}^{-3}$ ,  
(expect  $10^{21}$ - $10^{22} \text{ cm}^{-3}$ , for a 10 MeV proton [1],  
LET  $\sim 10$ - $15 \text{ MeV cm}^2/\text{mg} \sim 6 \times 10^{10} \text{ eV/cm}$ )
- Calculate generation rate of defects at the GaN-AlGaN interface of an n-channel HEMT at  $V_{\text{DS}} = 1 \text{ V}$

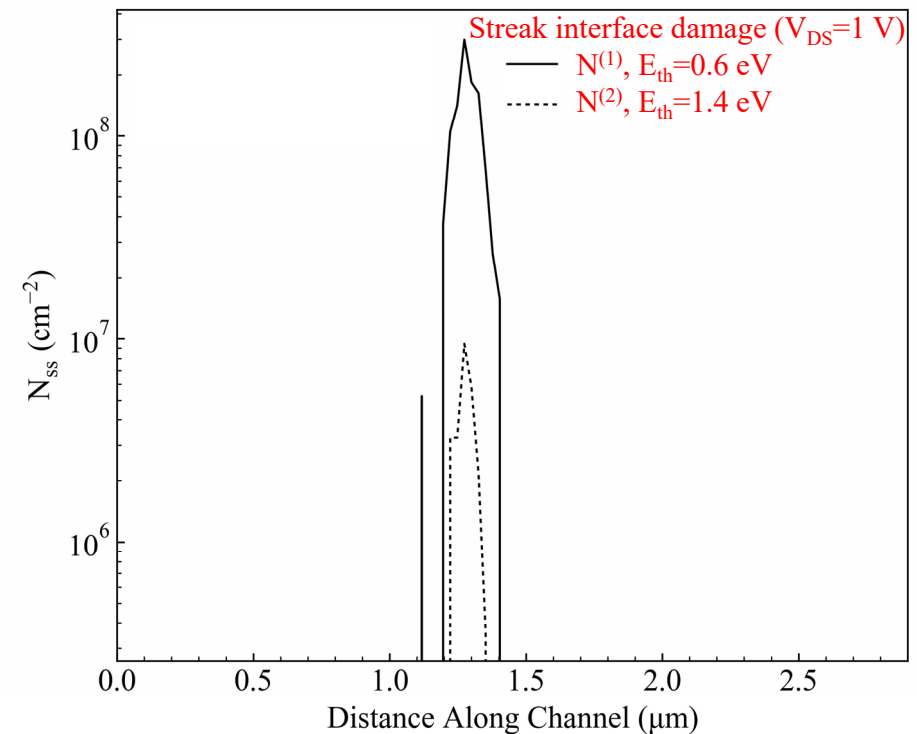


### 3. Damage induced by 100 eV electrons in a GaN/AlGaN HEMT

- Low  $V_{DS}$  (1 V) prevents damage from energetic electrons in channel
- Damage from streak confined to relatively narrow region ( $\sim 300$  nm); induced at early times ( $< 1$  ns)



Peak 'streak' density  $10^{18} \text{ cm}^{-2}$ , Gaussian width  $\sim 20$  nm



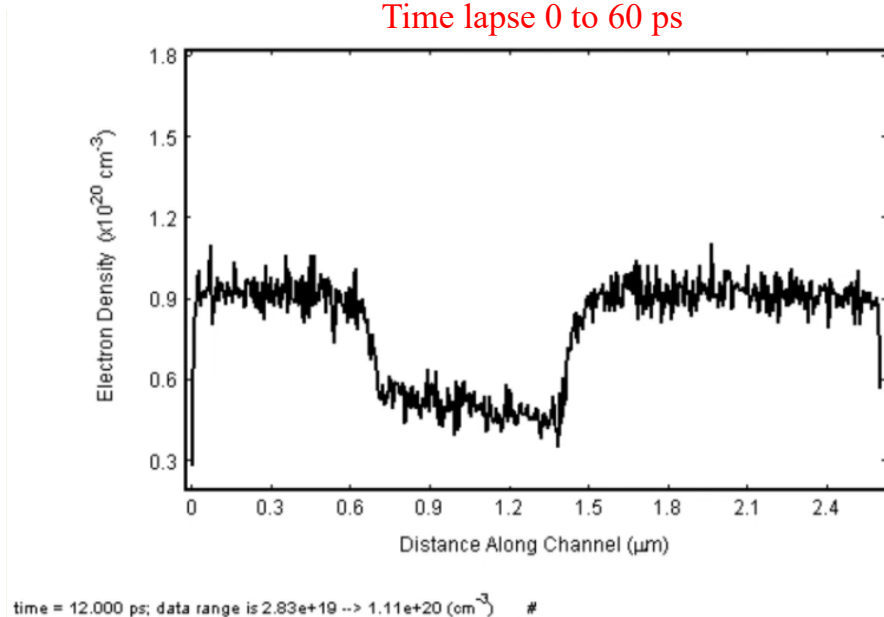
Issues: 1. Frozen field (no holes)  
2. Zero total momentum (need info from Geant4)  
... but likely already isotropic in  $\sim 100$  fs

### 3. Damage induced by 100 eV electrons in a GaN/AlGaN HEMT

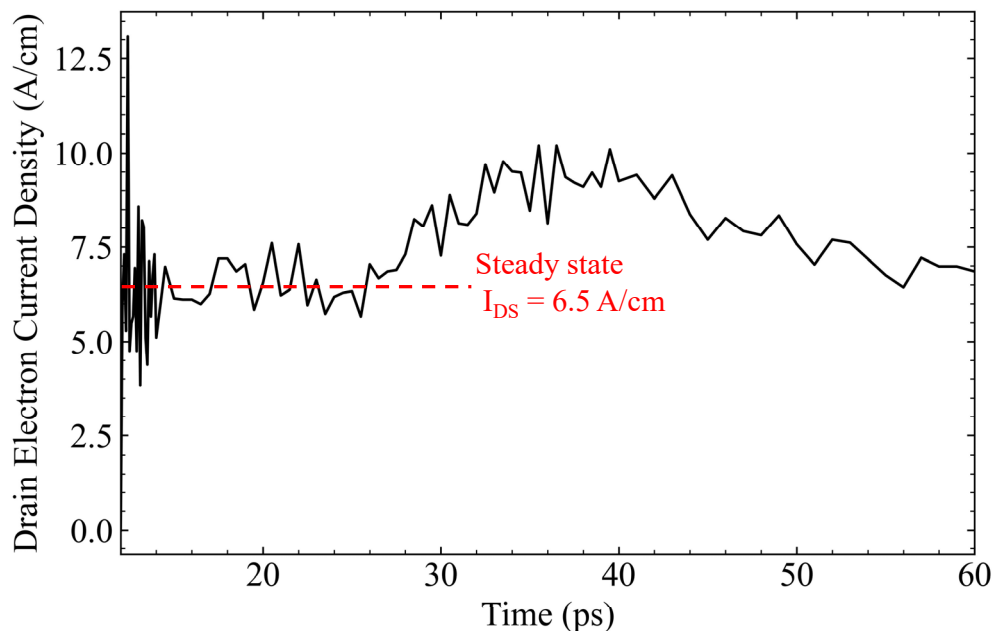


- Track radiation-induced excess charge and current at the contacts
- Video: Carrier density in channel as function of time
  - “Wave” of electrons builds near streak location
  - Moves toward drain contact and disappears after  $\sim 40$  ps
- This behavior is reflected in plot of current vs. time
  - Initially, current density remains near pre-streak levels
  - Apparent increase in current at about 25 ps signifies arrival of “wave”

Electron density along channel  
Time lapse 0 to 60 ps

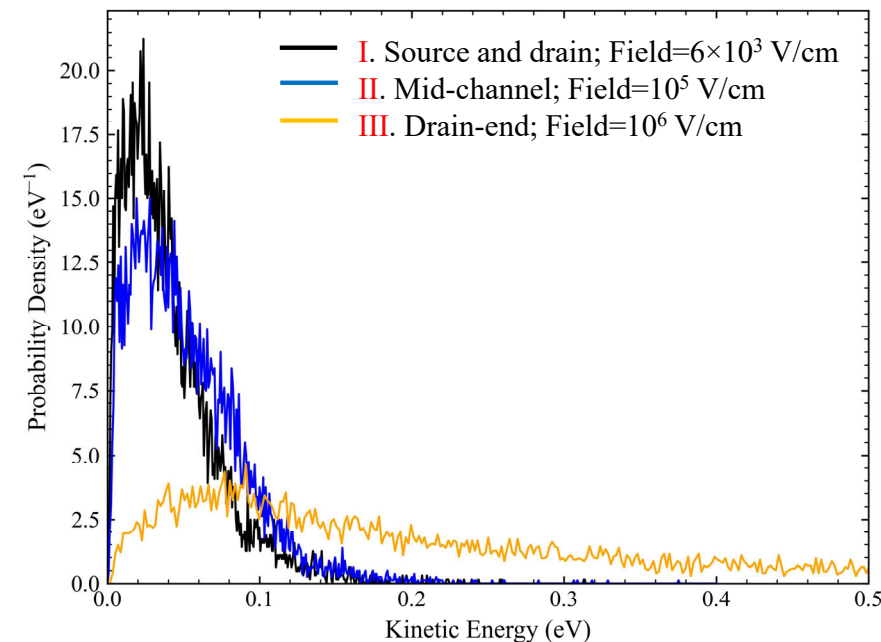
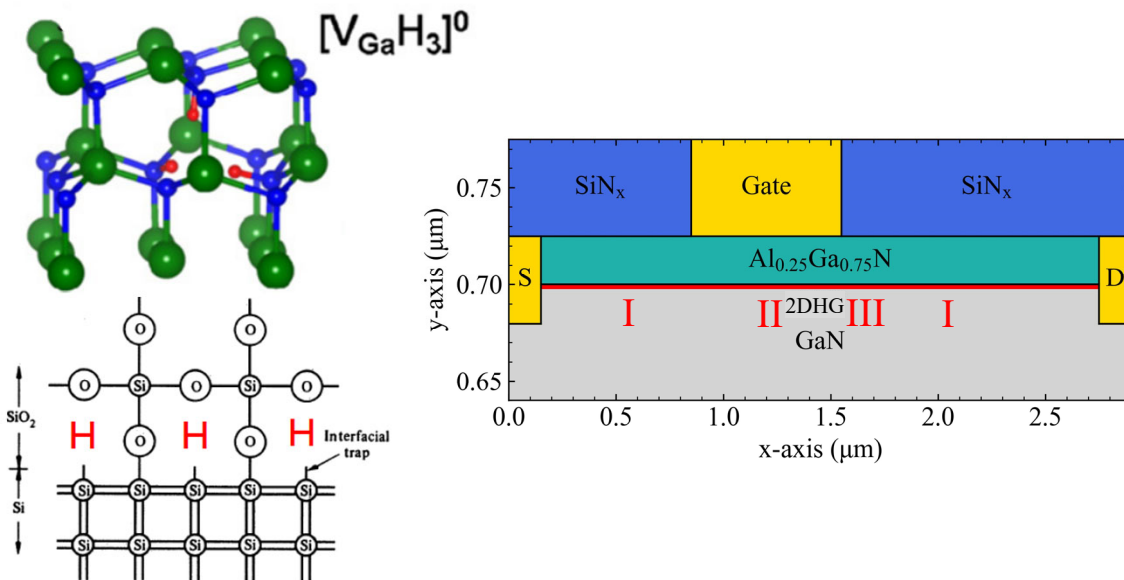


Displacement current not available (frozen field).  
Need holes and self-consistency with Poisson (Ramos theorem).



# 4. Dehydrogenation of $V_{\text{Ga}}\text{-H}_3$ complexes at the GaN-AlGaN interface

- Study of the dehydrogenation of  $V_{\text{Ga}}\text{-H}_3$  complexes at the GaN-AlGaN interface [1,2] of a p-channel HEMT. Holes are easier: higher DoS – Nichols, Pantelides (VU), and Zhang (UF)
- Monte Carlo hole energy distributions in GaN serve as input to calculate dehydrogenation rate
- ...in progress



[1] S. T. Pantelides *et al.*, Microelectron. Eng. **90**, 3 (2012)

[2] L. Nichols, unpublished and PhD Thesis (2024)

- First principles study of electron transport in the “10-100 eV gap” (wurtzite GaN, C-diamond,  $\beta$ -Ga<sub>2</sub>O<sub>3</sub>, Si, SiC)
  - Included band structure effects up to 100 eV
  - Studied plasmon and impact ionization in this regime
    - Very short plasmon lifetime
    - Fast ( $10^{-17} - 10^{-16}$  s) plasmon-induced energy losses; phonon-assisted thermalization in 1 ps or so
    - Spatial ‘spread’ of  $\sim 100$ -200 nm in the first ps (bad for devices?), highly anisotropic in  $\beta$ -Ga<sub>2</sub>O<sub>3</sub>
    - Average energy/pair ( $\lesssim 2 - 3E_{\text{gap}}$ ) in agreement with experimental values
- Ready (almost) to study SEEs/SEUs and radiation-induced device degradation
  - Studied relaxation of the ‘streak’ in GaN/AlGaIn HEMTs, but must implement MC holes
  - Calculation of transport characteristics of C,  $\beta$ -Ga<sub>2</sub>O<sub>3</sub>, ... (low-field mobility, velocity-field)
  - Collaborate with Vanderbilt Univ. (Fleetwood, Pantelides, Schrimpf) to account for hot-electron defect generation (e.g., H release) and for scattering with radiation-induced defects in FETs
- Study carrier relaxation in other materials (SiO<sub>2</sub>, AlN, AlGaIn, etc.)
- Investigate temperature effects (?)

More stuff



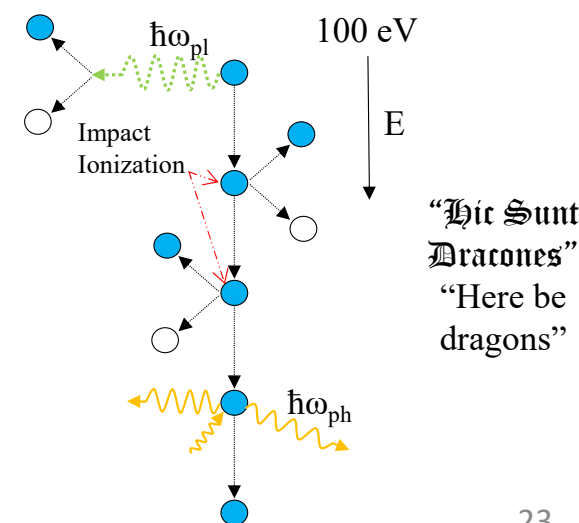
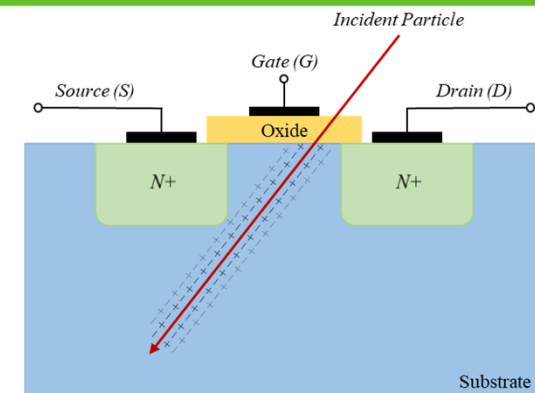
# Overview of the problem



**AFOSR**  
AIR FORCE OFFICE OF SCIENTIFIC RESEARCH



- Ionizing radiation in devices → high-energy charge carriers
  - Nuclear/particle physics community: Binary-collision codes [1,2]
    - Accurate down to  $\sim 100$  eV [3]
    - Band structure effects?
  - Electronic device community: Full-band Monte Carlo [4]
    - Region below  $\sim 5$ -10 eV: well studied
  - Intermediate energy range:  $\sim 10$ -100 eV
    - Energy-loss processes: free-electrons, plasmon emission -- mostly MFP in metals [5], simplified models in semiconductors [6,7] and  $\text{SiO}_2$  [8], no phonon losses, no time scale of the dynamics
- Close the ‘gap’ using *ab initio* methods
- Increasing interest in wide-bandgap materials
  - Focus on GaN (wurtzite), C (diamond), and  $\beta\text{-Ga}_2\text{O}_3$



- [1] R. A. Reed, *et al.*, IEEE Trans. Nucl. Sci. **62**, 1441 (2015).
- [2] S. Agostinelli, *et al.*, Nucl. Instrum. Methods Phys. Res. A: Accel. Spectrom. Detect. Assoc. Equip. **506**, 250 (2003).
- [3] C. M. Dozier and D. B. Brown, IEEE Trans. Nucl. Sci. **28**, 4137 (1981).
- [4] H. Shichijo and K. Hess, Phys. Rev. B **23**, 4197 (1981).
- [5] D. Pines, Rev. Mod. Phys. **28**, 184 (1956). J. J. Quin and R. A. Ferrel, Phys. Rev. **112**, 812 (1958); J. J. Quinn, Phys. Rev. **126**, 1453 (1962); D. R. Penn, Phys. Rev. B. **35**, 482 (1982).
- [6] R. C. Alig, S. Bloom, and C. W. Struck, Phys. Rev. B **22**, 5565 (1980).
- [7] A. Akkerman, J. Barak, and D. Emfietzoglou, Nucl. Inst. Meth. Phys. Res. B **227**, 319 (2005).
- [8] G. A. Ausman Jr. and F. B. McLean, Appl. Phys. Lett. **26**, 173 (1975).

# Scattering rates from the energy-loss function: Numerical details



- Discretize the Brillouin zone (as usual) into volume elements centered at  $\mathbf{k}_j$  with density of states  $\mathcal{D}_{jn}(E)$  for band  $n$  at energy  $E$  (from the Gilat-Raubenheimer [1] or tetrahedron [2] methods); tabulate  $E_n(\mathbf{k}_i)$
- Discretize the  $\omega$ -axis into elements of width  $\Delta\omega$  centered at  $\omega_r$  and tabulate  $\text{Im} \left[ \frac{-1}{\varepsilon(\mathbf{q}_i, \omega_r)} \right]$
- Evaluate energy-loss rate:

Replaced by  $|\mathbf{q}_i|$   
(isotropic)

as:

$$\frac{1}{\tau_n(\mathbf{k})} = \frac{2\pi}{\hbar} e^2 \sum_{n'} \int \frac{d\mathbf{q}}{(2\pi)^3} \frac{1}{q^2} \int d\omega \text{Im} \left[ \frac{-1}{\varepsilon(\mathbf{q}, \omega)} \right] \delta[E_n(\mathbf{k}) - E_{n'}(\mathbf{k} + \mathbf{q}) + \hbar\omega],$$

$$\frac{1}{\tau_n(\mathbf{k}_i)} = 2\pi e^2 \sum_{n', \omega_r} \Delta\omega \sum_{\mathbf{k}_j} \frac{1}{|\mathbf{k}_i - \mathbf{k}_j|^2} \text{Im} \left[ \frac{-1}{\varepsilon(|\mathbf{k}_i - \mathbf{k}_j|, \omega_r)} \right] \mathcal{D}_{jn'}[E_n(\mathbf{k}_i) - \hbar\omega_r].$$

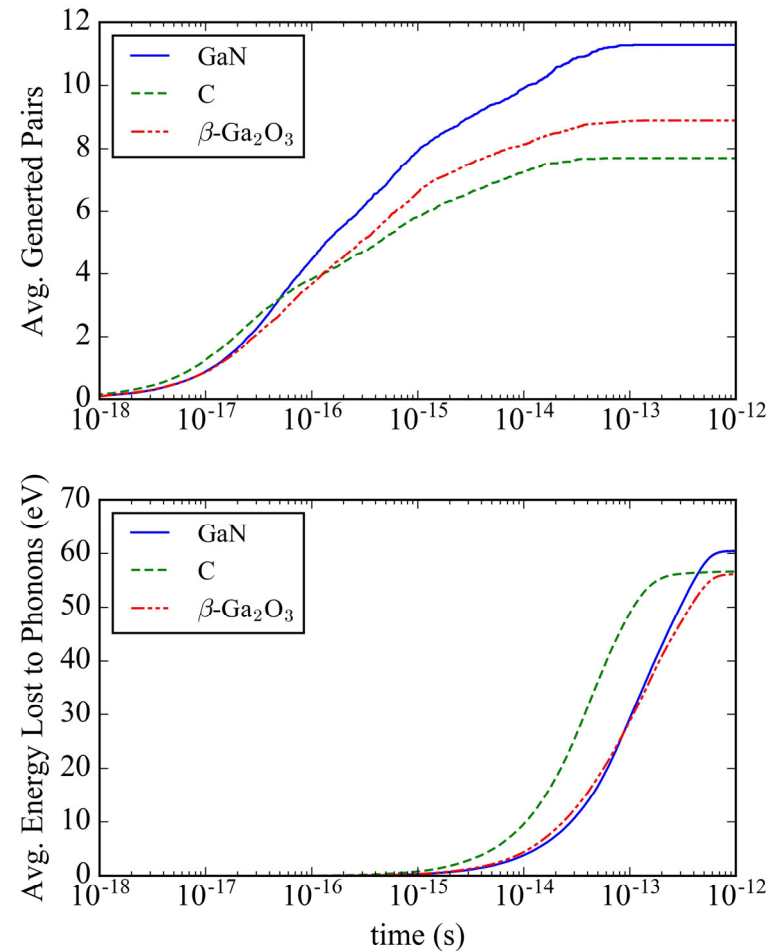
The sum over  $\mathbf{k}_j$  spans only ‘energy-conserving’ elements selected using the Gilat-Raubenheimer [1] or Blöchl’s tetrahedron method [2]

- In the Monte Carlo simulations, after a collision, select a random band  $n'$ , wavevector  $\mathbf{k}_j$ , and energy-loss  $\hbar\omega_r$  with probability distribution given by the ‘integrand’ of the equation above (using the ‘rejection technique’[3])
- Account for Landau damping by generating an electron-hole pair with total kinetic energy  $\hbar\omega_r - E_{\text{gap}}$

- [1] G. Gilat and L. J. Raubenheimer. Phys. Rev. **144**, 390 (1966).  
 [2] P. E. Blöchl, O. Jepsen, and O. K. Andersen, Phys. Rev. B **49**, 16223 (1994).  
 [3] Jacoboni and L. Reggiani, Rev. Mod. Phys. **55**, 645 (1983).

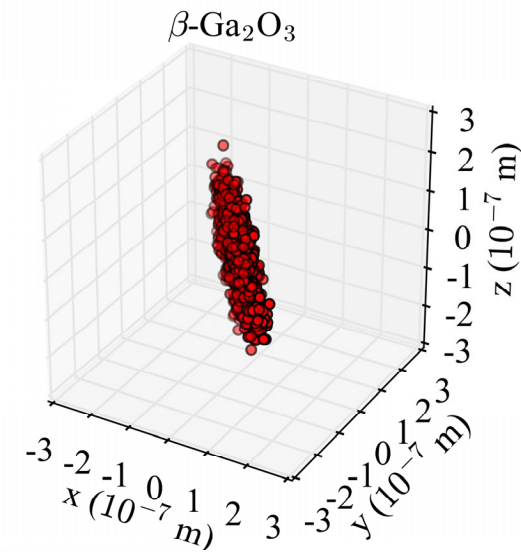
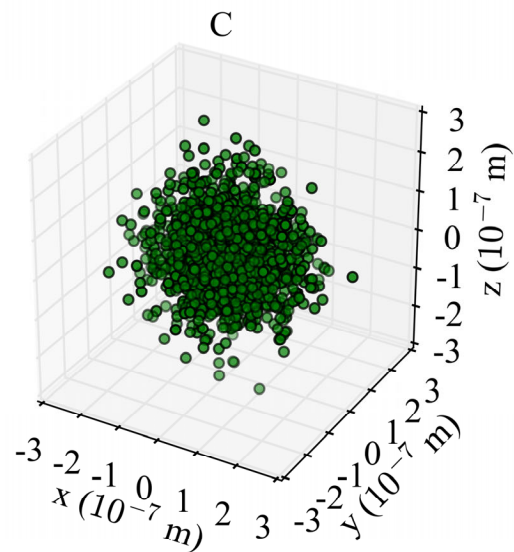
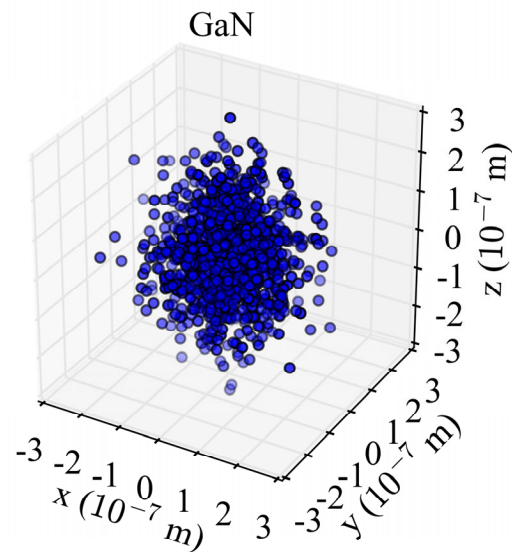
# Where does the energy go?

- At short times, for  $t < 10^{-14}$  s, energy distributed among e-h pairs
- At  $\sim 10^{-14}$  s, phonon emission increases significantly, pair generation flattens
- Somewhat more energy deposited into lattice for GaN than C and  $\text{Ga}_2\text{O}_3$  (60% vs. 56-57%) .
- Fewer pairs generated in C and  $\text{Ga}_2\text{O}_3$   
→ larger creation energy
- Left-over energy: Recombination [1], ignored here (small density for Auger, much longer times for SRH)
- Temperature effects
  - Ignore, for now: Assume low carrier/phonon density
  - No problems early on, may be a factor for  $t > 10^{-13}$  s

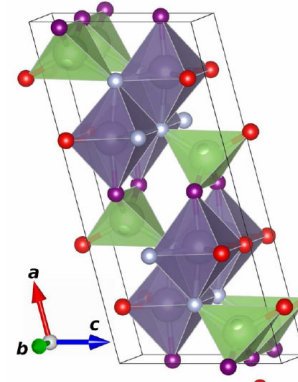


[1] S. Jursenas, *et al.*, J. Cryst. Growth **281**, 161 (2005).

# Diffusion in real space

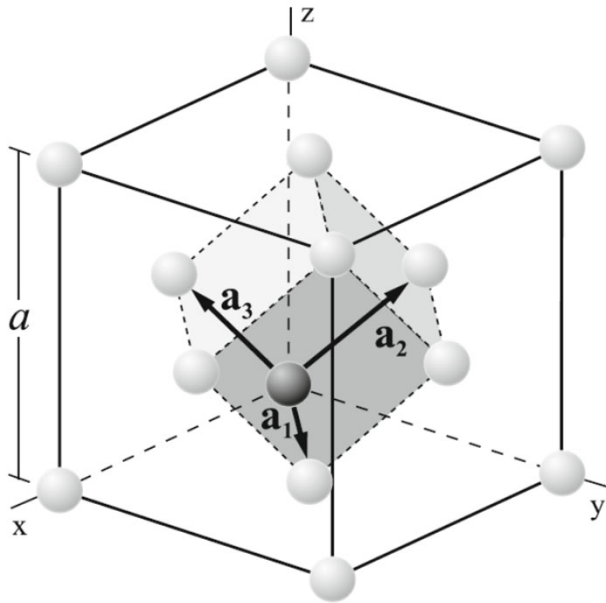


- After full thermalization,  $\sim 1$  ps, electrons diffuse over  $\sim 100$  nm
  - Approximately isotropic in C and GaN, strongly anisotropic in  $\beta$ -Ga<sub>2</sub>O<sub>3</sub> (following long edge of unit cell)

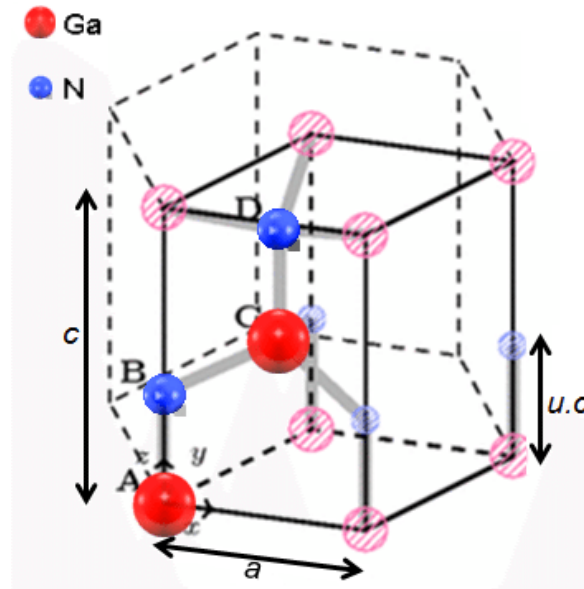


Unit cell of  $\beta$ -Ga<sub>2</sub>O<sub>3</sub>  
H. Peelaers and C. Van de Walle, Phys. Stat. Sol. B **252**, 828 (2015).

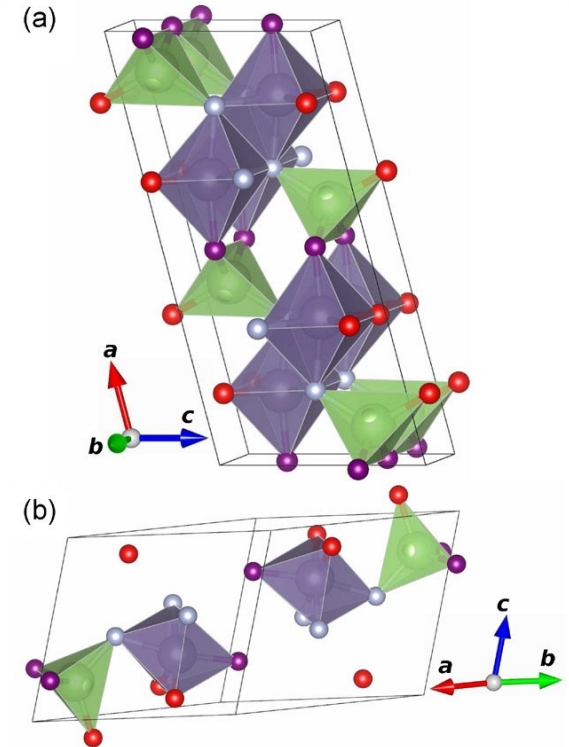
# Wigner-Seitz cells: C (diamond), GaN (wurtzite), $\beta$ -Ga<sub>2</sub>O<sub>3</sub>



C (diamond, Fd3m)  
fcc, 2 atoms/WS cell [1]



GaN (P6<sub>3</sub>mc)  
wurtzite, 4 atoms/WS cell [2]

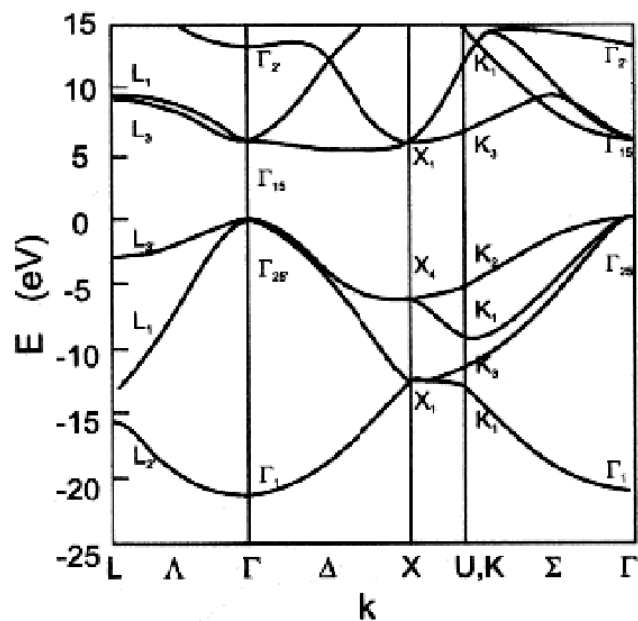


$\beta$ -Ga<sub>2</sub>O<sub>3</sub> (C2/m)  
monoclinic, 10 atoms/WS cell [3]

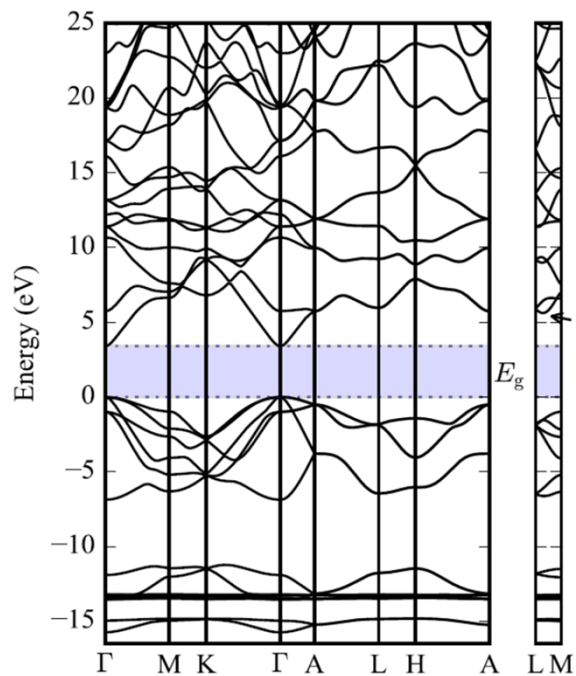
- [1] M. V. Fischetti and W. V. Vandenberghe, *Advanced Physics of Electron Transport in Semiconductors and Nanostructures* (Springer, 2016)  
[2] D. Andiwijayakusuma *et al.*, J. Phys.: Conf. Ser. **739**, 012027 (2016).  
[3] H. Peelaers and C. Van de Walle, Phys. Stat. Sol. B **252**, 828 (2015).



# Band structure: C (diamond), GaN (wurtzite), $\beta$ -Ga<sub>2</sub>O<sub>3</sub>

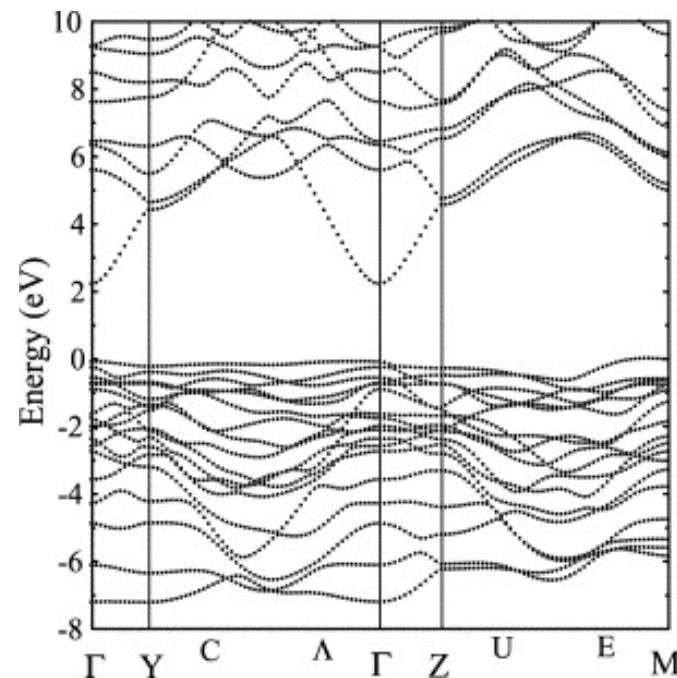


C (diamond, HSE-ONCV) [1]  
PBESOL-USSP for 'el-ph &  $\varepsilon(q, \omega)$   
 $E_{\text{gap}} = 5.45 \text{ eV}$



GaN (PBE-ONCV) [2]

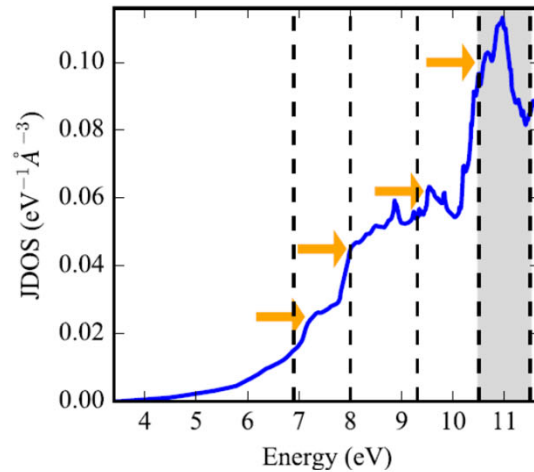
$E_{\text{gap}} = 1.78 \text{ eV} \rightarrow 3.4 \text{ eV}$



$\beta$ -Ga<sub>2</sub>O<sub>3</sub> (PBE-ONCV) [3]  
PBESOL-USSP for 'el-ph &  $\varepsilon(q, \omega)$   
 $E_{\text{gap}} = 2.63 \text{ eV} \rightarrow 4.8 \text{ eV}$

- [1] M. Dipalo, *Nanocrystalline diamond growth and device applications*, (Doctoral Thesis, Ulm University, 2008)  
[2] D. O. Nielsen *et al.*, Phys. Rev. B **108**, 155203 (2023).  
[3] K. Yamaguchi, Solid State Commun. **131**, 739 (2004) .

# Can we trust DFT up to 100 eV? The case of GaN



DFT-calculated JDOS vs.  $E$  of GaN [1]  
Dashed lines: Vertical transition energies [2]  
Orange arrows: JDOS peaks and

So: OK up to  $\sim 10$  eV  
More and more free-electron-like at higher energies?

	$U_{v_1-c_1}$	$M_{v_1-c_2}$	$L_{v_1-c_1}$	$K_{v_1-c_1}$	$K_{v_2-c_1}$	$A_{v_1-c_3}$	$L_{v_3-c_3}$
Our work (GGA-PBE) [1]	7.18	8.00	7.80	9.42	9.68	10.49	11.10
DFT-LDA [2]	6.87	7.65	7.64	9.57	9.68	10.53	11.05
Exp. [2]	6.9	8.0	8.0	9.3	9.3	10.5–11.5	10.5–11.5
Exp. [3]	7.1	8.1	8.1	9.2	9.2		
Exp [4]	7.0	7.9	7.9	9.0	9.0		

[1] ] D. O. Nielsen *et al.*, Phys. Rev. B **108**, 155203 (2023).

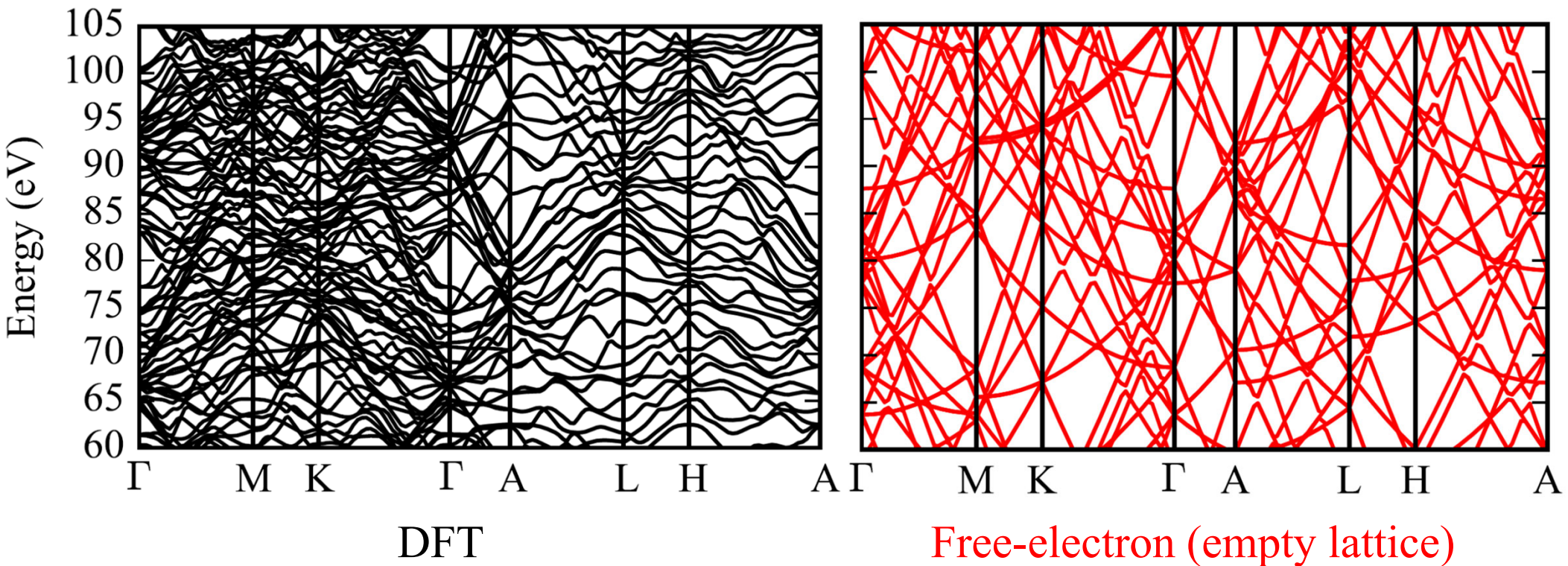
[2] W. R. L. Lambrecht *et al.*, Phys. Rev. B **51**, 13516 (1995).

[3] G. Brockett and H. Lakner, Micron **31**, 435 (2000).

[4] S. Logothetidis *et al.*, Phys. Rev. B **50**, 18017 (1994).



# Free electrons at 100 eV? Hm...



# Can we use 1<sup>st</sup>-order perturbation theory?

- Scattering rates as high as  $5 \times 10^{16} - 10^{17}/\text{s}$  for 100 eV electrons imply a broadening  $\Delta E \sim \hbar/\tau \sim 30\text{-}60$  eV
  - Problem discussed by Quinn [1]: “[...] *the quasiparticle concept is not too bad*”
  - **Concerns even worse at lower energy** ( $\sim 5$  eV):  
The Capasso-Hess controversy of the '80s [2,3]:  
For impact ionization in GaAs,  $\Delta E \gtrsim E$ !
  - The ‘solution’ (*i.e.*, collisional broadening) worse than the original problem: 2<sup>nd</sup>-order corrections ‘corrected’ by higher-order terms
  - Bethe-Salpeter equation (Coulomb e-h interaction, matters in low- $\epsilon$  materials) does not help ( $\rightarrow$ probably even higher rates). Need full quantum transport model (NEGF+DFT), unrealistic with up to 350 bands

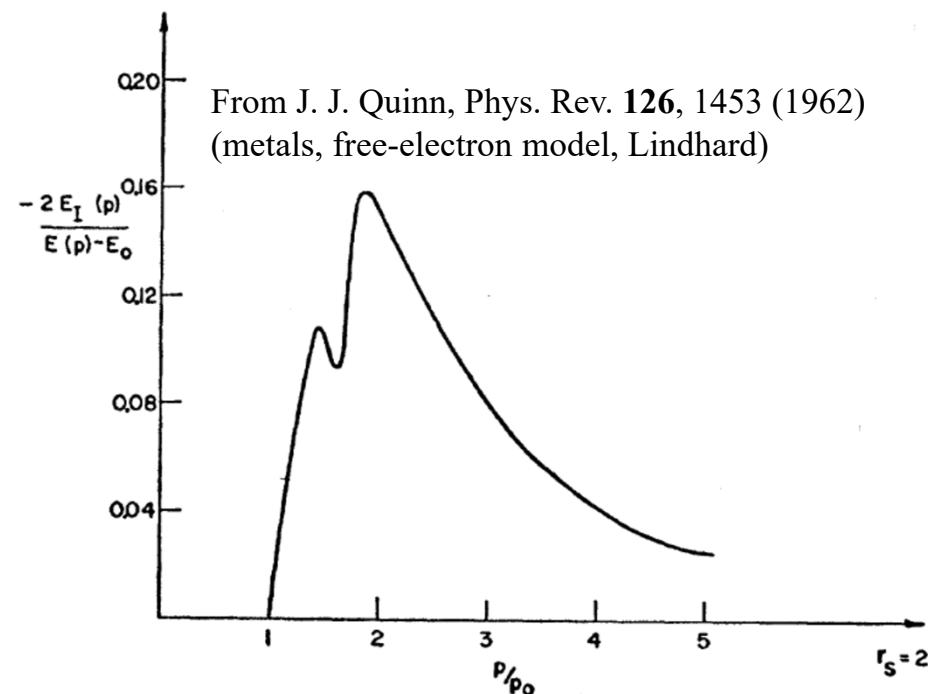
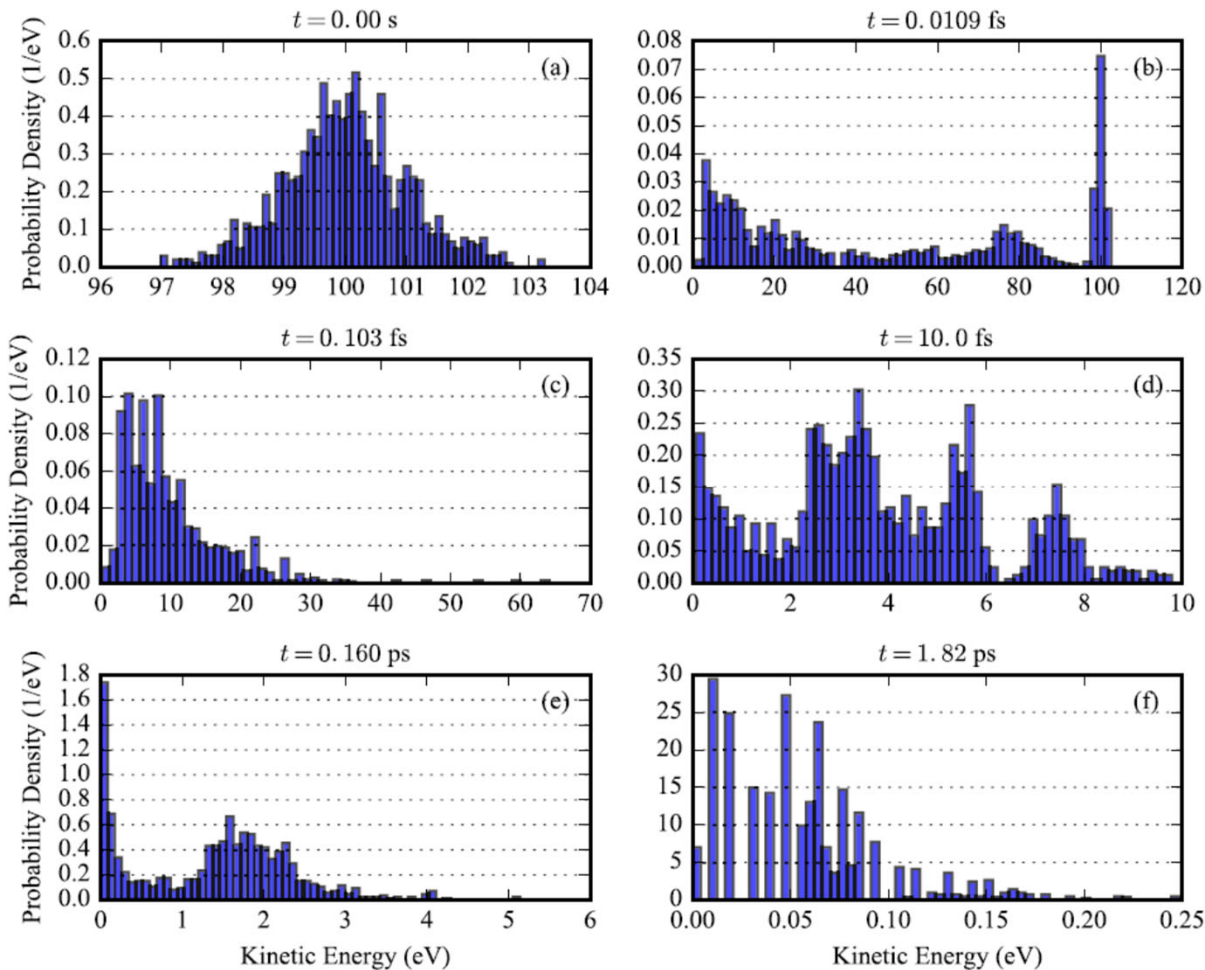


FIG. 3. The ratio of the uncertainty in the quasi-particle energy to its excitation energy as a function of  $p/p_0$ .

- [1] J. J. Quinn, Phys. Rev. **126**, 1453 (1962).  
[2] F. Capasso *et al.*, J. Appl. Phys. **53**, 3324–3326 (1982).  
[3] Y.-C. Chang *et al.*, Appl. Phys. Lett. **42**, 76–78 (1983).

# Time evolution of the electron-energy distribution function in GaN



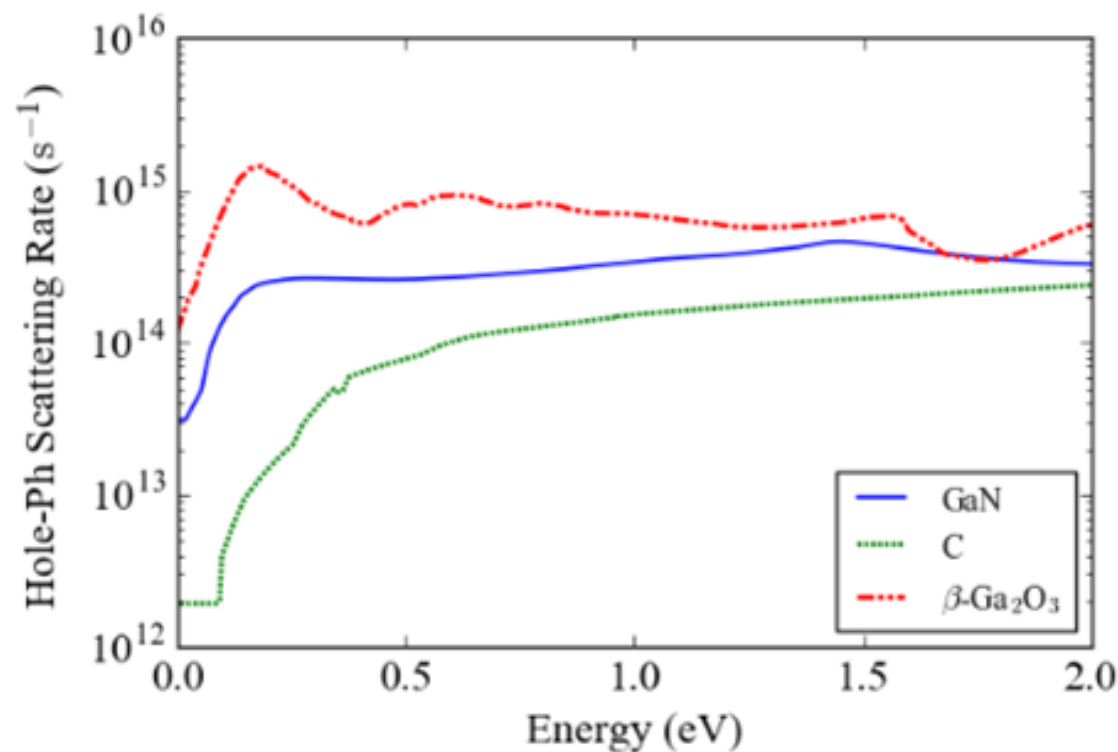
D. O. Nielsen *et al.*,  
Phys. Rev. B **108**, 155203 (2023).

# Hole-phonon scattering rates: Slow hole thermalization in C (diamond)



- Hole mobility (300K):
  - C (diamond, exp): 3,500 cm<sup>2</sup>/(Vs) [1]
  - GaN (wurtzite, theory): 52 cm<sup>2</sup>/(Vs) [2](?)  
(nearly intrinsic, exp): 170 cm<sup>2</sup>/(Vs) [3]
  - $\beta$ -Ga<sub>2</sub>O<sub>3</sub> (N doped, exp):  $\sim 23$  cm<sup>2</sup>/(Vs) [4]

Small hole mass in diamond, flat VBs in GaN and Ga<sub>2</sub>O<sub>3</sub>



- [1] J. Isberg *et al.*, Science **297**, 1670 (2002).  
[2] S. Poncé, D. Jena, and F. Giustino, Phys. Rev. B **100**, 085204 (2019).  
[3] M. Rubin *et al.*, Appl. Phys. Lett. **64**, 64 (1994).  
[4] C. Ma *et al.*, J. Mater. Chem. C **10**, 6673 (2022).