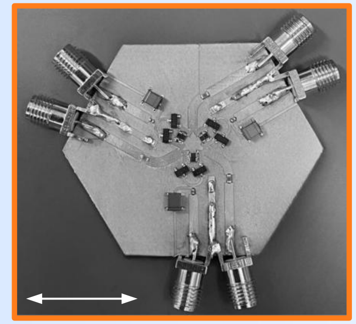

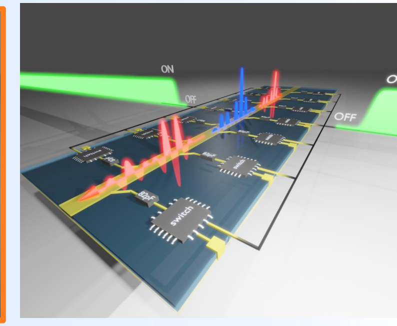

TIME METAMATERIALS

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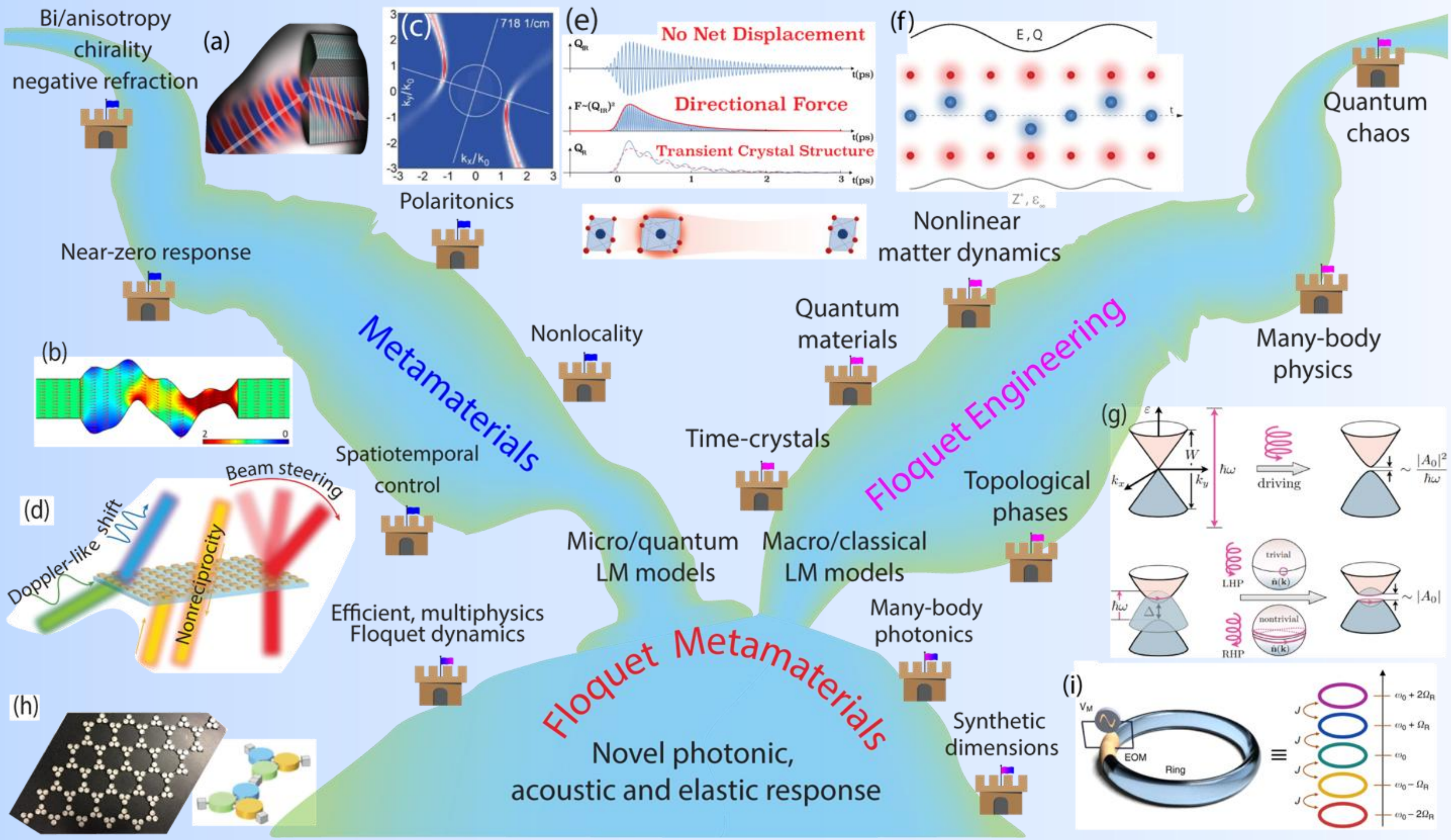
Supported by the ***Air Force Office of Scientific Research*** (***Dr. Arje Nachman***) through an ***SBIR program*** with ***Silicon Audio, Inc.***

CUNY The City University of New York ADVANCED SCIENCE RESEARCH CENTER

METAMATERIALS & PLASMONICS RESEARCH LABORATORY

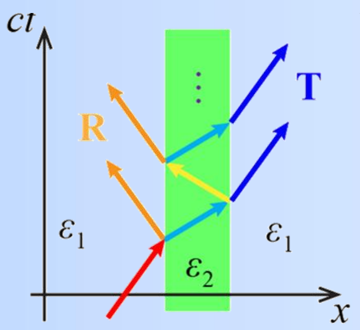
Floquet Metamaterials



S. Yin, E. Galiffi, A. Alù, *E-Light* **2**, 8 (2022)

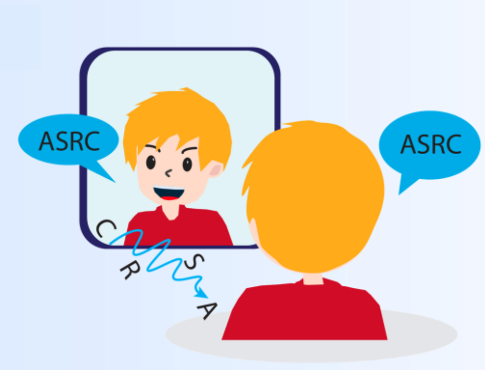
Time as a New Dimension for Wave Manipulation

Spatial reflections

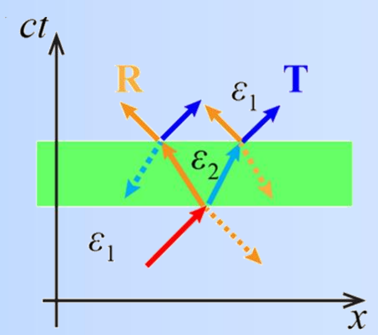


ω (frequency) is conserved

k (momentum) is reversed

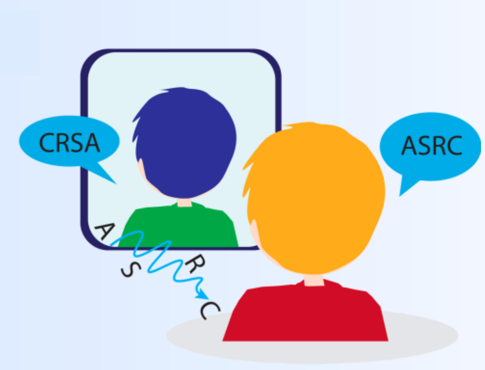


Temporal reflections



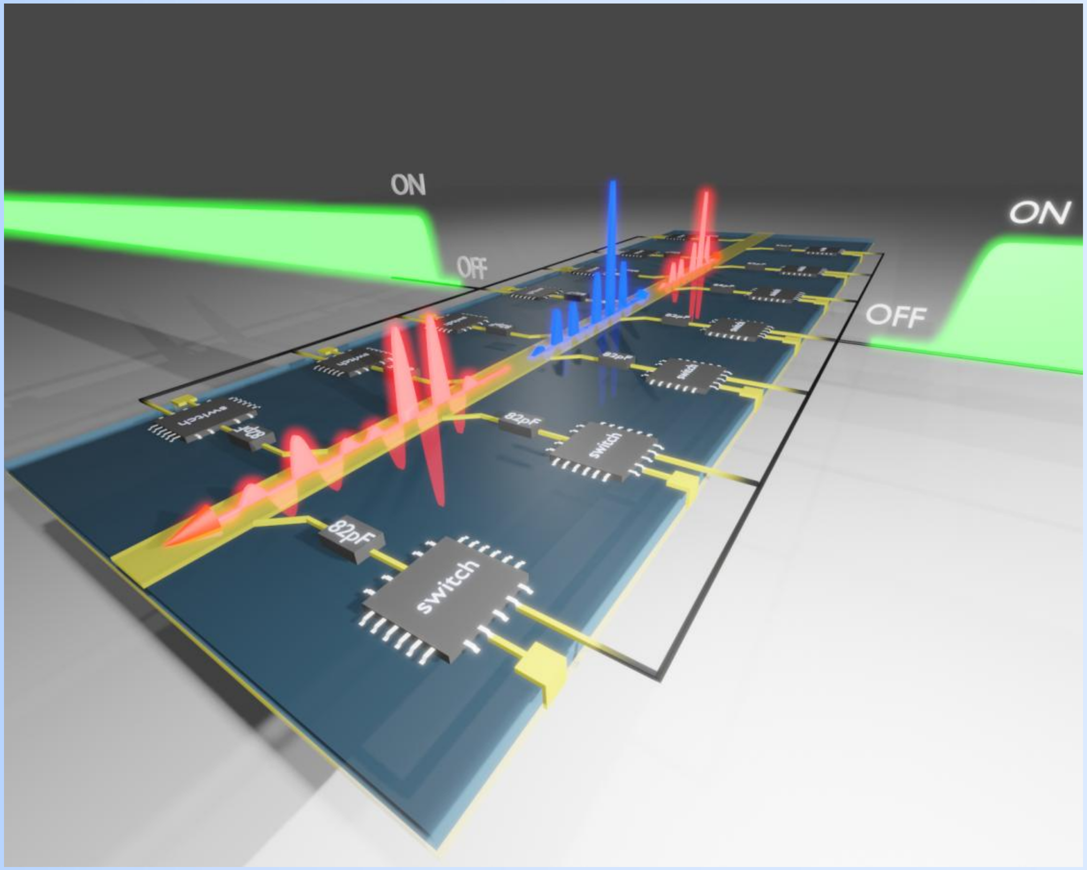
k (momentum) is conserved

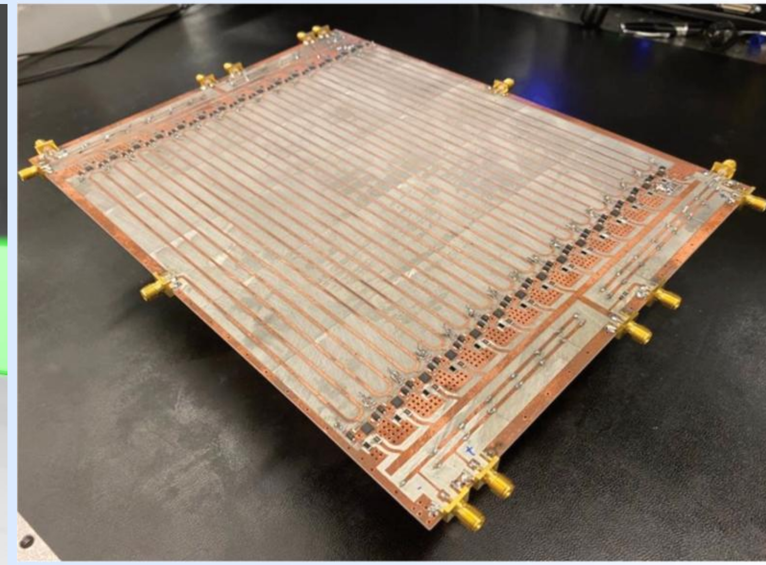
ω (frequency) changes

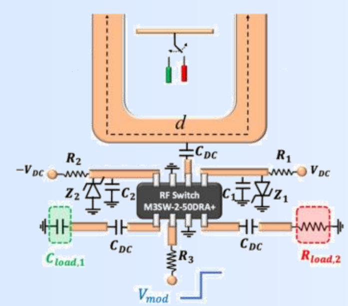


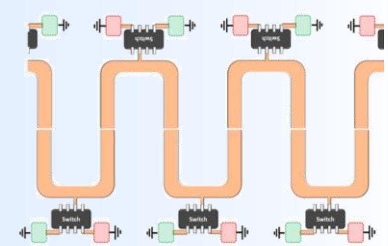
H. Moussa, et al., *Nature Physics* **19**, 863 (2023)

A Time Metamaterial



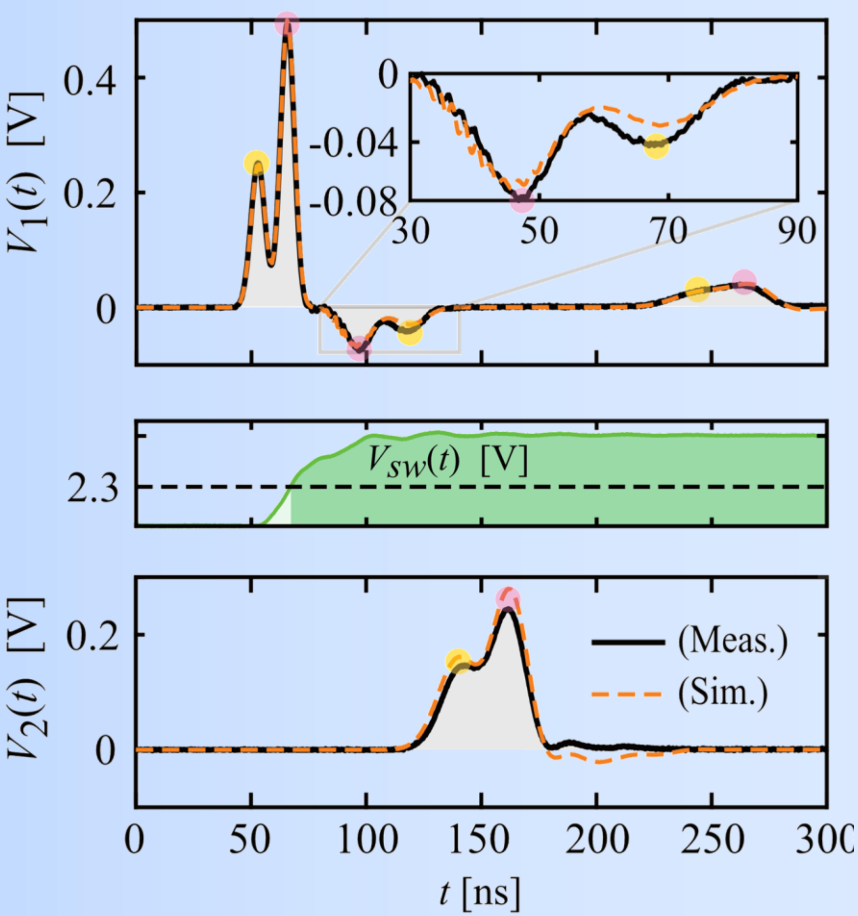


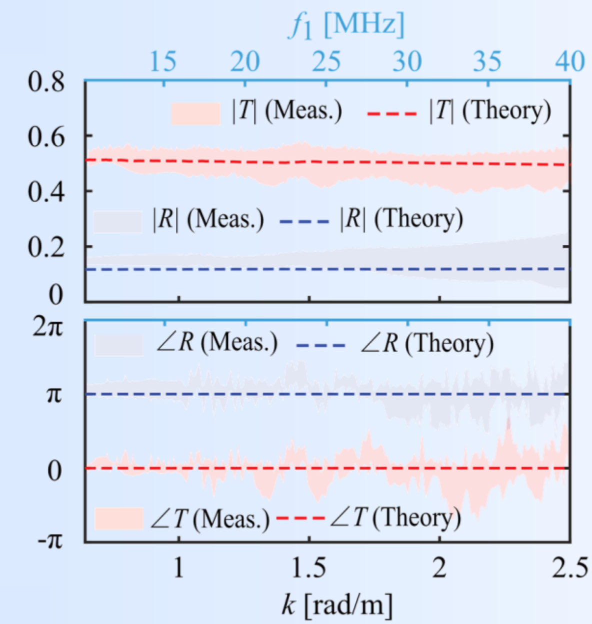


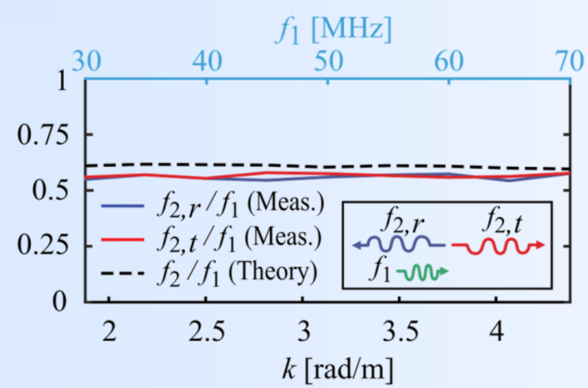


H. Moussa, G. Xu, S. Yin, et al., *Nature Physics* **19**, 863 (2023)

Observation of Broadband Time Reversal

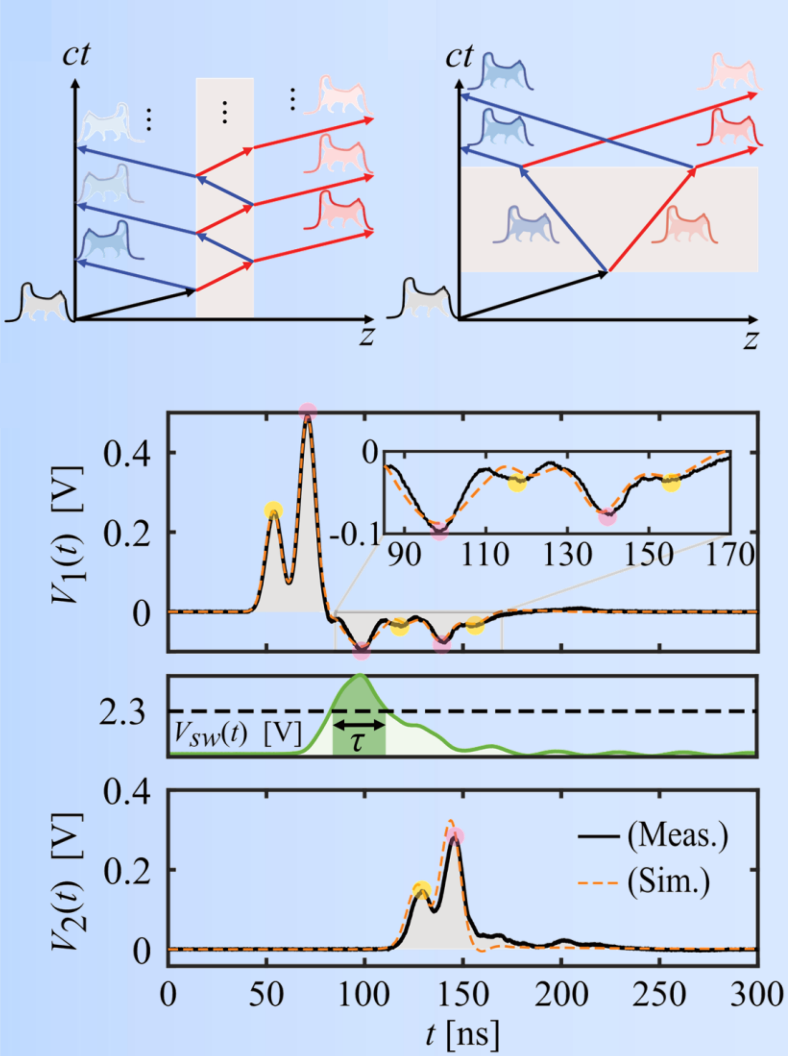


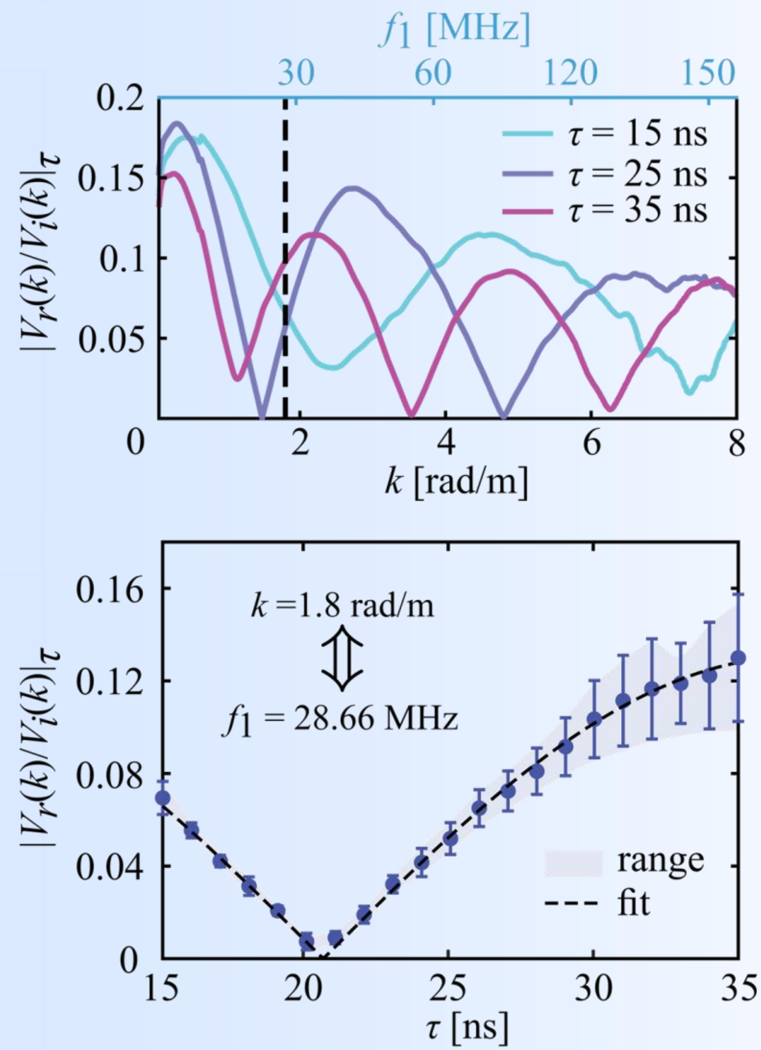




H. Moussa, G. Xu, S. Yin, et al., *Nature Physics* **19**, 863 (2023)

Scattering and Interference with a Temporal Slab

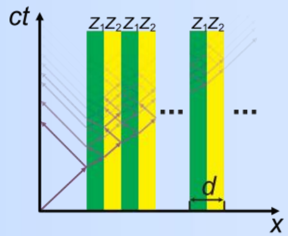




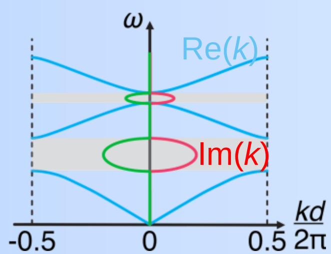
H. Moussa, et al., *Nature Physics* **19**, 863 (2023)

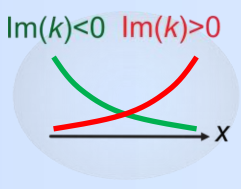
Time Crystals and K-Gaps

Photonic crystal

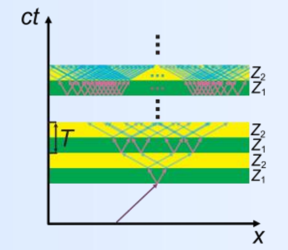


$\psi(x+d) = \psi(x)e^{-jkd}$ k: Bloch wavevector

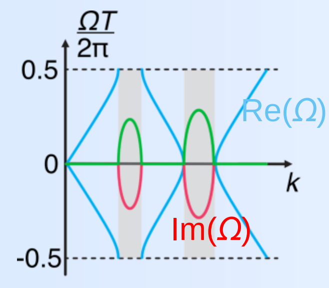


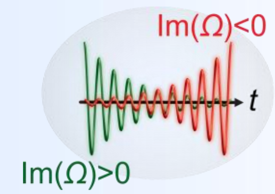


Photonic time crystal

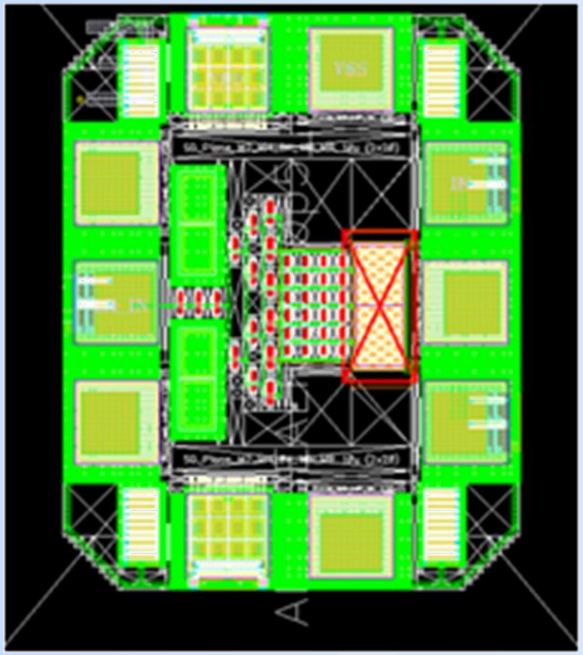


$\psi(t+T) = \psi(t)e^{j\Omega T}$ Ω: Floquet frequency

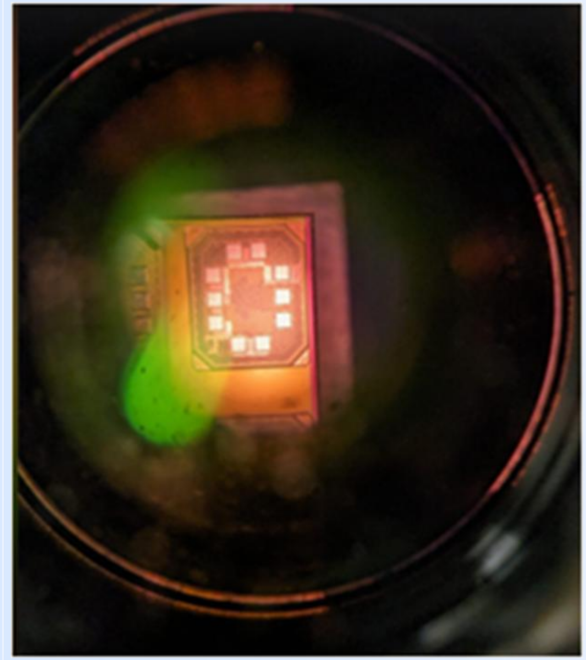




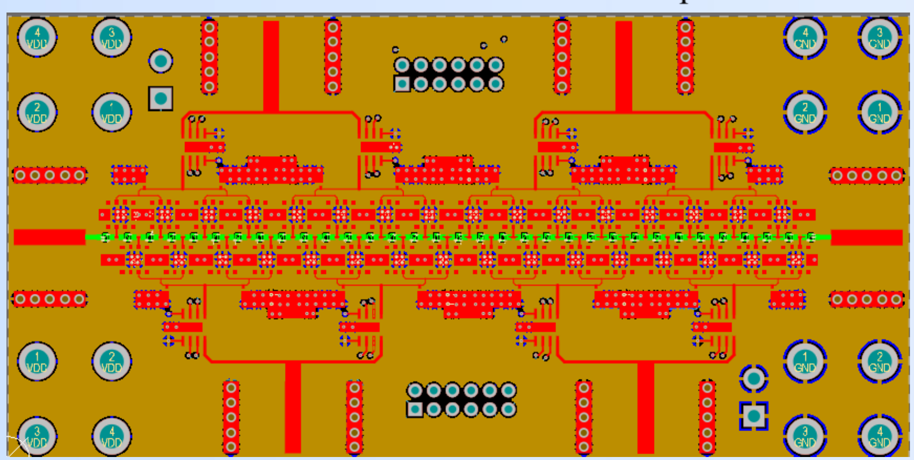
Extension to Higher Frequencies



Taped-out chip

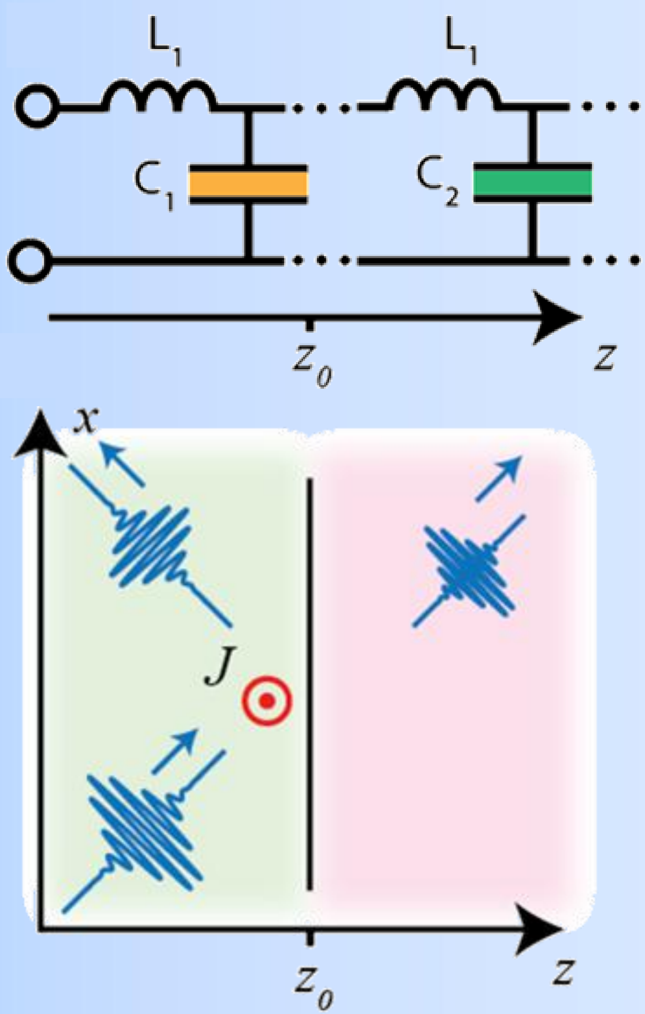


Microscopic view of the CMOS chip



with Silicon Audio

Boundary Conditions and Energy Requirements



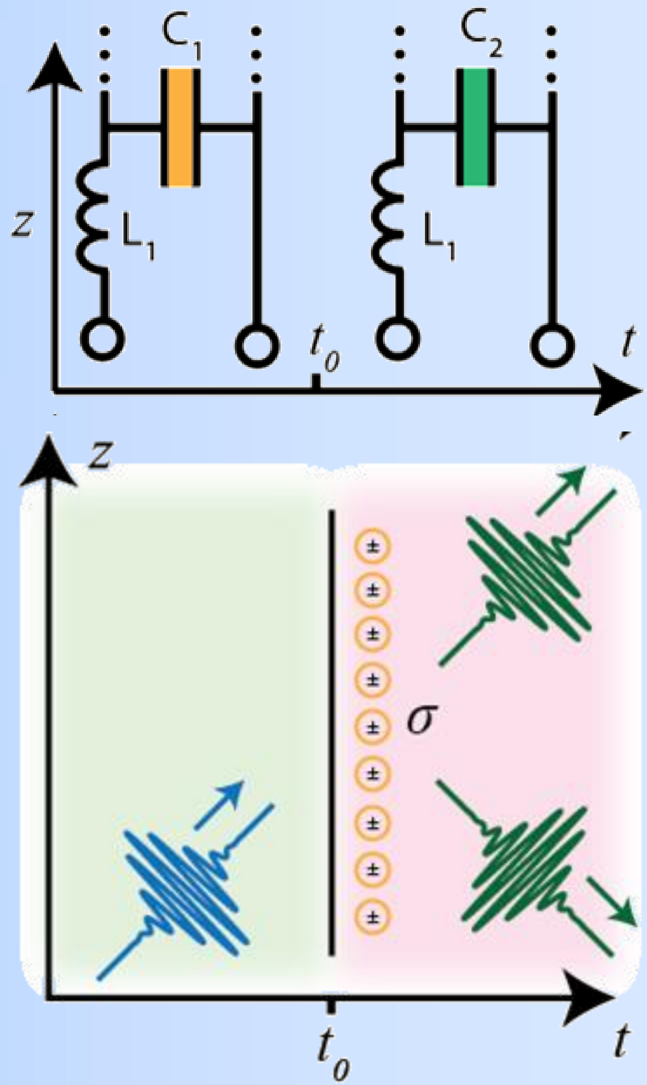
At a uniform spatial interface:

- ω is conserved
- energy is conserved
- spatial symmetry is broken (typically leading to a change in momentum)

$$E_{\tan}\left(z_o^-\right) - E_{\tan}\left(z_o^+\right) = -J_m$$

$$H_{\tan}\left(z_o^-\right) - H_{\tan}\left(z_o^+\right) = J_e$$

Boundary Conditions and Energy Requirements



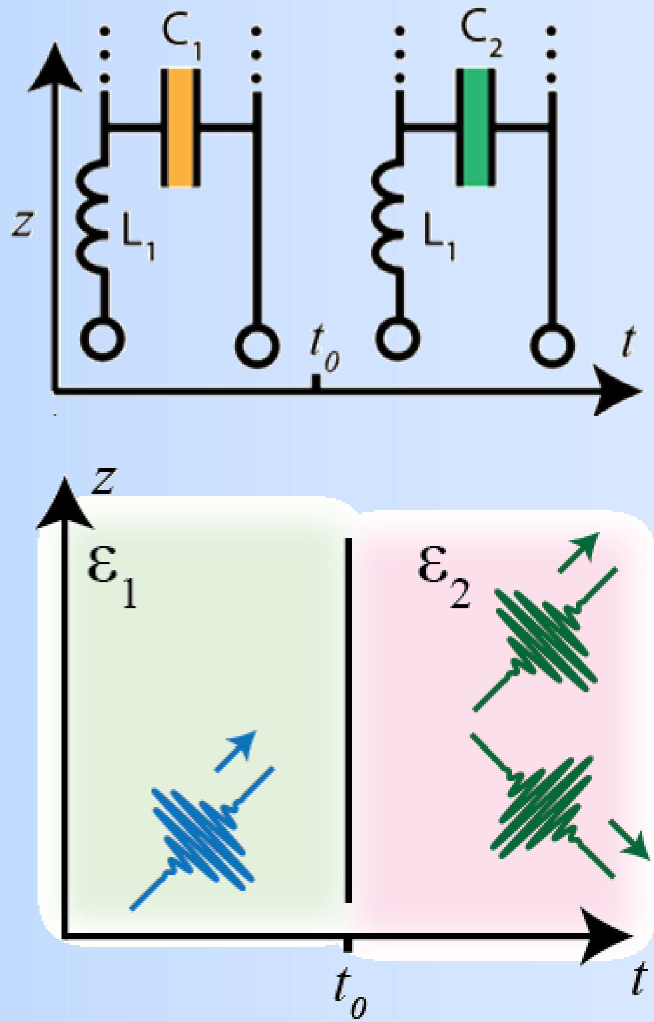
At a uniform time interface:

- ω is **not necessarily** conserved
- energy is **not necessarily** conserved
- spatial symmetry is preserved (conserving momentum)

$$D(t_o^-) - D(t_o^+) = \sigma$$

$$B(t_o^-) - B(t_o^+) = \sigma_m$$

Boundary Conditions and Energy Requirements



If the charge is conserved, the energy requirements may become very large:

$$P_{req} = \frac{D^2}{2\varepsilon_2} - \frac{D^2}{2\varepsilon_1}$$

$$\Delta U \sim N E_f \frac{\Delta\varepsilon}{\varepsilon_\infty}$$

$\Delta\varepsilon \sim 1$ requires $\Delta U \sim 10\ \mathrm{J/cm^3}$!!!

Z. Hayran, J. B. Khurgin, F. Monticone. *Opt. Mater. Express, OME*. **12**, 3904–3917 (2022)

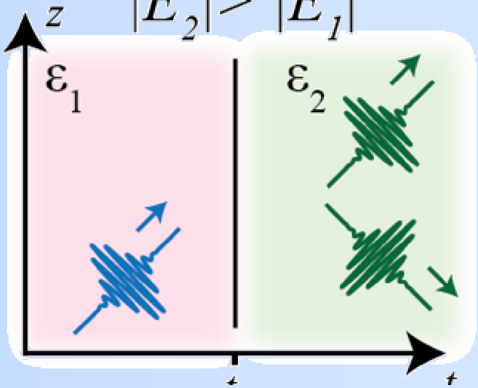
Energy Requirements

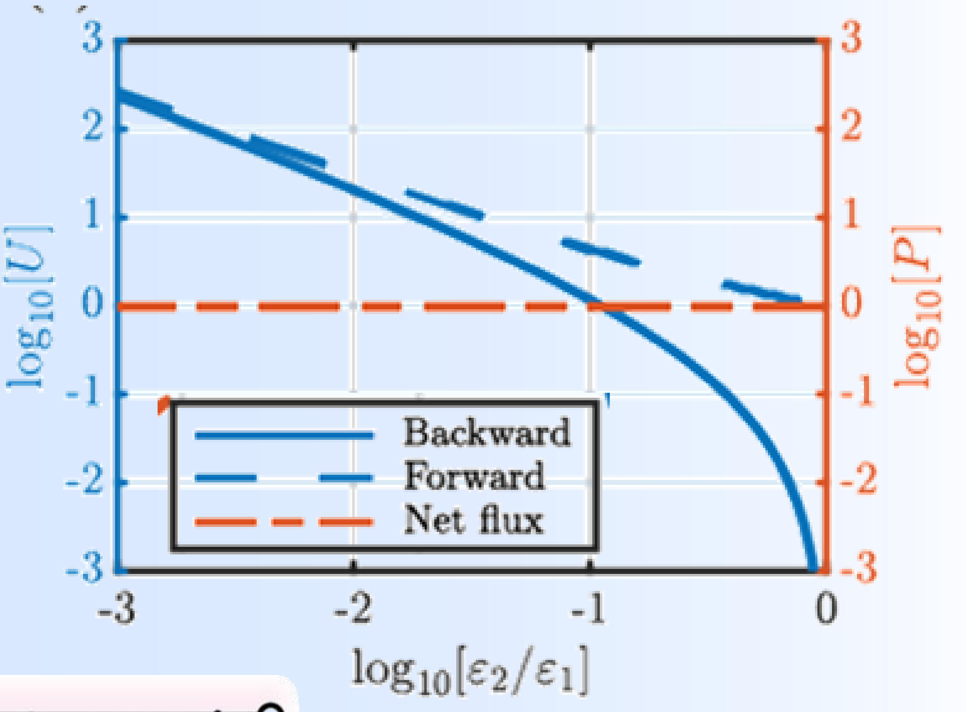
Pockels/Kerr mod. (on)

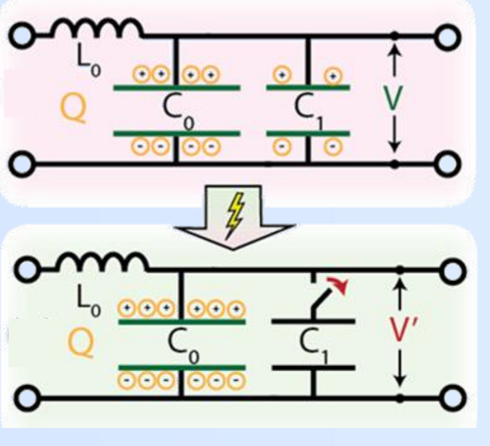
$$\varepsilon_2 < \varepsilon_1$$

$$|D_2| = |D_1|$$

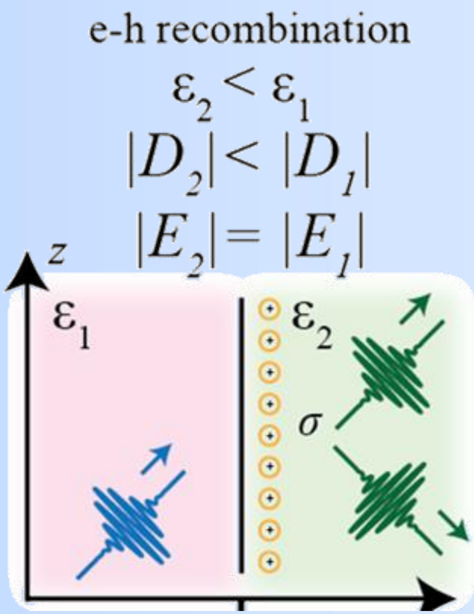
$$|E_2| > |E_1|$$

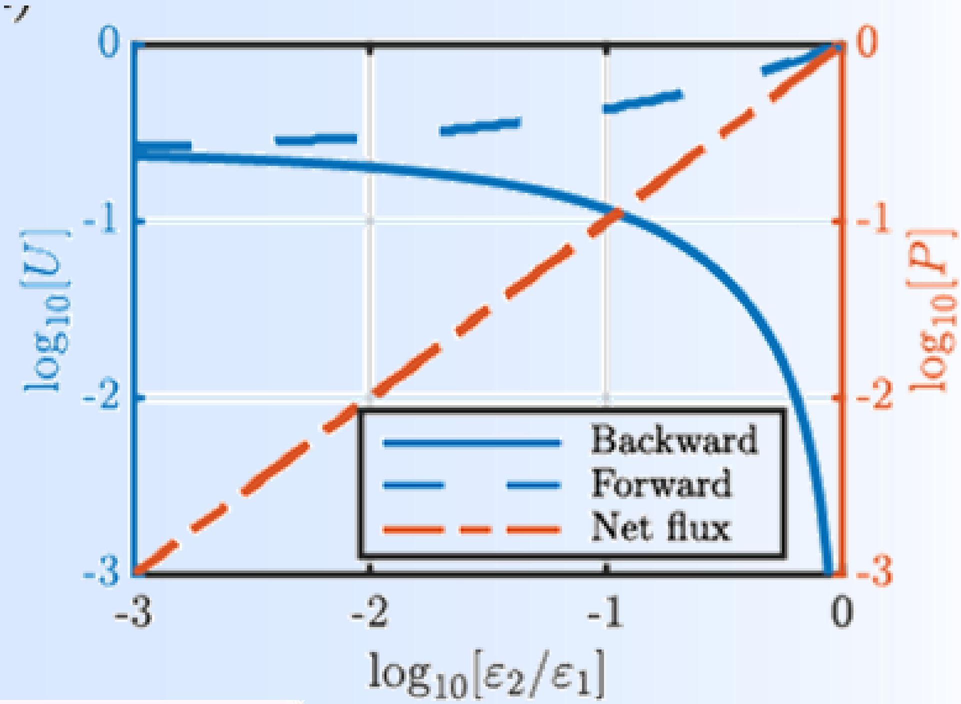


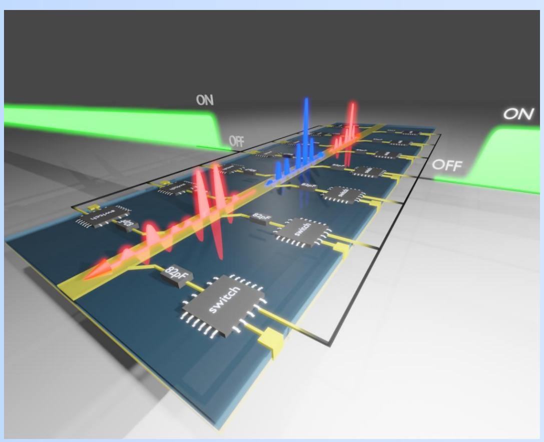


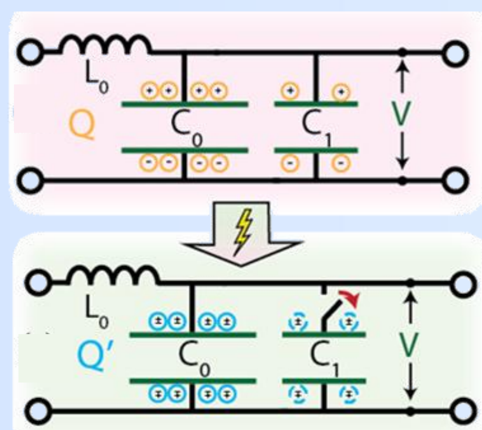


ENERGY REQUIREMENTS









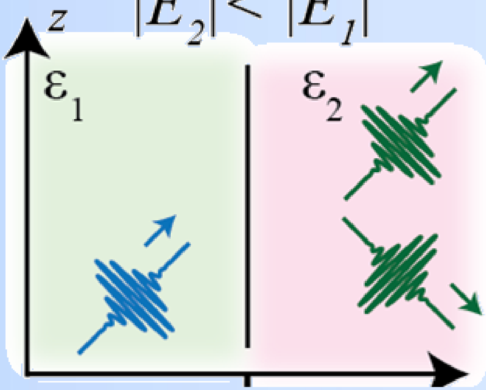
ENERGY REQUIREMENTS

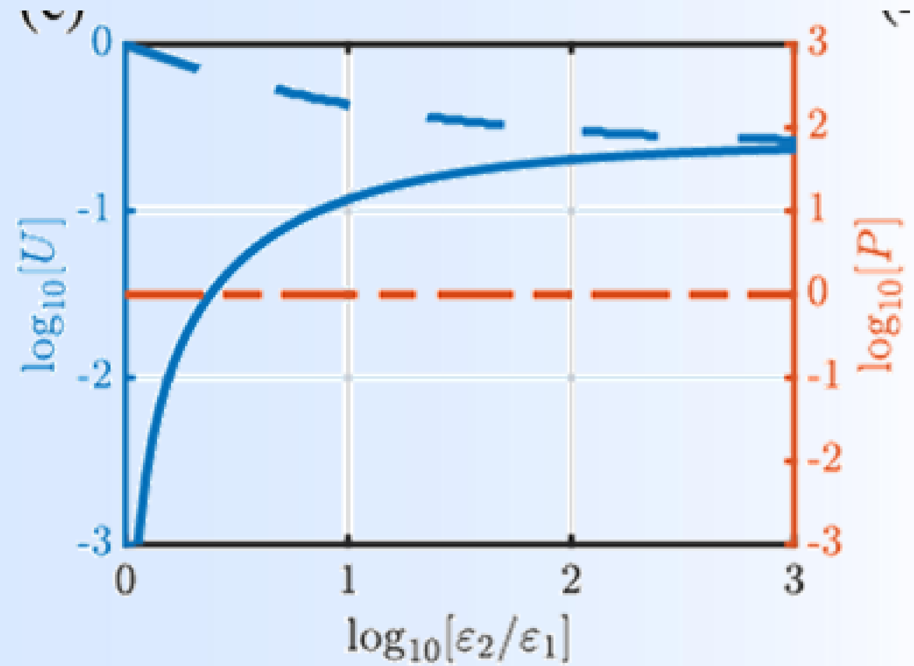
Pockels/Kerr mod. (off)

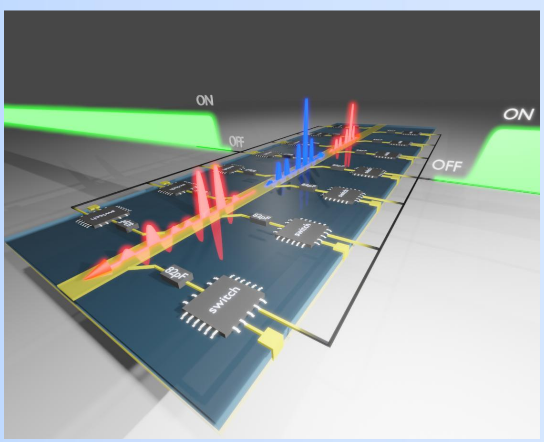
$\varepsilon_2 > \varepsilon_1$

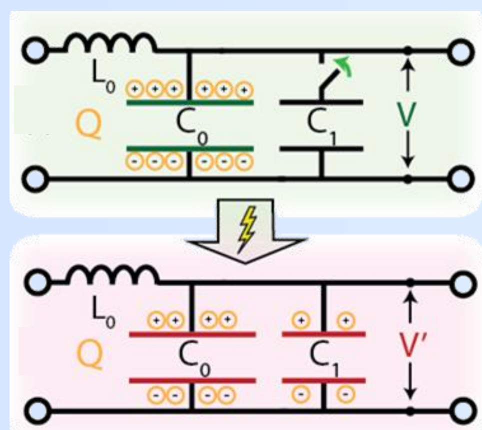
$|D_2| = |D_1|$

$|E_2| < |E_1|$



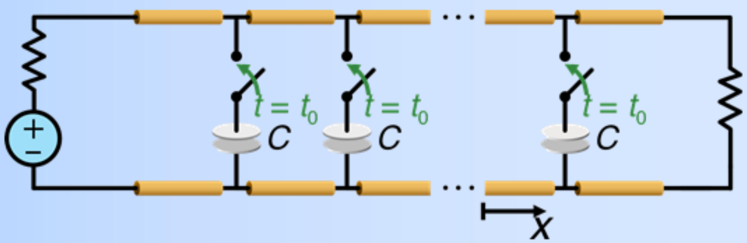




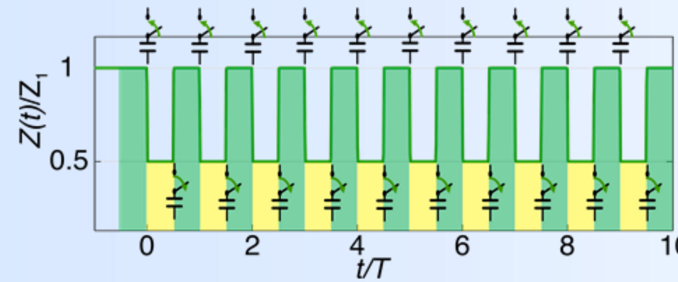


Passive Photonic Time Crystal

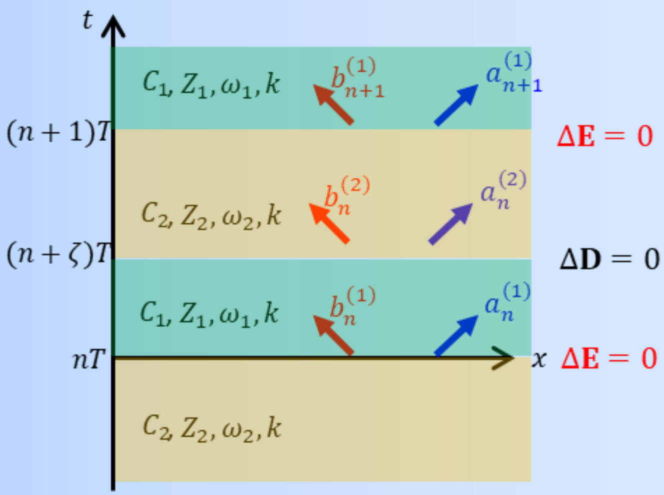
Transmission-line metamaterial



Switching scheme



Temporal transfer matrix method



$$\widetilde{V_n^{(1)}}(x,t) = V_n^{(1)}(t)e^{-jkx} = \left[a_n^{(1)}e^{j\omega_1(t-nT)} + b_n^{(1)}e^{-j\omega_1(t-nT)}\right]e^{-jkx}$$

$$\widetilde{I_n^{(1)}}(x,t) = I_n^{(1)}(t)e^{-jkx} = \frac{1}{Z_1}\left[a_n^{(1)}e^{j\omega_1(t-nT)} - b_n^{(1)}e^{-j\omega_1(t-nT)}\right]e^{-jkx}$$

$$\begin{bmatrix} V \\ I \end{bmatrix}_{n+1} = M\begin{bmatrix} V \\ I \end{bmatrix}_n = e^{j\Omega T}\begin{bmatrix} V \\ I \end{bmatrix}_n$$

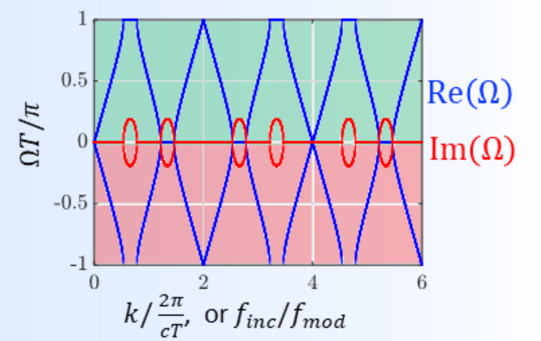
$$\mathrm{tr}(M) = 2\cos\Omega T - (1 - z^2)e^{-j\Omega T}$$

where $z = \frac{Z_2}{Z_1} = \sqrt{\frac{C_1}{C_2}} < 1$ is the ratio of impedances

Passive Photonic Time Crystal

Comparison to *active* photonic time crystals (PTC)

	Passive PTC	Active PTC
Dispersion	$\mathrm{tr}(M) = 2\cos\Omega T - (1 - z^2)e^{-j\Omega T}$	$\mathrm{tr}(M) = 2\cos\Omega T$
Momentum balance	$\det(M) = \frac{C_1}{C_2} = z^2 < 1$ (Possible momentum decay)	$\det(M) = 1$



From the transfer matrix M, it is easy to find

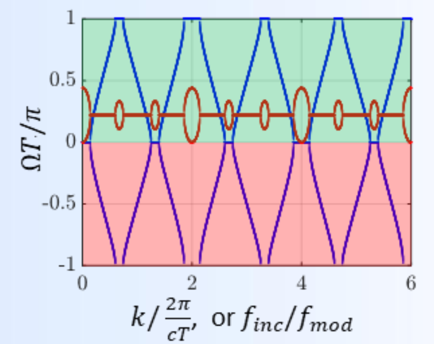
$\mathrm{tr}(M) \in \mathbb{R}$, for both passive and active PTCs

For an active PTC,

Ω is either real (in the bands) or purely imaginary (in the gaps).

For a passive PTC,

Ω is complex almost everywhere except for countably infinite momenta.



Passive Photonic Time Crystal

Passive PTC with stable *k*-gaps

Condition for the existence of stable k-gaps:

$$1 + \det(M) - |\mathrm{tr}(M)| = 0,$$

where the transfer matrix $M = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}$ with

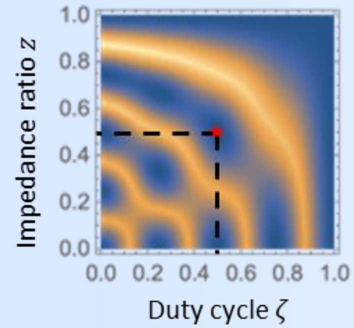
$$M_{11} = z^2 \cos\theta_1 \cos\theta_2 + z \sin\theta_1 \sin\theta_2,$$

$$M_{12} = jz^2 \cos\theta_1 \cos\theta_2 - jz \sin\theta_1 \sin\theta_2,$$

$$M_{21} = \frac{j(1+z)}{2} \sin(\theta_1 + \theta_2) + \frac{j(1-z)}{2} \sin(\theta_1 - \theta_2),$$

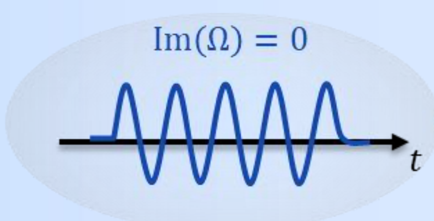
$$M_{22} = \frac{1+z}{2} \cos(\theta_1 + \theta_2) + \frac{1-z}{2} \cos(\theta_1 - \theta_2).$$

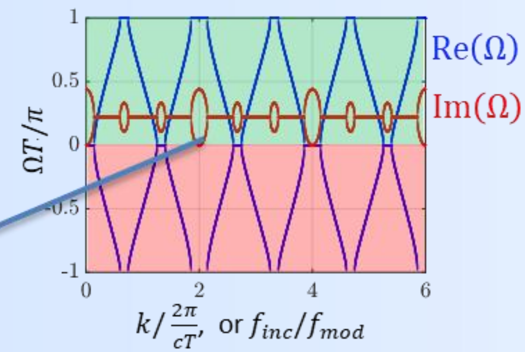
$|1 + \det(M) - |\mathrm{tr}(M)||$



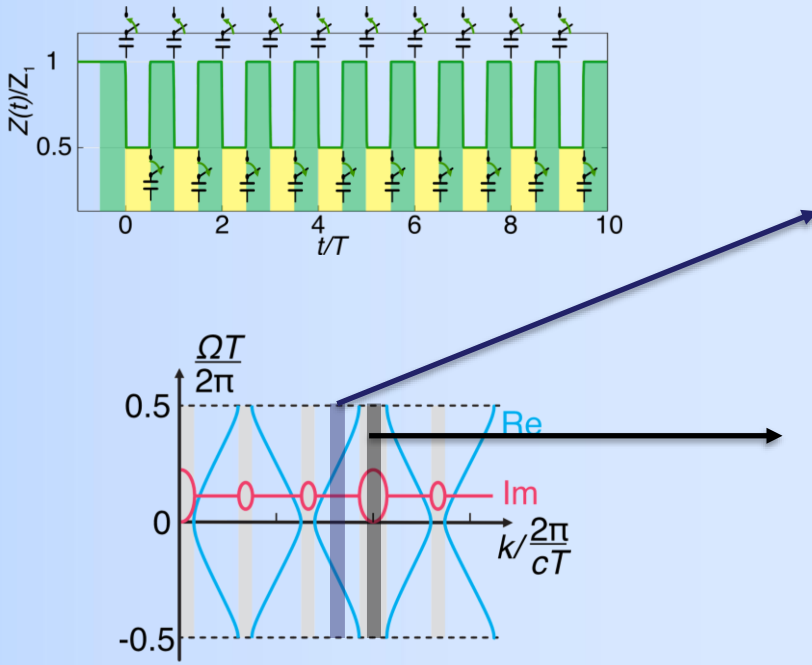
Three degrees of freedom:

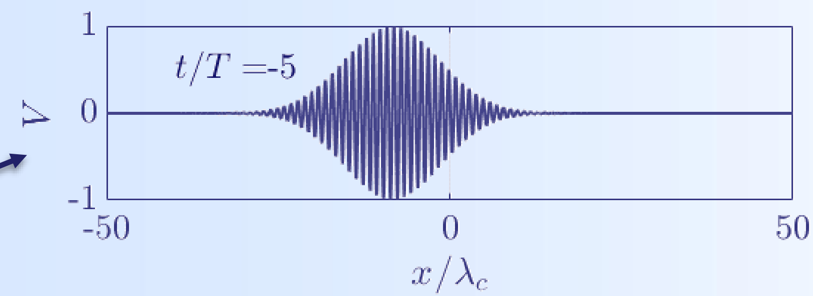
- Modulation frequency;
- Duty cycle;
- Impedance contrast.

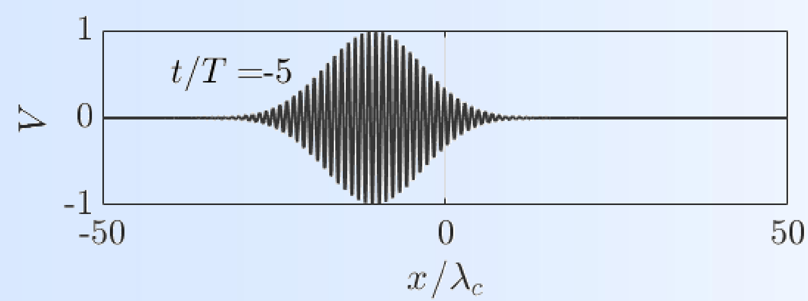




Momentum Selectivity

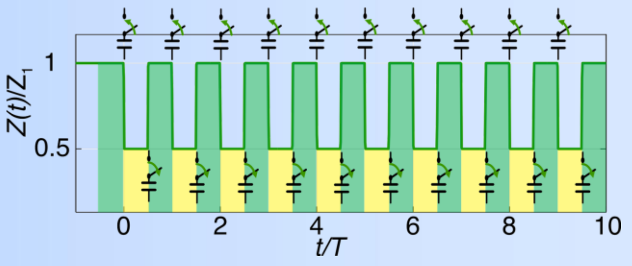


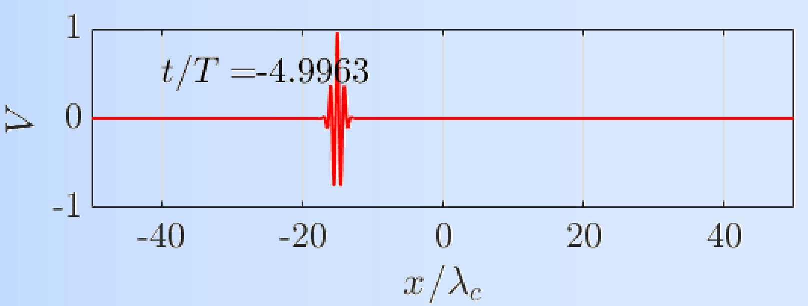


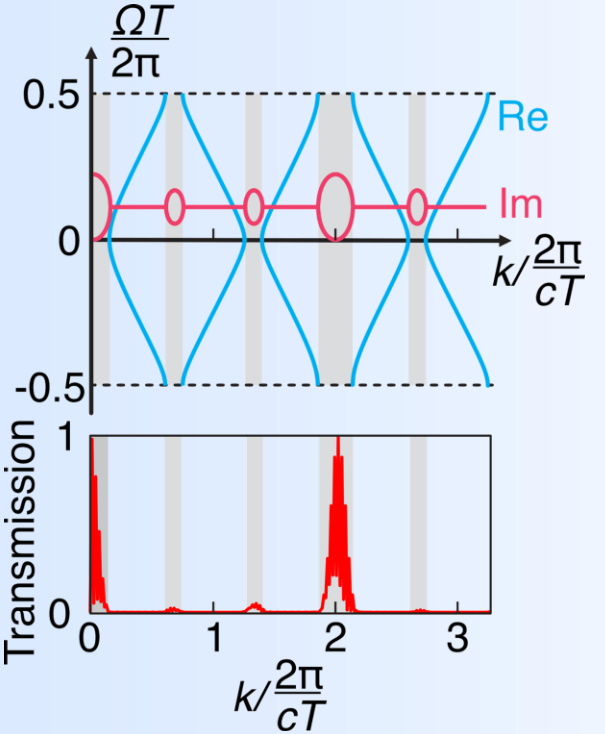


Momentum Selectivity

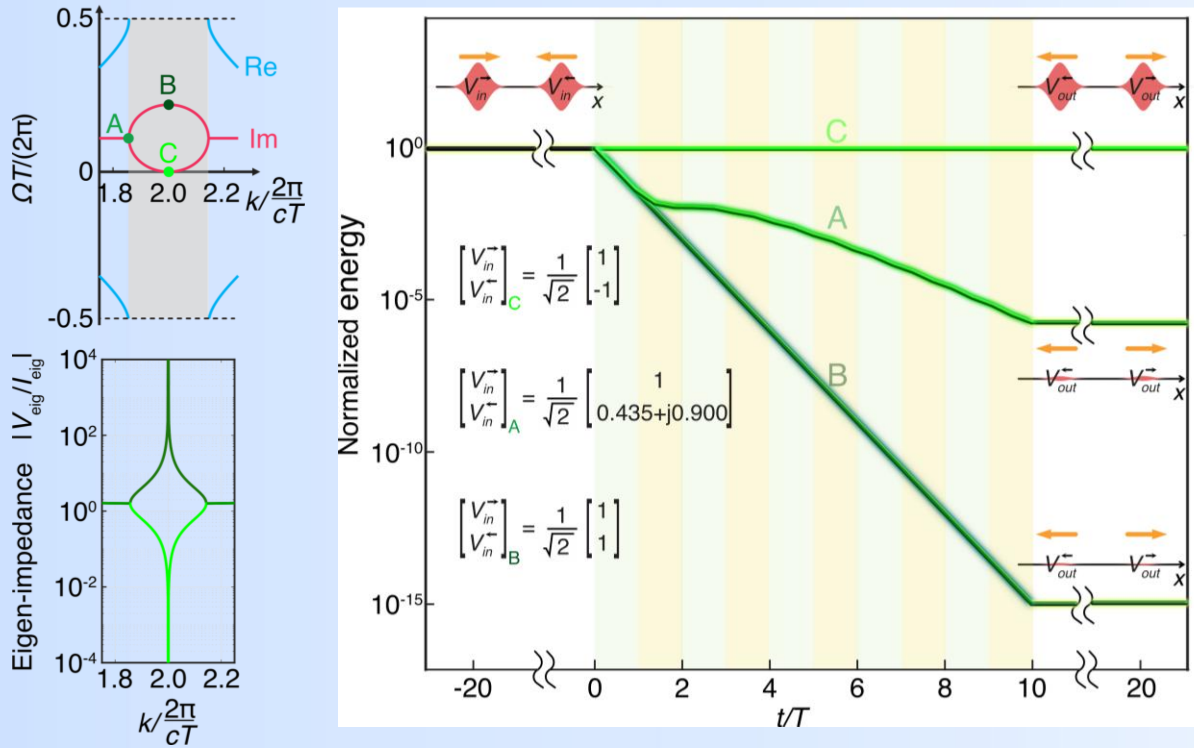
Broadband (short pulse) incidence





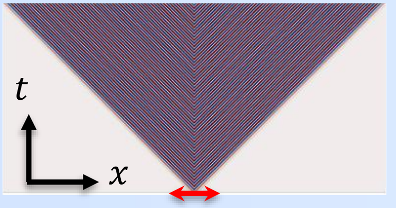


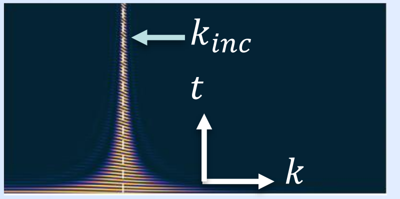
Coherent Passive K-Gaps



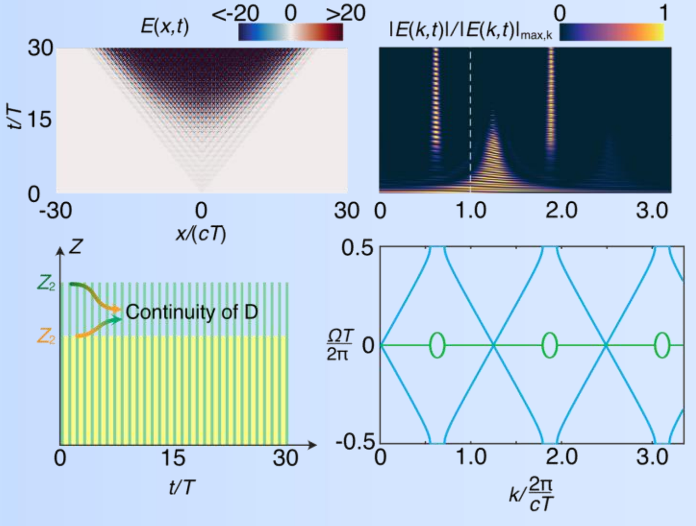
Emitter Coupled to a Passive Time Crystal

Radiation in free space

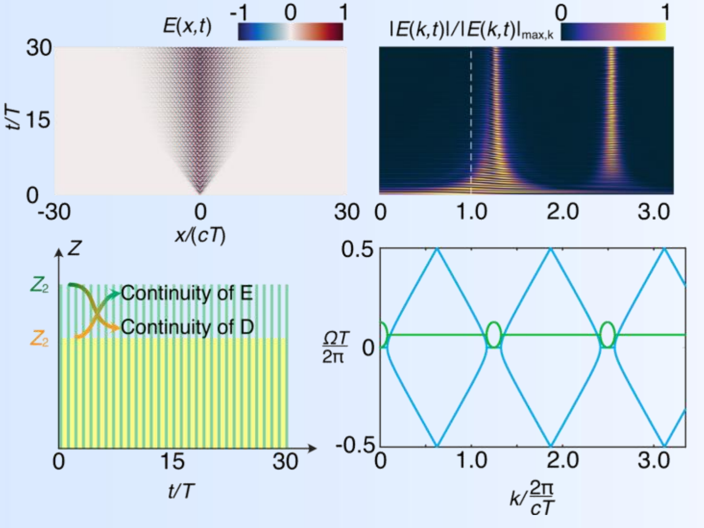




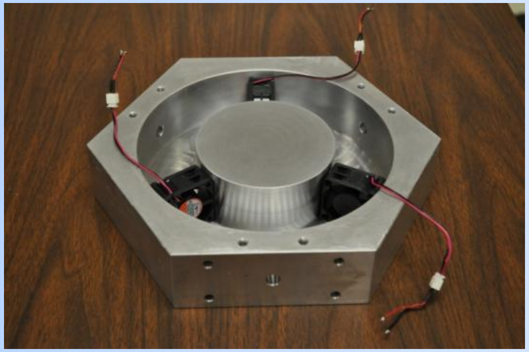
in an active PTC

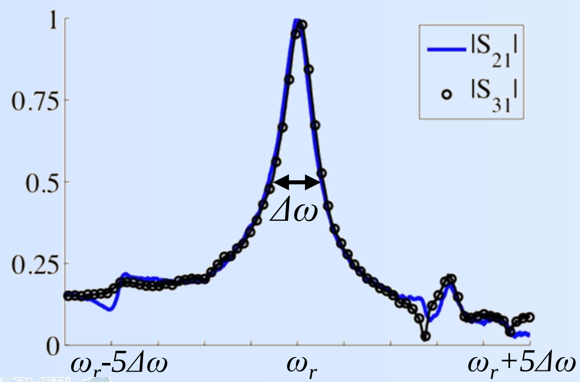


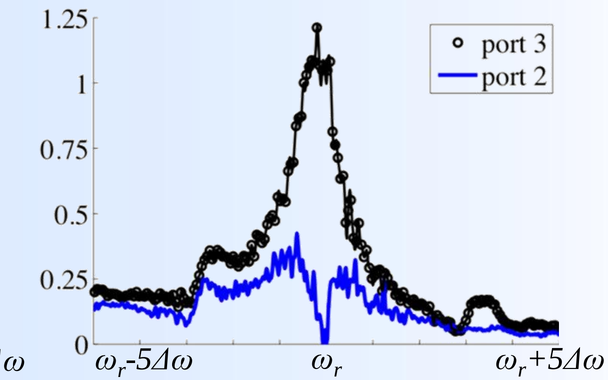
in a passive PTC

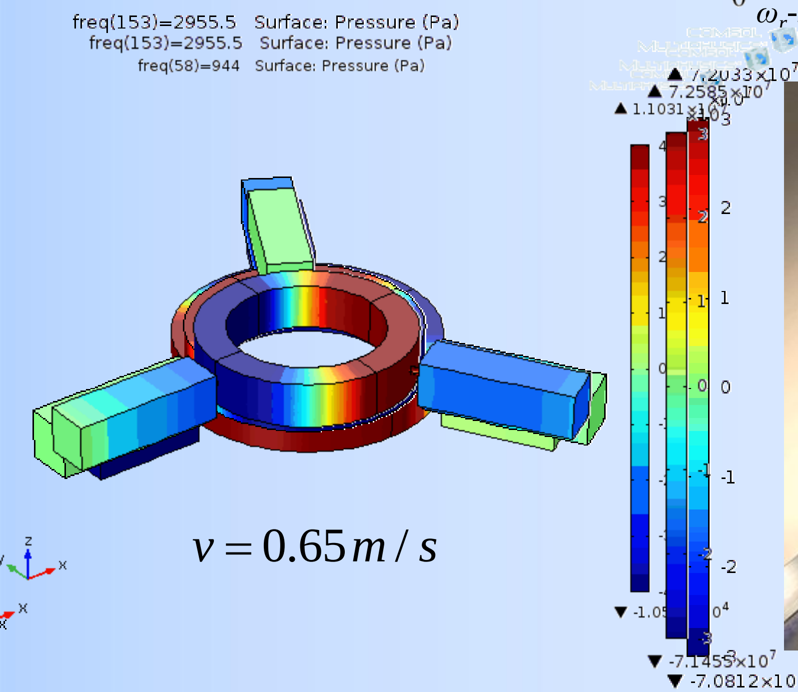


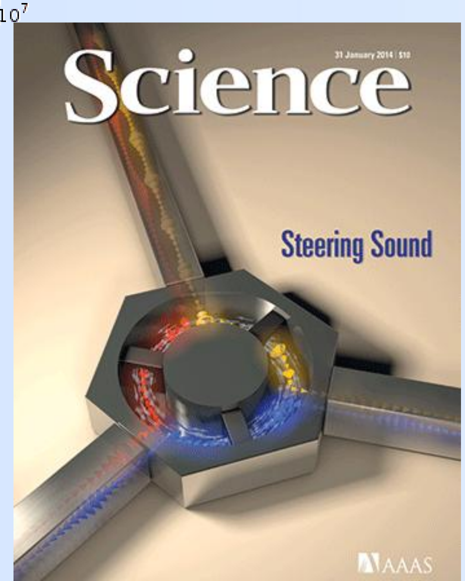
Angular-Momentum Bias

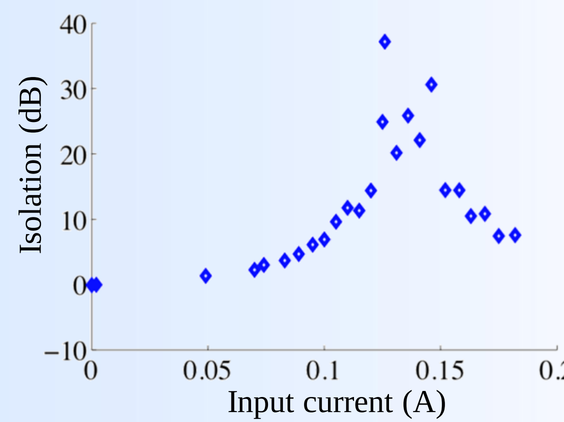






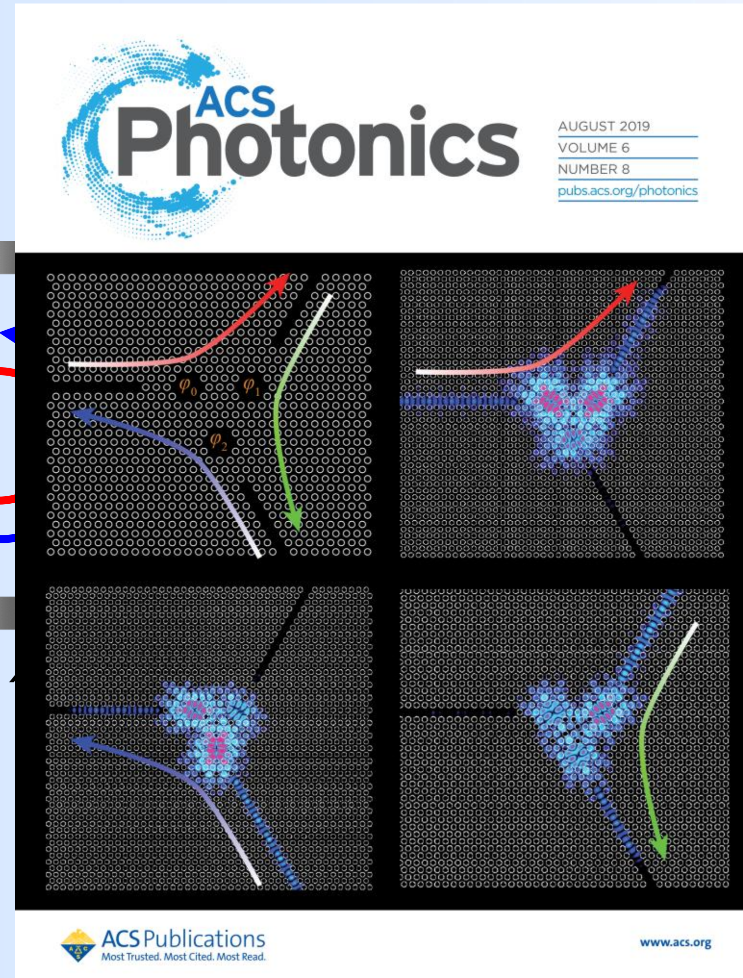






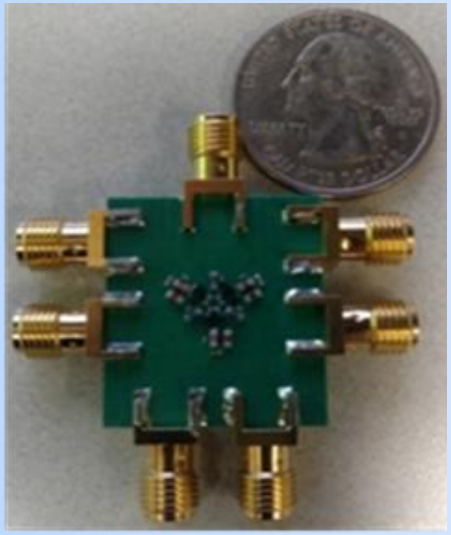
R. Fleury, D. L. Sounas, C. Sieck, M. Haberman, A. Alù, *Science* 343, 516 (2014)

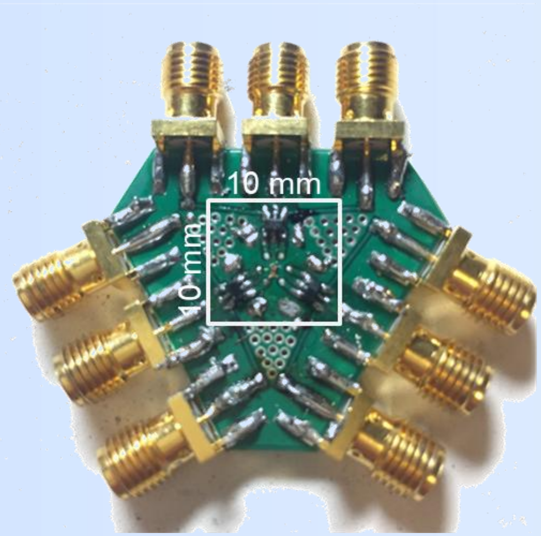
Synthetic Angular Momentum with Time Modulation

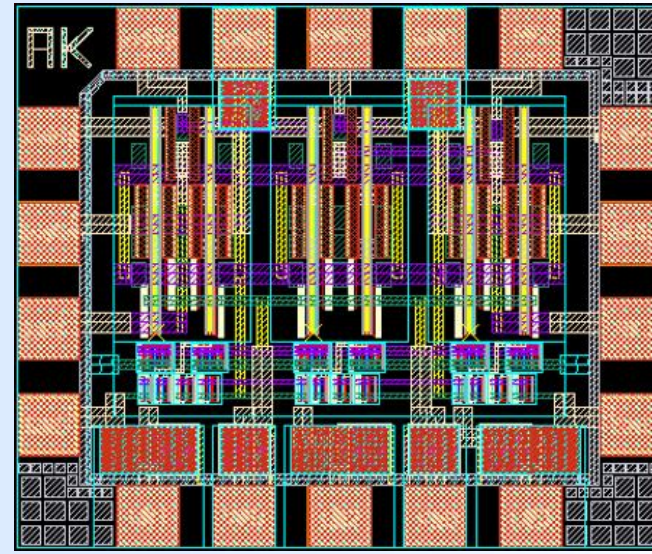


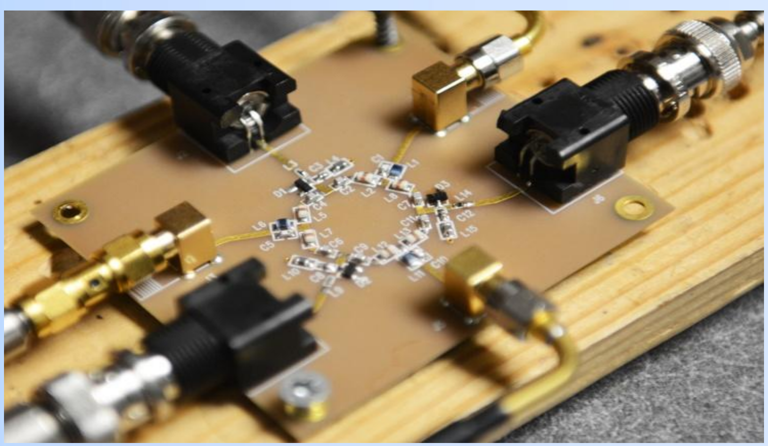
D. Sounas, A. Alù, *ACS Photonics* 1, 198 (2014)
A. Mock, D. Sounas, A. Alù, *ACS Photonics* 6, 2056 (2019)

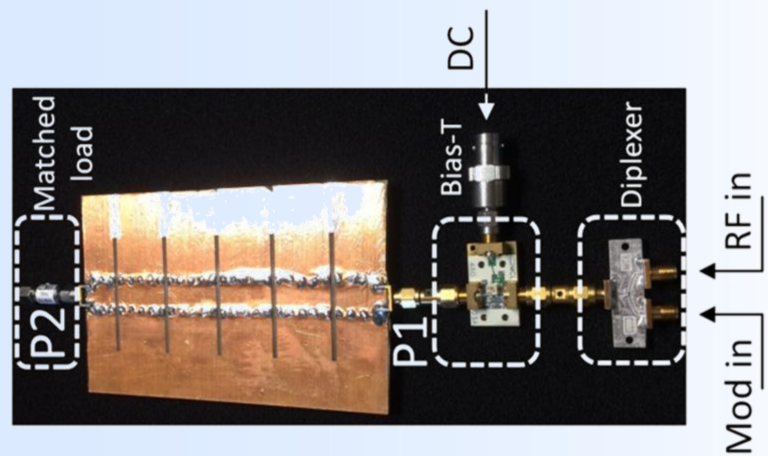
Compact, Magnet-Less Circulators for RF and Light



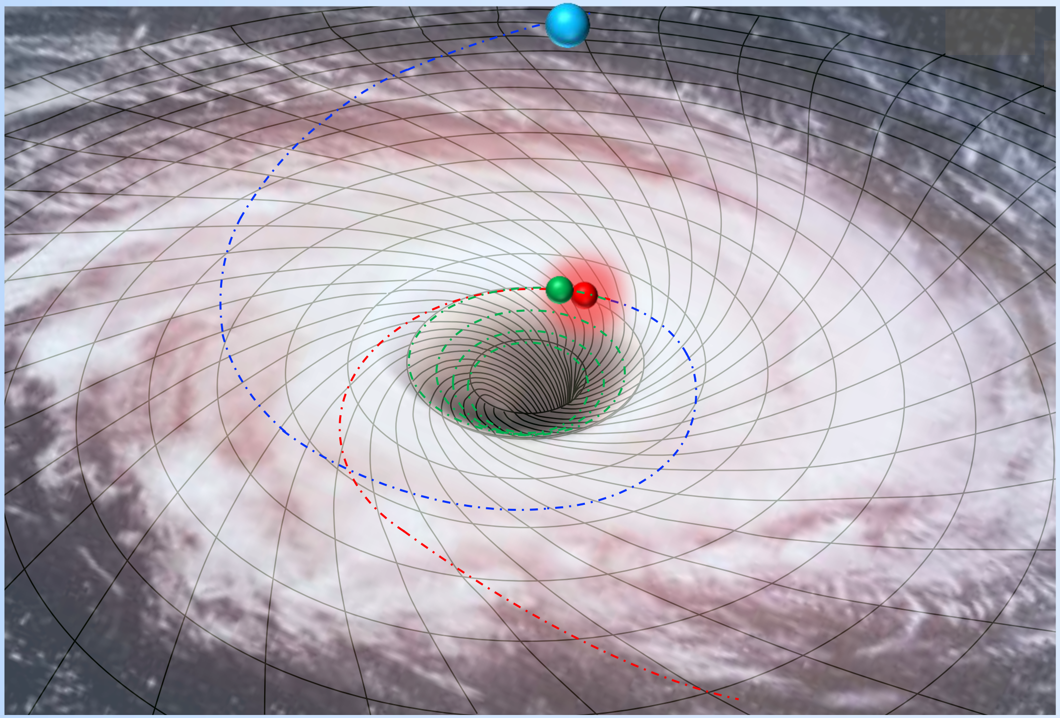




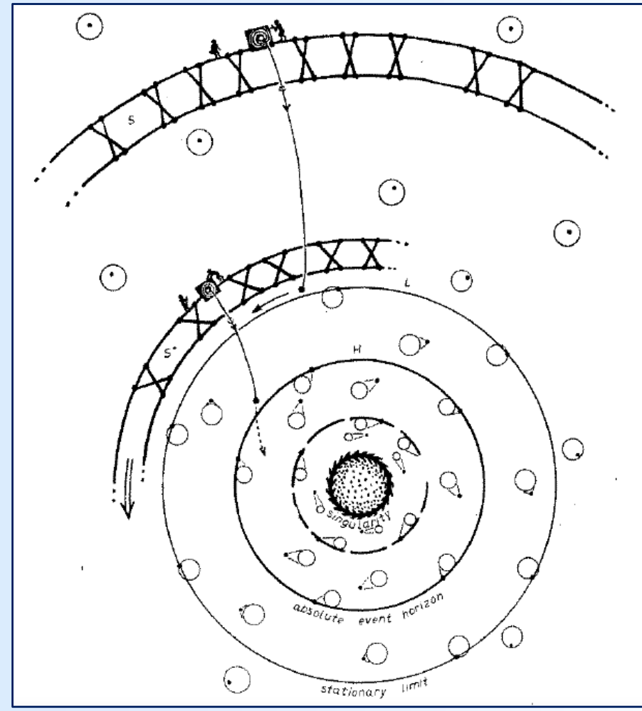




Extracting Energy From a Rotating Black Hole



Negative energy orbits



Extraction of Rotational Energy from a Black Hole

THERE has been considerable interest recently in the question of the gravitational collapse of a massive body and of the possible astrophysical consequences of the existence of the "black hole" which general relativity predicts should sometimes be the result of such a collapse. In particular, the question has arisen whether the mass–energy content of a black hole could, under suitable circumstances, be a source of available energy. We now consider the extraction of rotational energy from a black hole, not least because the rotational energy (defined appropriately) of a black hole should, in general, be comparable with its total mass–energy[1].

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London

Received December 16, 1970.

[1] Bardeen, J. M., *Nature*, **226**, 65 (1970).
[2] Israel, W., *General Relativity and Gravitation*, **2**, No. 1 (Plenum, in the press).

R. Penrose,
Riv. Nuovo Cim. Num. Spec. 1, 257 (1969)

Electromagnetic Penrose Super-Radiance

GENERATION OF WAVES BY A ROTATING BODY

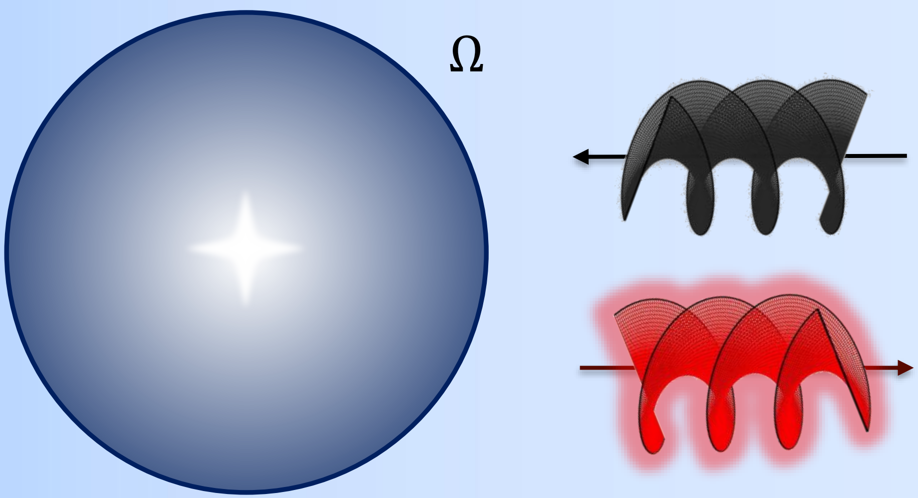
Ya.B. Zel'dovich
Institute of Applied Mathematics, USSR Academy of Sciences
Submitted 9 July 1971
ZhETF Pis. Red. 14, No. 4, 270 - 272 (20 August 1971)

An axially-symmetrical body rotating inside a resonator cavity is capable of amplifying definite oscillation modes inside the resonator, transferring the rotation energy to these oscillations.

SOVIET PHYSICS JETP VOLUME 35, NUMBER 6 DECEMBER, 1972

Amplification of Cylindrical Electromagnetic Waves Reflected from a Rotating Body

Ya. B. Zel'dovich
Institute of Applied Mathematics, USSR Academy of Sciences
Submitted December 10, 1971
Zh. Eksp. Teor. Fiz. 62, 2076–2081 (June, 1972)

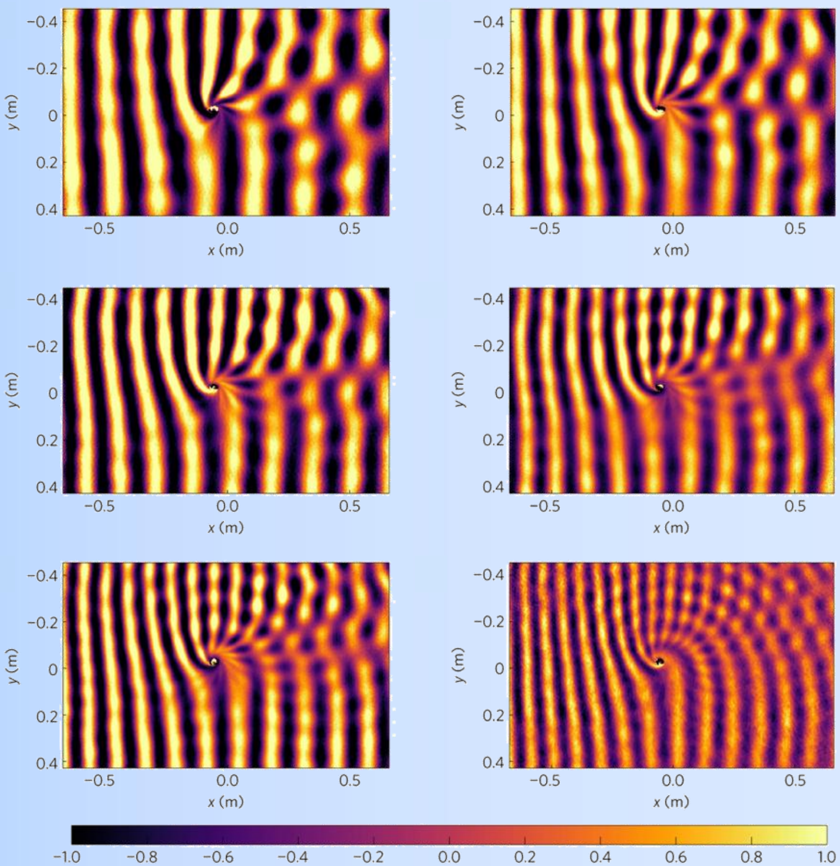


$$E(r,\varphi) = e^{-j\omega t} e^{jn\varphi}\, e^{-jkr}/\sqrt{r}$$

$$\omega(k) - v.k < 0$$

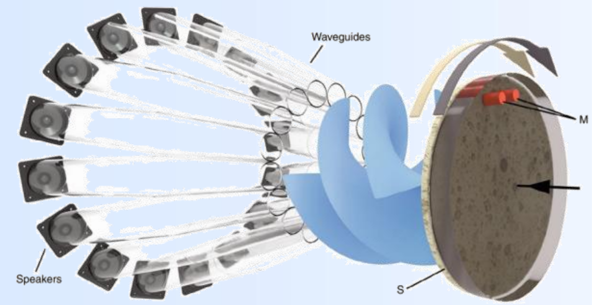
Wave Amplification From Rotating Bodies

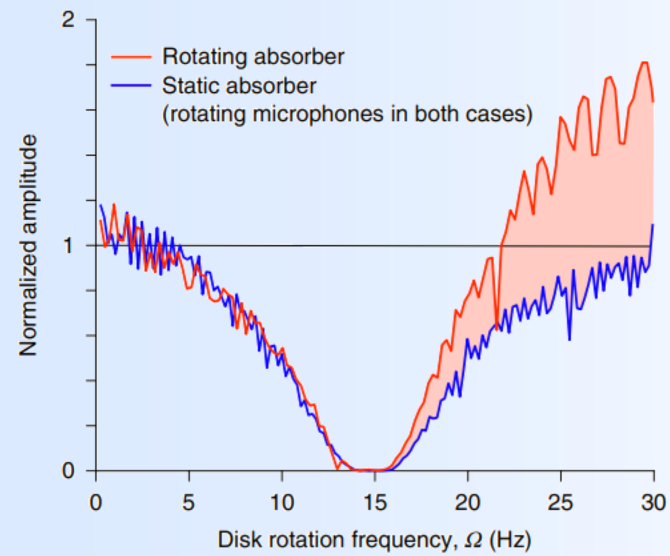
Rotational superradiant scattering in a *water vortex flow*



T. Torres et. al., *Nature Phys.* 13, 833–836 (2017)

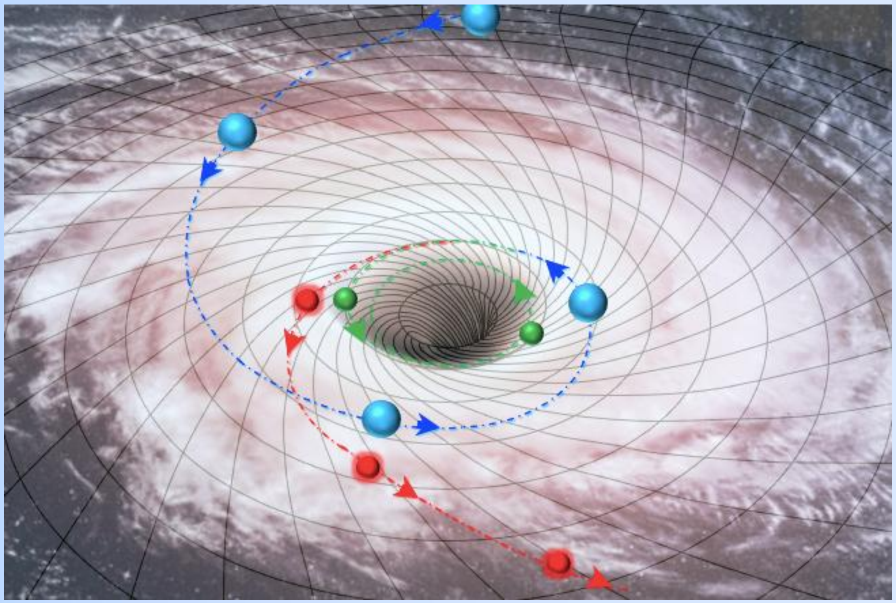
Amplification of acoustic waves from a rotating body

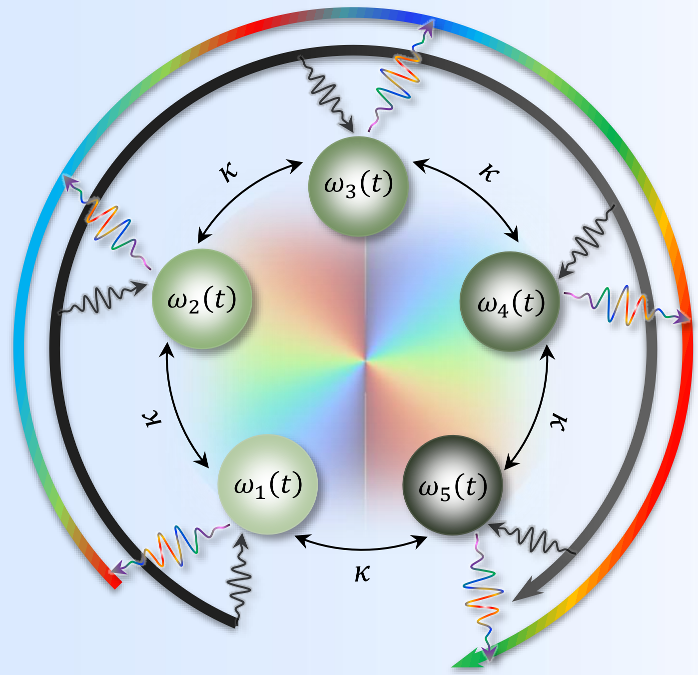




M. Cromb, et al., *Nature Physics* 16, 1069 (2020)

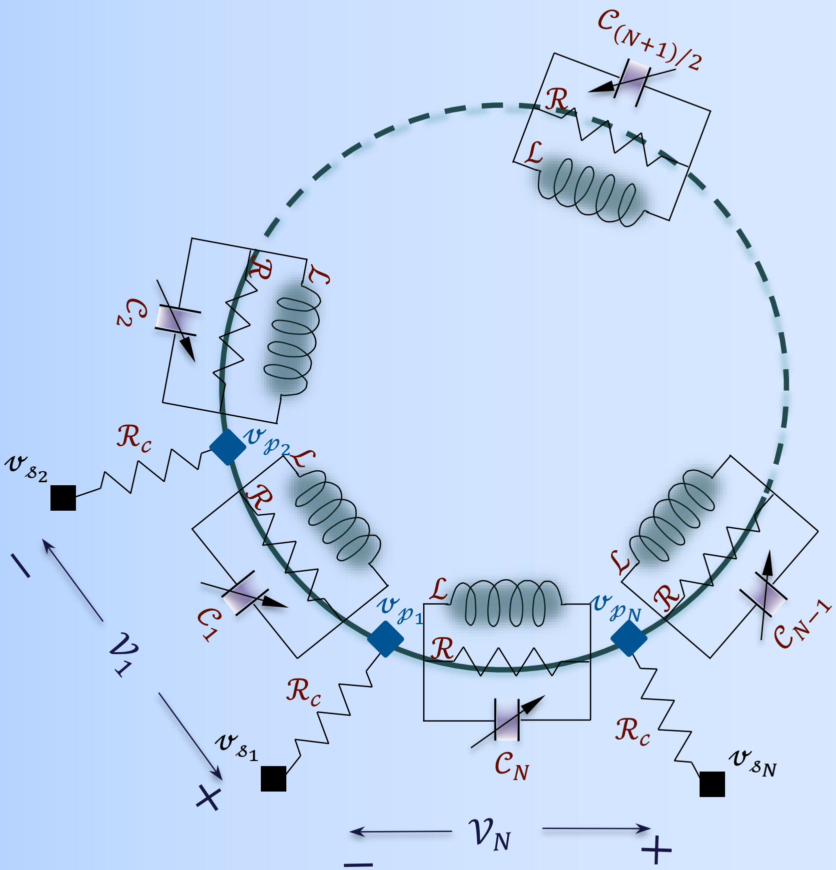
Synthetic Penrose Super-radiance





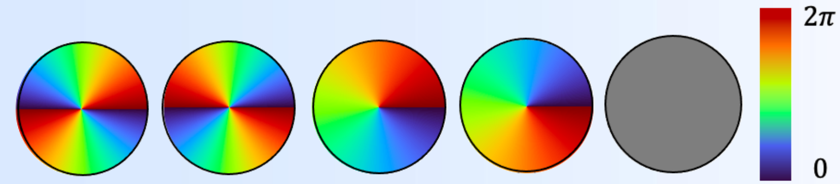
- Excitation with OAM signal
- Synthetic rotation based on Angular Momentum Biasing
- Monitoring the strength of output OAM signals

Governing Matrix Equation & Harmonic Analysis



$$\{\bar{\bar{A}}\,\omega^2+\bar{\bar{B}}\omega+\bar{\bar{D}}\}\bar{V}+\frac{\delta}{2}\left\{\omega^2\,\bar{\bar{C}}\bar{V}^{(+)}+\omega^2\,\bar{\bar{C}}^*\bar{V}^{(-)}\right\}=\bar{\bar{G}}\,\bar{V}_s$$

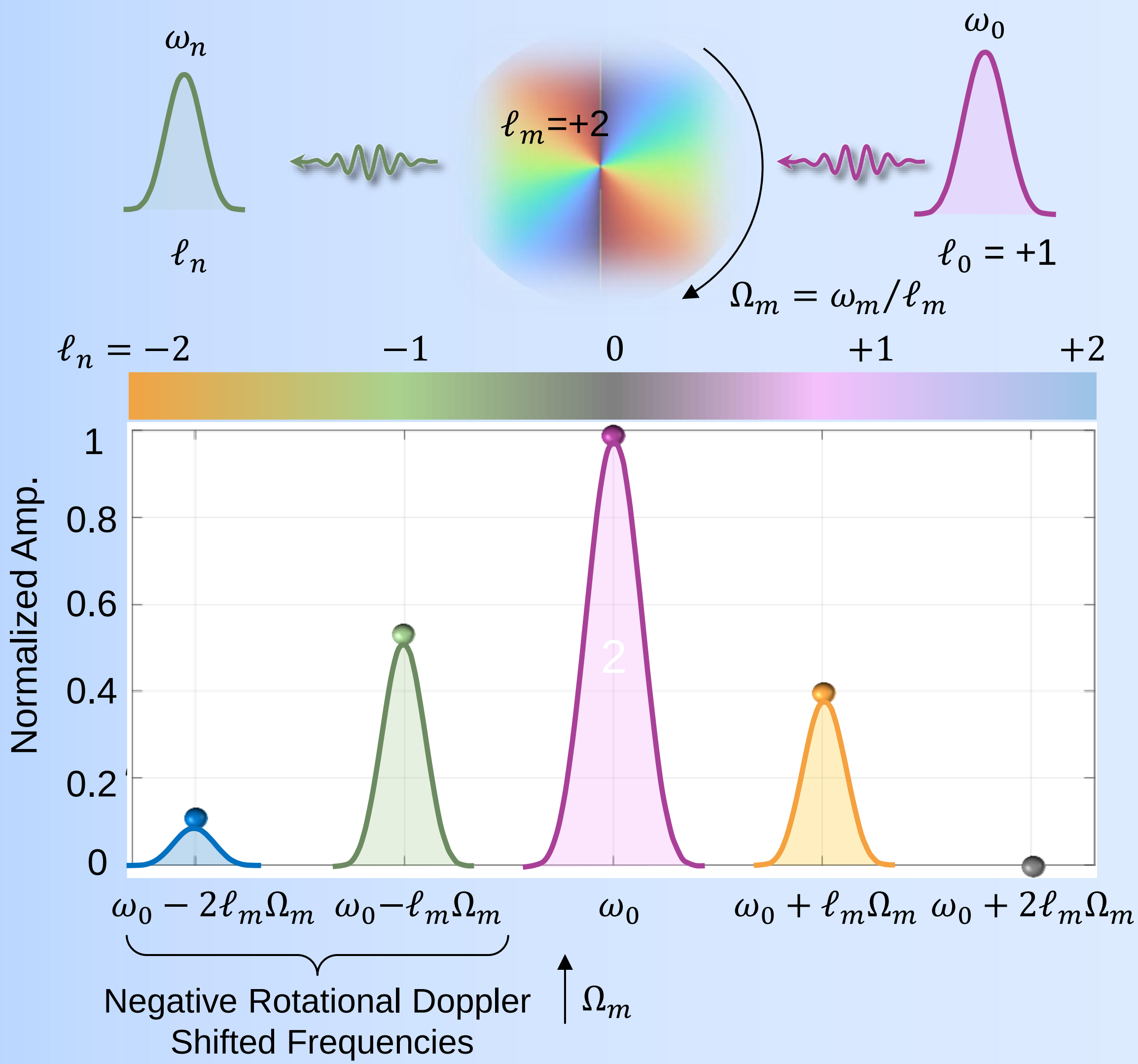
Eigenbasis of the static system

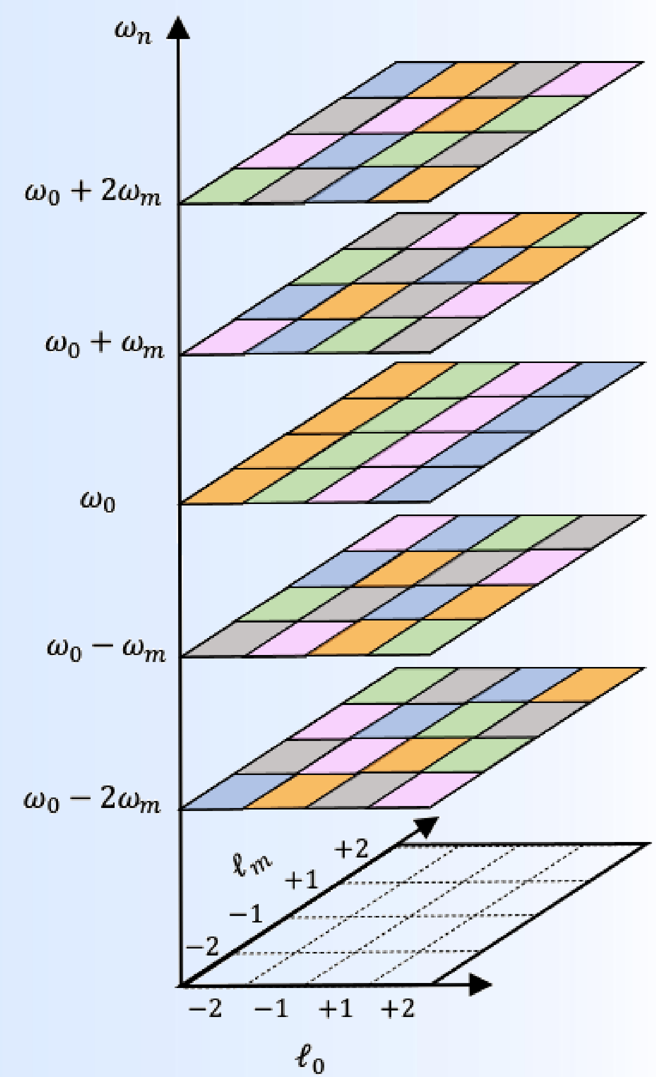


$$\bar{\bar{H}}\,\bar{V}_R+\frac{\delta}{2}\left\{\omega^2\,\bar{\bar{\bar{C}}}_{R(\ell_m)}\bar{V}_R^{\;(+)}+\omega^2\,\overline{\bar{\bar{C}}^*}_{R(\ell_m)}\,\bar{V}_R^{\;(-)}\right\}=\bar{\bar{\bar{G}}}_R\,\bar{V}_{s,R}$$

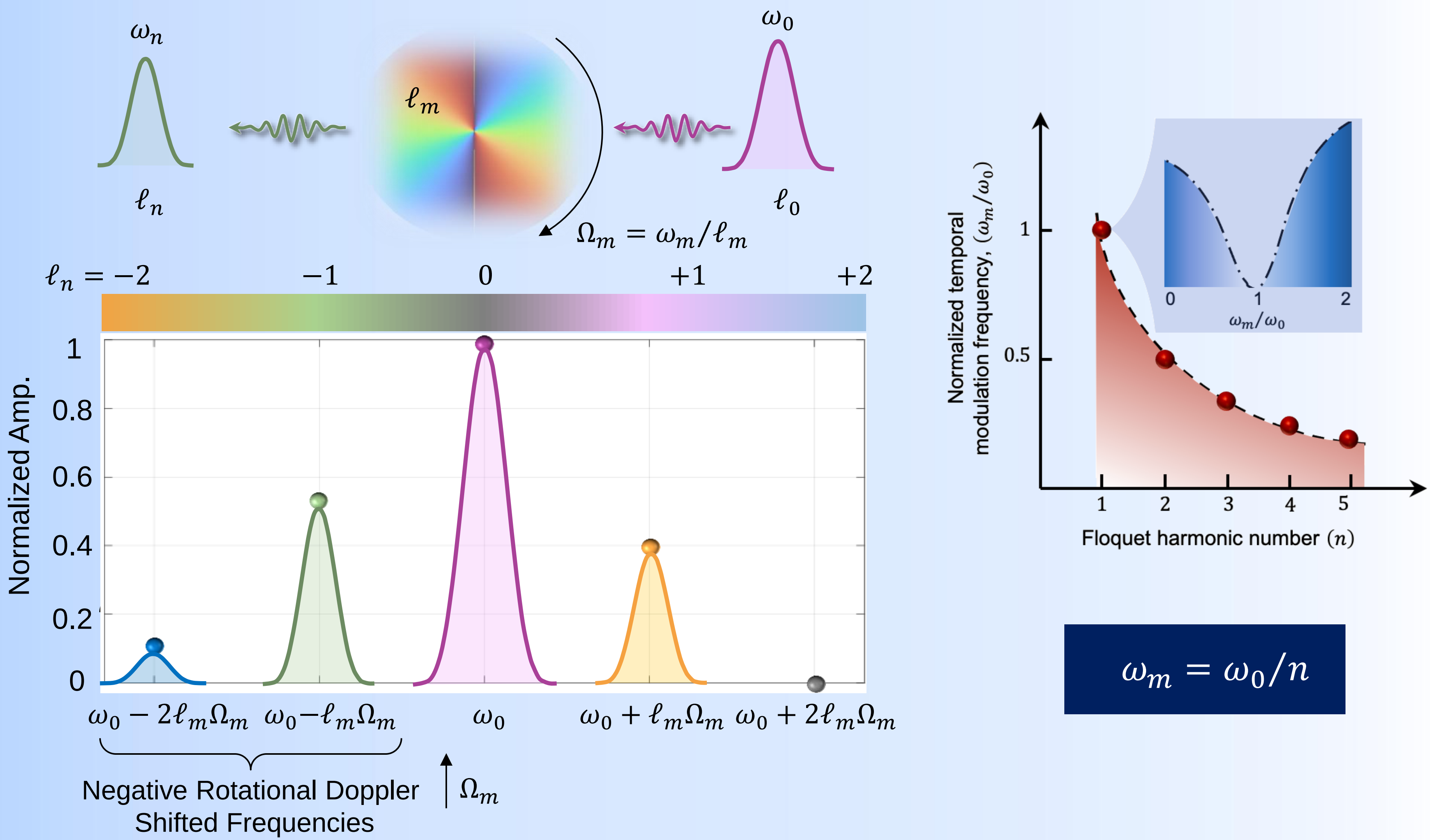
$$\bar{\bar{H}}=\bar{\bar{A}}_R\;\omega^2+\bar{\bar{B}}_R\;\omega+\bar{\bar{D}}_R$$

Synthetic Rotational Doppler Effect



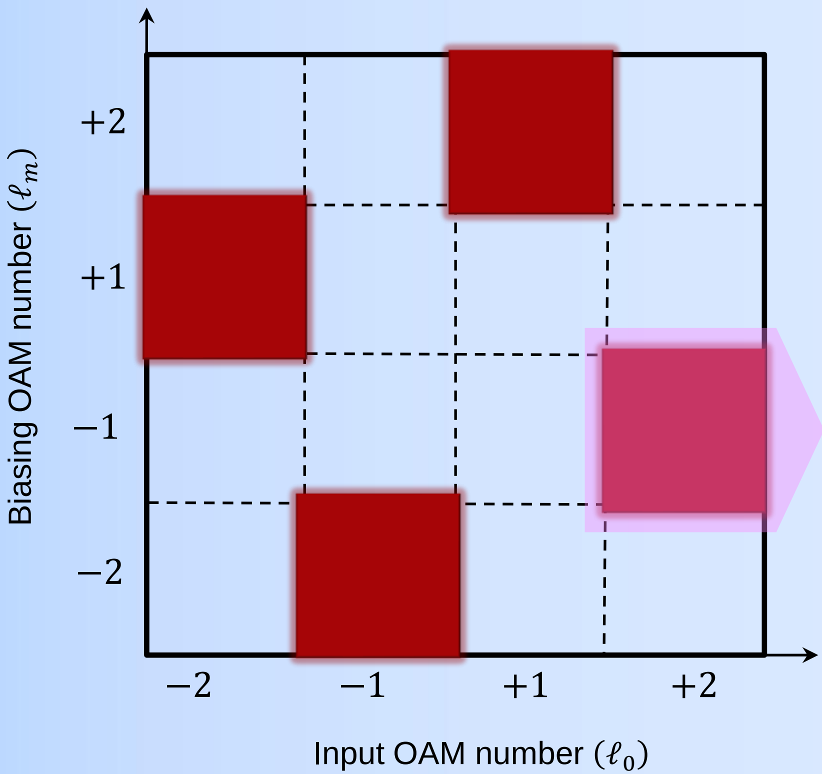


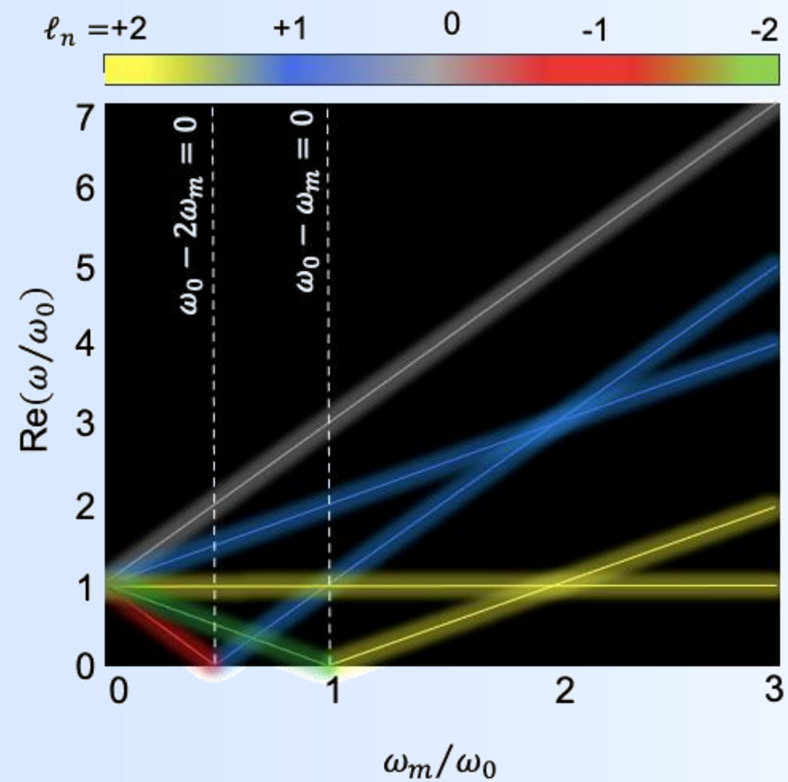
Synthetic Negative Rotation Doppler Shift



$$\omega_m = \omega_0 / n$$

Synthetic Penrose Super-Radiance

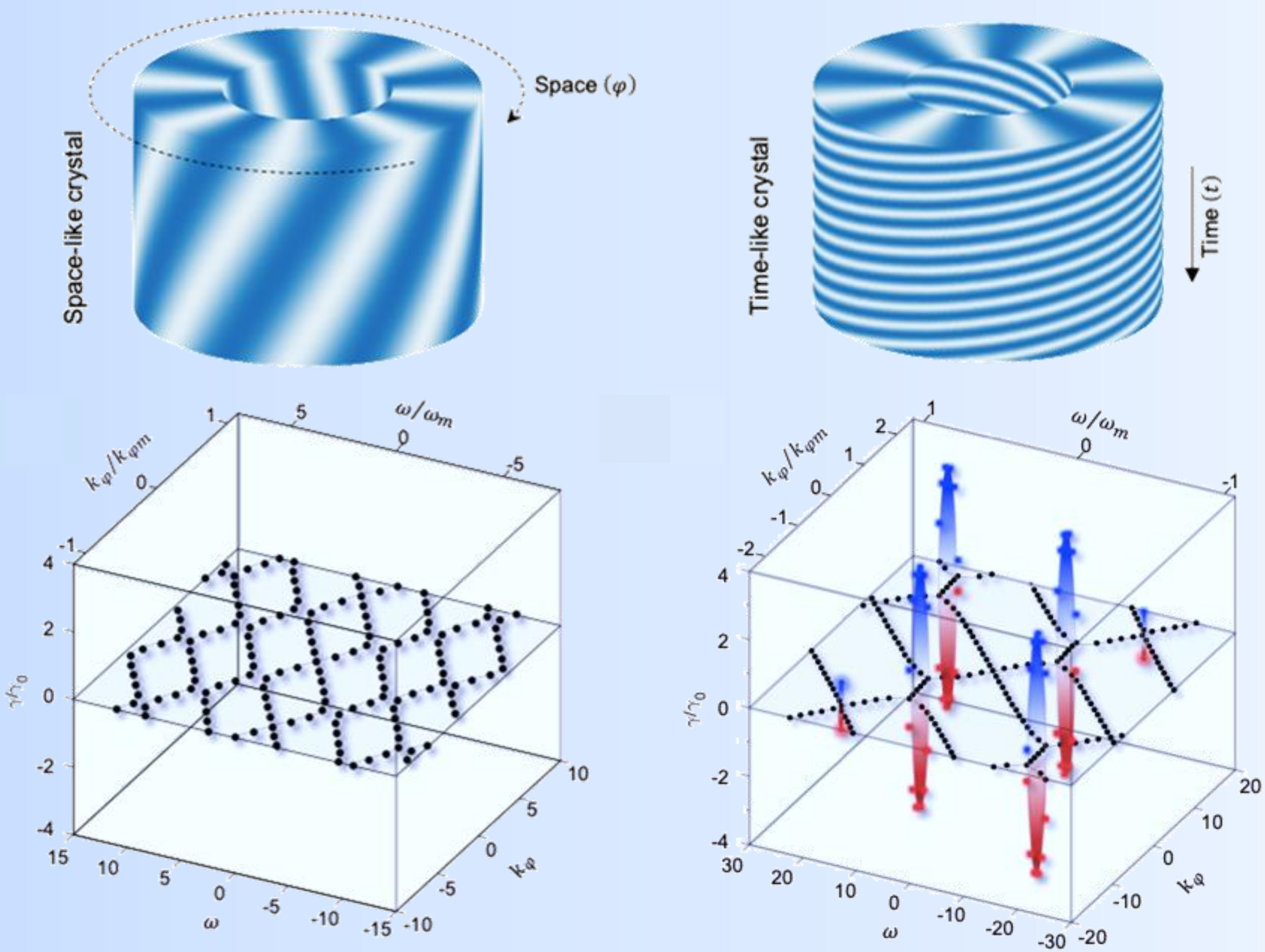




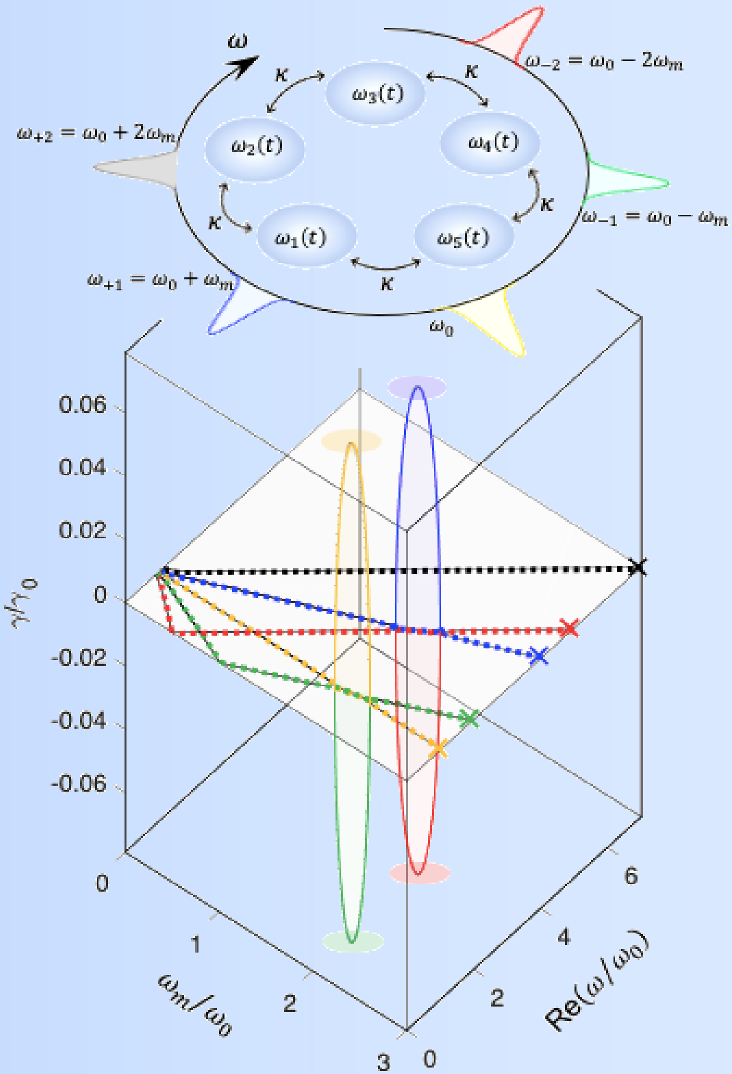
Phase matching between the

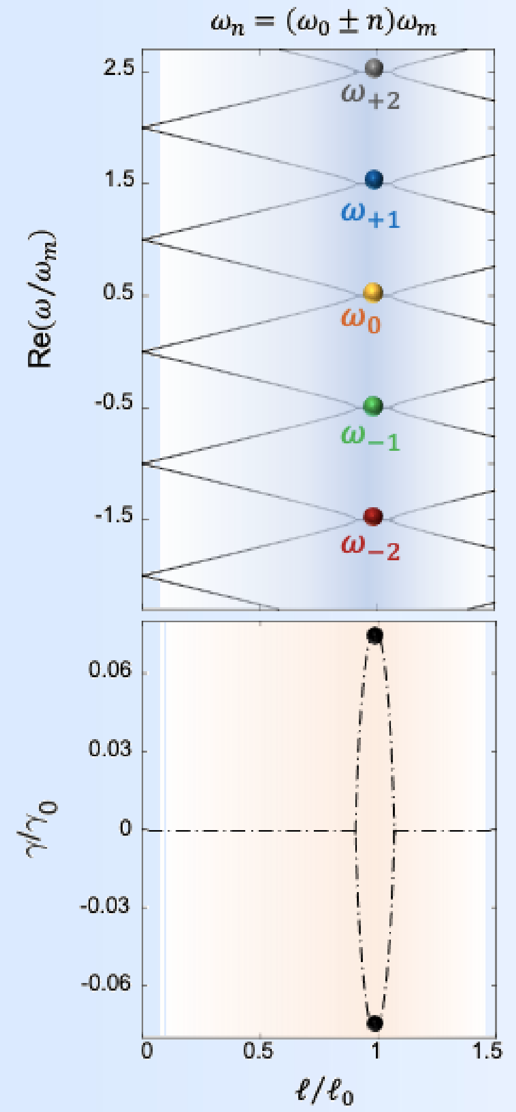
- Modulation and fundamental harmonic
- Interacting harmonics

Angular-Momentum Bandgaps

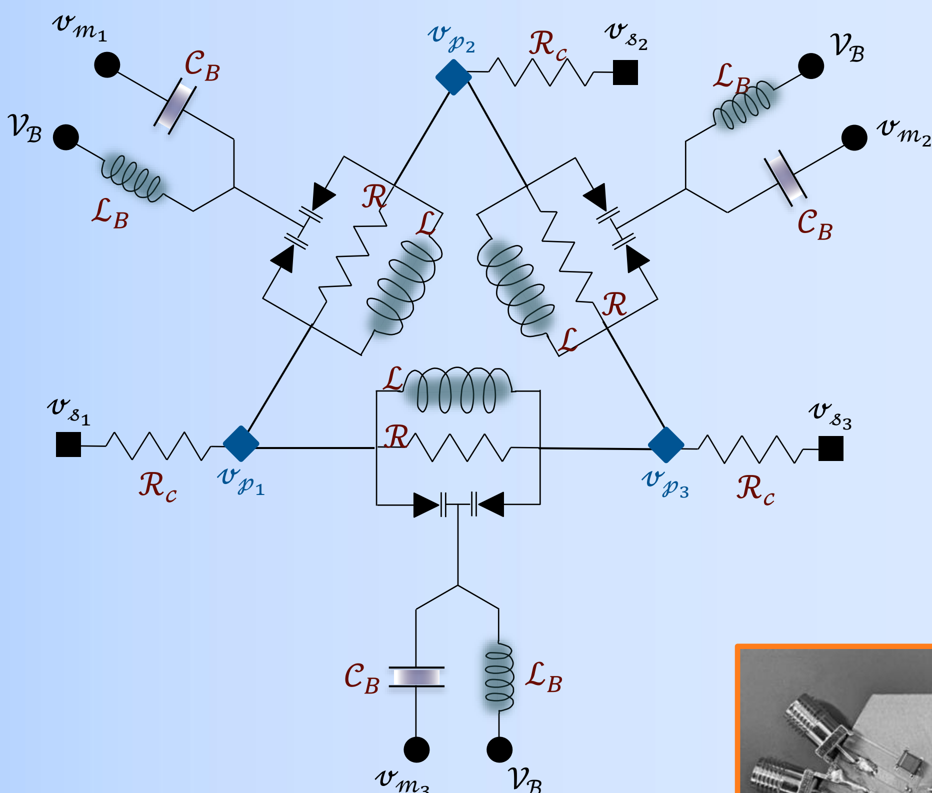


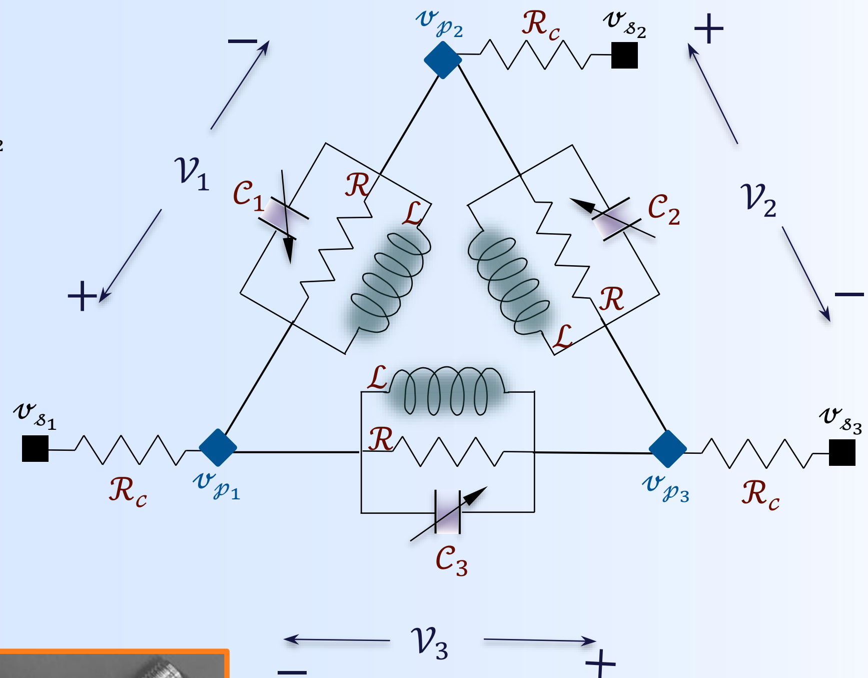
Angular-Momentum Bandgaps

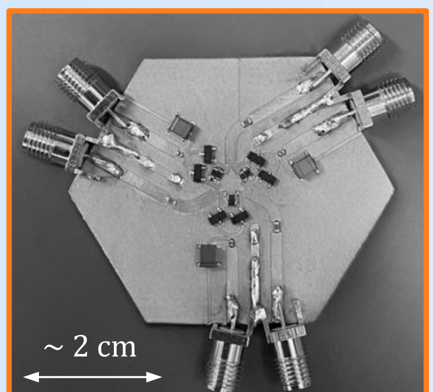




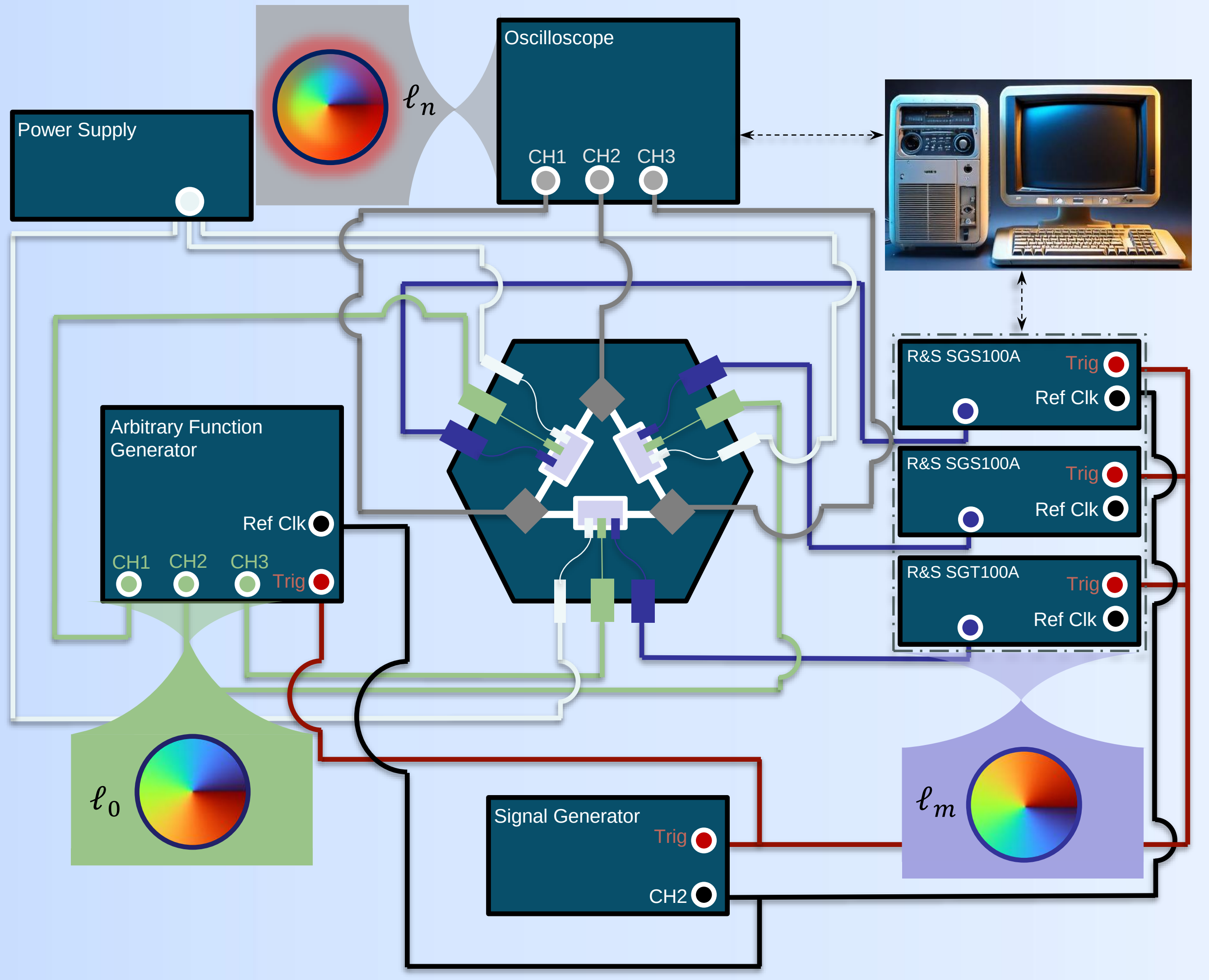
Synthetically Rotating Circuit



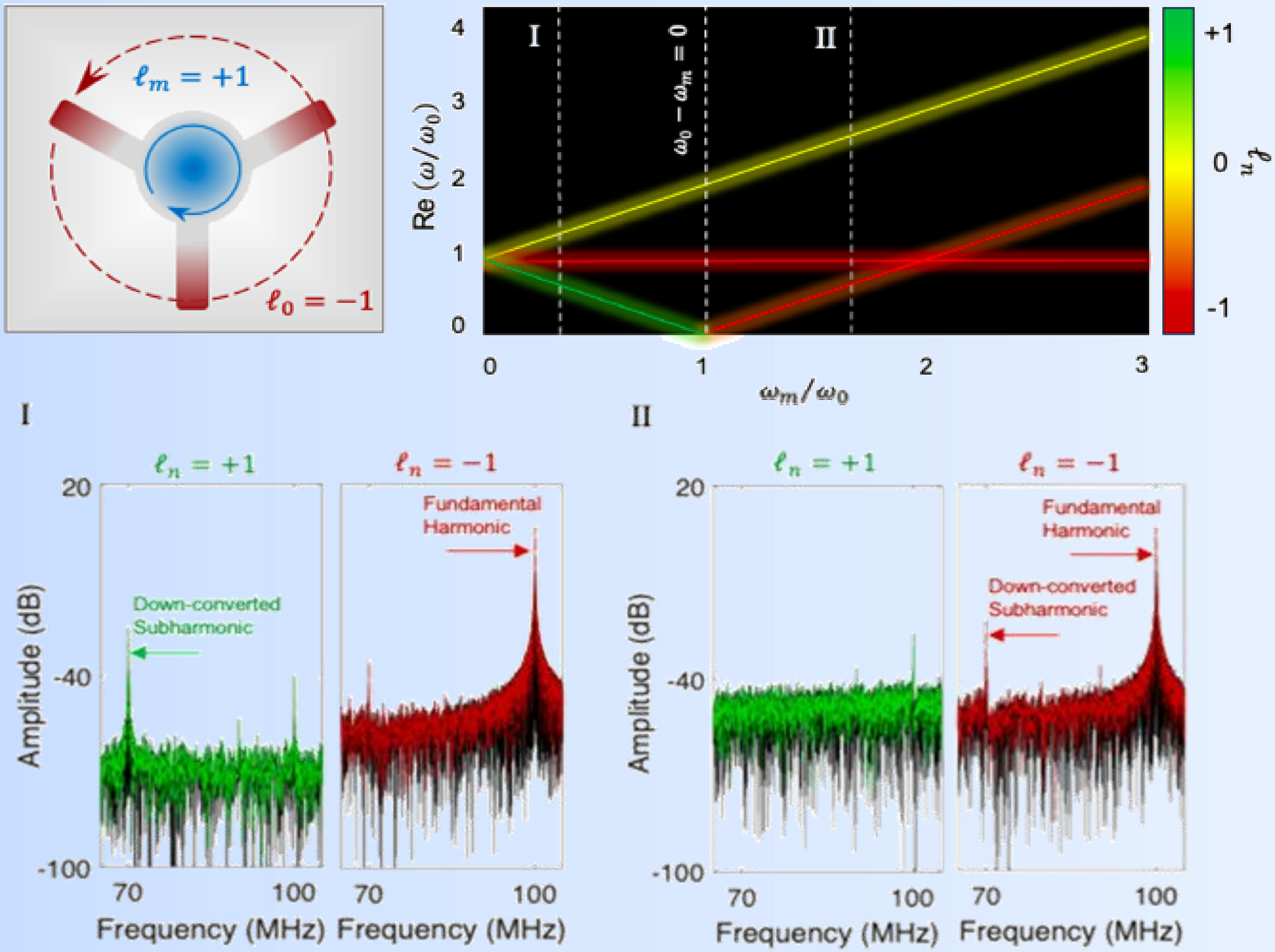




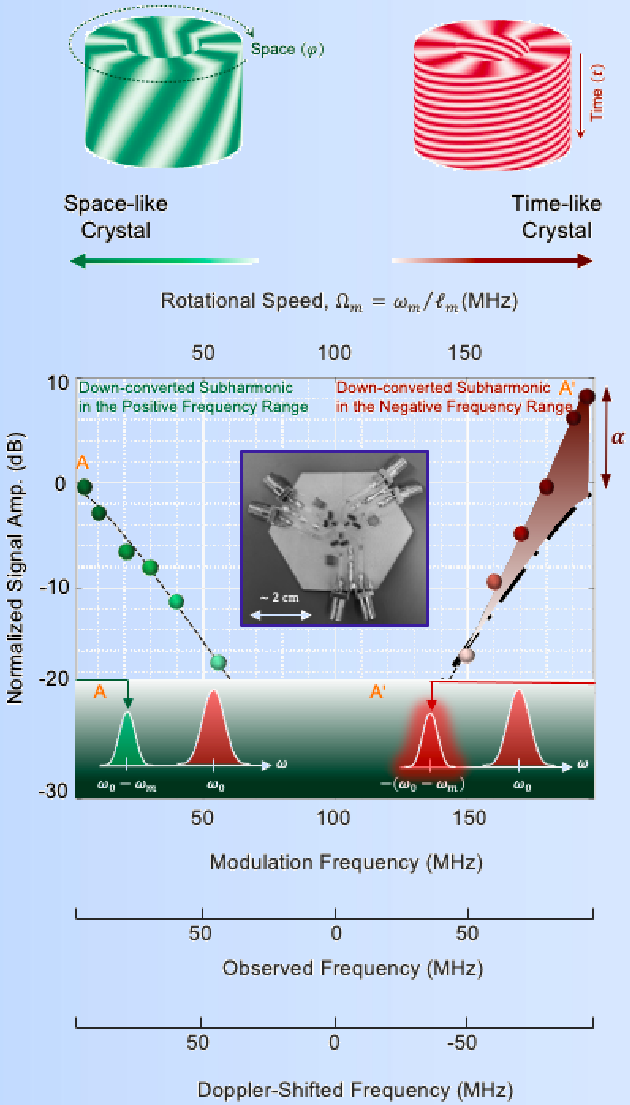
Experimental Setup

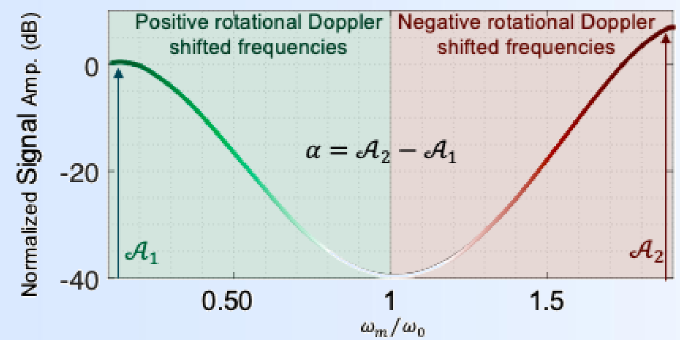


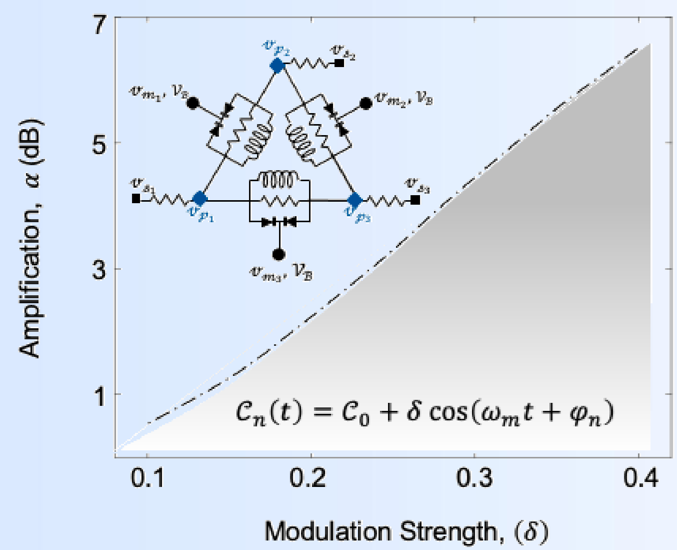
Observation of Synthetic Penrose Super-Radiance



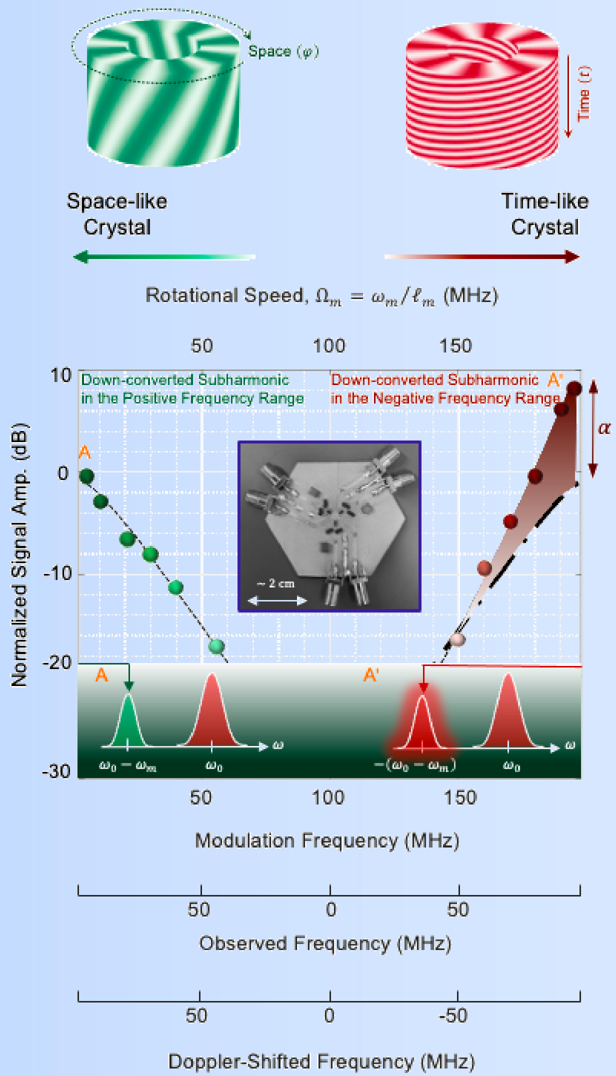
Observation of Synthetic Penrose Super-Radiance

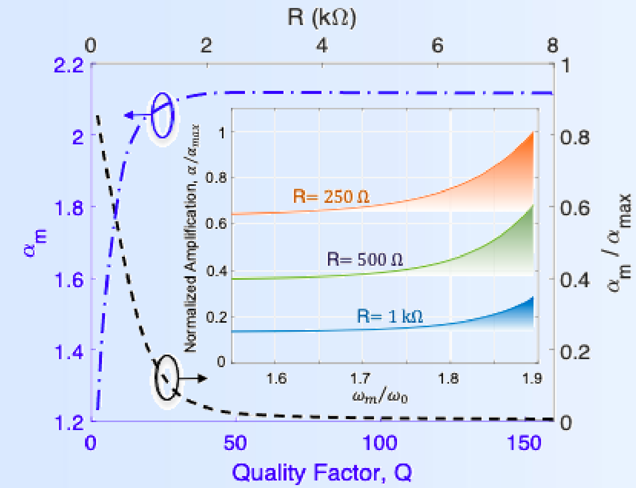


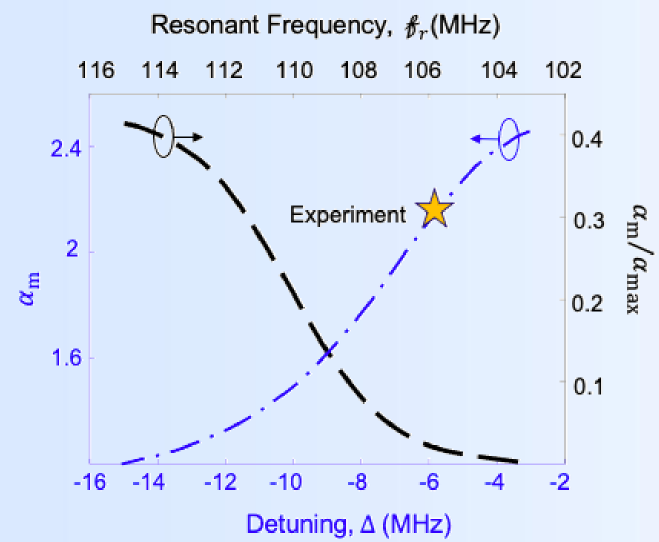




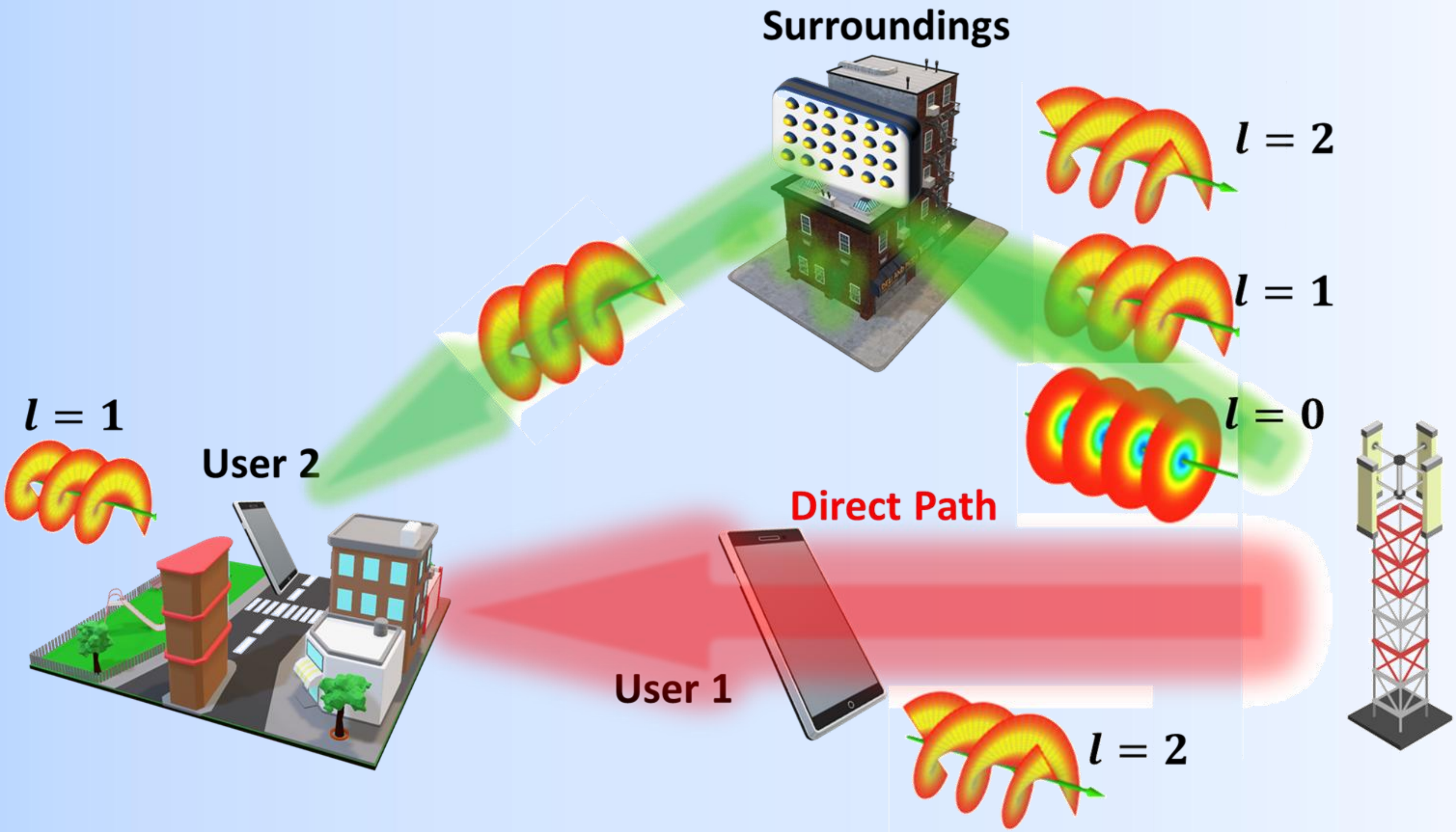
Observation of Synthetic Penrose Super-Radiance



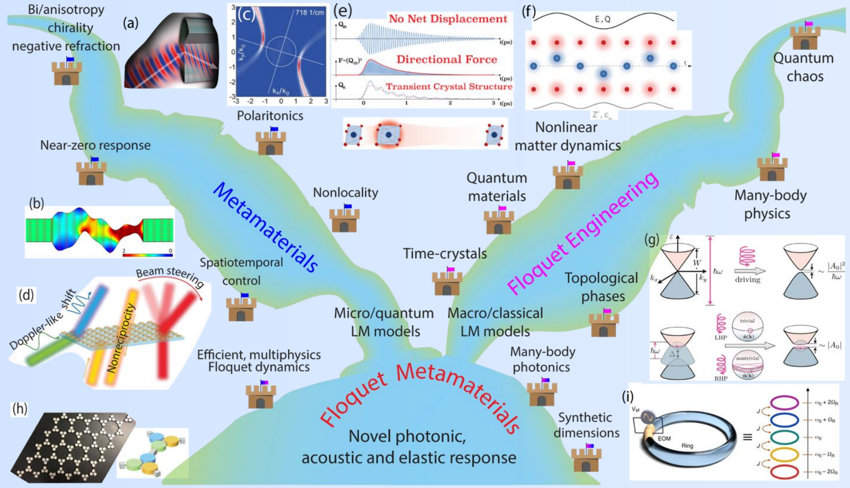




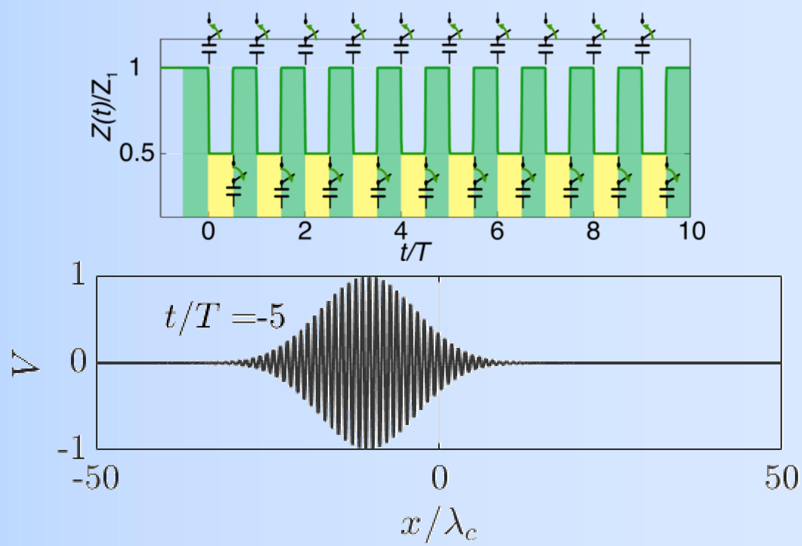
Penrose Super-Radiance for 6G Communications



TIME METAMATERIALS



Passive time crystals



Synthetic Penrose super-radiance