Discrete Huygens Representations Using Electric and Magnetic Dipoles with Application to 2D Grating Problems

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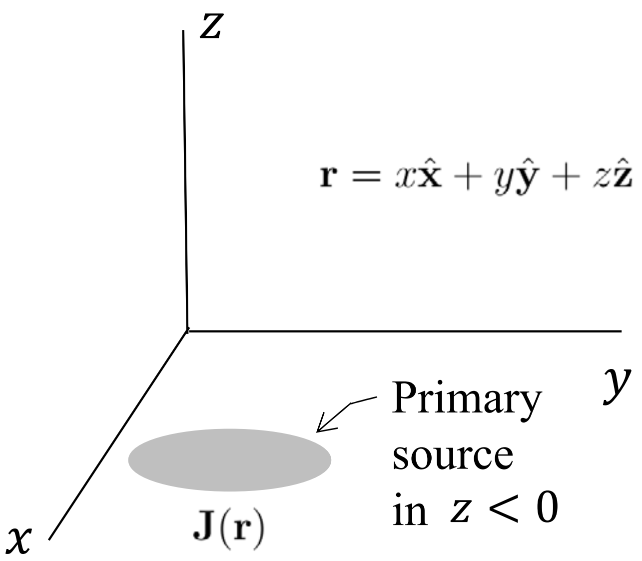
Discrete Huygens Source Solution



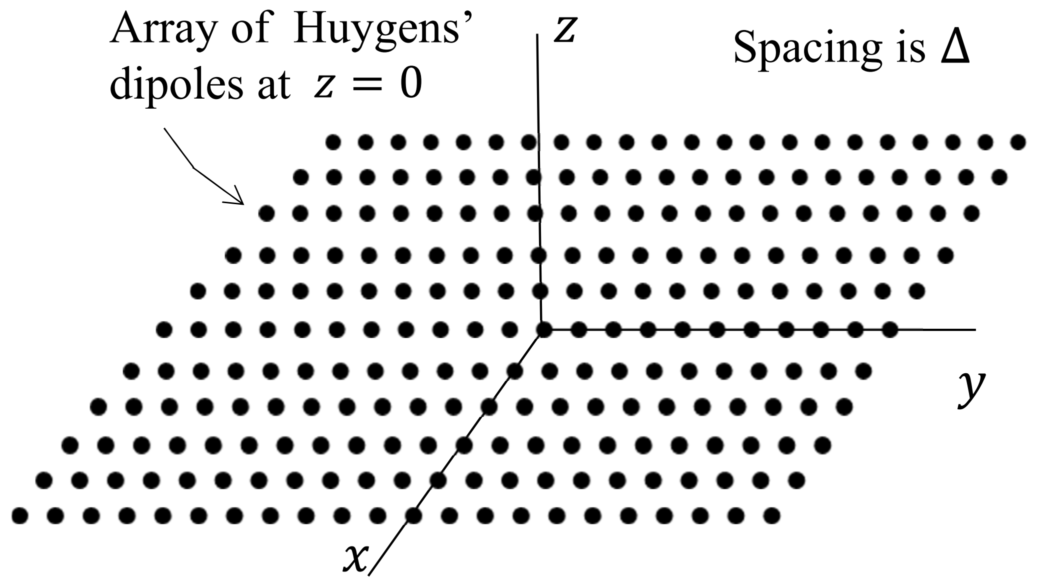
With $\mathbf{r}' = x'\hat{\mathbf{x}} + y'\hat{\mathbf{y}}$ and $z > 0$ [Kottler 1923, Hansen-Yaghjian 1999]

$$\mathbf{E}(\mathbf{r}) = i\omega\mu \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left[2\,\hat{\mathbf{z}}\times\mathbf{H}(\mathbf{r}')\right]\cdot\bar{\mathbf{G}}(\mathbf{r},\mathbf{r}')dx'dy'$$

which can be discretized as ($\mathbf{r}_{mn} = m\Delta\hat{\mathbf{x}} + n\Delta\hat{\mathbf{y}}$)

$$\mathbf{E}(\mathbf{r}) \simeq i\omega\mu \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} \left[2\Delta^2\,\hat{\mathbf{z}}\times\mathbf{H}(\mathbf{r}_{mn})\right]\cdot\bar{\mathbf{G}}(\mathbf{r},\mathbf{r}_{mn})$$

so that $\mathbf{p}_{mn} = 2\Delta^2\,\hat{\mathbf{z}}\times\mathbf{H}(\mathbf{r}_{mn})$.



Electric volume current

Electric dipole moments

$$\mathbf{J}^a(\mathbf{r}) = \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} \mathbf{p}_{mn}\,\delta(\mathbf{r}-\mathbf{r}_{mn}), \quad \mathbf{r}_{mn} = m\Delta\hat{\mathbf{x}} + n\Delta\hat{\mathbf{y}}$$

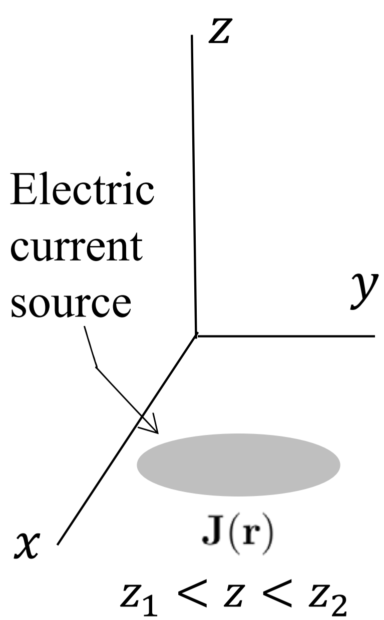
$$\mathbf{E}^a(\mathbf{r}) = i\omega\mu \int \mathbf{J}^a(\mathbf{r}')\cdot\bar{\mathbf{G}}(\mathbf{r},\mathbf{r}')\,dV'$$

$$= i\omega\mu \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} \mathbf{p}_{mn}\cdot\bar{\mathbf{G}}(\mathbf{r},\mathbf{r}_{mn})$$

Free-space electric dyadic Green's function

Plane-Wave Expansion in Terms of *x*-*y* Components (up)

[Weyl 1919, Kerns 1981, Hansen-Yaghjian 1999]



Electric and magnetic fields in region with $k_p = \sqrt{\omega^2 \mu \epsilon_p + i\omega\mu\sigma_p}$:

$$\mathbf{E}_p^{\uparrow}(\mathbf{r}) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \mathbf{S}_p^{\uparrow}(k_x, k_y) e^{i\mathbf{k}_p^{\uparrow}\cdot\mathbf{r}} dk_x dk_y, \quad z \geq z_2$$

and

$$\mathbf{H}_p^{\uparrow}(\mathbf{r}) = \frac{1}{2\pi\omega\mu} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \mathbf{k}_p^{\uparrow} \times \mathbf{S}_p^{\uparrow}(k_x, k_y) e^{i\mathbf{k}_p^{\uparrow}\cdot\mathbf{r}} dk_x dk_y, \quad z \geq z_2$$

where

$$\mathbf{k}_p^{\uparrow} = k_x\hat{\mathbf{x}} + k_y\hat{\mathbf{y}} + \gamma_p\hat{\mathbf{z}}, \quad \gamma_p = \sqrt{k_p^2 - k_x^2 - k_y^2}, \quad \mathrm{Re}(\gamma_p) \geq 0, \ \mathrm{Im}(\gamma_p) \geq 0$$

The spectrum satisfies $\mathbf{k}_p^{\uparrow} \cdot \mathbf{S}_p^{\uparrow}(k_x, k_y) = 0$ and is given by

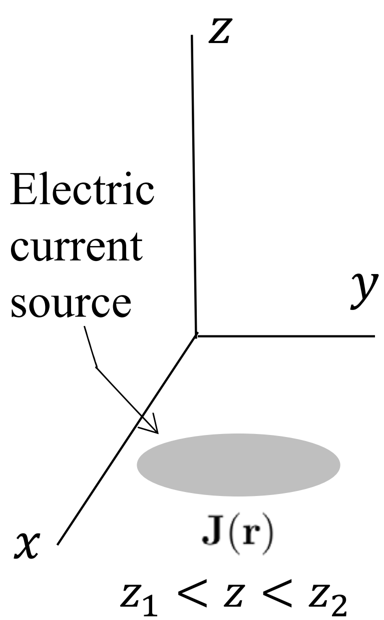
$$\mathbf{S}_p^{\uparrow}(k_x, k_y) = \frac{\omega\mu}{4\pi\gamma_p k_p^2} \mathbf{k}_p^{\uparrow} \times \left(\mathbf{k}_p^{\uparrow} \times \int_V \mathbf{J}(\mathbf{r}) e^{-i\mathbf{k}_p^{\uparrow}\cdot\mathbf{r}} dV \right), \qquad S_{pz}^{\uparrow}(k_x, k_y) = -\frac{k_x S_{px}^{\uparrow}(k_x, k_y) + k_y S_{py}^{\uparrow}(k_x, k_y)}{\gamma_p}$$

Hence, we need only compute x and y components:

$$\mathbf{s}_p^{\uparrow} = \begin{bmatrix} S_{px}^{\uparrow} \\ S_{py}^{\uparrow} \end{bmatrix}, \quad \mathbf{e}_p^{\uparrow} = \begin{bmatrix} E_{px}^{\uparrow} \\ E_{py}^{\uparrow} \end{bmatrix}, \quad \mathbf{e}_p^{\uparrow}(\mathbf{r}) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \mathbf{s}_p^{\uparrow}(k_x, k_y) e^{i\mathbf{k}_p^{\uparrow}\cdot\mathbf{r}} dk_x dk_y, \quad z \geq z_2$$

Plane-Wave Expansion in Terms of *x*-*y* Components (down)

[Weyl 1919, Kerns 1981, Hansen-Yaghjian 1999]



Electric and magnetic fields in region with $k_p = \sqrt{\omega^2 \mu \epsilon_p + i\omega\mu\sigma_p}$:

$$\mathbf{E}_p^{\downarrow}(\mathbf{r}) = \frac{1}{2\pi} \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \mathbf{S}_p^{\downarrow}(k_x, k_y) e^{i\mathbf{k}_p^{\downarrow}\cdot\mathbf{r}} dk_x dk_y, \quad z \le z_1$$

and

$$\mathbf{H}_p^{\downarrow}(\mathbf{r}) = \frac{1}{2\pi\omega\mu} \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \mathbf{k}_p^{\downarrow} \times \mathbf{S}_p^{\downarrow}(k_x, k_y) e^{i\mathbf{k}_p^{\downarrow}\cdot\mathbf{r}} dk_x dk_y, \quad z \le z_1$$

where

$$\mathbf{k}_p^{\downarrow} = k_x\hat{\mathbf{x}} + k_y\hat{\mathbf{y}} - \gamma_p\hat{\mathbf{z}}, \quad \gamma_p = \sqrt{k_p^2 - k_x^2 - k_y^2}, \quad \mathrm{Re}(\gamma_p) \ge 0, \;\; \mathrm{Im}(\gamma_p) \ge 0$$

The spectrum satisfies $\mathbf{k}_p^{\downarrow} \cdot \mathbf{S}_p^{\downarrow}(k_x, k_y) = 0$ and is given by

$$\mathbf{S}_p^{\downarrow}(k_x, k_y) = \frac{\omega\mu}{4\pi\gamma_p k_p^2} \mathbf{k}_p^{\downarrow} \times \left(\mathbf{k}_p^{\downarrow} \times \int_V \mathbf{J}(\mathbf{r}) e^{-i\mathbf{k}_p^{\downarrow}\cdot\mathbf{r}} dV \right), \qquad S_{pz}^{\downarrow}(k_x, k_y) = \frac{k_x S_{px}^{\downarrow}(k_x, k_y) + k_y S_{py}^{\downarrow}(k_x, k_y)}{\gamma_p}$$

Hence, we need only compute x and y components:

$$\mathbf{s}_p^{\downarrow} = \begin{bmatrix} S_{px}^{\downarrow} \\ S_{py}^{\downarrow} \end{bmatrix}, \quad \mathbf{e}_p^{\downarrow} = \begin{bmatrix} E_{px}^{\downarrow} \\ E_{py}^{\downarrow} \end{bmatrix}, \quad \mathbf{e}_p^{\downarrow}(\mathbf{r}) = \frac{1}{2\pi} \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \mathbf{s}_p^{\downarrow}(k_x, k_y) e^{i\mathbf{k}_p^{\downarrow}\cdot\mathbf{r}} dk_x dk_y, \quad z \le z_1$$

Huygens Solution from Plane-Wave Expansion

The plane-wave spectra for the primary source and the Huygens' array are

$$\mathbf{S}_0^{\uparrow}(k_x,k_y) = \frac{\omega\mu}{4\pi\gamma_0 k_0^2}\mathbf{k}_0^{\uparrow}\times\left(\mathbf{k}_0^{\uparrow}\times\int_V \mathbf{J}(\mathbf{r})e^{-i\mathbf{k}_0^{\uparrow}\cdot\mathbf{r}}dV\right), \quad \mathbf{S}_0^{\uparrow a}(k_x,k_y) = \frac{\omega\mu}{4\pi\gamma_0 k_0^2}\mathbf{k}_0^{\uparrow}\times\mathbf{k}_0^{\uparrow}\times\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\mathbf{p}_{mn}\, e^{-imk_x\Delta-ink_y\Delta}$$

Let $\hat{\mathbf{z}}\cdot\mathbf{p}_{mn}=0$ to get

$$\mathbf{s}_0^{\uparrow a}(k_x,k_y) = \bar{\mathcal{K}}_0(k_x,k_y)\cdot\begin{bmatrix} A_x^a(k_x,k_y) \\ A_y^a(k_x,k_y)\end{bmatrix}, \quad \mathbf{A}^a(k_x,k_y) = \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\mathbf{p}_{mn}e^{-imk_x\Delta-ink_y\Delta}$$

$$\bar{\mathcal{K}}_0(k_x,k_y) = \frac{\omega\mu}{4\pi\gamma_0 k_0^2}\begin{bmatrix} k_x^2-k_0^2 & k_xk_y \\ k_xk_y & k_y^2-k_0^2\end{bmatrix}, \quad \bar{\mathcal{K}}_0^{-1}(k_x,k_y) = \frac{4\pi}{\omega\mu\gamma_0}\begin{bmatrix} k_y^2-k_0^2 & -k_xk_y \\ -k_xk_y & k_x^2-k_0^2\end{bmatrix}$$

Requiring $\mathbf{s}_0^{\uparrow a}(k_x,k_y) = \mathbf{s}_0^{\uparrow}(k_x,k_y)$ gives

$$\begin{bmatrix} A_x^a(k_x,k_y) \\ A_y^a(k_x,k_y)\end{bmatrix} = \begin{bmatrix} A_x(k_x,k_y) - \dfrac{k_x}{\gamma_0}A_z(k_x,k_y) \\ A_y(k_x,k_y) - \dfrac{k_y}{\gamma_0}A_z(k_x,k_y)\end{bmatrix}, \quad \mathbf{A}(k_x,k_y) = \int_V \mathbf{J}(\mathbf{r})e^{-i\mathbf{k}_0^{\uparrow}\cdot\mathbf{r}}dV$$

Huygens Solution from Plane-Wave Expansion

We note that

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathbf{A}^a(k_x,k_y)e^{im_0k_x\Delta+in_0k_y\Delta}dk_xdk_y = \frac{4\pi^2}{\Delta^2}\,\mathbf{p}_{m_0n_0}\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\delta(m_0-m)\,\delta(n_0-n)$$

Based on this expression with $\hat{\mathbf{z}}\cdot\mathbf{p}_{mn}=0$ we try

$$\hat{\mathbf{x}}\cdot\mathbf{p}_{mn} = \frac{\Delta^2}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left(A_x(k_x,k_y)-\frac{k_x}{\gamma_0}A_z(k_x,k_y)\right)e^{imk_x\Delta+ink_y\Delta}dk_xdk_y$$

$$\hat{\mathbf{y}}\cdot\mathbf{p}_{mn} = \frac{\Delta^2}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left(A_y(k_x,k_y)-\frac{k_y}{\gamma_0}A_z(k_x,k_y)\right)e^{imk_x\Delta+ink_y\Delta}dk_xdk_y$$

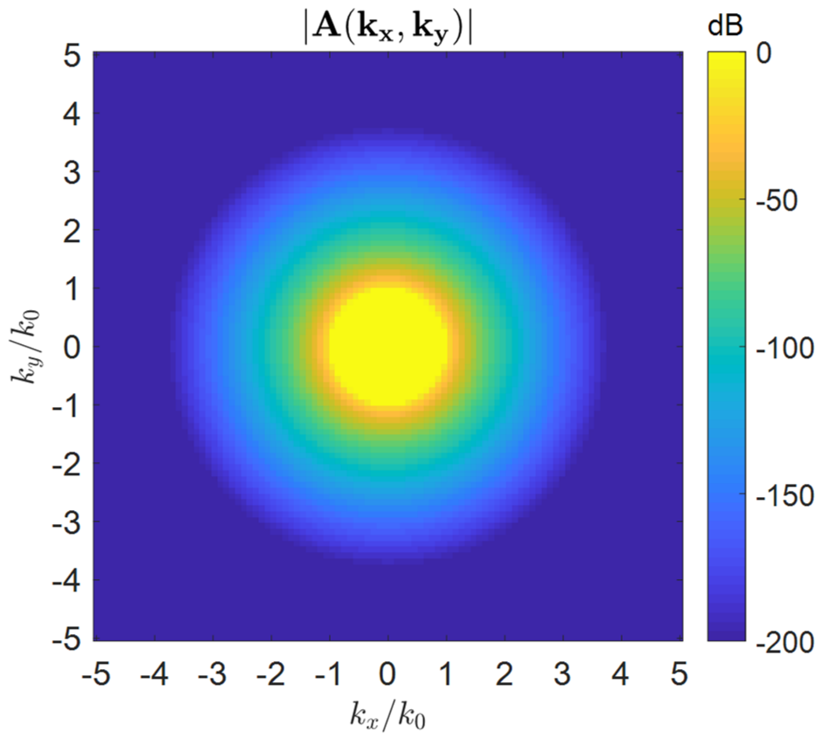
Inserting $\mathbf{A}(k_x,k_y)=\int_V \mathbf{J}(\mathbf{r})e^{-i\mathbf{k}_0^{\uparrow}\cdot\mathbf{r}}dV$ and using the Weyl identity gives $\mathbf{p}_{mn}=2\Delta^2\,\hat{\mathbf{z}}\times\mathbf{H}(\mathbf{r}_{mn})$.

Inserting our expression for $\mathbf{p}_{mn}$ in terms of $\mathbf{A}$ into the expression for the spectrum shows that when $A_z=0$

$$\mathbf{A}^a(k_x,k_y)=\mathbf{A}(k_x,k_y)+\sum_{\substack{m=-\infty \\ (m,n)\neq(0,0)}}^{\infty}\sum_{n=-\infty}^{\infty}\mathbf{A}(k_x-2\pi m/\Delta,\,k_y-2\pi n/\Delta)$$

Desired

"Higher-order" modes

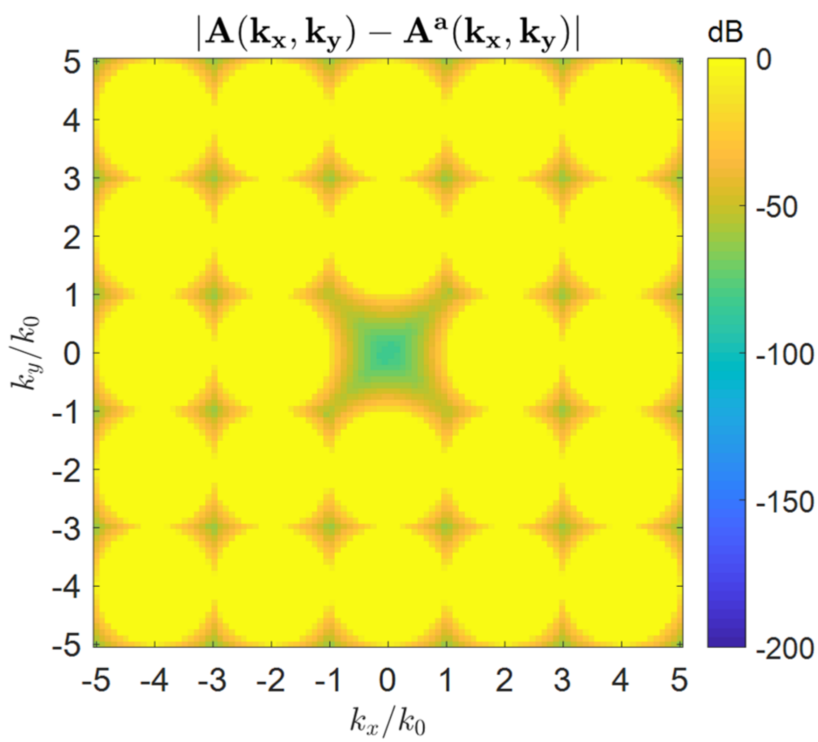


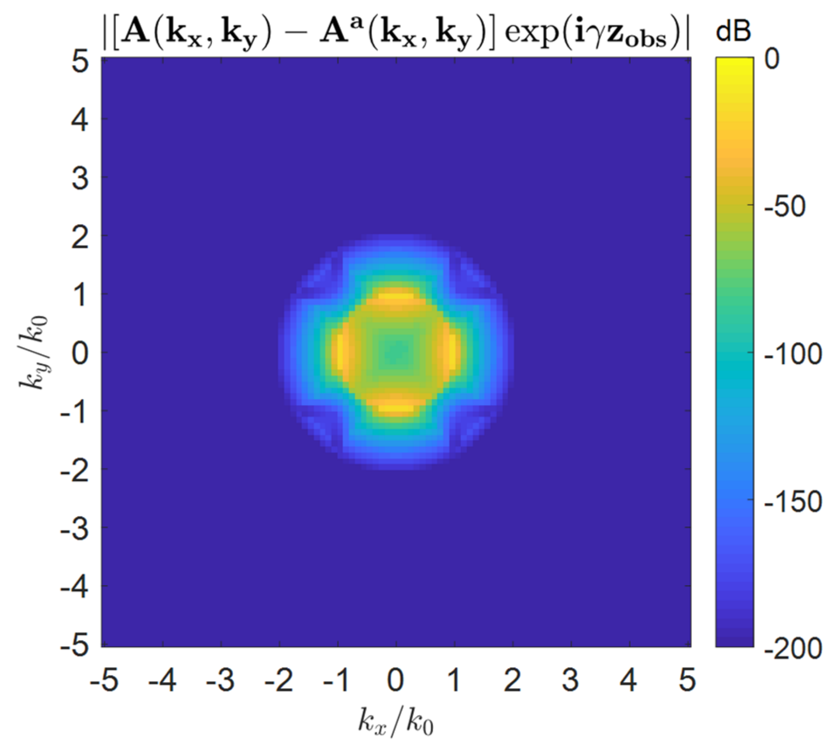
$$\mathbf{A}(k_x, k_y) = \mathbf{u}\, e^{-i\mathbf{k}_0 \cdot \mathbf{r}_s}, \quad |\mathbf{u}| = 1$$

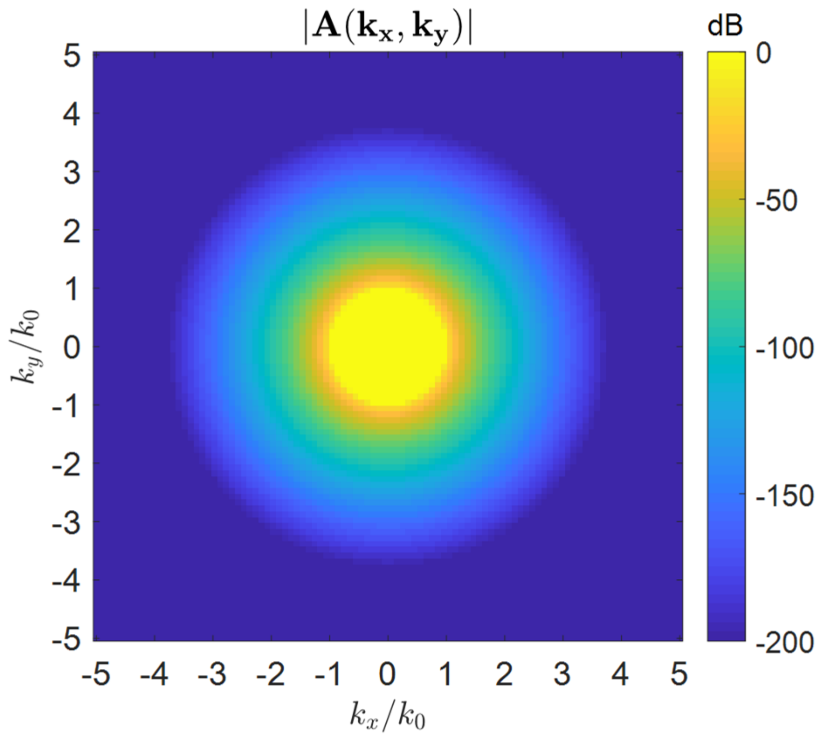
$$\boxed{\Delta = 0.5\lambda_0}$$

$$\mathbf{r}_s = [0.8\hat{\mathbf{x}} - 3.8\hat{\mathbf{y}} - \hat{\mathbf{z}}]\lambda_0$$

$$z_{obs} = 2\lambda_0, \quad \text{Field error level at } -10\,\text{dB}$$





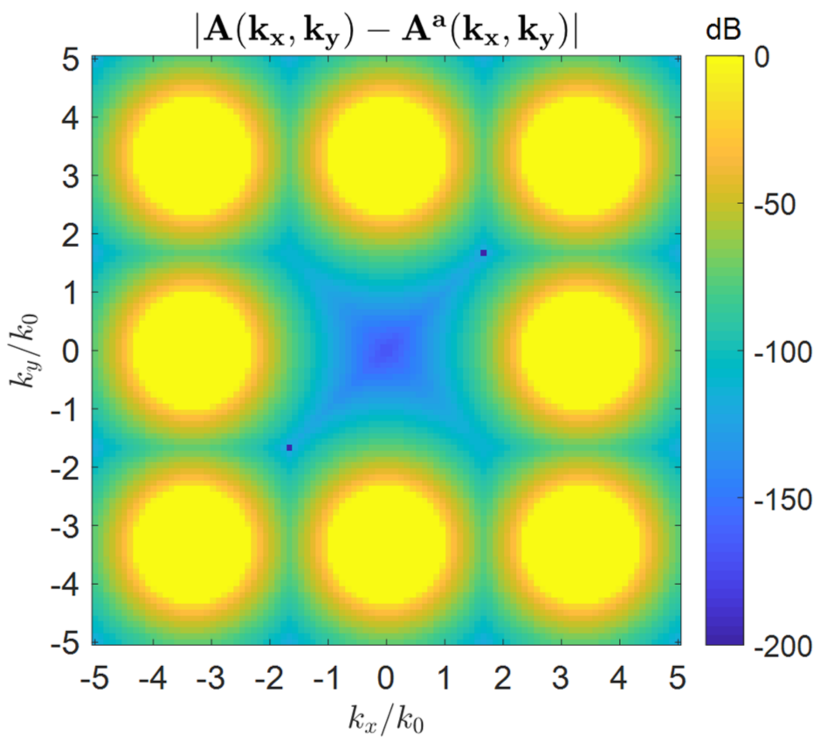


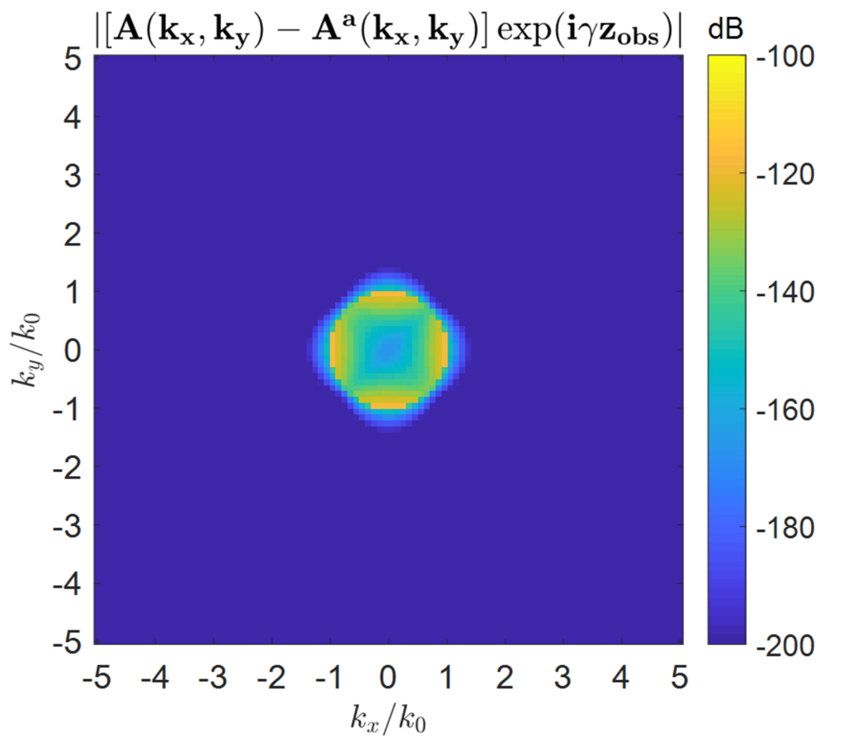
$$\mathbf{A}(k_x, k_y) = \mathbf{u}\, e^{-i\mathbf{k}_0 \cdot \mathbf{r}_s}, \quad |\mathbf{u}| = 1$$

$$\boxed{\Delta = 0.3\lambda_0}$$

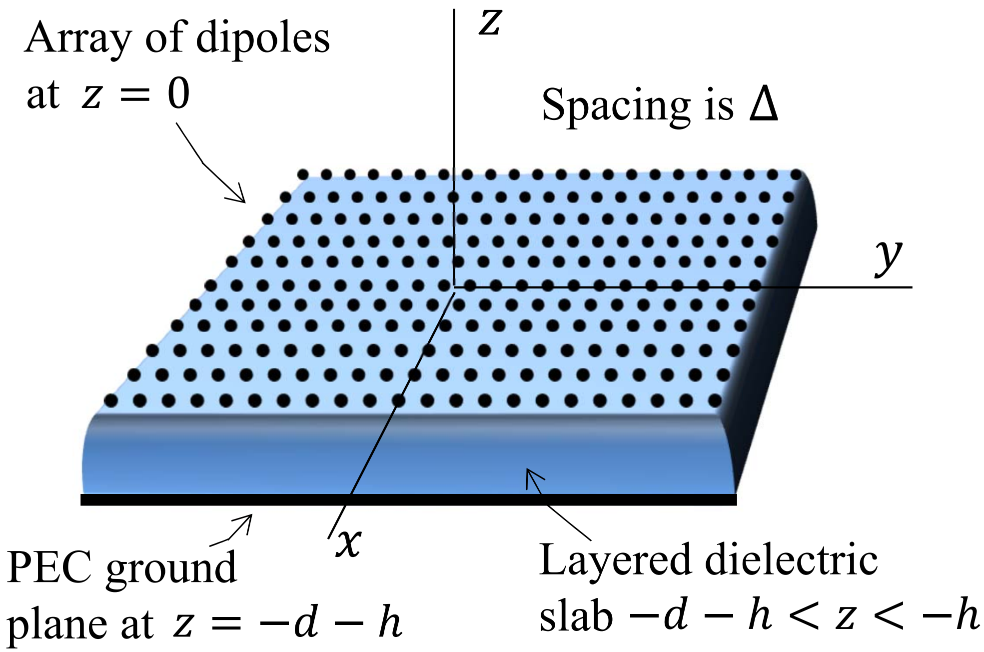
$$\mathbf{r}_s = [0.8\hat{\mathbf{x}} - 3.8\hat{\mathbf{y}} - \hat{\mathbf{z}}]\lambda_0$$

$$z_{obs} = 2\lambda_0, \quad \text{Field error level at } -100\,\text{dB}$$





Dielectric Slab Backed by PEC Ground Plane



Propagation constants are k_0 for $z > -h$ and k_1 for $-h > z > -d - h$ with $h > 0$ and $d > 0$.

$$\mathbf{A}^a(k_x, k_y) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \mathbf{p}_{mn} e^{-imk_x\Delta - ink_y\Delta}, \quad \hat{\mathbf{z}} \cdot \mathbf{p}_{mn} = 0$$

Spectra for the array radiating in free space:

$$\mathbf{s}_0^{\uparrow a}(k_x, k_y) = \mathbf{s}_0^{\downarrow a}(k_x, k_y) = \bar{\mathcal{K}}_0(k_x, k_y) \cdot \begin{bmatrix} A_x^a(k_x, k_y) \\ A_y^a(k_x, k_y) \end{bmatrix}$$

$$\bar{\mathcal{K}}_0(k_x, k_y) = \frac{\omega\mu}{4\pi\gamma_0 k_0^2} \begin{bmatrix} k_x^2 - k_0^2 & k_x k_y \\ k_x k_y & k_y^2 - k_0^2 \end{bmatrix}$$

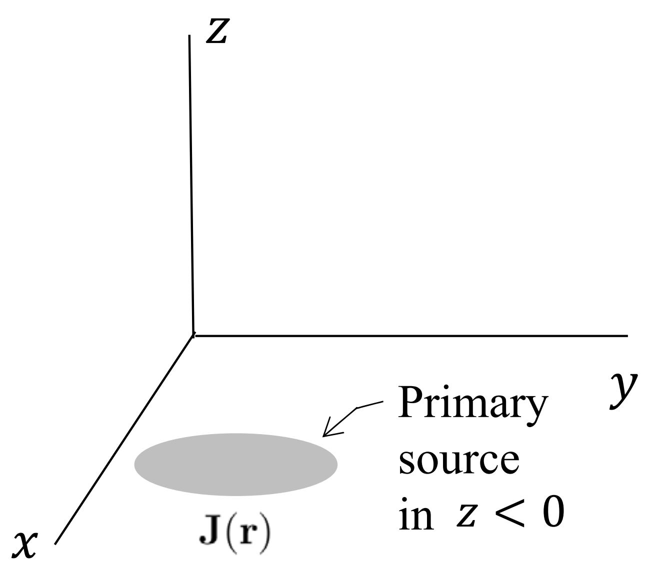
Total up spectrum for the array and its interaction with the slab:

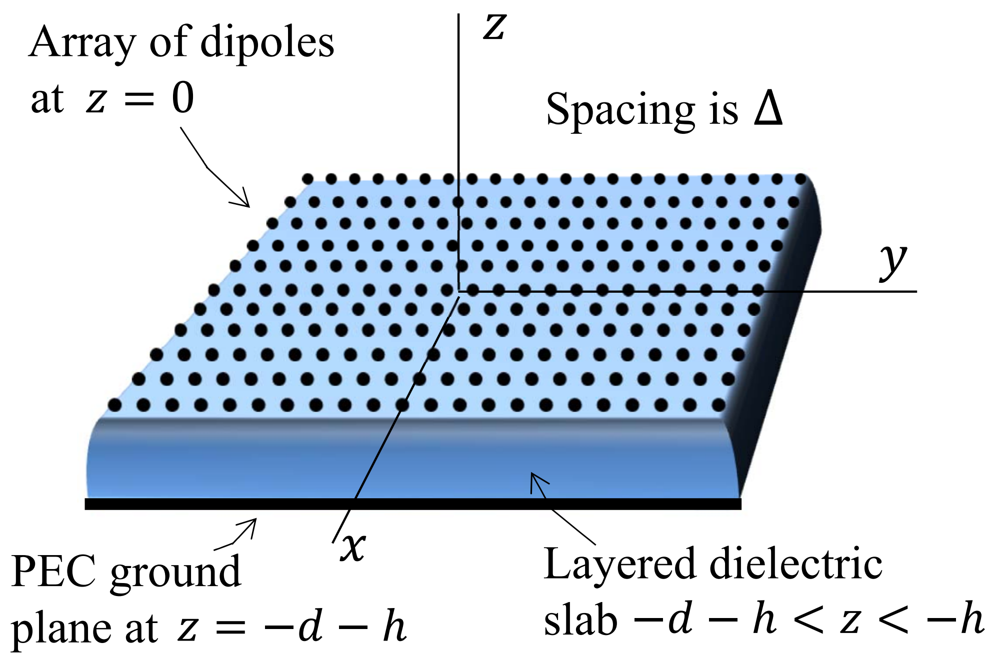
$$\mathbf{s}_0^{\uparrow a}(k_x, k_y) = \left[\bar{\mathbf{I}} + \bar{\mathbf{R}}_{012}^{\uparrow}(k_x, k_y)\right] \cdot \bar{\mathcal{K}}_0(k_x, k_y) \cdot \begin{bmatrix} A_x^a(k_x, k_y) \\ A_y^a(k_x, k_y) \end{bmatrix}$$

$$\bar{\mathbf{R}}_{012}^{\uparrow} = e^{2i\gamma_0 h}\left[\bar{\mathbf{R}}_{01}^{\uparrow} - e^{2i\gamma_1 d}\, \bar{\mathbf{T}}_{10}^{\uparrow} \cdot \left(\bar{\mathbf{M}}^{\downarrow}\right)^{-1} \cdot \bar{\mathbf{T}}_{01}^{\downarrow}\right], \quad \bar{\mathbf{M}}^{\downarrow} = \bar{\mathbf{I}} + e^{2i\gamma_1 d}\, \bar{\mathbf{R}}_{10}^{\downarrow}$$

$$\bar{\mathbf{T}}_{pq}^{\downarrow} = \frac{2}{(\gamma_p + \gamma_q)(\gamma_p\gamma_q + k_x^2 + k_y^2)} \begin{bmatrix} (\gamma_q\gamma_p^2 + \gamma_q k_x^2 + \gamma_p k_y^2) & k_x k_y(\gamma_q - \gamma_p) \\ k_x k_y(\gamma_q - \gamma_p) & (\gamma_q\gamma_p^2 + \gamma_q k_y^2 + \gamma_p k_x^2) \end{bmatrix}$$

Discrete Huygens Source Problem





$$\mathbf{S}_0^{\uparrow}(k_x, k_y) = \frac{\omega\mu}{4\pi\gamma_0 k_0^2}\mathbf{k}_0^{\uparrow}\times\left(\mathbf{k}_0^{\uparrow}\times\int\limits_V \mathbf{J}(\mathbf{r})e^{-i\mathbf{k}_0^{\uparrow}\cdot\mathbf{r}}dV\right)$$

$$\mathbf{s}_0^{\uparrow} = \begin{bmatrix} S_{0x}^{\uparrow} \\ S_{0y}^{\uparrow} \end{bmatrix}, \qquad \mathbf{e}_0^{\uparrow} = \begin{bmatrix} E_{0x}^{\uparrow} \\ E_{0y}^{\uparrow} \end{bmatrix}$$

$$\mathbf{e}_0^{\uparrow}(\mathbf{r}) = \frac{1}{2\pi}\int\limits_{-\infty}^{+\infty}\int\limits_{-\infty}^{+\infty}\mathbf{s}_0^{\uparrow}(k_x, k_y)e^{i\mathbf{k}_0^{\uparrow}\cdot\mathbf{r}}dk_x dk_y, \;\; z \geq 0$$

$$\mathbf{A}^a(k_x, k_y) = \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\mathbf{p}_{mn}e^{-imk_x\Delta - ink_y\Delta}, \qquad \hat{\mathbf{z}}\cdot\mathbf{p}_{mn} = 0$$

$$\mathbf{s}_0^{\uparrow a}(k_x, k_y) = \left[\bar{\mathbf{I}} + \bar{\mathbf{R}}_{012}^{\uparrow}(k_x, k_y)\right]\cdot\boldsymbol{\mathcal{K}}_0(k_x, k_y)\cdot\begin{bmatrix} A_x^a(k_x, k_y) \\ A_y^a(k_x, k_y) \end{bmatrix}$$

$$\mathbf{e}_0^{\uparrow a}(\mathbf{r}) = \frac{1}{2\pi}\int\limits_{-\infty}^{+\infty}\int\limits_{-\infty}^{+\infty}\mathbf{s}_0^{\uparrow a}(k_x, k_y)e^{i\mathbf{k}_0^{\uparrow}\cdot\mathbf{r}}dk_x dk_y, \;\; z > 0$$

Discrete Huygens Source Solution

Matching the plane-wave spectra and using the approach developed for free-space, we get

$$\mathbf{p}_{mn} = \frac{\Delta^2}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\bar{\boldsymbol{\mathcal{K}}}_0^{-1}(k_x,k_y)\cdot\Big[\bar{\mathbf{I}}+\bar{\mathbf{R}}_{012}^{\uparrow}(k_x,k_y)\Big]^{-1}\cdot\mathbf{s}_0^{\uparrow}(k_x,k_y)e^{imk_x\Delta+ink_y\Delta}dk_x dk_y,$$

Primary source spectrum

When $||\bar{\mathbf{R}}_{012}^{\uparrow}(k_x,k_y)|| < 1$ the Neumann formula gives

$$\Big[\bar{\mathbf{I}}+\bar{\mathbf{R}}_{012}^{\uparrow}(k_x,k_y)\Big]^{-1} = \sum_{m=0}^{\infty}\Big[-\bar{\mathbf{R}}_{012}^{\uparrow}(k_x,k_y)\Big]^{m}$$

To investigate the accuracy of the solution for $\mathbf{p}_{mn}$, let

$$\bar{\mathbf{V}}(k_x,k_y) = \Big[\bar{\mathbf{I}}+\bar{\mathbf{R}}_{012}^{\uparrow}(k_x,k_y)\Big]\cdot\bar{\boldsymbol{\mathcal{K}}}_0(k_x,k_y),\quad \bar{\mathbf{V}}^{-1}(k_x,k_y) = \bar{\boldsymbol{\mathcal{K}}}_0^{-1}(k_x,k_y)\cdot\Big[\bar{\mathbf{I}}+\bar{\mathbf{R}}_{012}^{\uparrow}(k_x,k_y)\Big]^{-1}$$

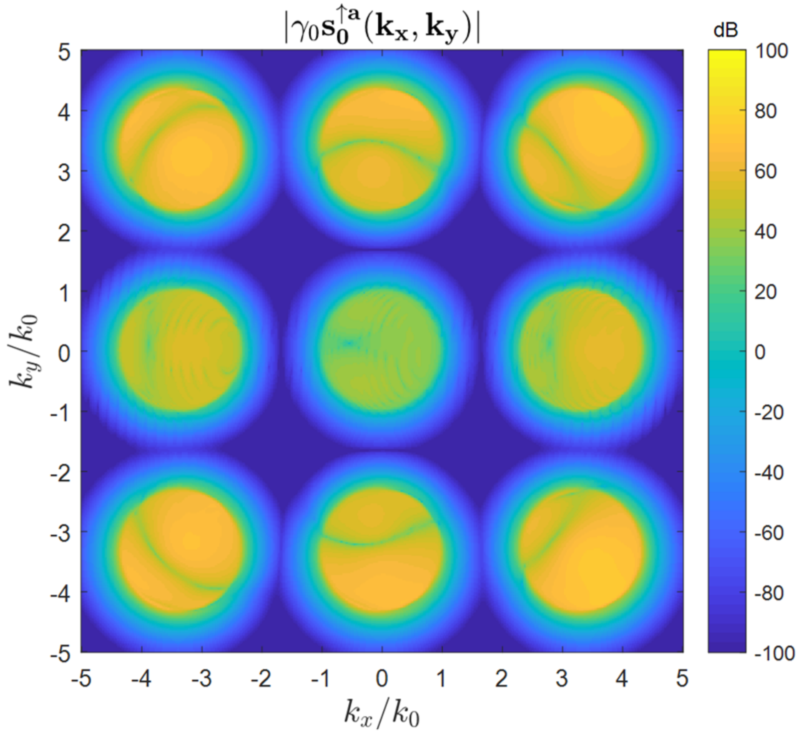
and insert these dipole moments into the expression for $\mathbf{A}^a(k_x,k_y)$ to get

$$\mathbf{s}_0^{\uparrow a}(k_x,k_y) = \mathbf{s}_0^{\uparrow}(k_x,k_y)+\sum_{\substack{m=-\infty \\ (m,n)\neq(0,0)}}^{\infty}\sum_{n=-\infty}^{\infty}\bar{\mathbf{V}}(k_x,k_y)\cdot\bar{\mathbf{V}}^{-1}(k_x-2\pi m/\Delta,k_y-2\pi n/\Delta)\cdot\mathbf{s}_0^{\uparrow}(k_x-2\pi m/\Delta,k_y-2\pi n/\Delta)$$

Desired spectrum

“Higher-order” modes

Primary Dipole Source

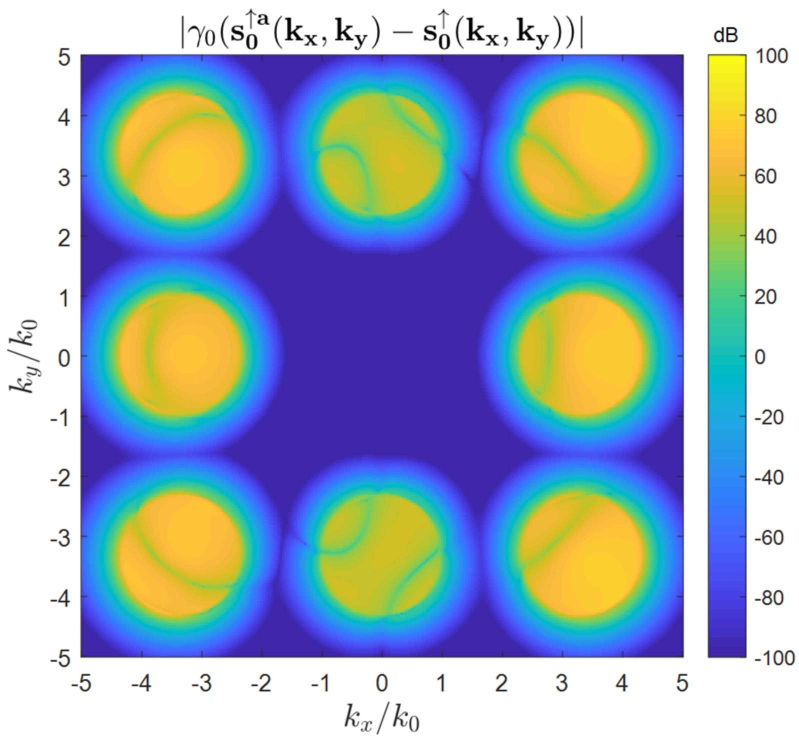


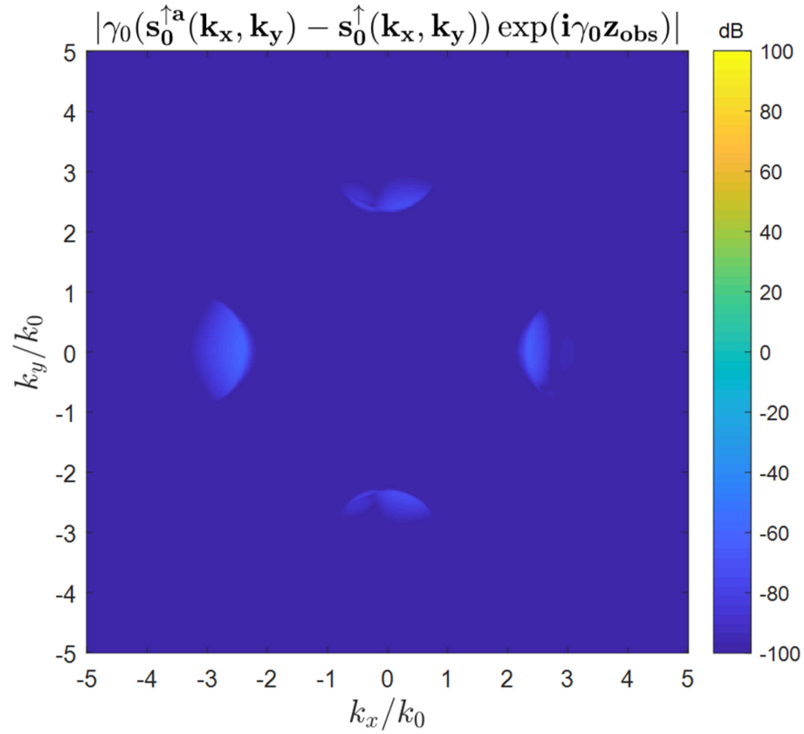
Primary source $\mathbf{J}(\mathbf{r}) = \mathbf{u}\,\delta(\mathbf{r} - \mathbf{r}_s)$

$\mathbf{r}_s = [-3\hat{\mathbf{x}} + 0.3\hat{\mathbf{y}} - 2\hat{\mathbf{z}}]\lambda_0$, $\mathbf{u} = (-1, 0.2, 1.6)$

$\boxed{\Delta = 0.3\lambda_0,}$ $k_1 = 1.2k_0$, $d = 0.34\lambda_0$, $h = 0.05\lambda_0$

$z_{obs} = 1\lambda_0$



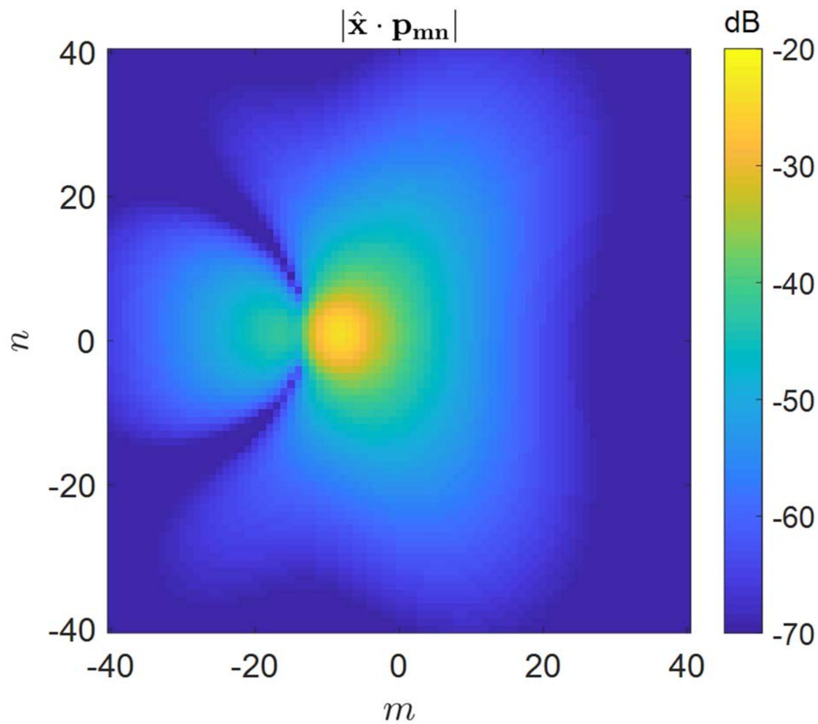


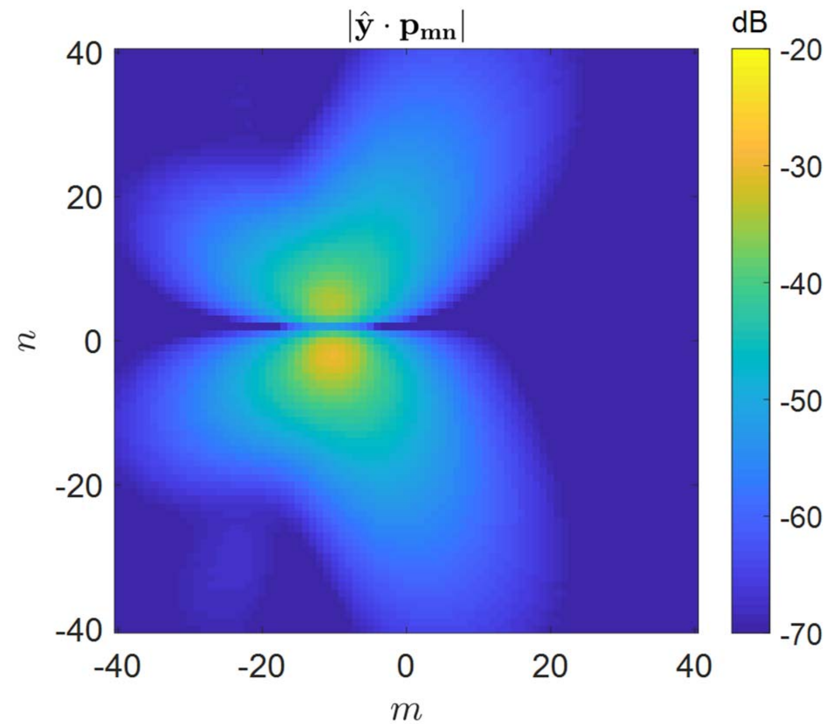
Dipole Moments

Primary source $\mathbf{J}(\mathbf{r}) = \mathbf{u}\,\delta(\mathbf{r} - \mathbf{r}_s)$

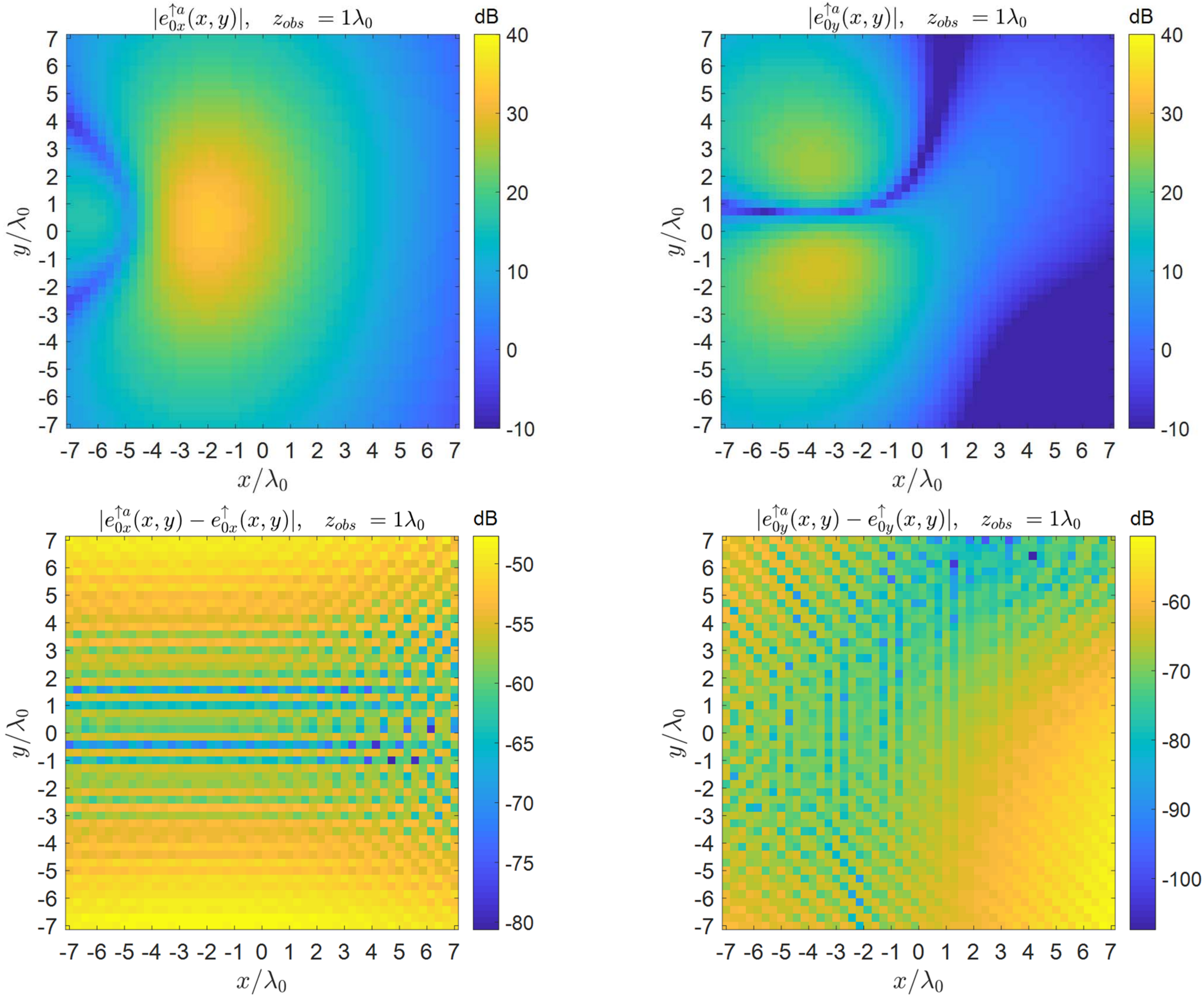
$\mathbf{r}_s = [-3\hat{\mathbf{x}} + 0.3\hat{\mathbf{y}} - 2\hat{\mathbf{z}}]\lambda_0,\ \mathbf{u} = (-1, 0.2, 1.6)$

$\Delta = 0.3\lambda_0, \quad k_1 = 1.2k_0, \quad d = 0.34\lambda_0, \quad h = 0.05\lambda_0$



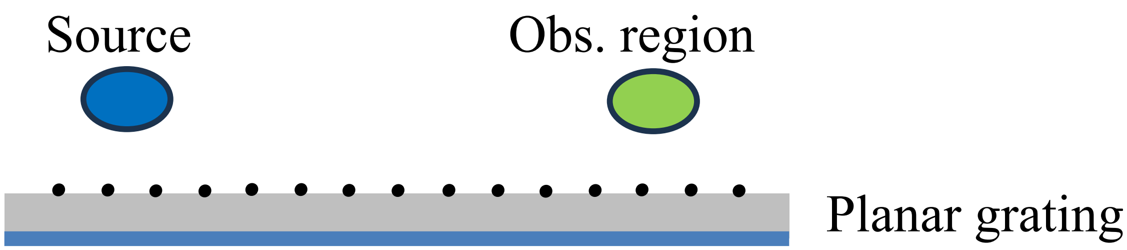


Field Calculation



Metagratings in Near-Field Applications

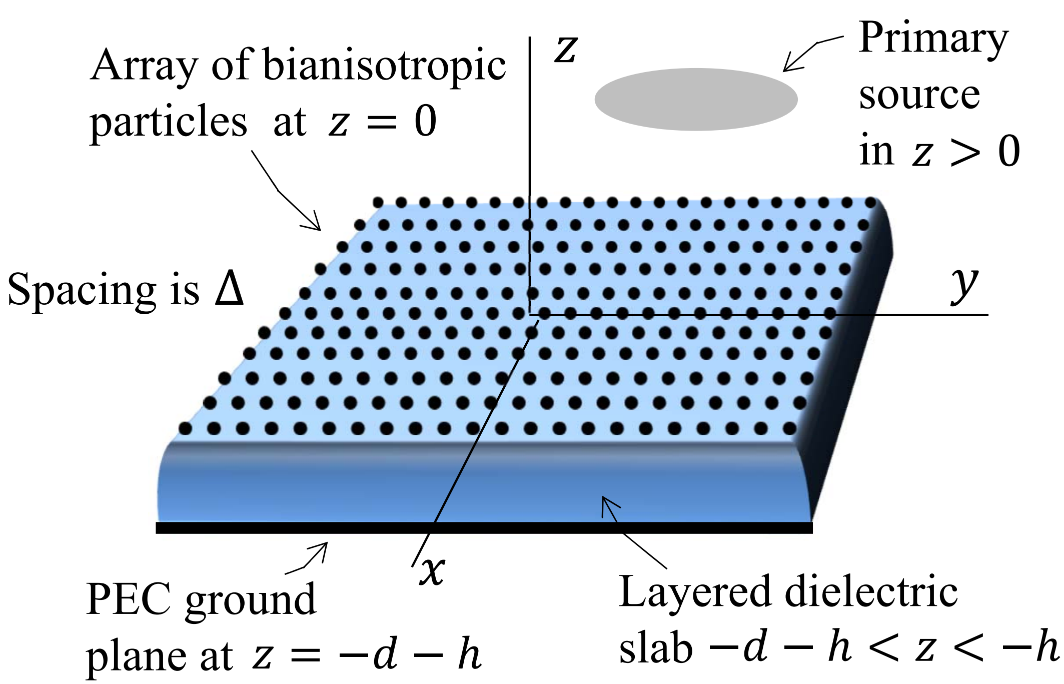
Metagratings, with their ability to manipulate both propagating and evanescent waves, offer unique opportunities for near-field applications: Sensing, Optical Trapping, Subwavelength Imaging, Spectroscopy, Guided Optics and Super-resolution Microscopy, Near-field Focusing, Nonradiative Field Patterns,



V. Popov, F. Boust, and S. N. Burokur, "Constructing the near field and far field with reactive metagratings: Study on the degrees of freedom," *Phys. Rev. Appl.*, vol. 11, no. 2, 2019, Art. no. 024074,

Y. Ra'di, and A. Alù, "Metagratings for Efficient Wavefront Manipulation," *IEEE Photonics Journal*, vol. 14, no. 1, February 2022, paper # 2207513.

2D Grating with Bianisotropic Particles



Incident field from primary source: $\mathbf{s}_0^{\downarrow i}$

Reflected field from slab alone: $\bar{\mathbf{R}}_{012}^{\uparrow} \cdot \mathbf{s}_0^{\downarrow i}$

Exterior field: $\mathbf{s}_0^{\downarrow i} + \bar{\mathbf{R}}_{012}^{\uparrow} \cdot \mathbf{s}_0^{\downarrow i}$

Desired total up field (excl. primary): $\mathbf{s}_0^{\uparrow d}$

Particle field: $\mathbf{s}_0^{\uparrow d} - \bar{\mathbf{R}}_{012}^{\uparrow} \cdot \mathbf{s}_0^{\downarrow i}$

Required electric dipole moments ($\hat{\mathbf{z}} \cdot \mathbf{p}_{mn} = 0$)

$$\mathbf{p}_{mn} = \frac{\Delta^2}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \bar{\mathcal{K}}_0^{-1}(k_x, k_y) \cdot \left[\bar{\mathbf{I}} + \bar{\mathbf{R}}_{012}^{\uparrow}(k_x, k_y) \right]^{-1} \cdot \left[\mathbf{s}_0^{\uparrow d}(k_x, k_y) - \bar{\mathbf{R}}_{012}^{\uparrow}(k_x, k_y) \cdot \mathbf{s}_0^{\downarrow i}(k_x, k_y) \right] e^{imk_x\Delta + ink_y\Delta} dk_x dk_y$$

Bianisotropic particles modeled by polarizability dyadics (only “transverse *ee*” included) [Tretyakov 2003]

$$\mathbf{p}_{mn} = \bar{\boldsymbol{\alpha}}_{mn}^{(ee)} \cdot \mathbf{e}_o^{local}(\mathbf{r}_{mn}), \qquad \mathbf{e}_o^{local} = \mathbf{e}_o^{exterior} + \mathbf{e}_o^{dipoles}$$

Determining the Polarizability Dyadics

The local field at each dipole can be computed from

$$\mathbf{e}_o^{exterior}(\mathbf{r}_{m_0 n_0}) = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\left[\mathbf{s}_0^{\downarrow i}(k_x,k_y)+\bar{\mathbf{R}}_{012}^{\uparrow}(k_x,k_y)\cdot\mathbf{s}_0^{\downarrow i}(k_x,k_y)\right]e^{im_0 k_x\Delta + in_0 k_y\Delta}dk_x dk_y$$

$$\mathbf{e}_o^{dipoles}(\mathbf{r}_{m_0 n_0}) = i\omega\mu \sum_{\substack{m=-\infty \\ (m,n)\neq(m_0,n_0)}}^{\infty}\sum_{n=-\infty}^{\infty}\mathbf{p}_{mn}\cdot\bar{\mathbf{G}}(\mathbf{r}_{m_0 n_0},\mathbf{r}_{mn})$$
$$+\frac{1}{2\pi}\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\left[\bar{\mathbf{R}}_{012}^{\uparrow}(k_x,k_y)\cdot\bar{\boldsymbol{\mathcal{K}}}_0(k_x,k_y)\right]\cdot\mathbf{p}_{mn}\,e^{i(m_0-m)k_x\Delta+i(n_0-n)k_y\Delta}dk_x dk_y$$

With $\bar{\boldsymbol{\alpha}}_{mn}^{(ee)} = \alpha_{xx,mn}^{(ee)}\hat{\mathbf{x}}\hat{\mathbf{x}} + \alpha_{yy,mn}^{(ee)}\hat{\mathbf{y}}\hat{\mathbf{y}}$ we get

$$\alpha_{xx,mn}^{(ee)} = \frac{\hat{\mathbf{x}}\cdot\mathbf{p}_{mn}}{\hat{\mathbf{x}}\cdot\mathbf{e}_o^{exterior}(\mathbf{r}_{mn})+\hat{\mathbf{x}}\cdot\mathbf{e}_o^{dipoles}(\mathbf{r}_{mn})},\qquad \alpha_{yy,mn}^{(ee)} = \frac{\hat{\mathbf{y}}\cdot\mathbf{p}_{mn}}{\hat{\mathbf{y}}\cdot\mathbf{e}_o^{exterior}(\mathbf{r}_{mn})+\hat{\mathbf{y}}\cdot\mathbf{e}_o^{dipoles}(\mathbf{r}_{mn})}$$

Conclusions

- Derived approximate discrete Huygens representations with electric dipole sources determined by the plane-wave spectrum of the primary source for both free-space and dielectric-slab configurations.

- Formulation employs x-y components of the plane-wave spectrum and holds for arbitrary sources without dividing fields into TE and TM modes.

- Gamma singularity occurring in these plane-wave expressions is explicit in expressions. Discrete and single plane-wave configurations are contained in the theory. Low frequency scenarios included.

- Similar representation can be obtained with magnetic dipoles and combinations of electric and magnetic dipoles.

- Error of the representation is expressed in terms of the spectrum of the primary source.

- Huygens representation determine the polarizability dyadics for grating consisting of bianisotropic particles above dielectric layer and PEC ground plane.