
A Novel Approach to Imaging Moving Targets in Complex Stationary Scenes

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Motivation

Goal: image moving targets in a complex stationary scenery consisting of numerous, randomly placed targets aka, clutter.

Existing approaches: mostly for SAR

- **Extract motion info directly from data:** Moving target indicator radars that detect Doppler shifts and newer methods that utilize optimization (e.g. robust principal component analysis).

Implementation can be tricky: careful phase synchronization and various signal processing techniques. Slow movers hard to see.

- **Work with formed images:** Autofocus methods estimate phase errors that cause smearing; Optimization methods search in dictionaries of atoms of position-velocity combinations and impose constraints, like sparsity.

Expensive computing and may not work well in strong clutter.

Motivation

MIMO radar is becoming popular. Examples are:

- Various transmission schemes e.g., orthogonal waveforms.
- Matrix filtering (SVD) methods like Space Time Adaptive Processing (STAP) use heuristic clutter response models (Brennan's rule \rightsquigarrow Gaussian and low rank noise).

Our result: Introduce a different SVD approach to imaging with MIMO (array) radar:

- Starting from first principles, we show how to get from array data a matrix that is amenable to SVD analysis.
- SVD separates moving targets from clutter, it estimates their velocity and images them.
- We give a resolution analysis and illustrate the approach with a few simulations.

Setup

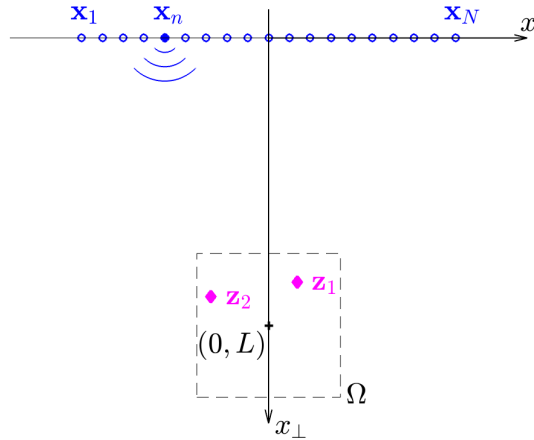
To focus on the main ideas and avoid technicalities, we introduce the following simplifications:

- Work in 2-D with a linear array (extension to 3-D) is easy.
- Born approximation (typical in radar). Not too strong multiple scattering can be handled.
- Use start-stop approximation (Doppler effects can be put in).
- Each antenna emits the probing pulse (chirps can be used)

$$f(t) = e^{-i\omega_o t} B\varphi(Bt) + \text{c.c.}$$

modulated at frequency ω_o , with bandwidth B & envelope φ .

Setup



- Antennas located at $\mathbf{x}_n = (x_n, 0)$
 $x_n = (n - N/2)\Delta a, \quad n = 1, \dots, N$

- Unknown reflectivity in Ω

$$\rho(t, \mathbf{x}) = \sum_{j=1}^M \rho_j(\mathbf{x} - \mathbf{v}_j t) + \underbrace{\gamma(\mathbf{x})}_{\text{clutter}}$$

- Data:** array response matrix $\mathbf{R}(t) = (R_{r,s}(t))_{r,s=1}^N$

Emit $f(t - T_s)$, for $T_s = s\Delta\tau$ from \mathbf{x}_s . Recordings at all antennas $\rightsquigarrow s^{\text{th}}$ column of $\mathbf{R}(t)$.

Can think of T_s as slow time and $V = \frac{\Delta a}{\Delta\tau}$ is velocity scale.

Data model

- Response matrix model

$$R_{r,s}(t) = \int_{\mathbb{R}} \frac{d\omega}{2\pi} k^2(\omega) e^{-i\omega(t-T_s)} \hat{f}(\omega) \int_{\mathbb{R}^2} d\mathbf{z} \rho(T_s, \mathbf{z}) \\ \times \hat{G}(\omega, \mathbf{z}, \mathbf{x}_r) \hat{G}(\omega, \mathbf{z}, \mathbf{x}_s) + \text{noise}$$

where $k(\omega)$ = wavenumber at frequency ω and

$$\hat{G}(\omega, \mathbf{x}, \mathbf{z}) = \frac{i}{4} H_0^{(1)}[k(\omega)|\mathbf{z} - \mathbf{x}|] \approx \frac{e^{\frac{i\pi}{4} + ik(\omega)|\mathbf{x} - \mathbf{z}|}}{2\sqrt{2\pi k(\omega)|\mathbf{x} - \mathbf{z}|}},$$

- Long range and high frequency regime:

$$L \gg a = N\Delta a \gg \lambda_o = \frac{2\pi}{k(\omega_o)}, \quad \left(\frac{a}{L}\right)^2 \ll \frac{B}{\omega_o} \ll 1$$

Standard space-velocity imaging function

- Synchronize data using matched filter

$$H_{r,s}(t; \mathbf{y}, \mathbf{u}) = \int_{\mathbb{R}} \frac{d\omega}{2\pi} e^{-i\omega t} \hat{f}(\omega) \hat{G}(\omega, \mathbf{x}_r, \mathbf{y} + \mathbf{u}T_s) \hat{G}(\omega, \mathbf{x}_s, \mathbf{y} + \mathbf{u}T_s)$$

and sum

$$I^{\text{MF}}(\mathbf{y}, \mathbf{u}) = \frac{1}{N^2} \sum_{r,s=1}^N \int_{\mathbb{R}} dt R_{r,s}(t) \overline{H_{r,s}(t - T_s; \mathbf{y}, \mathbf{u})}$$

where the bar denotes the complex conjugate.

- Without moving targets $I^{\text{MF}}(\mathbf{y}, \mathbf{u} = 0)$ peaks at points near stationary targets
- If there are moving targets, image deteriorates (blur, targets off-track). Difficult to analyze $I^{\text{MF}}(\mathbf{y}, \mathbf{u})$ because search point \mathbf{y} & velocity \mathbf{u} are tangled.

How to untangle the space and velocity variables?

- Consider a target with trajectory $\mathbf{z} + \mathbf{v}t$. With search trajectory $\mathbf{y} + \mathbf{u}t$ we have the phase*

$$\begin{aligned}
 \sum_{j \in \{s, r\}} \left(|\mathbf{x}_j - \mathbf{z} - T_s \mathbf{v}| - |\mathbf{x}_j - \mathbf{y} - T_s \mathbf{u}| \right) &\approx 2 \left[z_{\perp} - y_{\perp} + \left(\frac{z^2}{2z_{\perp}} - \frac{y^2}{2y_{\perp}} \right) \right] \\
 &+ 2x_s \left[\frac{v_{\perp} - u_{\perp}}{V} + \left(\frac{z}{z_{\perp}} \frac{v}{V} - \frac{y}{y_{\perp}} \frac{u}{V} \right) \right] - (x_s + x_r) \left[\frac{z}{z_{\perp}} - \frac{y}{y_{\perp}} \right] \\
 &+ x_s^2 \left[\frac{1}{z_{\perp}} - \frac{1}{y_{\perp}} + \left(\frac{v^2}{z_{\perp} V^2} - \frac{u^2}{y_{\perp} V^2} \right) \right] \\
 &- x_s (x_s + x_r) \left[\frac{1}{z_{\perp}} - \frac{1}{y_{\perp}} + \left(\frac{v}{z_{\perp} V} - \frac{u}{y_{\perp} V} \right) \right] \\
 &+ (x_s + x_r)^2 \frac{1}{2} \left[\frac{1}{z_{\perp}} - \frac{1}{y_{\perp}} \right].
 \end{aligned}$$

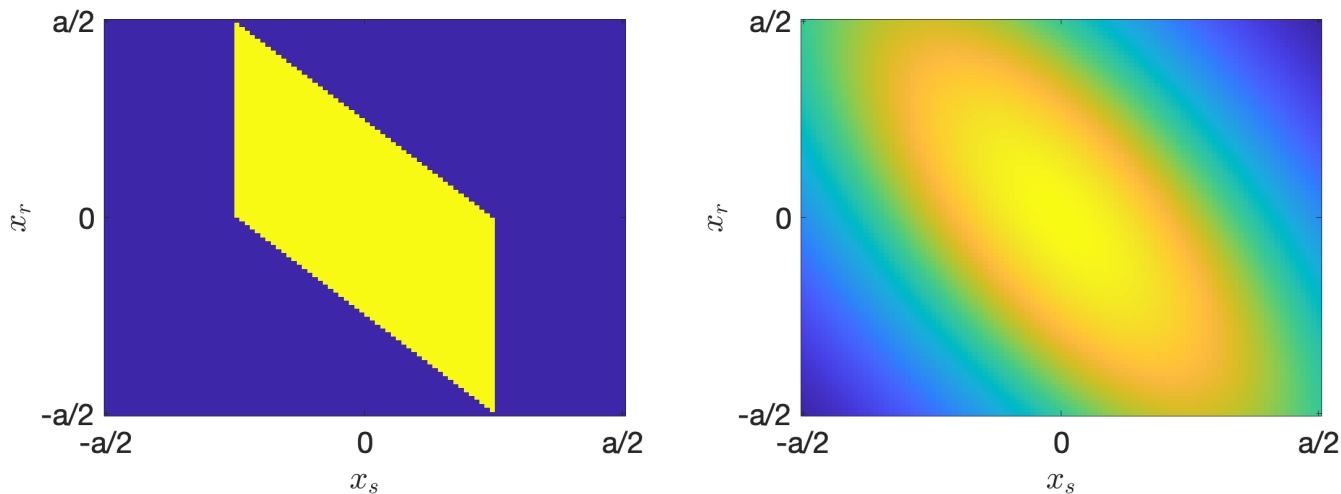
- Green terms are smaller (at least for $|v|, |u| < V$).
- Summing over x_s and mid-point $(x_r + x_s)/2 \rightsquigarrow$ a space-velocity imaging function that is separable.

*System of coordinates $\mathbf{z} = (z, z_{\perp})$ with range z_{\perp} orthogonal to array.

Proposed space-velocity imaging function

$$I(\mathbf{y}, \mathbf{u}) = \frac{4}{N(N+2)} \sum_{s=N/4}^{3N/4-1} \sum_{r=3N/4-s}^{5N/4-s} \int_{\mathbb{R}} dt R_{r,s}(t) \overline{H_{r,s}(t - T_s; \mathbf{y}, \mathbf{u})},$$

Assume that N is a multiple of 4 and sum over the pairs (r, s) illustrated in the plot



- In analysis we use Gaussian appodization:

$$(x_s, x_r) \mapsto \exp[-2x_s^2/a^2 - 2(x_s + x_r)^2/a^2]$$

Analysis of space-velocity imaging function

Theorem: Consider a moving target with reflectivity $\rho(\mathbf{x} - \mathbf{v}t)$ in an otherwise empty imaging scene. If cross-range components of target and search velocity satisfy

$$\frac{|v - u|}{V} \ll \frac{\lambda_o L}{a^2} \quad \text{and} \quad \frac{|u|}{V}, \frac{|v|}{V} = O(1)$$

then

$$I(\mathbf{y}, \mathbf{u}) \approx \frac{1}{64\pi^2 y_{\perp}^2} e^{-\frac{k_o^2 a^2}{2} \left(\frac{v_{\perp} - u_{\perp}}{V} \right)^2} \int_{\mathbb{R}^2} d\mathbf{z} \rho(\mathbf{z}) e^{-\frac{(k_o a)^2}{8} \left(\frac{z - y}{y_{\perp}} \right)^2} \\ \times F \left[\frac{2}{c} \left(y_{\perp} - z_{\perp} + \frac{y^2}{2y_{\perp}} - \frac{z^2}{2z_{\perp}} \right) \right]$$

where $k_o = k(\omega_o)$ and $F(t) = B\Phi(Bt)e^{-i\omega_o t} + \text{c.c.}$ with envelope

$$\Phi(h) = \int_{\mathbb{R}} dh' \overline{\varphi(h')} \varphi(h' + h)$$

Analysis of space-velocity imaging function

- Function $I(\mathbf{y}, \mathbf{u})$ is separable in space and velocity variables.
- $I(\mathbf{y}, \mathbf{u})$ is imaging function because it peaks at $\mathbf{y} = \mathbf{z} \in \text{supp}(\rho)$ and $u_{\perp} = v_{\perp}$. Resolution of localization in cross-range is $\sim \lambda_o L/a$ and in range is $\sim c/B$. The range velocity resolution is $\sim V\lambda_o/a$.
- If cross-range velocity does not satisfy assumption, statement of Theorem is modified: $I(\mathbf{y}, (0, u_{\perp}))$ peaks at $\mathbf{y} = \mathbf{z}$ and at

$$u_{\perp} = v_{\perp} + \frac{z}{z_{\perp}}v$$

In our long range regime $z \ll y_{\perp} \sim L$ so the bias is small.

- Because of paraxial regime, cannot determine cross-range velocity so we search at $\mathbf{u} = (0, u_{\perp})$. Secondary array with aperture oblique to range direction can give cross-range velocity.

Selective imaging

- Use superposition for: $\rho(t, \mathbf{x}) = \sum_{j=1}^M \rho_j(\mathbf{x} - \mathbf{v}_j t) + \underbrace{\gamma(\mathbf{x})}_{\text{clutter}}$
- Sampling at $(\mathbf{y}_p)_{p=1}^P$ in Ω and $(u_{\perp,q})_{q=1}^Q$ in $[V_{\min}, V_{\max}] \rightsquigarrow$

$$\mathbf{I} = \left(I(\mathbf{y}_p, (0, u_{\perp,q})) \right)_{1 \leq p \leq P, 1 \leq q \leq Q} = \mathbf{\Gamma} \mathbf{\Sigma} \mathbf{\Theta}^*$$

This is the SVD with singular values σ_j stored in $\mathbf{\Sigma}$, left singular vectors stored in $\mathbf{\Gamma}$ and right singular vectors in $\mathbf{\Theta}$

Selective imaging

Scenario 1: Clutter is stronger than M moving targets and these have different range velocity:

- Leading σ_1 is for clutter and left singular vector Γ_1 , with rows corresponding to $(y_p)_{p=1}^P$, contains the image of clutter. The right singular vector Θ_1 peaks at $u_{\perp} = 0$.

In practice, neglected corrections to formula in theorem may give more than one leading singular value due to clutter. When searching for moving targets we can get rid of these as they do not display clear peaks at search velocity $u_{\perp} \neq 0$

- Other M non-negligible singular values are for moving targets. Right singular vectors peak at target velocity and left singular vectors give the images.

Numerical simulations

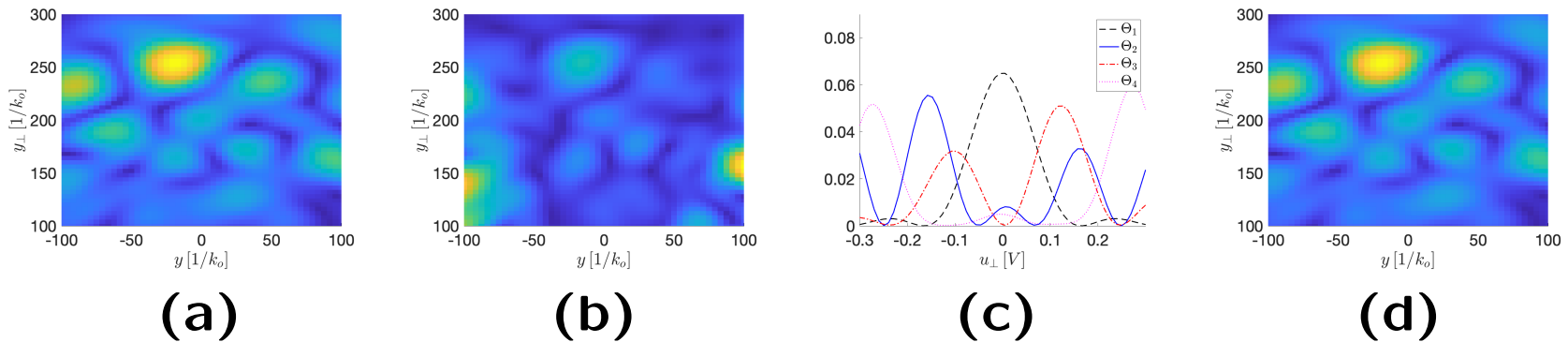
- Clutter consists of 1000 stationary, point scatterers independent and identically distributed with the uniform distribution in Ω . Their reflectivities are random, with zero-mean normal distribution and standard deviation equal to a one tenth of the reflectivity of the moving targets. Their cumulative effect gives a speckled image that masks the moving targets.

- Simulation parameters:

$k_o L = 200, k_o a = 60$ with $N = 40$ antennas and $V = 1.5$. Envelope of pulse is $\hat{\varphi}(w) = \exp(-4w^2)1_{[-1,1]}(w)$ and $B = \omega_o/10$.

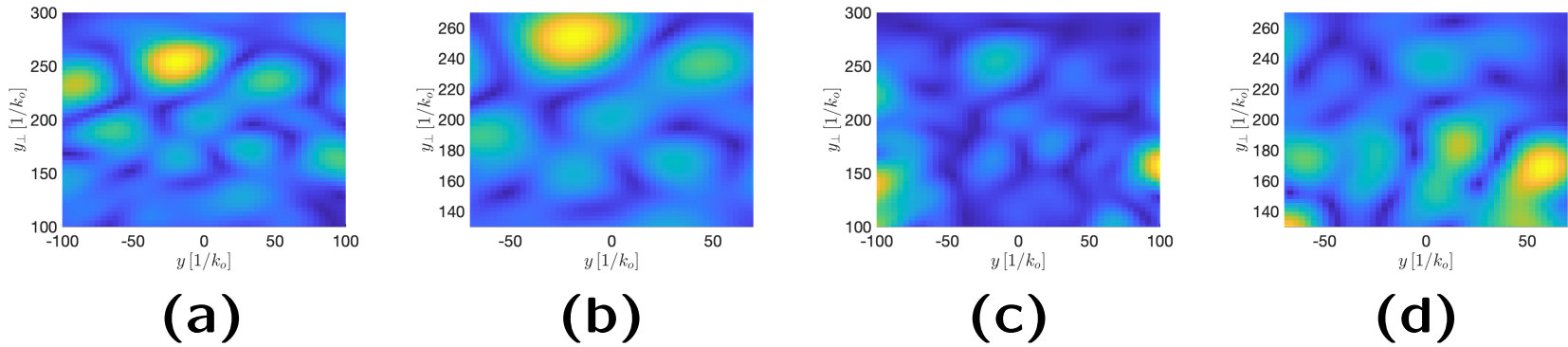
- Method is very robust to additive noise $< 200\%$ (standard deviation $\lesssim 2$ maximum of modulus of entries in $\hat{R}(\omega)$).
- We do not use Gaussian apodization in the simulations, so we have sinc-like lobes.

Scenario 1: One moving target. Search at small u_{\perp}



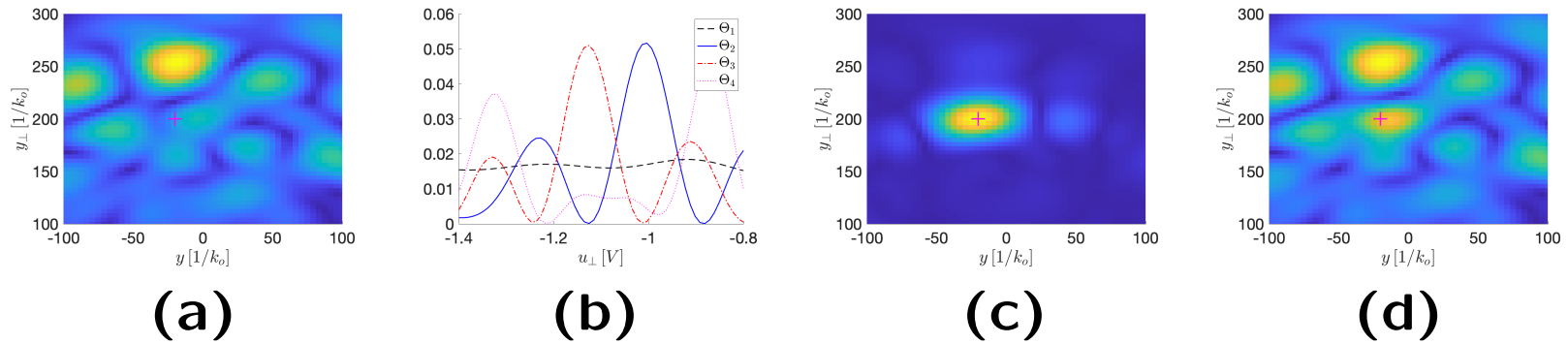
- When we sample near zero velocity, we see leading right singular vector Θ_1 peaked at $u_{\perp} = 0$ (Fig. (c)). The image stored in left singular vector Γ_1 (Fig. (a)) is basically the same as $I(y, (0, u_{\perp}))$ (Fig. (d)) because clutter dominates.
- Second singular value $\sigma_2 = 0.06\sigma_1$ is due to corrections to formula in Theorem. Image (b) given by Γ_2 does not correspond to a target. To make sure, we sample on a sub-domain (zoom) and see if we observe a zoom of an image or something unrelated.

Scenario 1: ZOOM search



- Zoom search reveals that image given by Γ_1 is real (Fig. (a-b)).
- Image given by Γ_2 is not for a real target. It is due to corrections and can be discarded. Fig (d) is not a zoom of Fig. (c).

Scenario 1: Single moving target



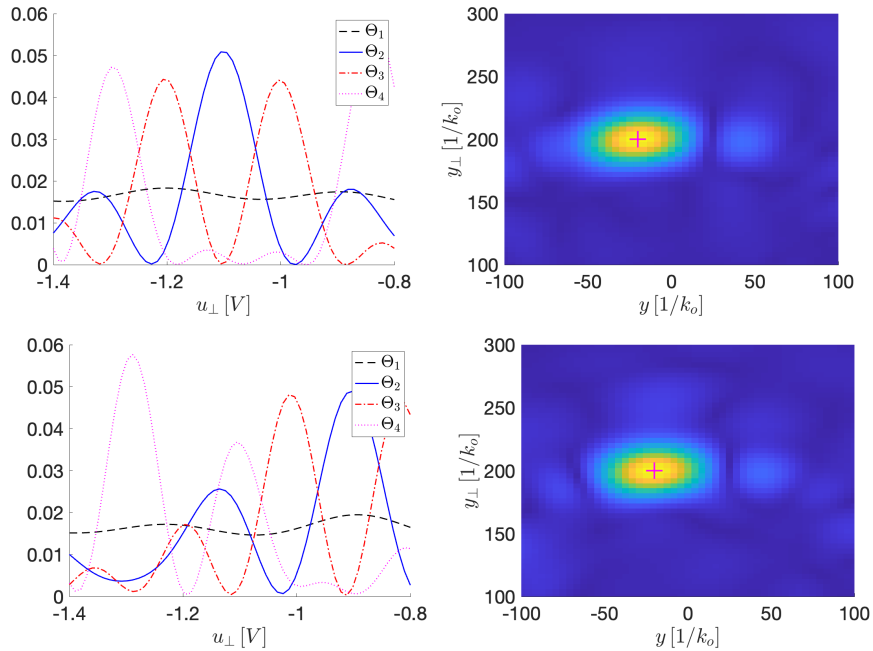
(a) Clutter image given by first left singular vector Γ_1 . The right singular vector Θ_1 has no peak (Fig (b))

(b) Second right singular vector (blue line) peaks at true target velocity $\mathbf{v} = (0, -1)V$. Here $\sigma_2 = 0.129\sigma_1$.

(c) Image contained in second left singular vector Γ_2 peaks at the true target location.

(d) Image $I(\mathbf{y}, \mathbf{v})$ is not as good due to clutter \rightsquigarrow optimization cannot be expected to do as well.

Scenario 1: Single target that moves in cross-range

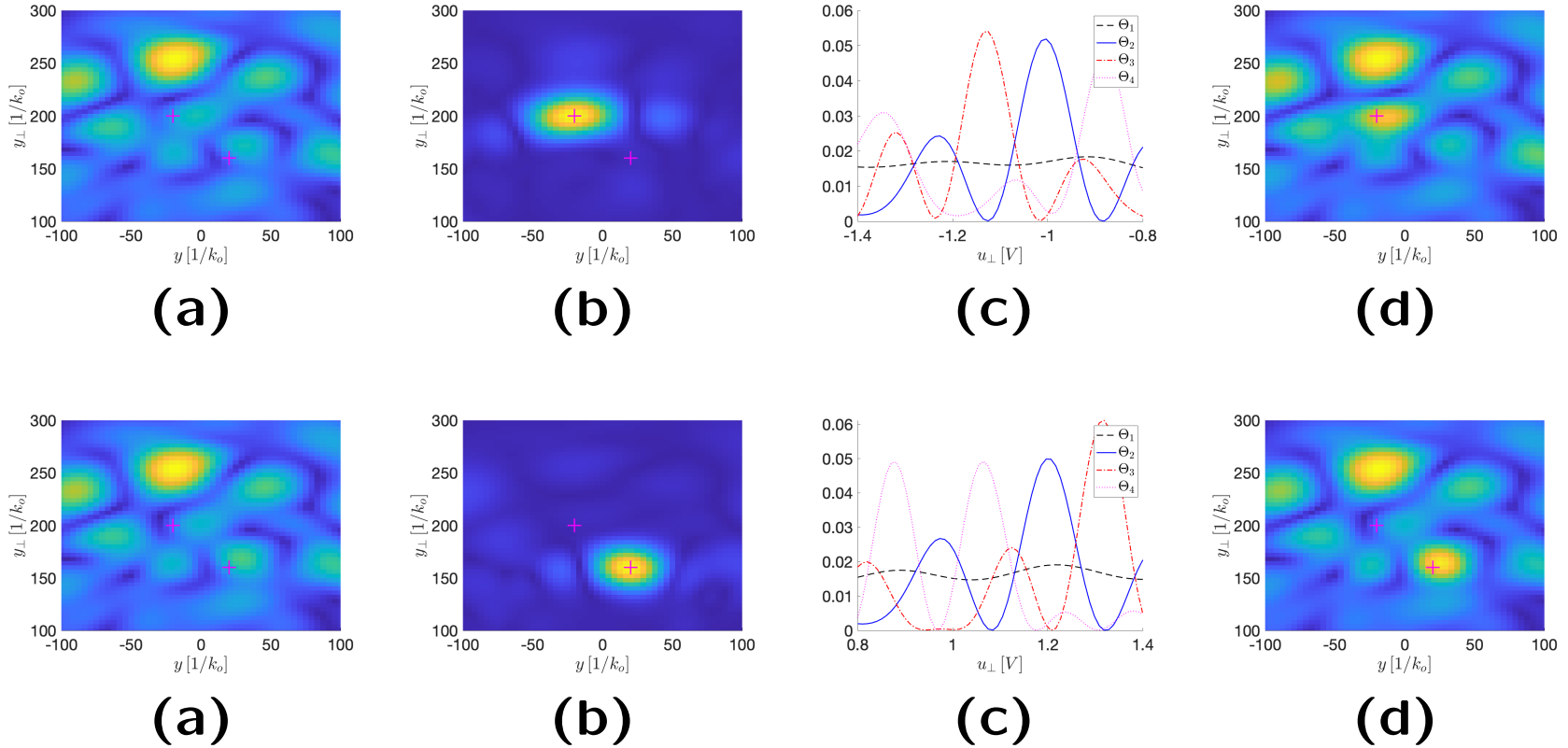


$$\mathbf{v} = (1, 1)V$$

$$\mathbf{v} = (1, -1)V$$

- Images are focused in the right place.
- Small bias in the velocity estimation, as predicted by theory.

Scenario 1: Two moving targets



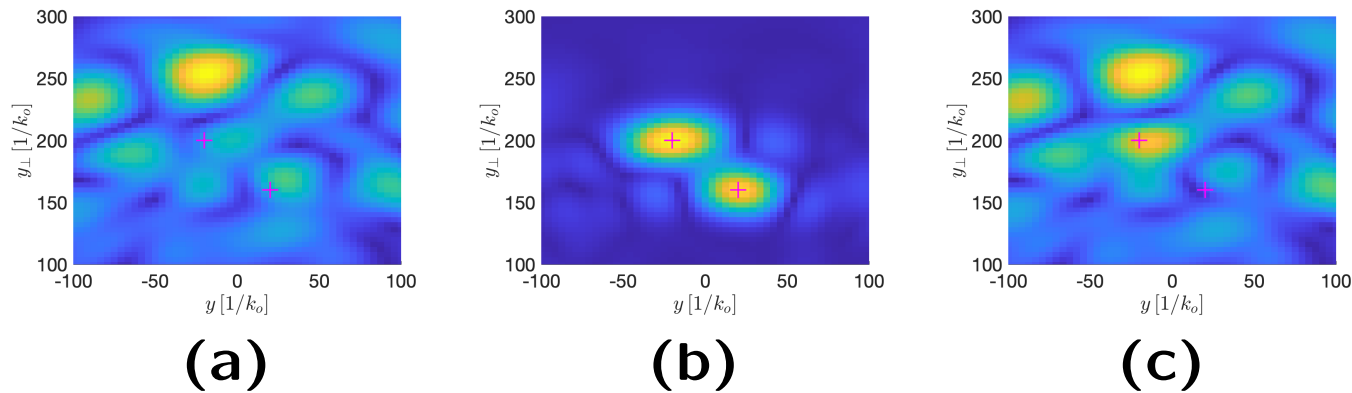
- Top plots show target with $\mathbf{v} = (0, -V)$ and the bottom ones are for target with speed $\mathbf{v} = (0, 1.2V)$.

- Fig. (a) shows image of clutter in Γ_1 . Fig. (b) shows image of moving target in Γ_2 . Fig. (c) shows right singular vectors. Fig. (d) shows $I(\mathbf{y}, \mathbf{v})$.

Selective imaging

Scenario 2: Moving targets have nearby range velocity:

- Leading σ_1 is for clutter, that dominates. Moving targets with nearby velocity should be represented by the same singular value.



- Fig. (a) is clutter image in Γ_1 .
- Fig. (b) is image in Γ_2 of two moving targets with $\mathbf{v}_1 = (0, -V)$ and $\mathbf{v}_2 = (0, -1.2V)$.
- Fig. (c) is $I(\mathbf{y}, \mathbf{u} = \mathbf{v}_2)$

Publications in 2024

- L Borcea, J Garnier, AV Mamonov, J Zimmerling, *When data driven reduced order modeling meets full waveform inversion*, SIAM Review, Vol. 66, No. 3 (2024), p. 501-532.
- L Borcea, Y Liu, J Zimmerling, *Electromagnetic inverse wave scattering in anisotropic media via reduced order modeling*, Journal of Computational Physics, Volume 515 (2024), 113272.
- L Borcea, J Garnier, *Enhanced wave transmission in random media with mirror symmetry*, Proceedings of the Royal Society A, Vol. 480, Issue 2292 (2024), 20240073.
- L Borcea, J Garnier, *Moving targets imaging by SVD of a space-velocity MIMO radar data driven matrix*, submitted to IEEE Transactions on computational imaging.