

2025 Annual EM Portfolio Review
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RANDOM MAGNETS AS MICROWAVE ABSORBERS: DYNAMICAL SCALING AND COHERENT ANISOTROPY

by

Eugene M. Chudnovsky
CUNY Lehman College and Graduate School



AFOSR AWARD NUMBER
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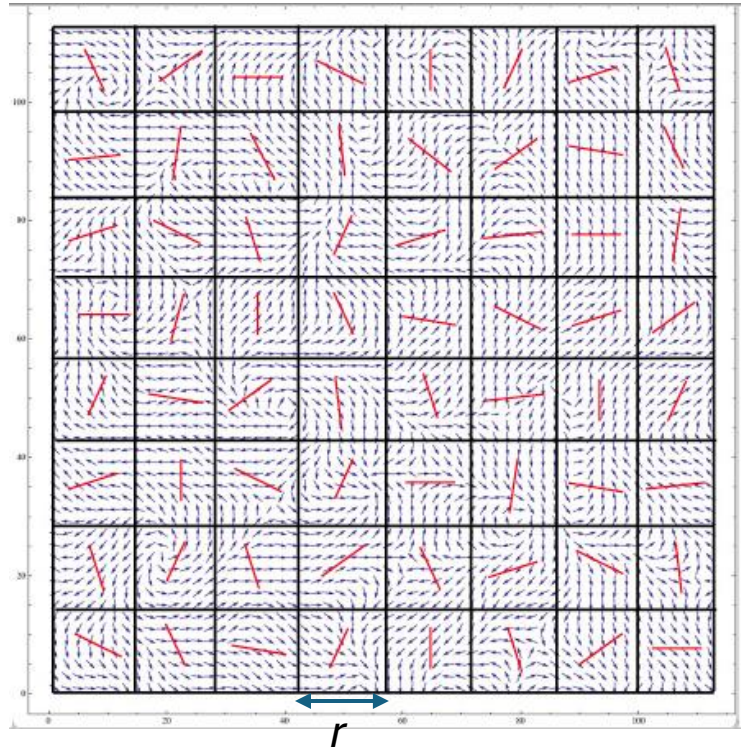
CUNY Researchers Supported:

Prof. E. M. Chudnovsky - PI
Prof. D. A. Garanin
Postdoc J. F. Soriano



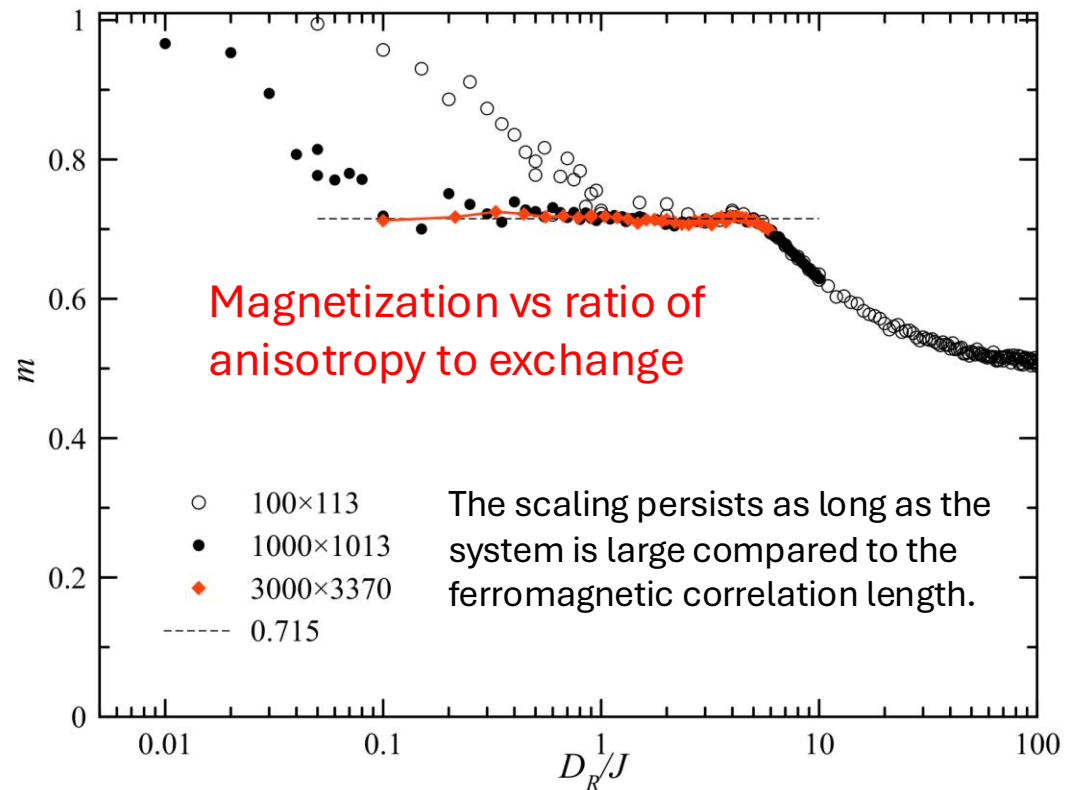
Scaling of Static Randomness

EC & Garanin, Scaling Theory of Magnetic Order and Microwave Absorption in Amorphous and Granular Ferromagnets, Physical Review B **109**, 054429 (2024)



$$J_{eff} = J \left(\frac{r}{a} \right)^{d-2}, \quad D_{eff} = D_R \left(\frac{r}{a} \right)^{d/2}$$

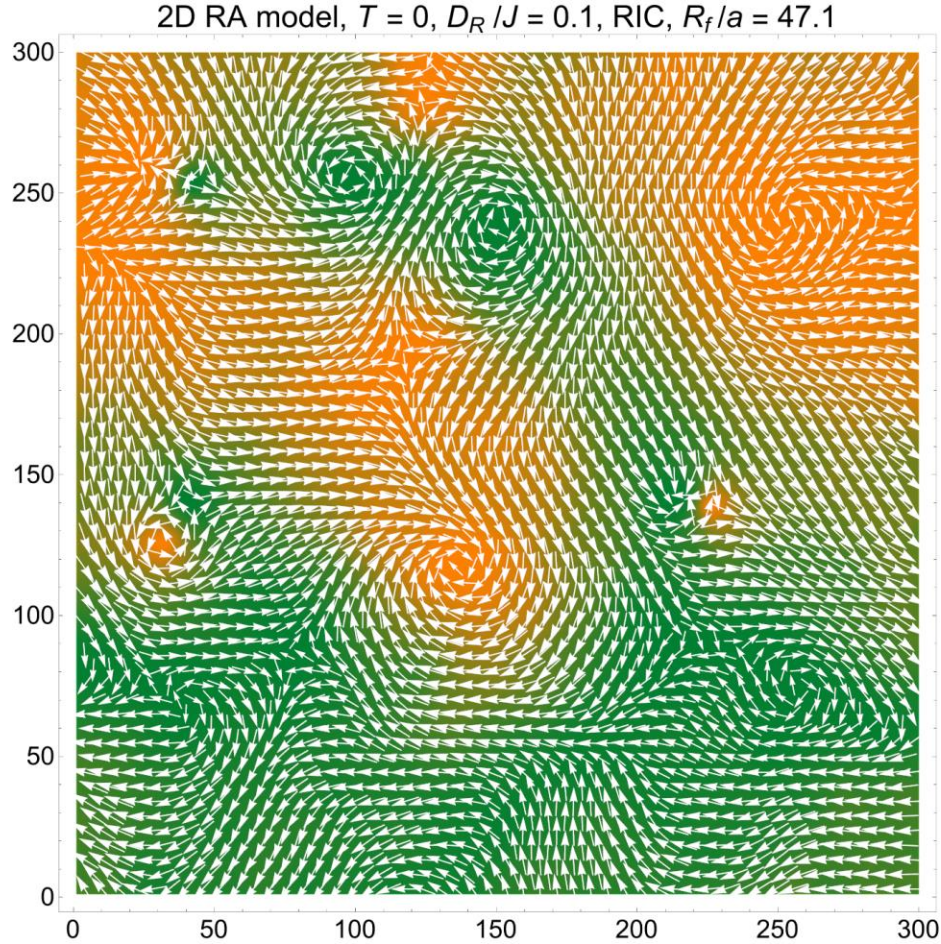
RA model: $\mathcal{H} = \frac{1}{2} \int \frac{d^d r}{a^d} [J a^2 (\nabla \mathbf{s})^2 - D_R (\mathbf{n} \cdot \mathbf{s})^2]$



$$R_f = k_d \left(\frac{J_{eff}}{D_{eff}} \right)^{2/(4-d)} \quad r = k_d \left(\frac{J}{D_R} \right)^{2/(4-d)} a$$

The random-anisotropy model with $D_R \ll J$ is mathematically equivalent to the model with $D_R \sim J$, which permits a drastic reduction of the computing time in the numerical work. Scaling breaks at $D_R \sim 5J$.

DYNAMICAL SCALING (Garanin & EC, Europhysics Letters **148**, 26001 (2024))



$$\mathcal{H} = -\frac{1}{2} \sum_{ij} J_{ij} \mathbf{s}_i \cdot \mathbf{s}_j - \frac{1}{2} D_R \sum_i (\mathbf{n}_i \cdot \mathbf{s}_i)^2$$

$$\hbar \frac{\partial \mathbf{s}_i}{\partial t} = \mathbf{s}_i \times \mathbf{H}_{\text{eff},i}, \quad \mathbf{H}_{\text{eff},i} = -\frac{\partial \mathcal{H}}{\partial \mathbf{s}_i} = \sum_j J_{ij} \mathbf{s}_j + D_R (\mathbf{n}_i \cdot \mathbf{s}_i) \mathbf{n}_i.$$

Switching to blocks of n_b spins of size $b \ll R_f$:

$$\hbar \frac{\partial \mathbf{s}_l}{\partial t} = \mathbf{s}_l \times \bar{\mathbf{H}}_{\text{eff},l}, \quad \bar{\mathbf{H}}_{\text{eff},l} = -\frac{1}{n_b} \frac{\partial \mathcal{H}}{\partial \mathbf{s}_l} = \frac{1}{n_b} \left[\sum_k J'_{lk} \mathbf{s}_k + D'_R \mathbb{T}_l \cdot \mathbf{s}_l \right]$$

$$(\mathbb{T}_l \cdot \mathbf{s}_l)_\alpha = T_{\alpha\beta,l} s_{l\beta}, \quad T_{\alpha\beta,l} \equiv \left(\frac{a}{b}\right)^{d/2} \sum_{i \in l} n_{l\alpha} n_{l\beta}, \quad \bar{\mathbf{H}}_{\text{eff},l} = \sum_k \bar{J}_{lk} \mathbf{s}_k + \bar{D}_R \mathbb{T}_l \cdot \mathbf{s}_l$$

$$\bar{J} = \left(\frac{a}{b}\right)^2 J, \quad \bar{D}_R = \left(\frac{a}{b}\right)^{d/2} D_R, \quad \frac{\bar{J}}{\bar{D}_R} = \left(\frac{a}{b}\right)^{(4-d)/2} \frac{J}{D_R},$$

The exchange to anisotropy ratio is effectively multiplied by a factor $(a/b)^{(4-d)/2} \ll 1$, which represents significant advantage for the numerical work. Since $\bar{J} \ll J$, the scaled integration step $\Delta t = 0.1\hbar/\bar{J}$ is large compared to the unscaled step $\Delta t = 0.1\hbar/J$, which speeds up the computation. Even greater advantage is achieved for sintered magnets made of ferromagnetically ordered grains of size b , for which $\bar{D}_R = D_R$.

Applying Scaling to Microwave Absorption

To compute the microwave absorption at any temperature T , one can use the fluctuation-dissipation theorem (FDT) which provides the following expression [40] for the absorbed power of microwaves of frequency ω and the amplitude of the magnetic field h_0 ,

$$P = \frac{\omega^2 h_0^2}{6Nk_B T} \text{Re} \left[\int_0^\infty dt e^{i\omega t} \langle \mathbf{S}(0) \cdot \mathbf{S}(t) \rangle \right], \quad (27)$$

where $\mathbf{S}(t) = \sum_i \mathbf{s}_i$ is the total spin of the system.

Expressing \mathbf{S} via spins of the spin blocks as

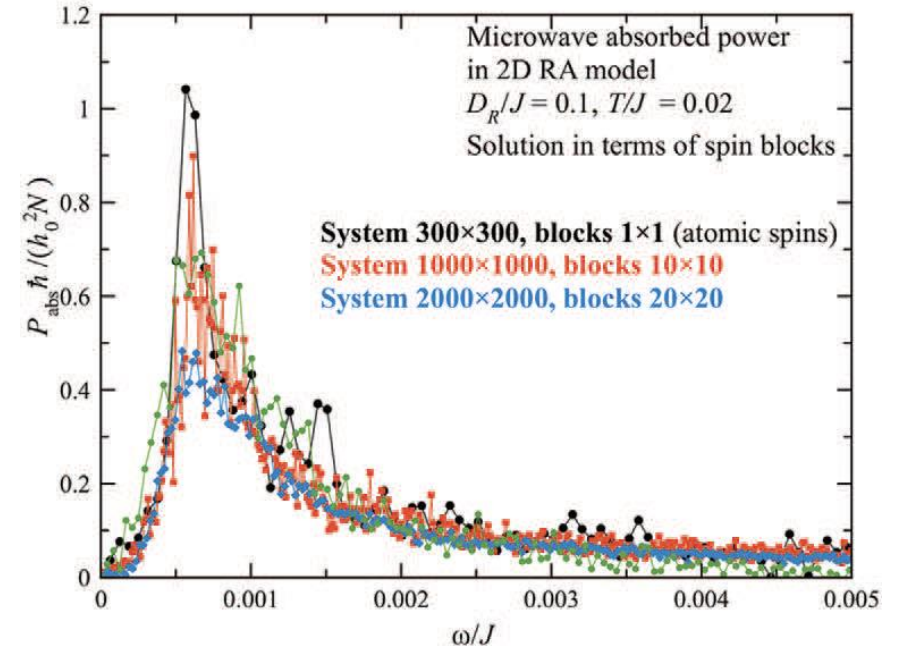
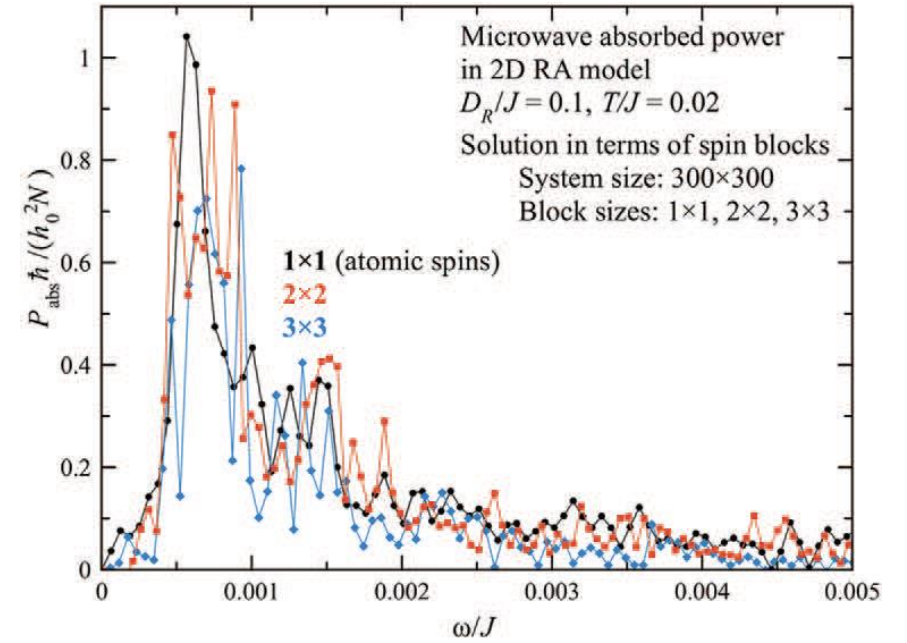
$$\mathbf{S} = n_b \sum_l \mathbf{s}_l = n_b N_b \mathbf{m}_b = N \mathbf{m}_b, \quad (28)$$

where

$$\mathbf{m}_b \equiv \frac{1}{N_b} \sum_l \mathbf{s}_l \quad (29)$$

is the average spin of the block, one can rewrite the absorbed power per spin as

$$P = \frac{\omega^2 N h_0^2}{6k_B T} \text{Re} \left[\int_0^\infty dt e^{i\omega t} \langle \mathbf{m}_b(0) \cdot \mathbf{m}_b(t) \rangle \right]. \quad (30)$$



Static and Microwave Properties of Amorphous Magnets Near Saturation

(EC & Garanin, European Physics Journal B **97**, 186 (2024))

Random-field model with external field:

$$\mathcal{H} = -\frac{1}{2} \sum_{ij} J_{ij} \mathbf{s}_i \cdot \mathbf{s}_j - \sum_i \mathbf{h}_i \cdot \mathbf{s}_i - \mathbf{H} \cdot \sum_i \mathbf{s}_i,$$

Rigorous results: $\frac{1}{k_H} = R_H = a \sqrt{\frac{J_S}{H}}$

$$3D: \quad \langle \mathbf{s}_\perp(\mathbf{r}_1) \cdot \mathbf{s}_\perp(\mathbf{r}_2) \rangle = \frac{h^2}{12\pi J^2 a^7 k_H} e^{-k_H |\mathbf{r}_1 - \mathbf{r}_2|}$$

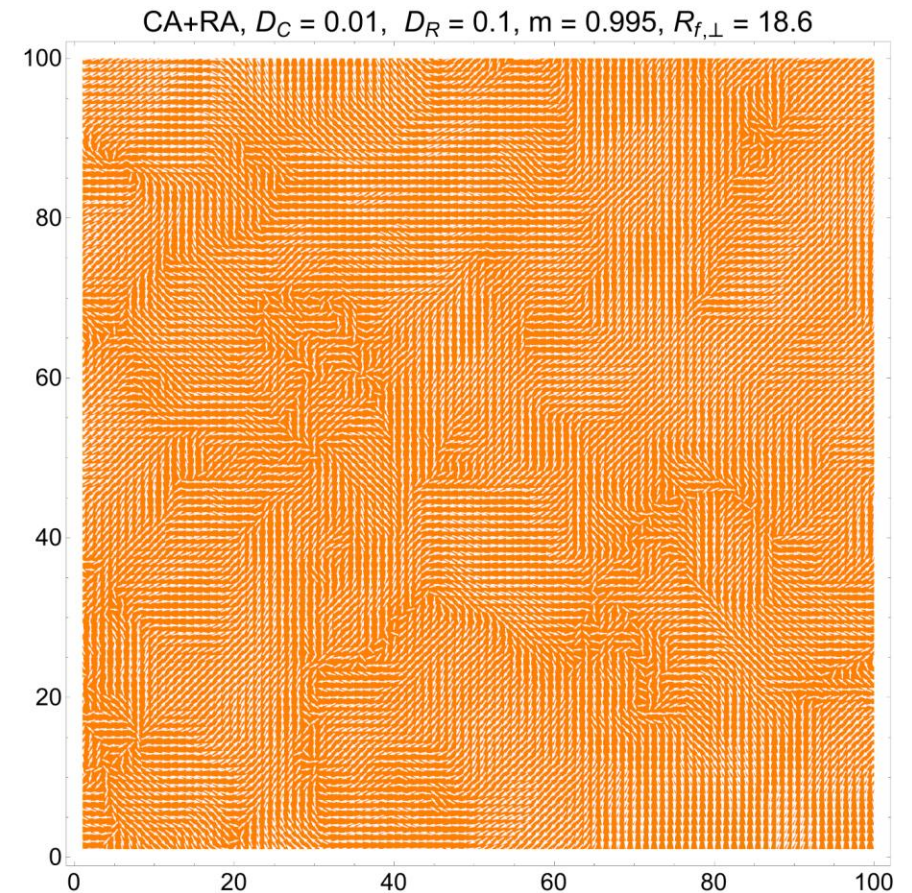
$$1 - \frac{S_z}{S} = \frac{h^2}{24\pi J^2 S^2 a^7 k_H} = \frac{1}{24\pi} \left(\frac{h}{J_S} \right)^2 \sqrt{\frac{J_S}{H}}$$

$$2D: \quad \langle \mathbf{s}_\perp(\mathbf{r}_1) \cdot \mathbf{s}_\perp(\mathbf{r}_2) \rangle = \frac{h^2}{6\pi J^2 a^6 k_H} |\mathbf{r}_1 - \mathbf{r}_2| K_1(k_H |\mathbf{r}_1 - \mathbf{r}_2|)$$

$$1 - \frac{S_z}{S} = \frac{h^2}{12\pi J^2 S^2 a^6 k_H^2} = \frac{1}{12\pi} \left(\frac{h}{J_S} \right)^2 \left(\frac{J_S}{H} \right)$$

Random-anisotropy model with coherent anisotropy:

$$\mathcal{H} = -\frac{1}{2} \sum_{ij} J_{ij} \mathbf{s}_i \cdot \mathbf{s}_j - \frac{D_R}{2} \sum_i (\mathbf{n}_i \cdot \mathbf{s}_i)^2 - \frac{D_C}{2} \sum_i (\mathbf{n}_C \cdot \mathbf{s}_i)^2.$$



Numerical simulation on a spin lattice

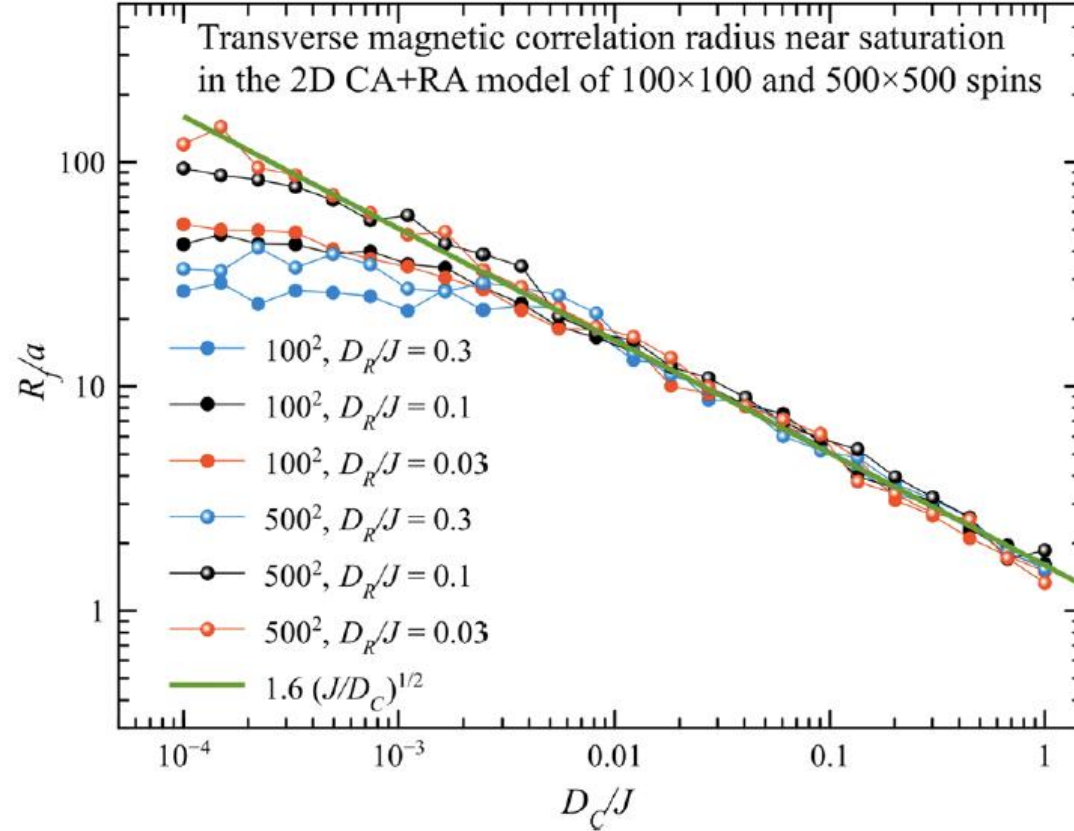


Fig. 3 Dependence of the transversal correlation length $R_{f,\perp}$ on coherent anisotropy D_C in a 2D RA model. It follows theoretical prediction $R_{f,\perp} \propto (J/D_C)^{1/2}$ [Eq. (17) for R_H with $h \Rightarrow D_C$, the straight line in the figure] as soon as $R_{f,\perp}$ becomes small compared to the linear size of the system

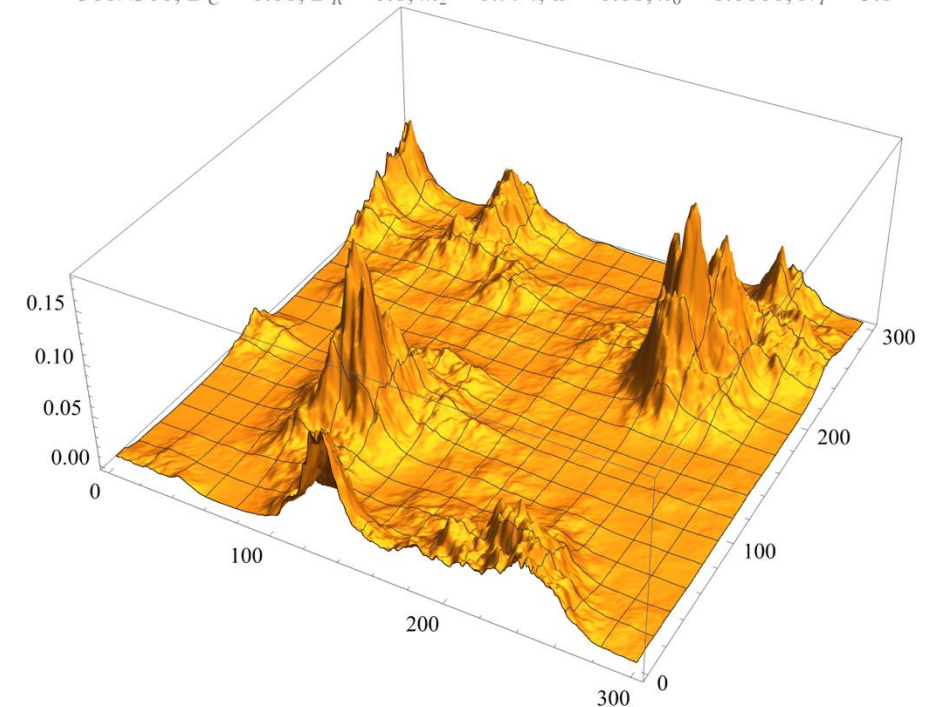
Localization of spin excitations

$$3D: \quad \langle \sigma(\mathbf{r}_1) \cdot \sigma(\mathbf{r}_2) \rangle = \langle |\sigma|^2 \rangle e^{-k_\omega |\mathbf{r}_1 - \mathbf{r}_2|}.$$

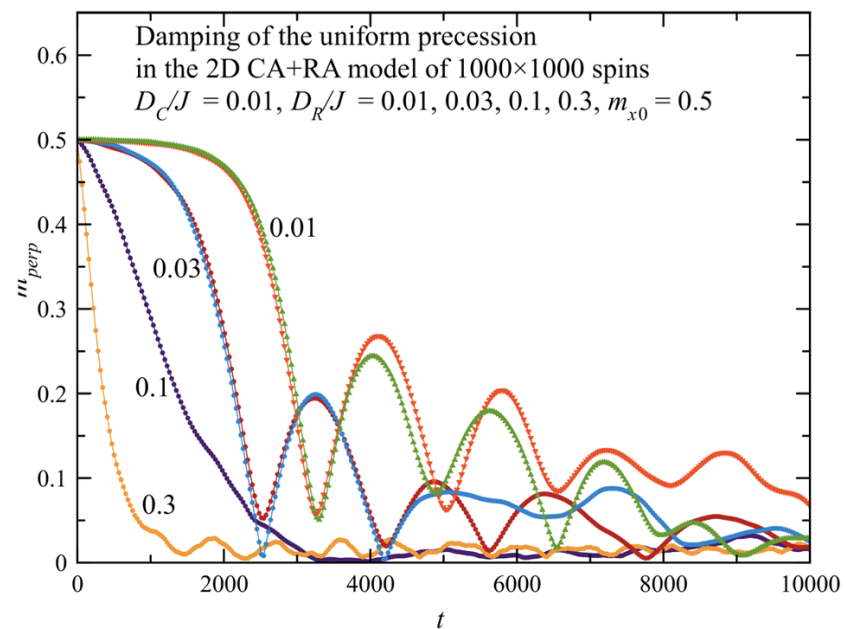
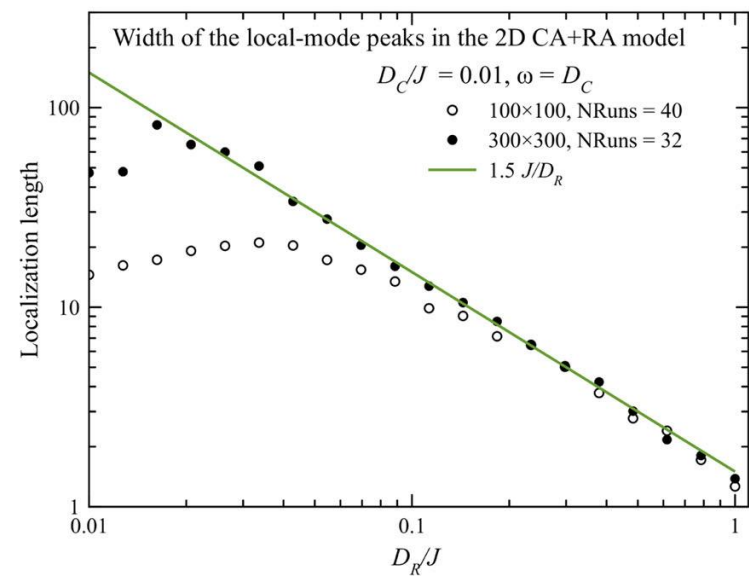
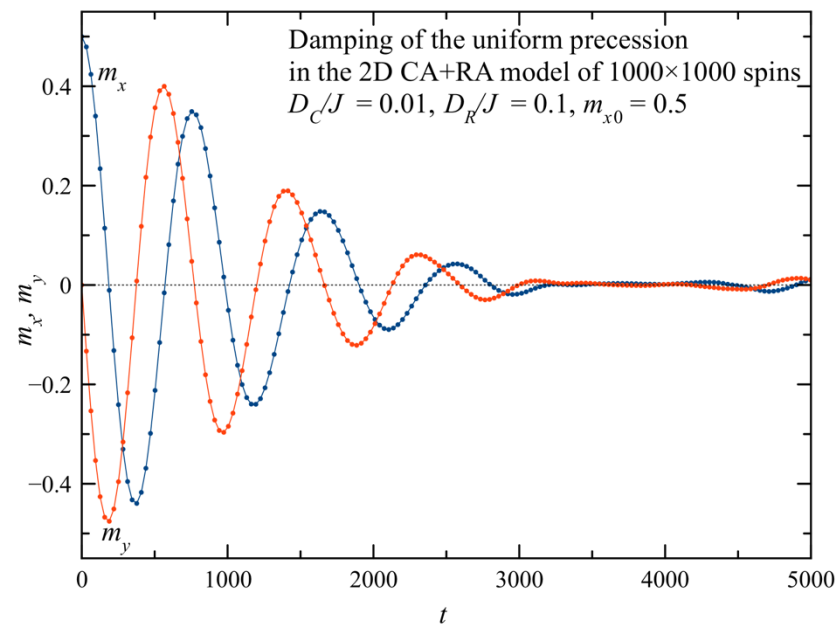
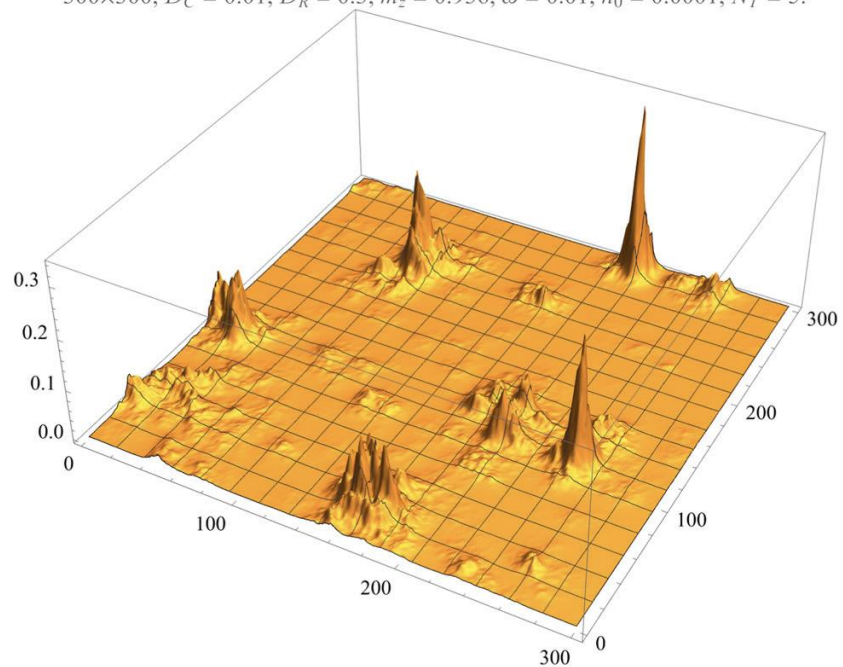
$$2D: \quad \langle \sigma(\mathbf{r}_1) \cdot \sigma(\mathbf{r}_2) \rangle = \langle |\sigma|^2 \rangle k_\omega |\mathbf{r}_1 - \mathbf{r}_2| K_1(k_\omega |\mathbf{r}_1 - \mathbf{r}_2|)$$

$$k_\omega^2 = \frac{H - \hbar\omega}{Jsa^2} = \frac{4}{R_f^2}, \quad \hbar\omega = H \left[1 - \left(\frac{2R_H}{R_f} \right)^2 \right]$$

$$300 \times 300, D_C = 0.01, D_R = 0.1, m_z = 0.994, \omega = 0.01, h_0 = 0.0001, N_T = 5.1$$

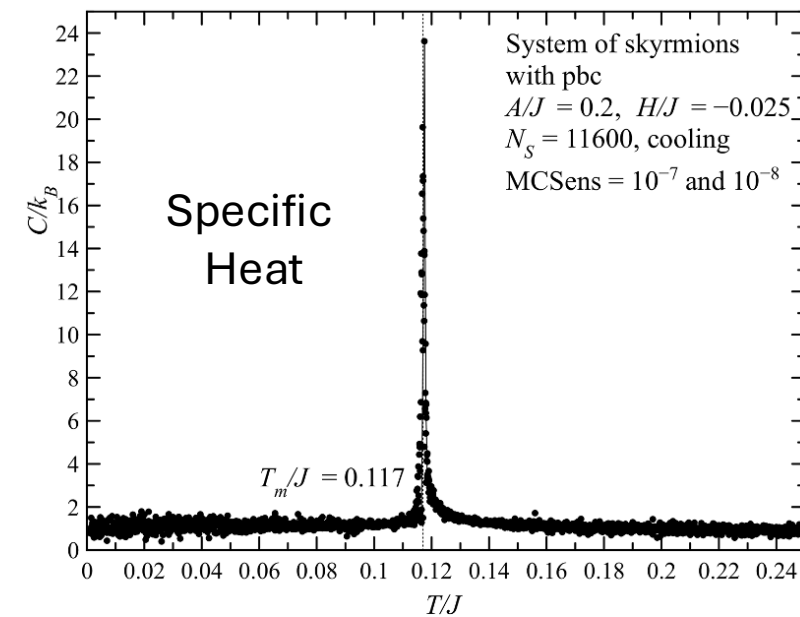
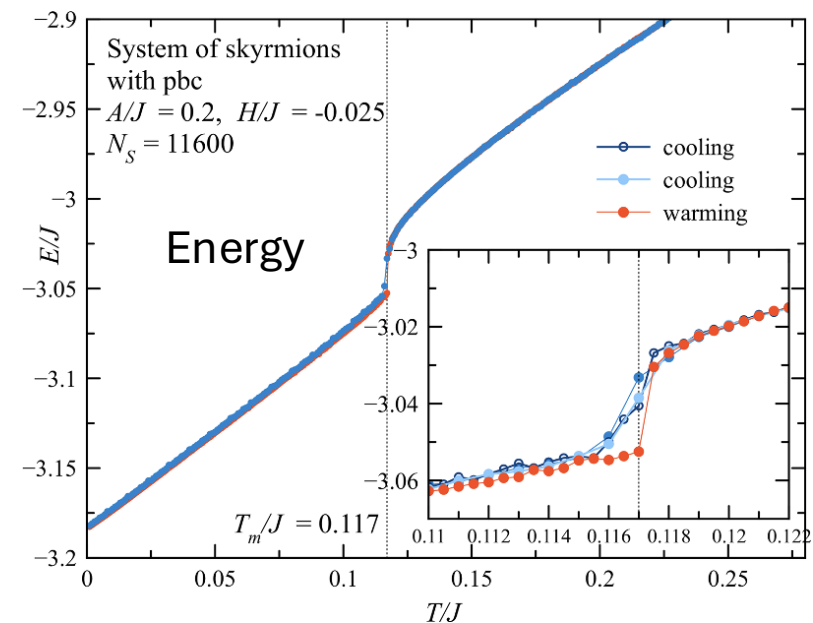
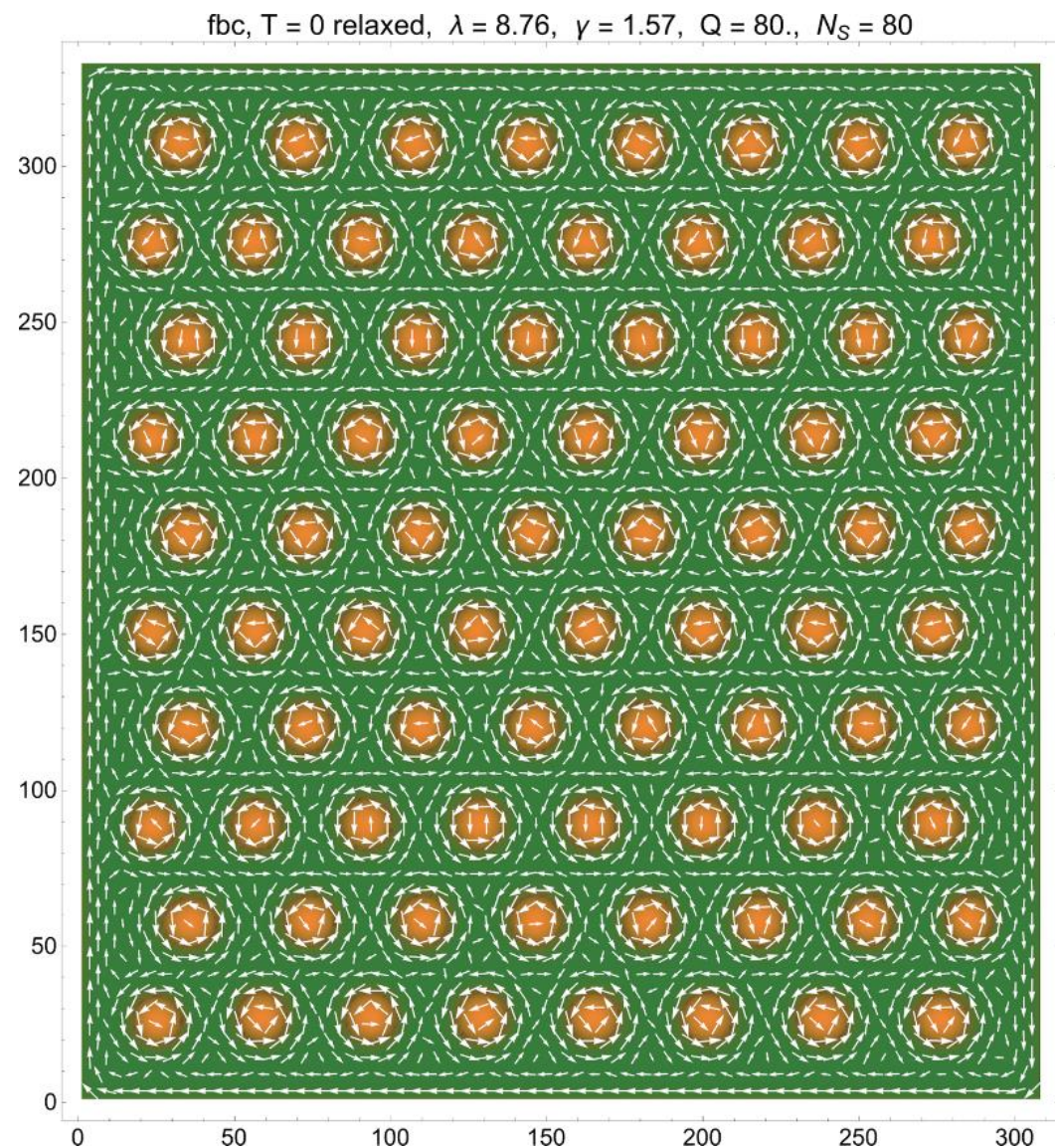


300×300 , $D_C = 0.01$, $D_R = 0.3$, $m_z = 0.936$, $\omega = 0.01$, $h_0 = 0.0001$, $N_T = 5$.



2D Magnets with Skyrmion Lattices

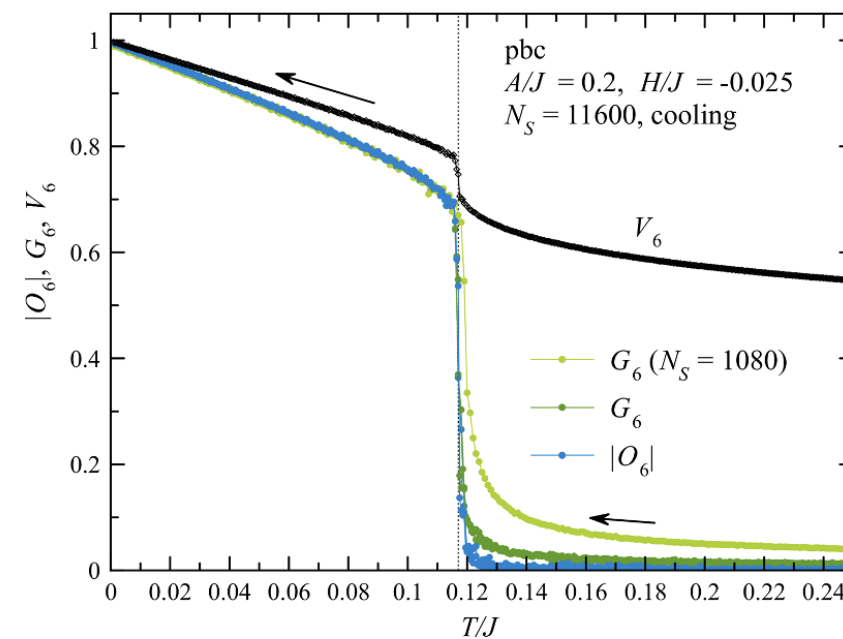
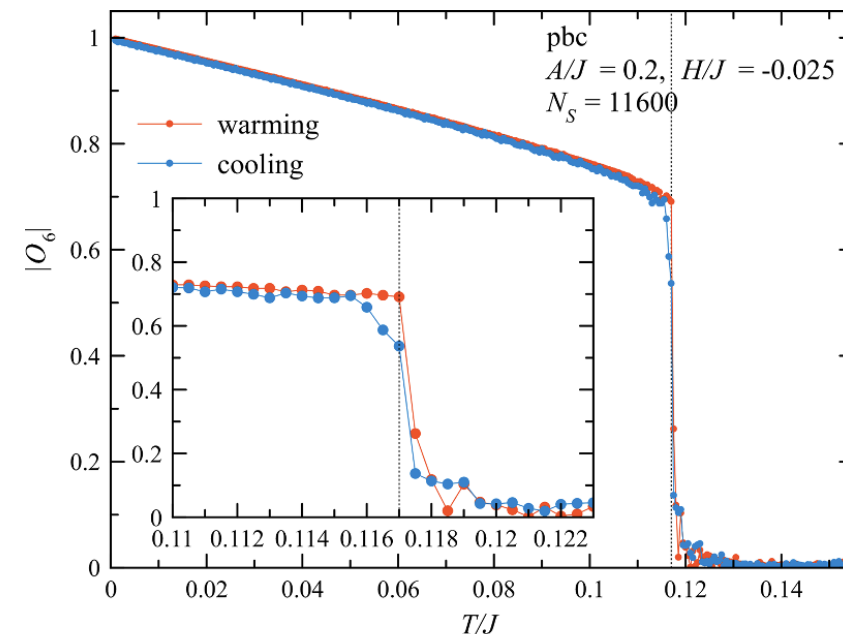
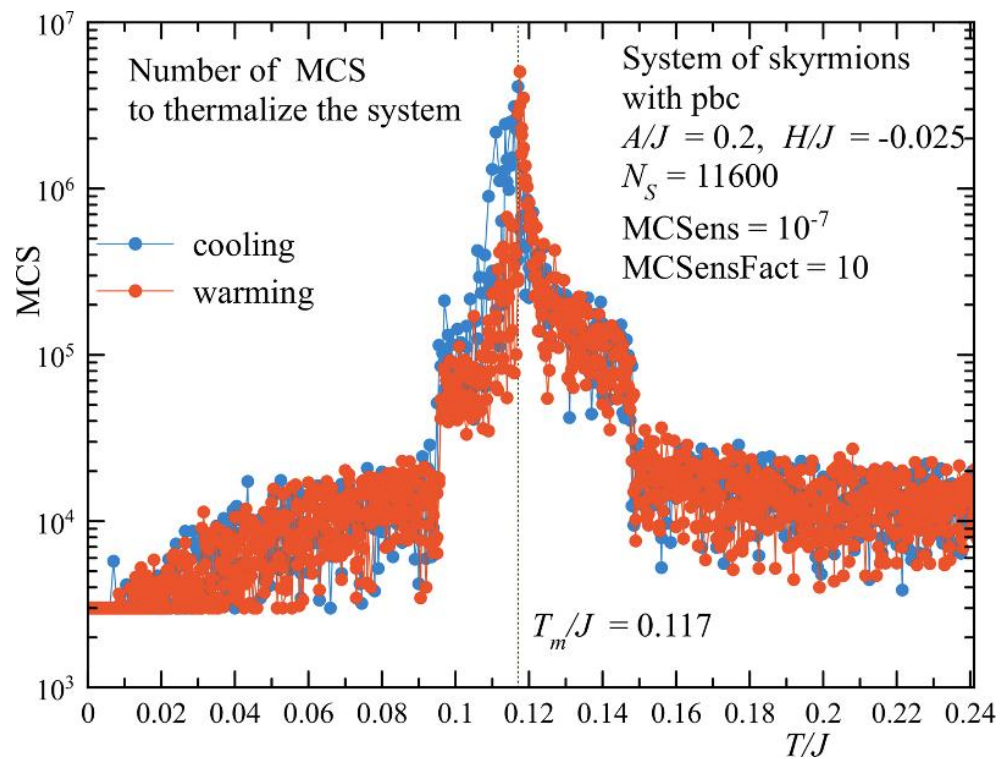
(Garanin, Soriano, & EC, Journal of Physics C **36**, 475802 (2024))

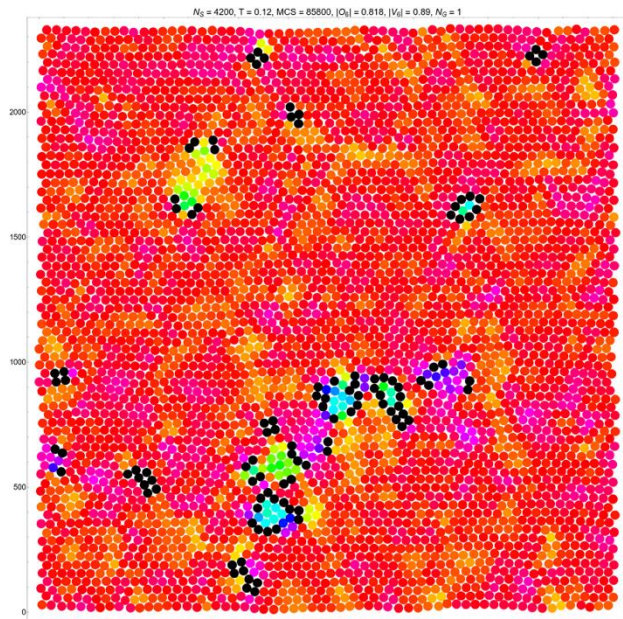


Orientational order in a skyrmion lattice

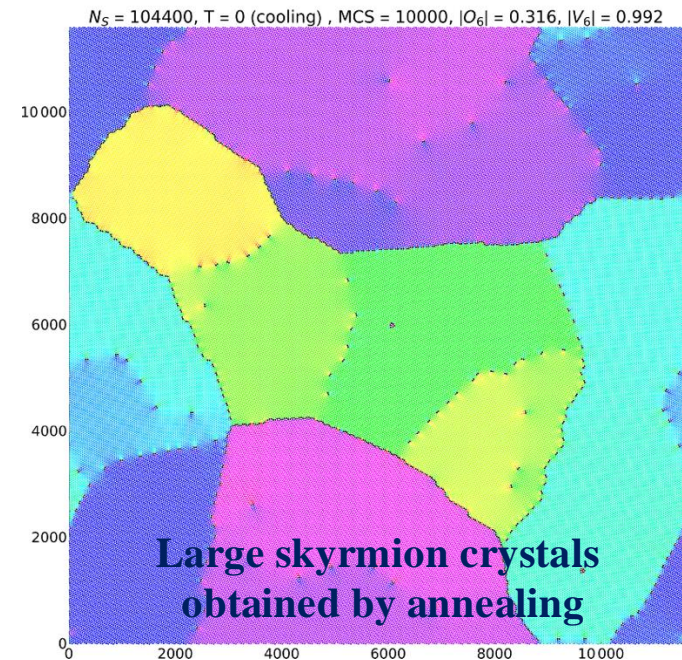
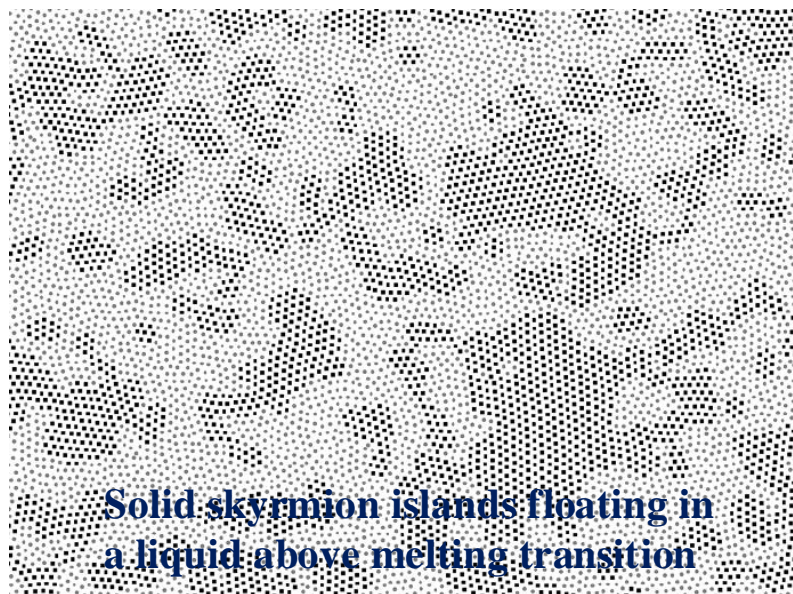
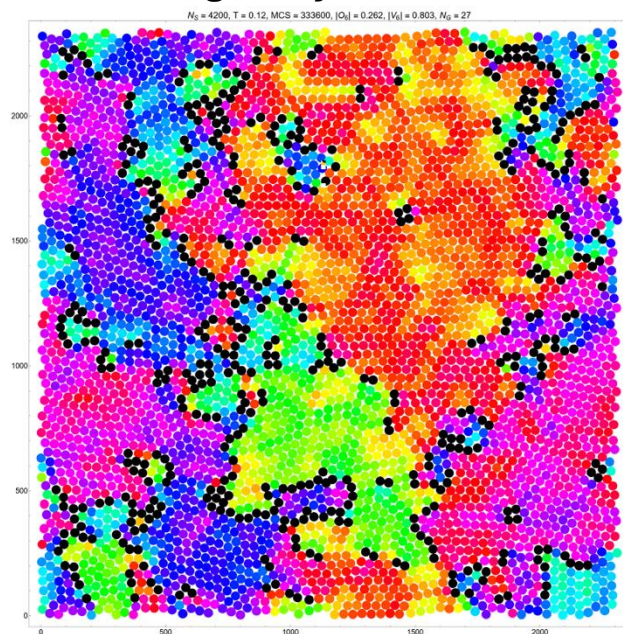
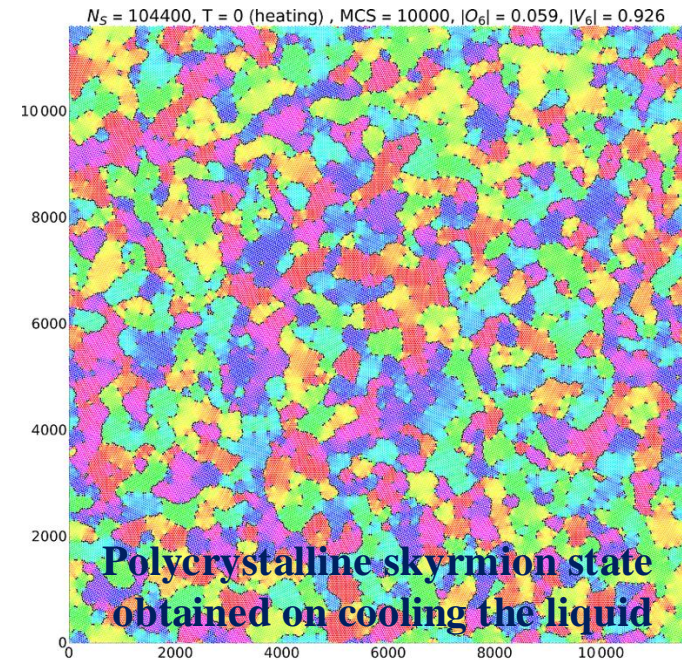
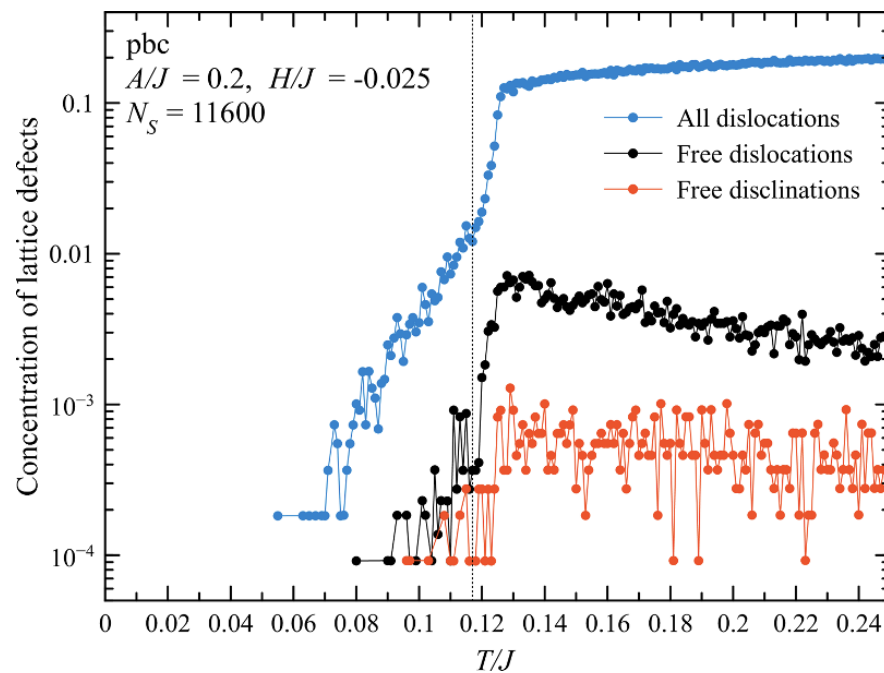
$$\Psi_i = \frac{1}{6} \sum_j \exp(6i\theta_{ij}) \quad V_6 \equiv \sqrt{\frac{1}{N_S} \sum_i |\Psi_i|^2} \quad \Psi = \frac{1}{N_S} \sum_i \Psi_i$$

$$V_6 \equiv \left\langle \sqrt{\frac{1}{N_S} \sum_i |\Psi_i|^2} \right\rangle \quad O_6 = \langle \Psi \rangle \quad G_6 = \sqrt{\langle |\Psi|^2 \rangle}$$

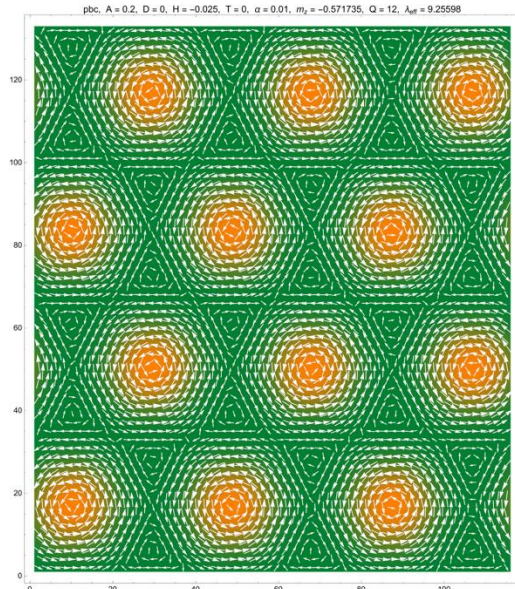




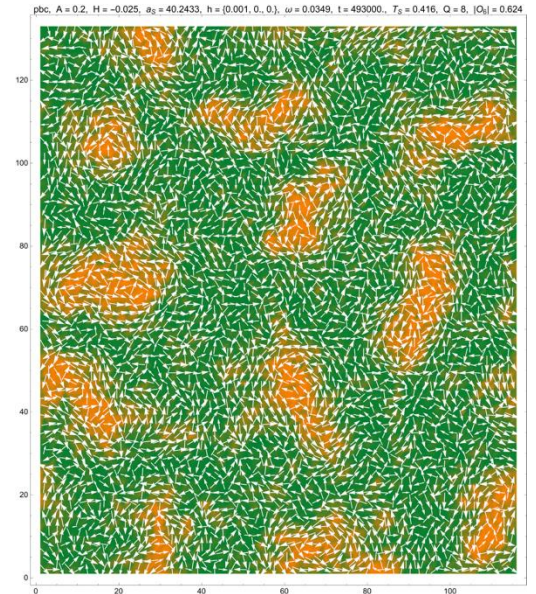
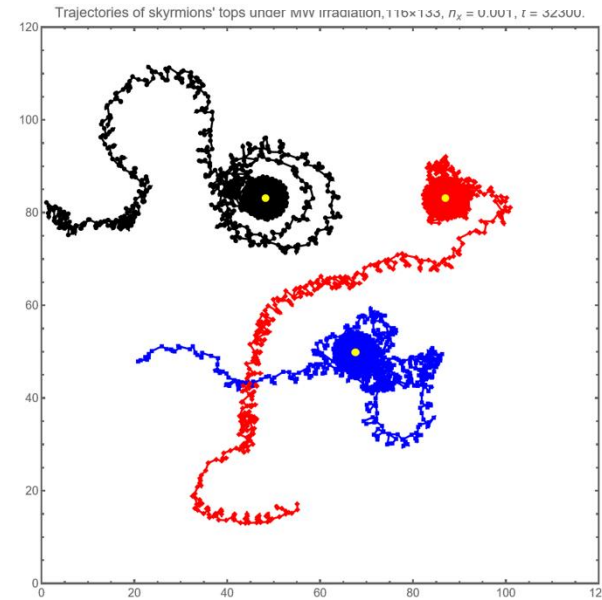
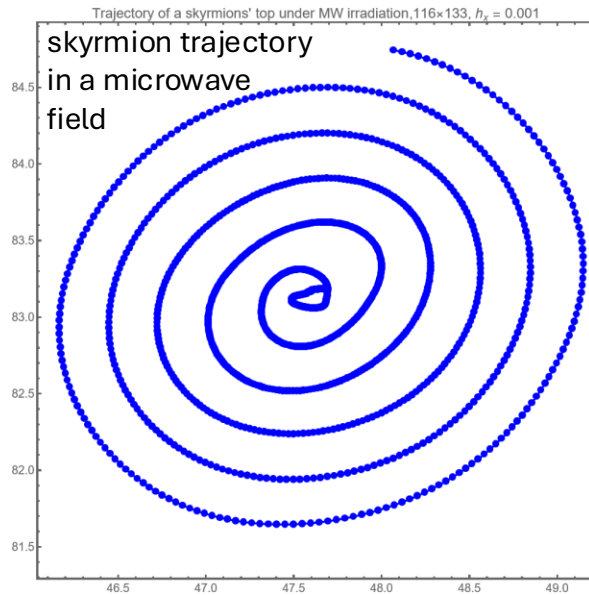
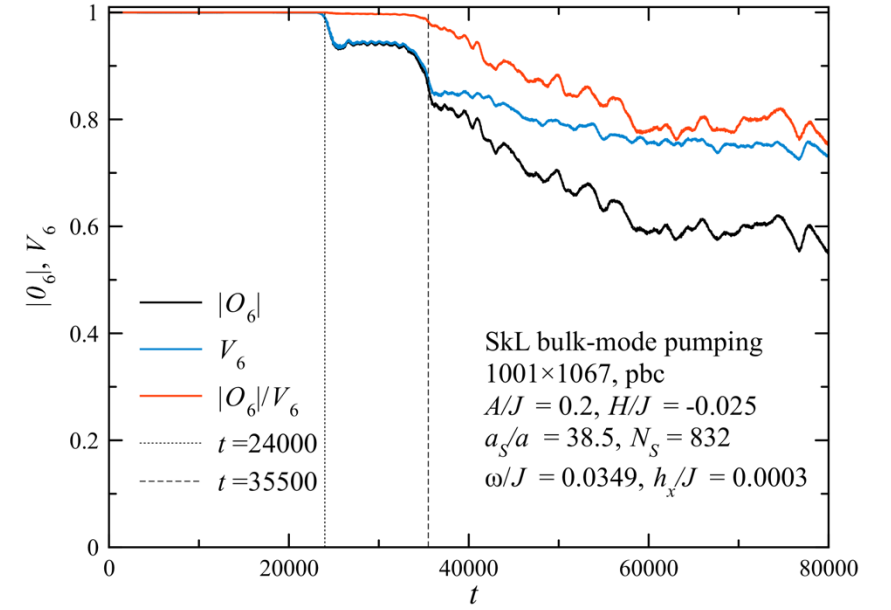
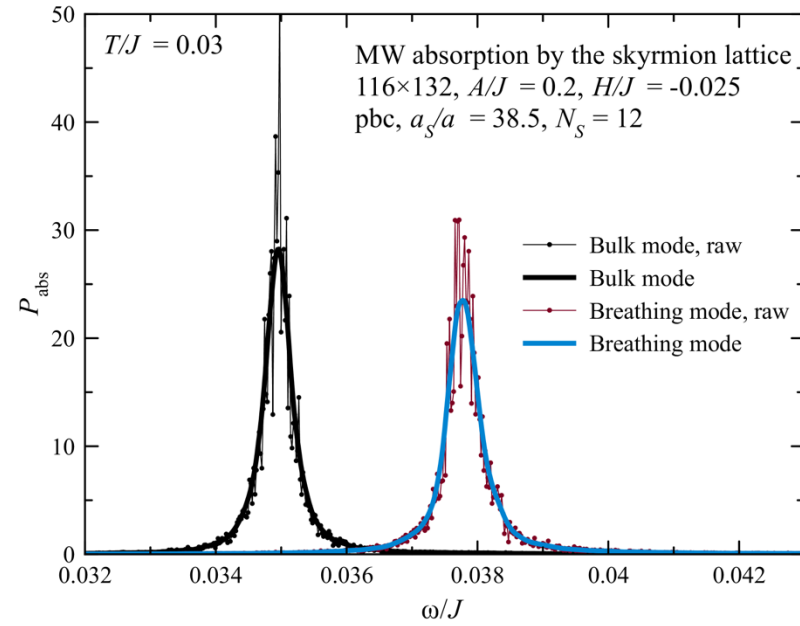
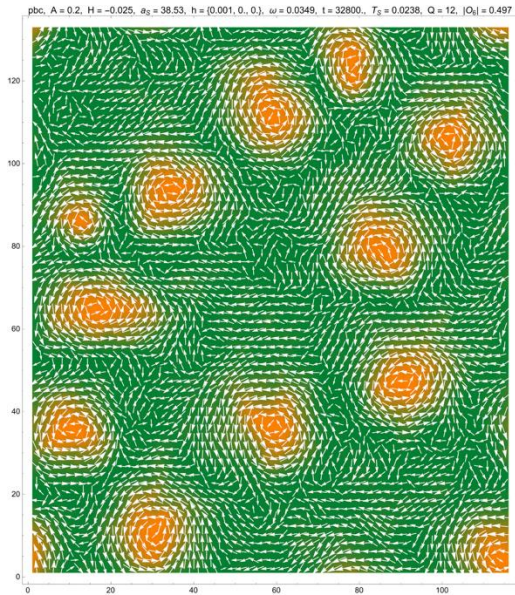
melting of skyrmion lattice



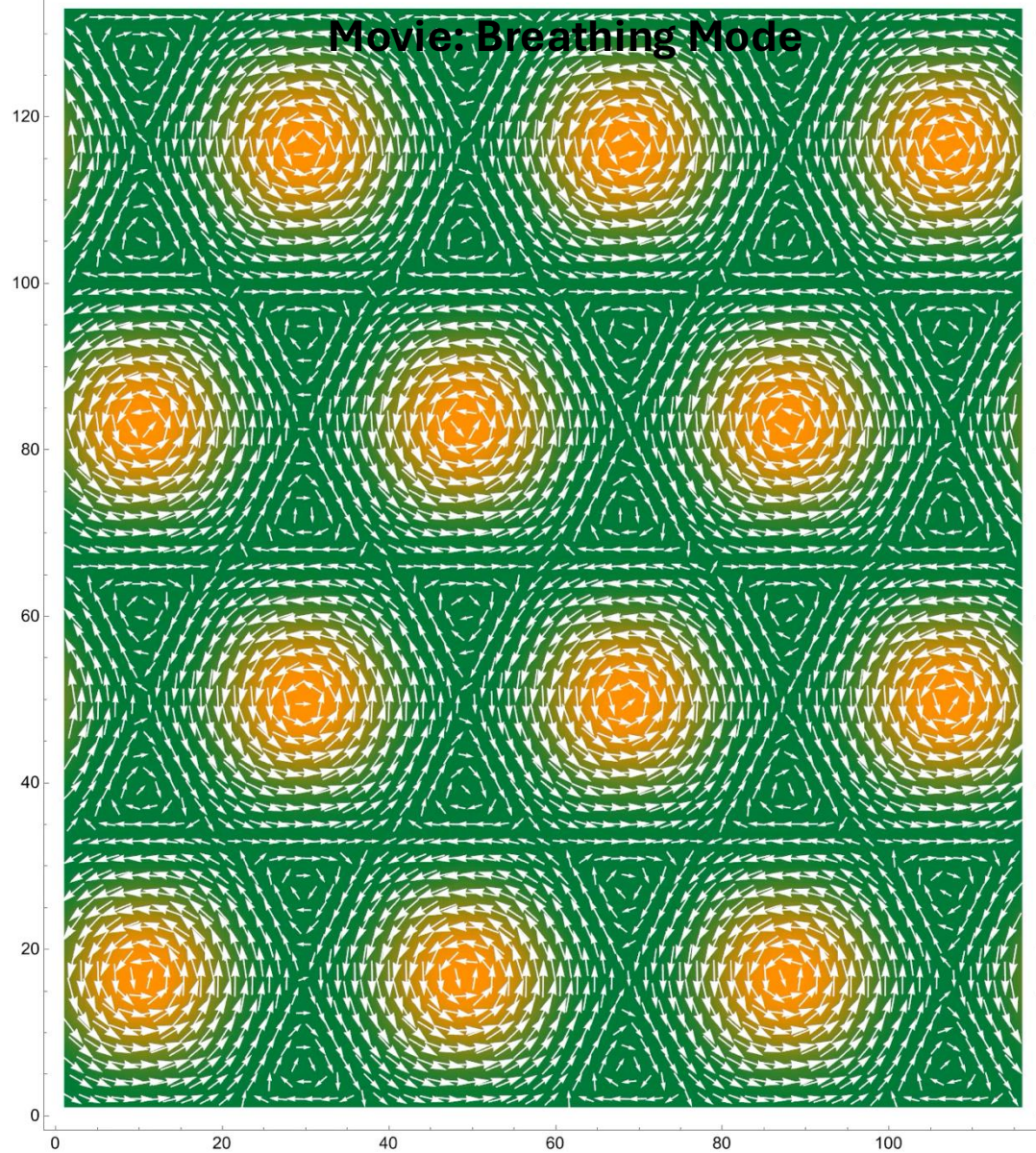
Microwave Melting of the Skyrmion Lattice



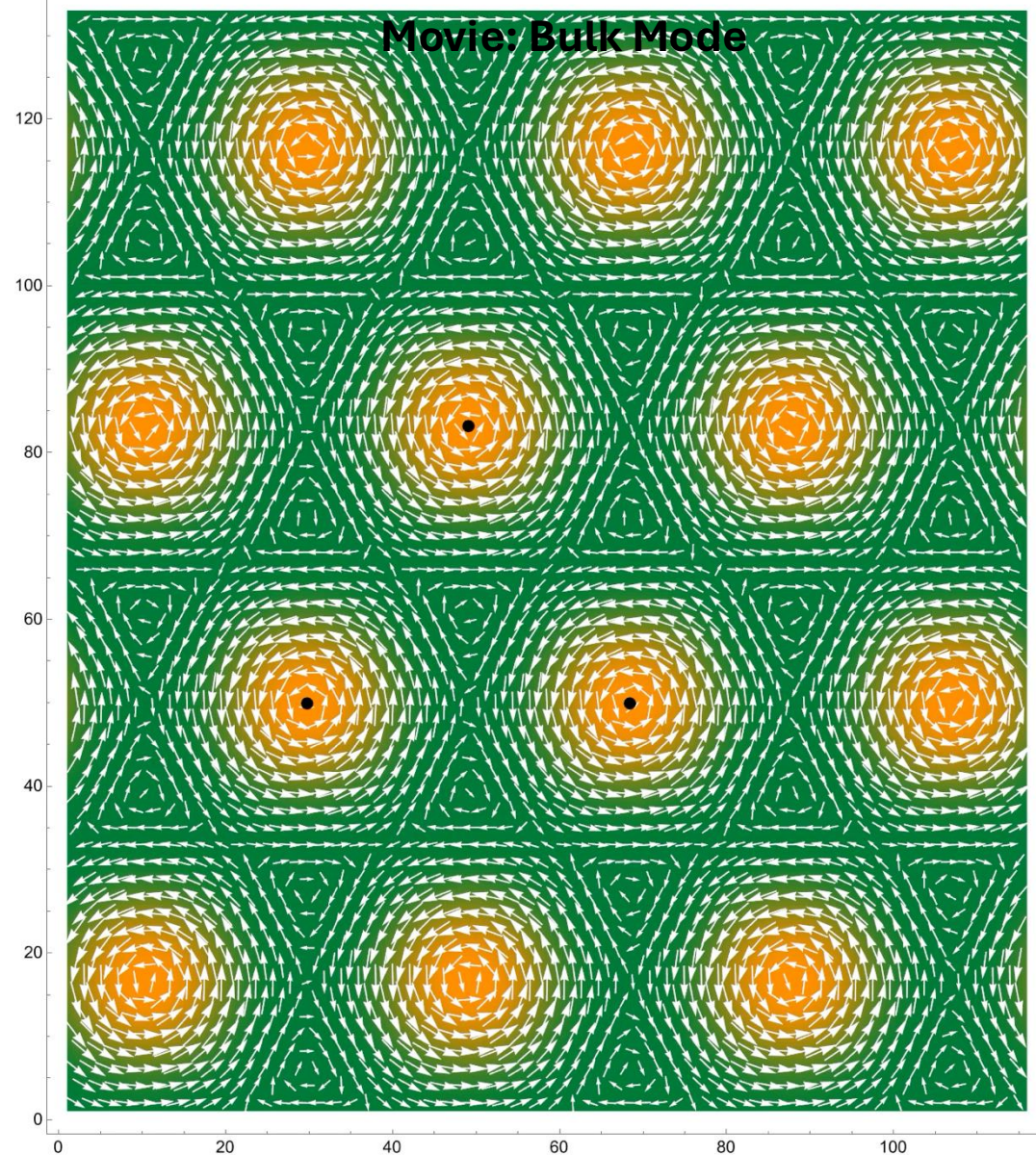
$s_z = -1$ green, $s_z = 1$ orange.



116×133, pbc, $A = 0.2$, $H = -0.025$, $\mathbf{h} = \{0., 0., 0.001\}$, $\omega = 0.0378$, $T_S = 0$, $Q = 12$, $t = 0$.



116×133, pbc, $A = 0.2$, $H = -0.025$, $\mathbf{h} = \{0.001, 0., 0.\}$, $\omega = 0.0349$, $T_S = 0$, $|\phi_6| = 1.$, $t = 0$.



RECENT ARTICLES ACKNOWLEDGING AFOSR SUPPORT:

1. E.M. Chudnovsky and D. A. Garanin, Scaling theory of magnetic order and microwave absorption in amorphous and granular ferromagnets, *Physical Review B* **109**, 054429 (2024).
2. D. A. Garanin, J. F. Soriano, and E. M. Chudnovsky, Melting and freezing of a skyrmion lattice, *Journal of Physics: Condensed Matter* **36**, 475802 (2024).
3. D. A. Garanin and E. M. Chudnovsky, Scaling of static and dynamical properties of random anisotropy magnets, *Europhysics Letters* **148**, 26001 (2024).
4. E. M. Chudnovsky and D. A. Garanin, Static and microwave properties of amorphous magnets near saturation, *European Physics Journal B* **97**, 186 (2024).