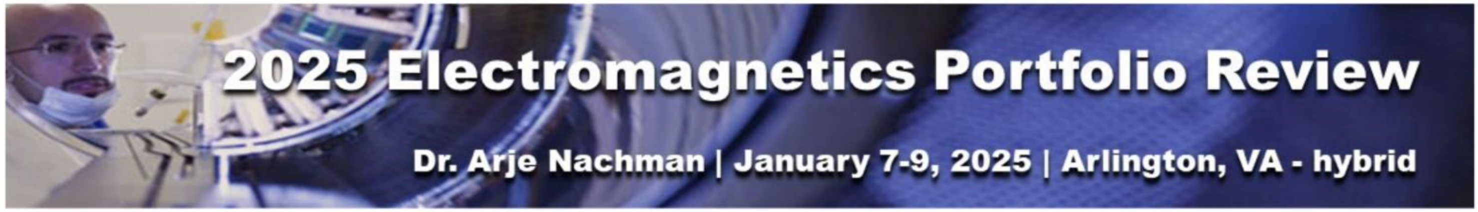

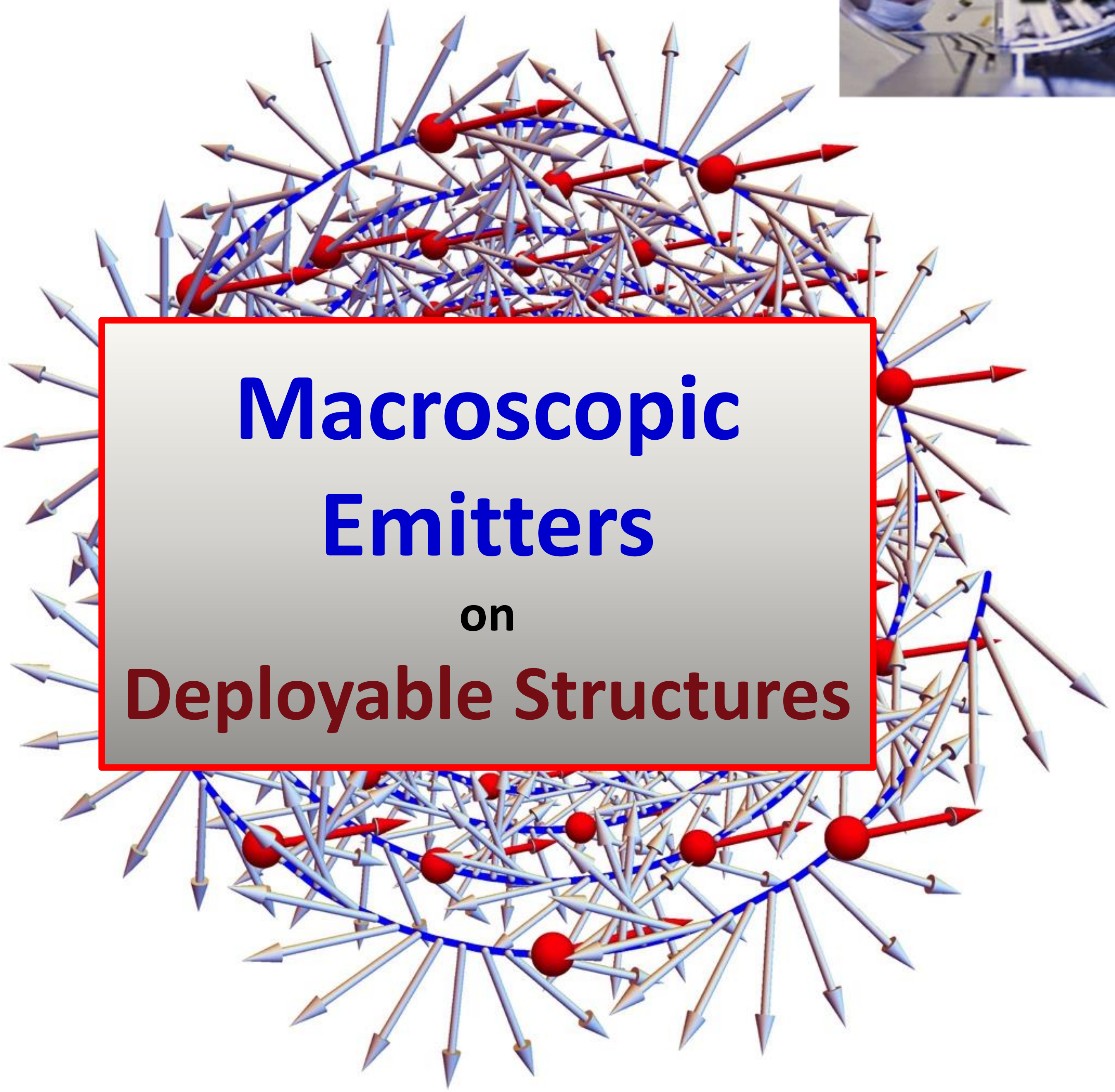

Macroscopic Emitters on Deployable Structures

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Supported by AFOSR

Outline

1) Antenna array on deployable structures
   a. Far-field formula
   b. Constructive interference
   c. Applications with illustrative examples

2) General time-harmonic solution of Maxwell's equation
   a. Trapezoidal rule
   b. Constructive/destructive interference
   c. Structure determination

Background: Maxwell's equation

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0} \qquad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \cdot \mathbf{B} = 0 \qquad \nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

Two relevant results in monochromatic regime $\mathbf{E}(\mathbf{x}, t) = e^{-i\omega t}\mathbf{E}_0(\mathbf{x})$

Field due to oscillating charge at mean position $\bar{\mathbf{y}}$ (Antenna radiation)

$$\mathbf{E}_0(\mathbf{x}) = k_{el} \frac{1}{|\mathbf{x} - \bar{\mathbf{y}}|} \left(\mathbf{I} - \frac{\mathbf{x} - \bar{\mathbf{y}}}{|\mathbf{x} - \bar{\mathbf{y}}|} \otimes \frac{\mathbf{x} - \bar{\mathbf{y}}}{|\mathbf{x} - \bar{\mathbf{y}}|} \right) \ddot{\bar{\mathbf{y}}} e^{i(\omega/c)|\mathbf{x} - \bar{\mathbf{y}}|}$$

Radiation in free space

$$\Delta \mathbf{E}_0 = -\frac{\omega^2}{c^2} \mathbf{E}_0$$

$$\nabla \cdot \mathbf{E}_0 = 0$$

Antenna model using oscillating charge

- For single antenna (modelled by an oscillating charge),

$$\mathbf{E}_0(\mathbf{x}) = k_{el}\frac{1}{|\mathbf{x}-\bar{\mathbf{y}}|}\left(\mathbf{I} - \frac{\mathbf{x}-\bar{\mathbf{y}}}{|\mathbf{x}-\bar{\mathbf{y}}|}\otimes\frac{\mathbf{x}-\bar{\mathbf{y}}}{|\mathbf{x}-\bar{\mathbf{y}}|}\right)\ddot{\bar{\mathbf{y}}}e^{i(\omega/c)|\mathbf{x}-\bar{\mathbf{y}}|}$$

$\bar{\mathbf{y}}$: Mean position of charge

k_{el} : Electronic constant

$\ddot{\bar{\mathbf{y}}}$: Mean acceleration of charge

ω : Time frequency of the radiation

- For multiple antennas (using superposition),

$$\mathbf{E}_0(\mathbf{x}) = k_{el}\frac{1}{|\mathbf{x}-\mathbf{y}_s|}\left(\mathbf{I} - \frac{\mathbf{x}-\mathbf{y}_s}{|\mathbf{x}-\mathbf{y}_s|}\otimes\frac{\mathbf{x}-\mathbf{y}_s}{|\mathbf{x}-\mathbf{y}_s|}\right)\sum_j \ddot{\bar{\mathbf{y}}}_j e^{i\phi_j} e^{i(\omega/c)|\mathbf{x}-\mathbf{y}_s|}$$

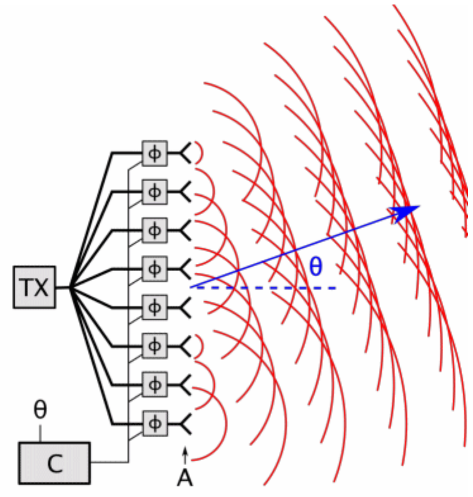
$\mathbf{y}_s$: Mean position of antenna array

ϕ_j : Relative phase between antenna

Radiation generated by antenna array

Using simple substitution : $\phi_j + (\omega/c)\,|\mathbf{x} - \mathbf{y}_j| = \varphi_j$

$$\mathbf{E}_0(\mathbf{x}) = k_{el}\frac{1}{|\mathbf{x}-\mathbf{y}_s|}\left(\mathbf{I} - \frac{\mathbf{x}-\mathbf{y}_s}{|\mathbf{x}-\mathbf{y}_s|} \otimes \frac{\mathbf{x}-\mathbf{y}_s}{|\mathbf{x}-\mathbf{y}_s|}\right)\sum_j \ddot{\bar{\mathbf{y}}} e^{i\varphi_j}$$



Relative phase between antennas parameterized by k_1 and k_2 : $\varphi_j = f_j\,\underbrace{(k_1, k_2)}_{}$

Free parameters (related to group)

f_j's are chosen as to get the desired interference pattern in parameter space

Isometry groups

$$g = (\mathbf{Q}|\mathbf{c}), \quad \mathbf{Q} \in O(3), \quad \mathbf{c} \in \mathbb{R}^3$$

$$g(\mathbf{x}) = \mathbf{Q}\mathbf{x} + \mathbf{c}, \quad \mathbf{x} \in \mathbb{R}^3$$

Group product

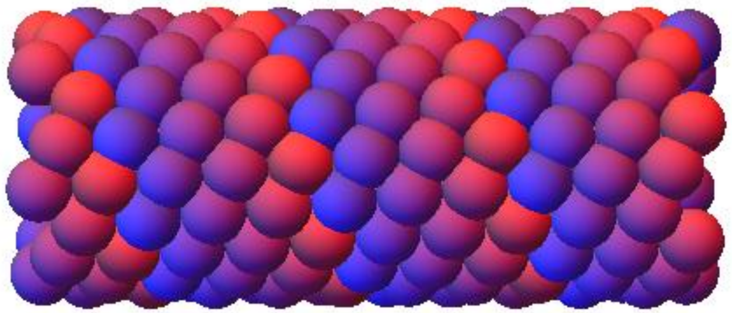
Identity

$$g_1 = (\mathbf{R}_1|\mathbf{c}_1) \quad g_2 = (\mathbf{R}_2|\mathbf{c}_2) \qquad g_1 g_2 = (\mathbf{R}_1\mathbf{R}_2|\mathbf{c}_1 + \mathbf{R}_1\mathbf{c}_2) \qquad id = (\mathbf{I}|\mathbf{0})$$

Helical group (Abelian, two generators)

$$\{h^i g^j : i \in \mathbb{Z}, j = 1, \ldots, n\}$$

$$h = (\mathbf{R}_\theta|\tau\mathbf{e} + (\mathbf{R}_\theta - \mathbf{I})\mathbf{x}_0\}$$
$$g = (\mathbf{R}_{2\pi/n}|(\mathbf{R}_{2\pi/n} - \mathbf{I})\mathbf{x}_0\}$$



Antenna position given by group orbits

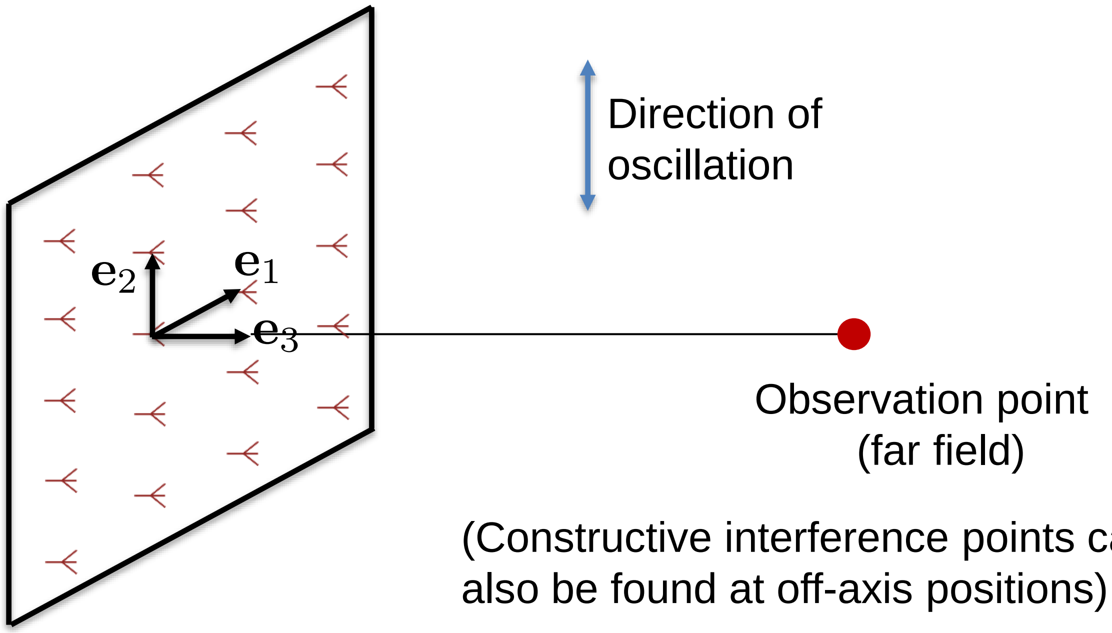
Translation group

$$g_1 = (\mathbf{I} \,|\, a\mathbf{e}_1)$$

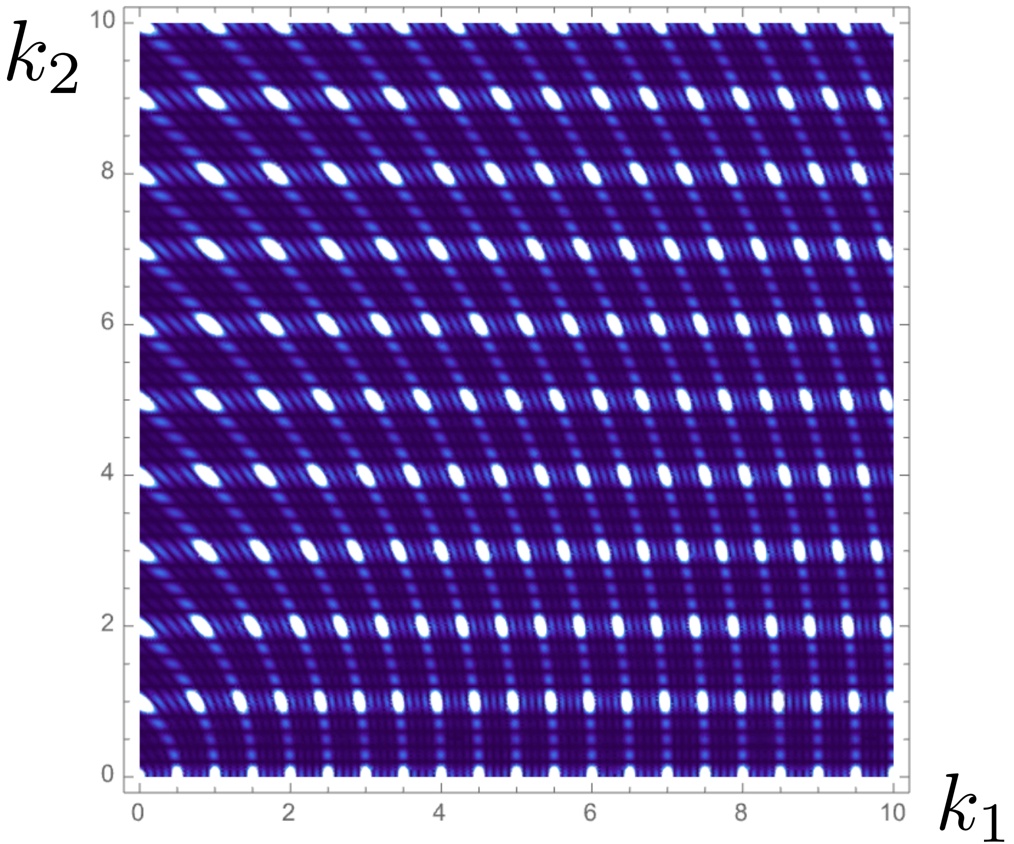
$$g_2 = (\mathbf{I} \,|\, b\mathbf{e}_2)$$

Position of antenna $\ell\, m$ $\quad \mathbf{y}_{\ell m} = g_1^{\ell} g_2^{m}(\mathbf{y}_0)$

Phase of antenna $\ell\, m$ $\quad \varphi_{\ell m} = 2\pi\left(\ell k_1 + m k_2 + \ell\sqrt{k_1^2 + k_2^2}\right)$



(Constructive interference points can also be found at off-axis positions)



Helical group

$$g = (\mathbf{R}_\theta | \tau \mathbf{e})$$

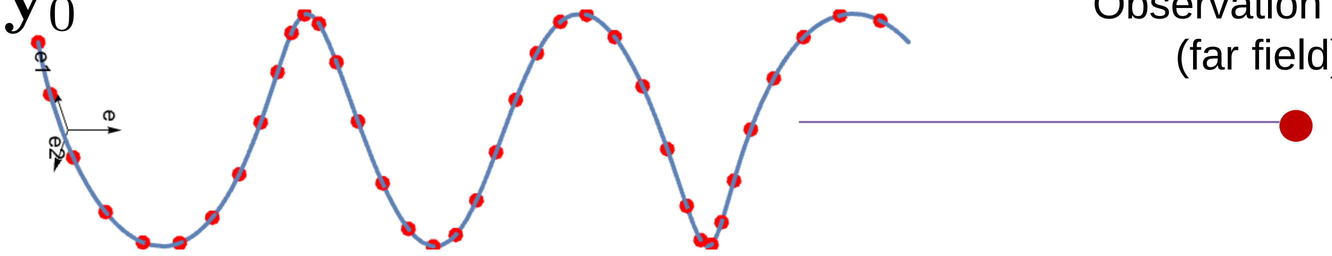
$$\mathbf{y}_j = g^j (\mathbf{y}_0)$$

$$\varphi_j = 2\pi \left(k_1 + j\tau \left(k_2 - \sqrt{k_1^2 + k_2^2} \right) \right)$$

Direction of oscillation:
into the page $\otimes$

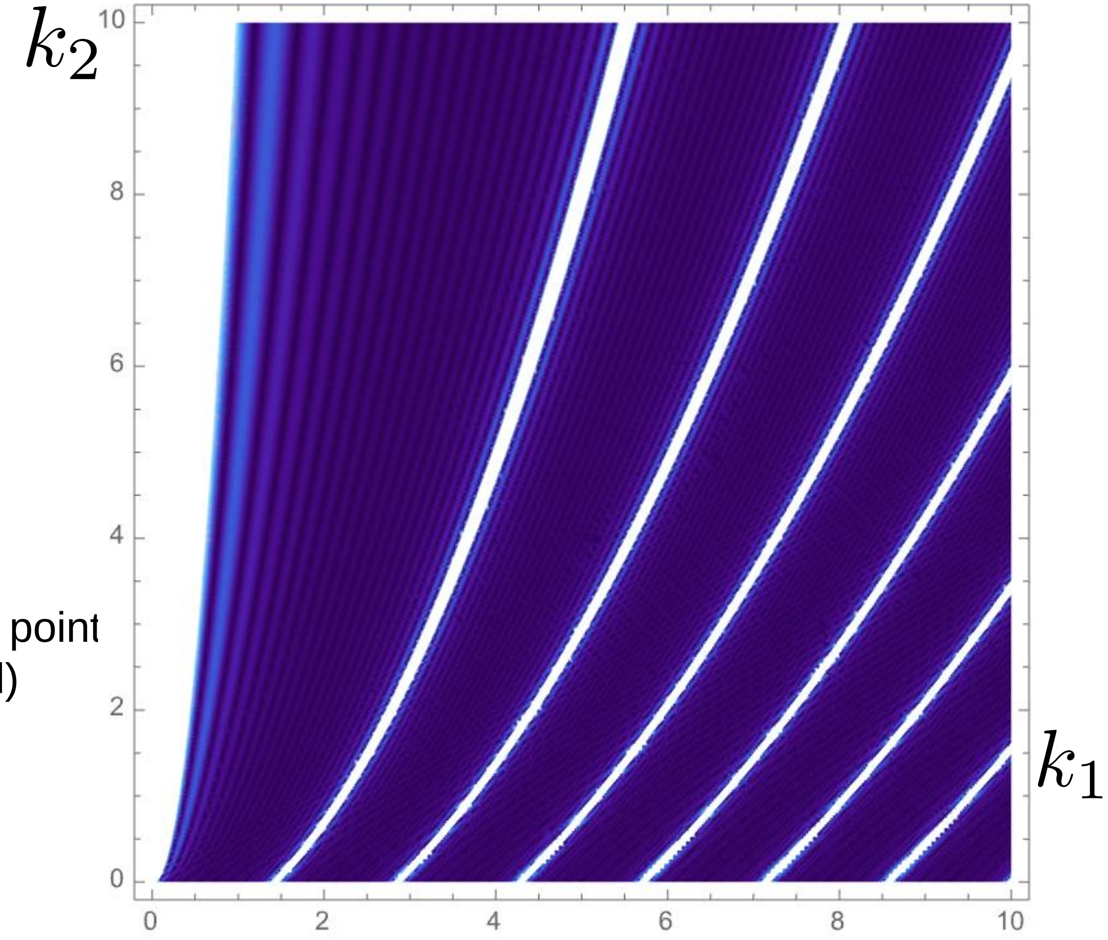
$\longrightarrow$
$\mathbf{e}$

$\mathbf{y}_0$



Observation point
(far field)

Intensity



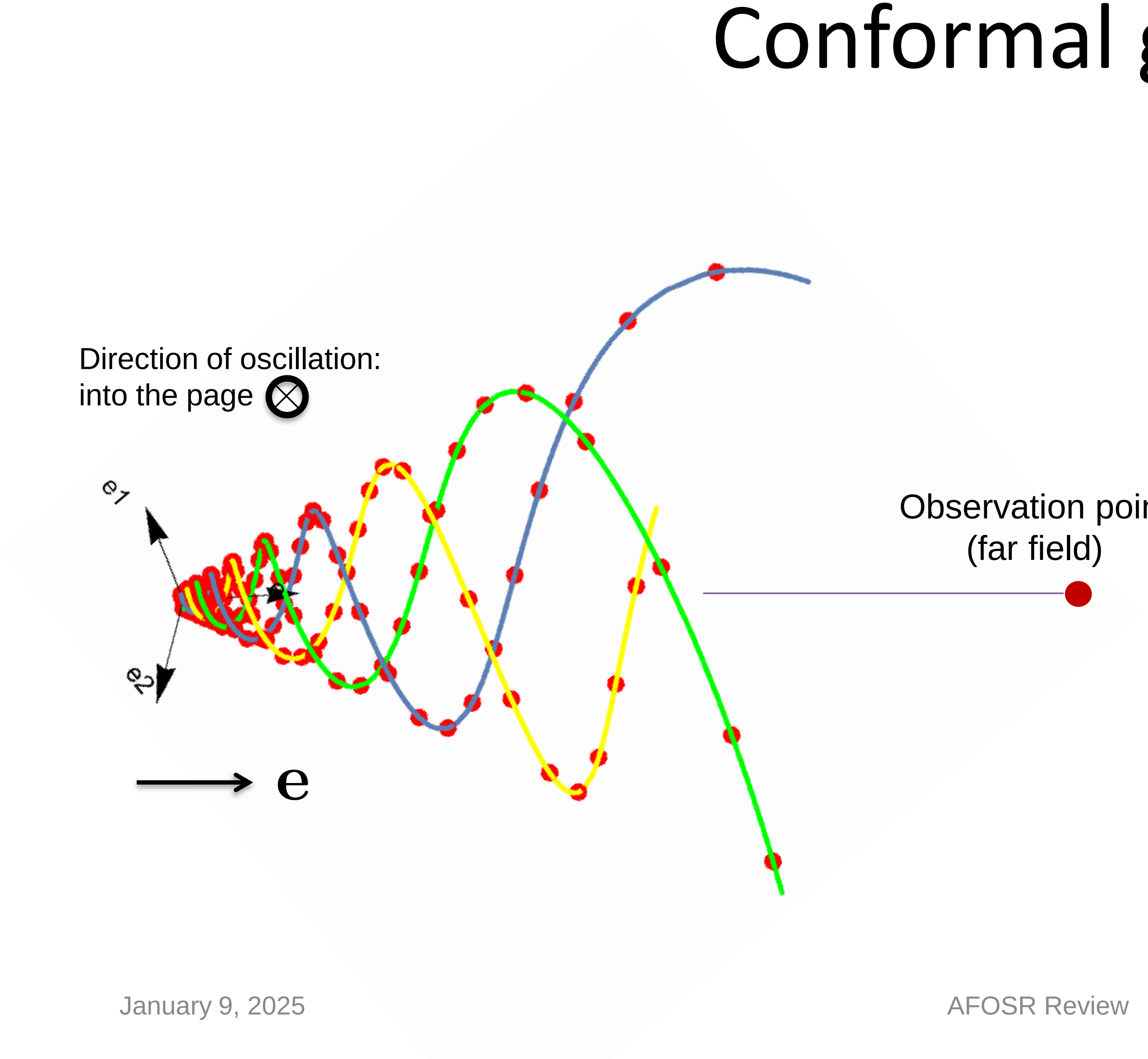
Conformal group

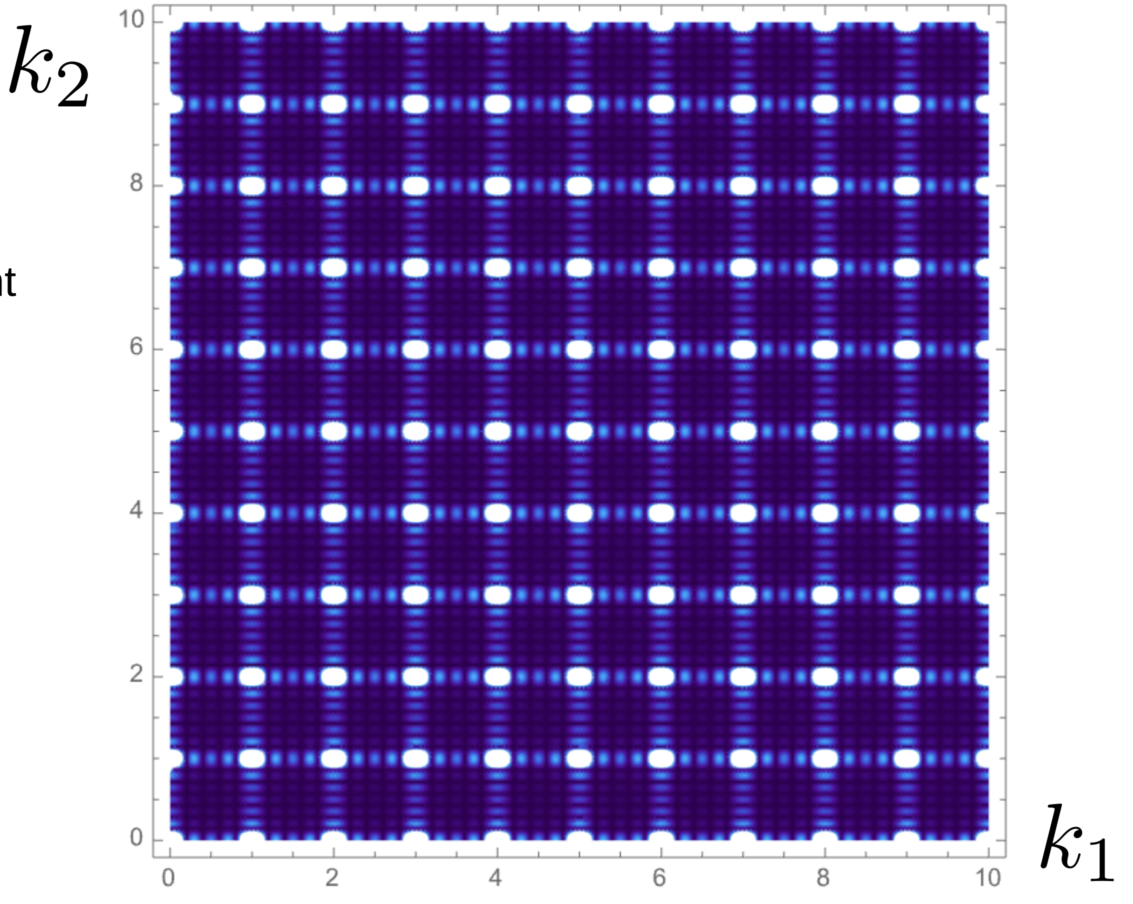
$$g_1 = (\lambda \mathbf{R}_\theta \,|\, \tau \mathbf{e})$$

$$g_2 = (\mathbf{R}_{2\pi/n} \,|\, 0)$$

$$\mathbf{y}_{\ell m} = g_1^{\ell} g_2^{m}(\mathbf{y}_0)$$

$$\varphi_{\ell m} = 2\pi\,(\ell k_1 + m k_2)$$





Summary

- ❖ If the antennas are placed at positions defined by a group orbit and suitable phases are assigned, strong constructive interference can be achieved in specific (and assignable) directions in the far field.

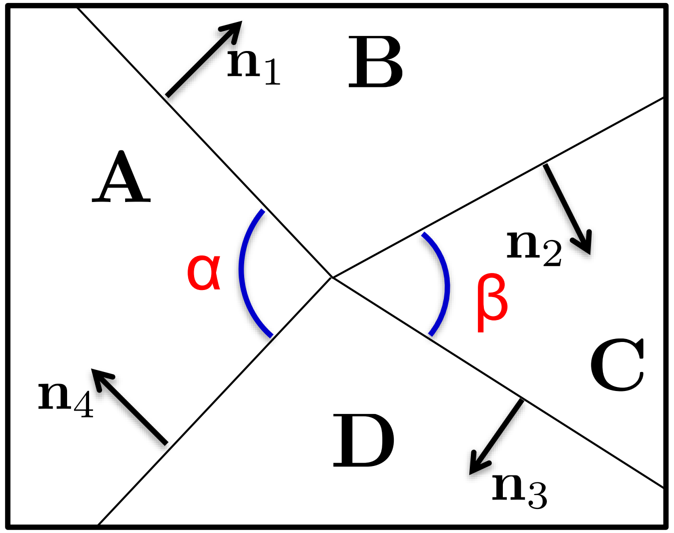
- ❖ The structure supporting the antennas may have additional functional requirements: reconfigurability, functionality.

- ❖ A natural method: origami design by the group orbit method. Match the groups.

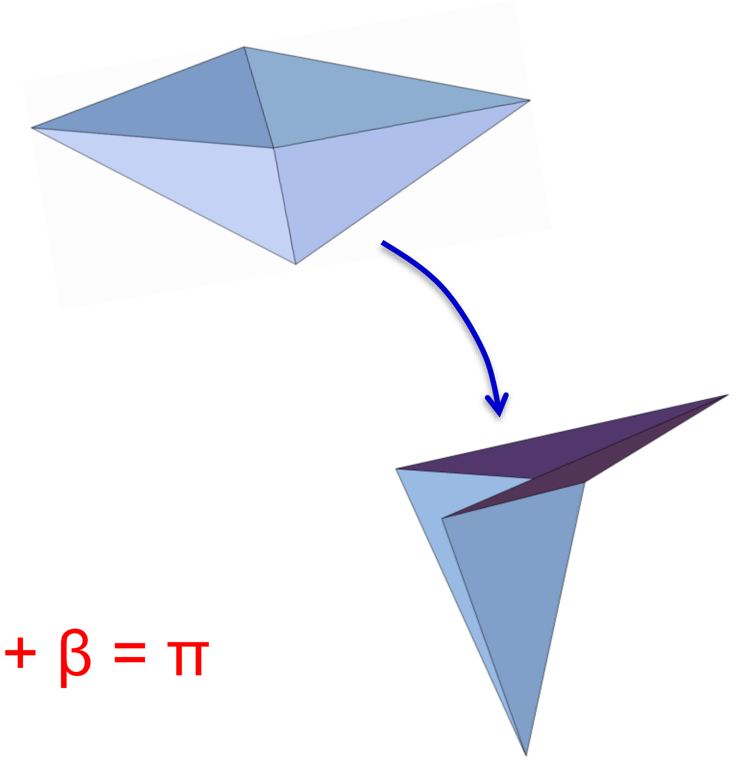
A classic theorem in origami

Theorem. A continuous y satisfies

$$\nabla \mathbf{y} =$$



(two families of solutions)

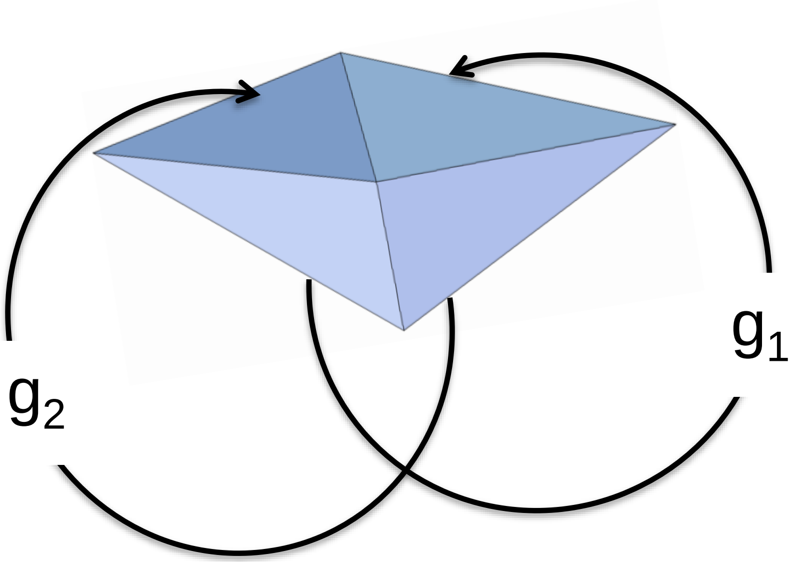


for A, C in SO(2) and B, D in SO(2)P if and only if α + β = π

In these cases, there exists a continuous folding

$\nabla \mathbf{y}_\omega : \Omega \to \mathbb{R}^3, \quad -\pi \le \omega < \pi$ where $\nabla \mathbf{y}_\omega \in \mathrm{O}(2,3)$

Where does the group come from?



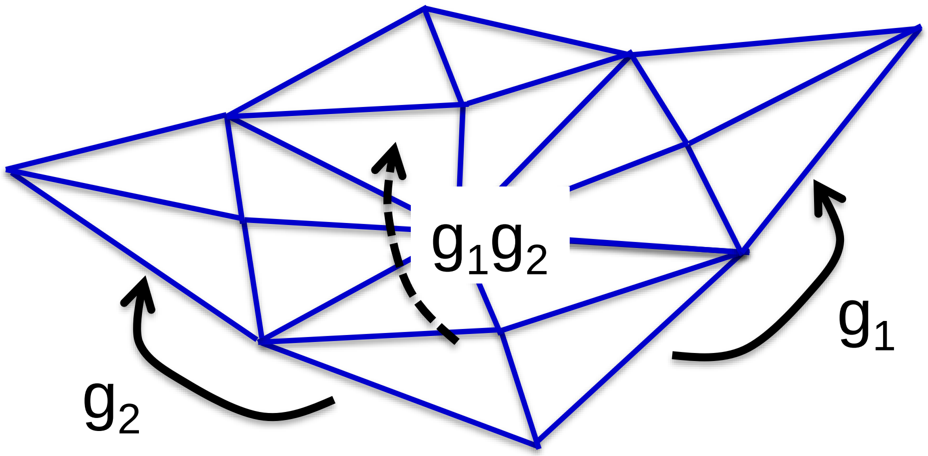
Two isometries :

$g_1 = (\mathbf{Q}_1|\mathbf{c}_1)$ and $g_2 = (\mathbf{Q}_2|\mathbf{c}_2)$,

$g_1 g_2 = g_2 g_1$

$$G = \{g_1^i g_2^j : i, j \in \mathbb{Z}\}$$

Get a compatible origami structure:



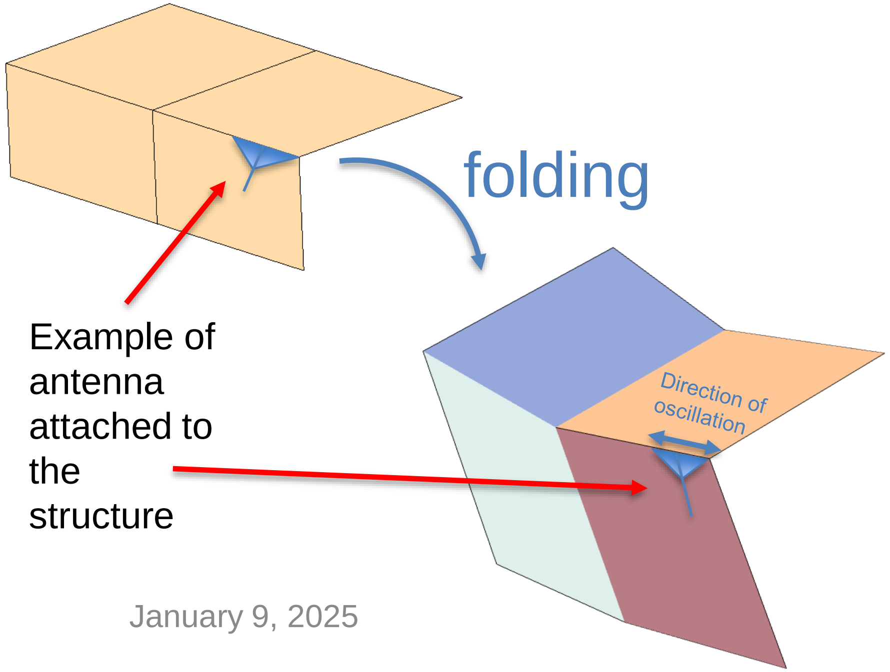
compatibilty = commutivity

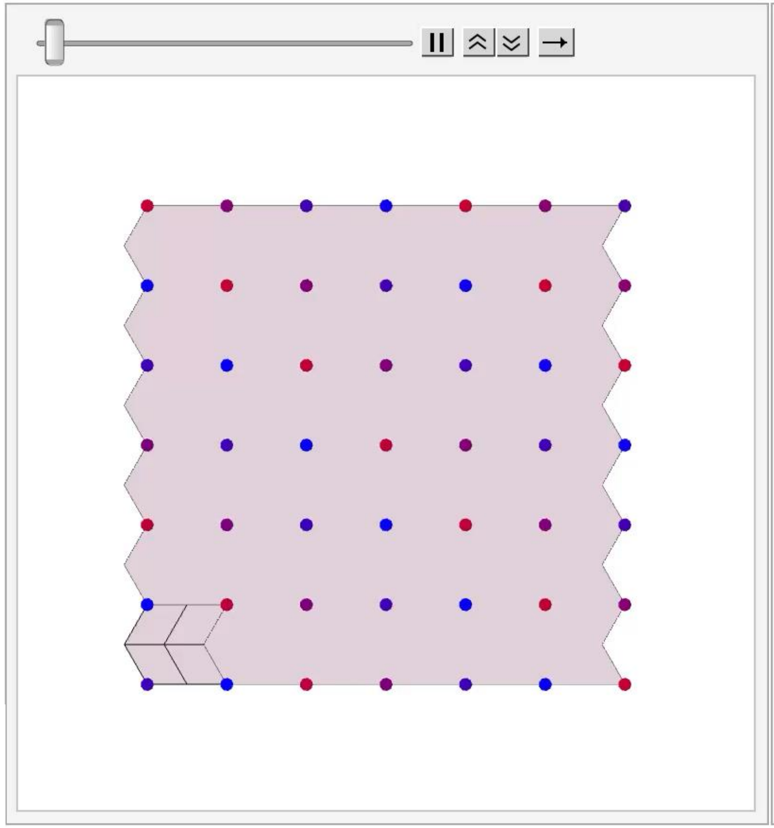
Application 1 : Monitoring deployable structure

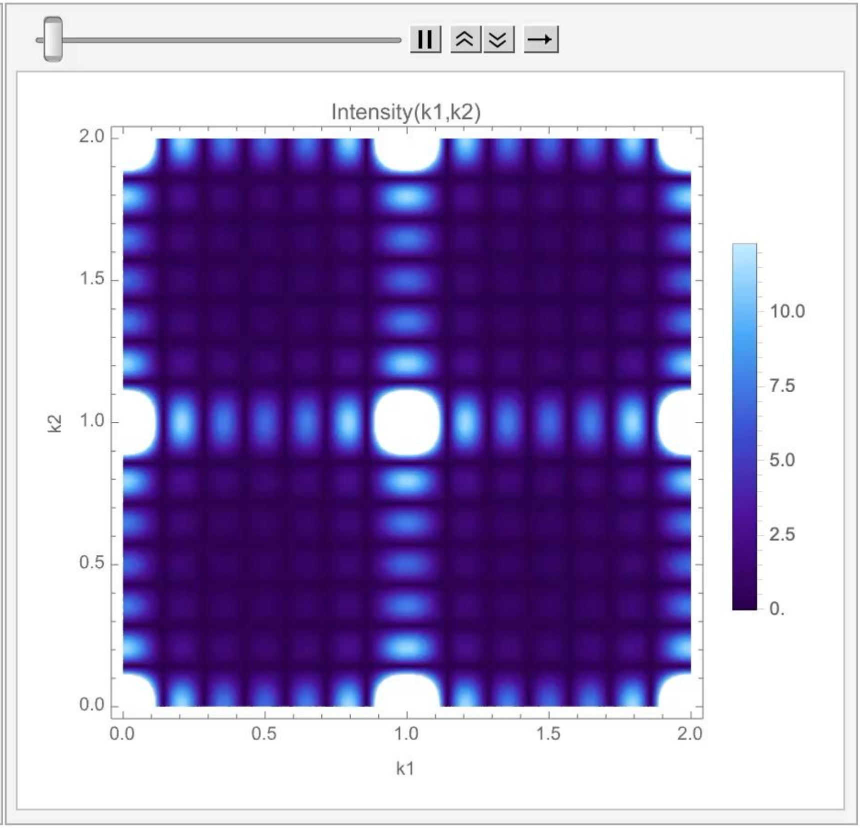
$$\varphi_{\ell m} = 2\pi\left(\ell k_1 + m k_2\right)$$

Color of the antenna represents its relative phase

Observation point is along a direction perpendicular to the structure in the far field



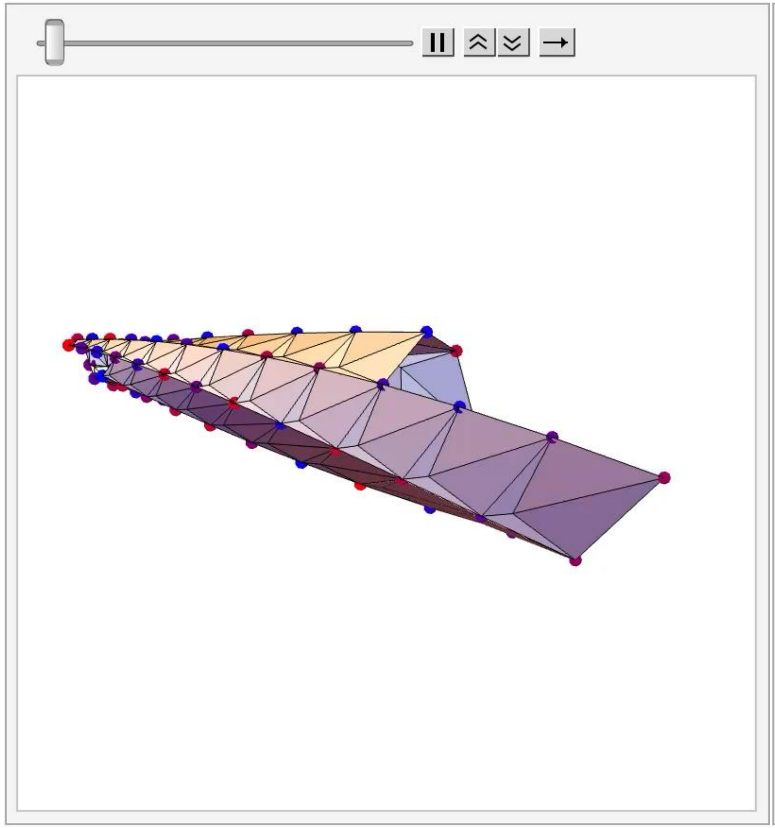


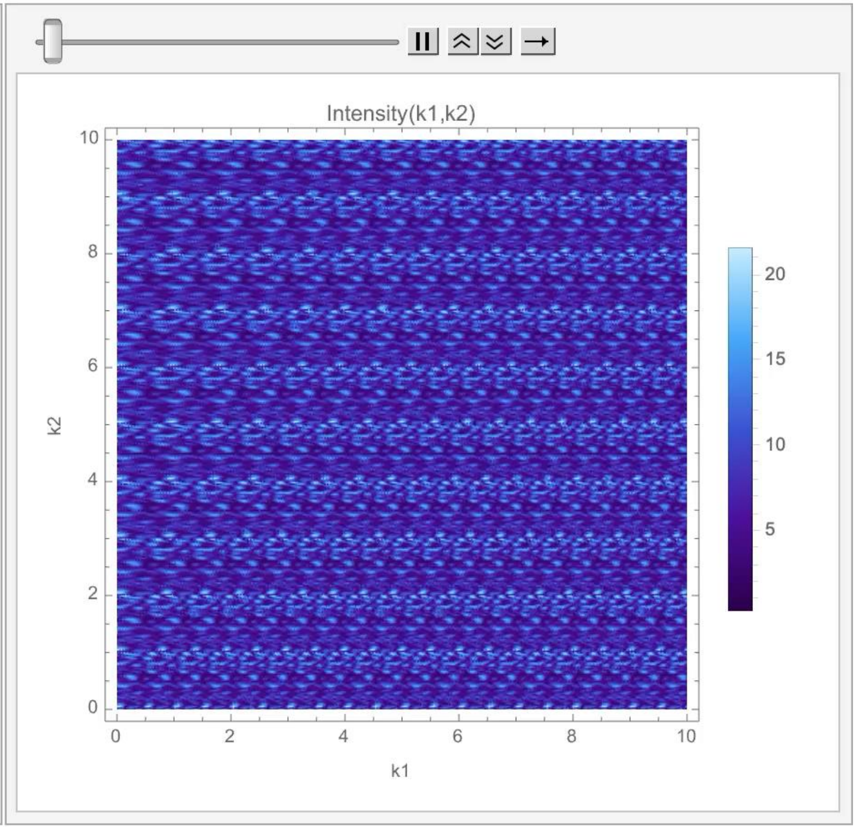


Based on the interference pattern in parameter space, progress of deployment can be determined.

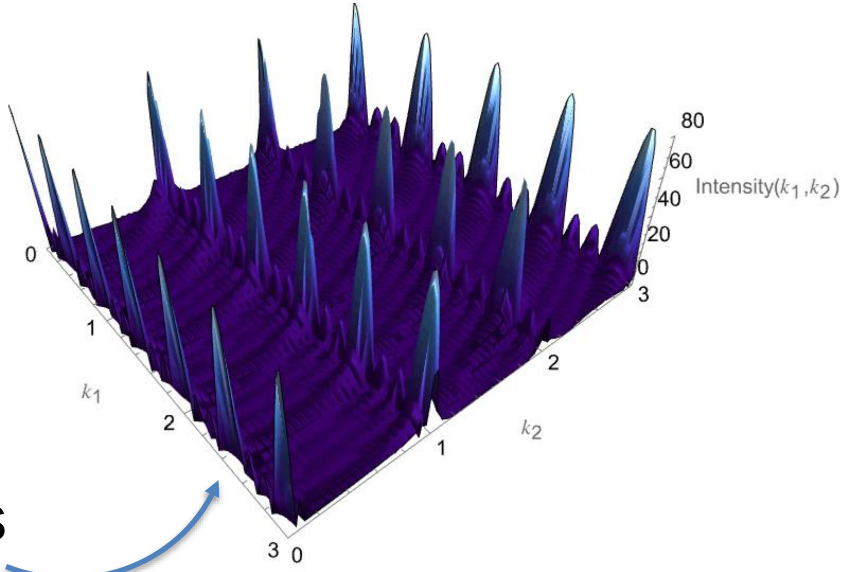
Kalpesh Jaykar, Richard D. James (2025), Macroscopic emitters on deployable structures : a group theoretic approach (preprint)

Application 2 : Validating the deployment





Observation point is along a direction perpendicular to screen in far field



We get the constructive interference when the structure is completely undeformed to a flat state

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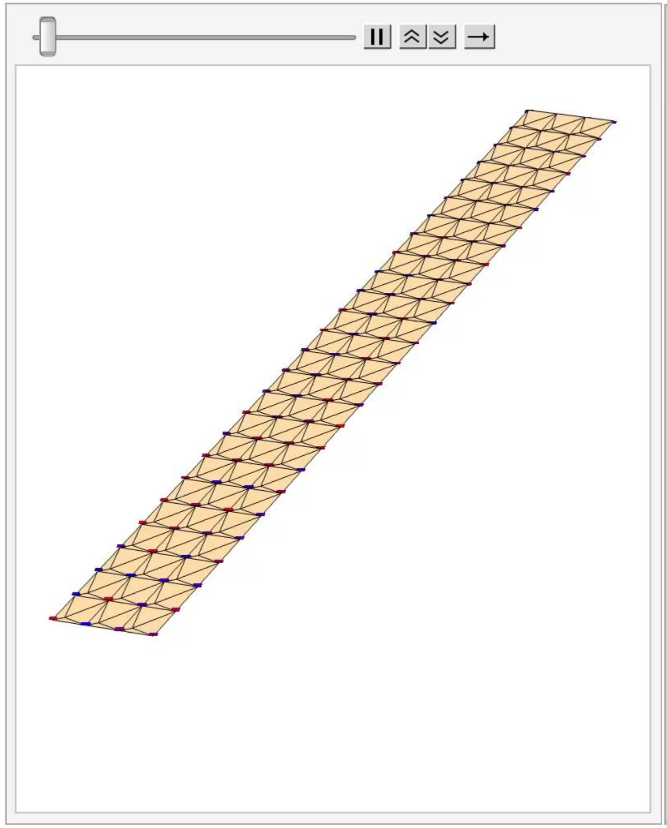
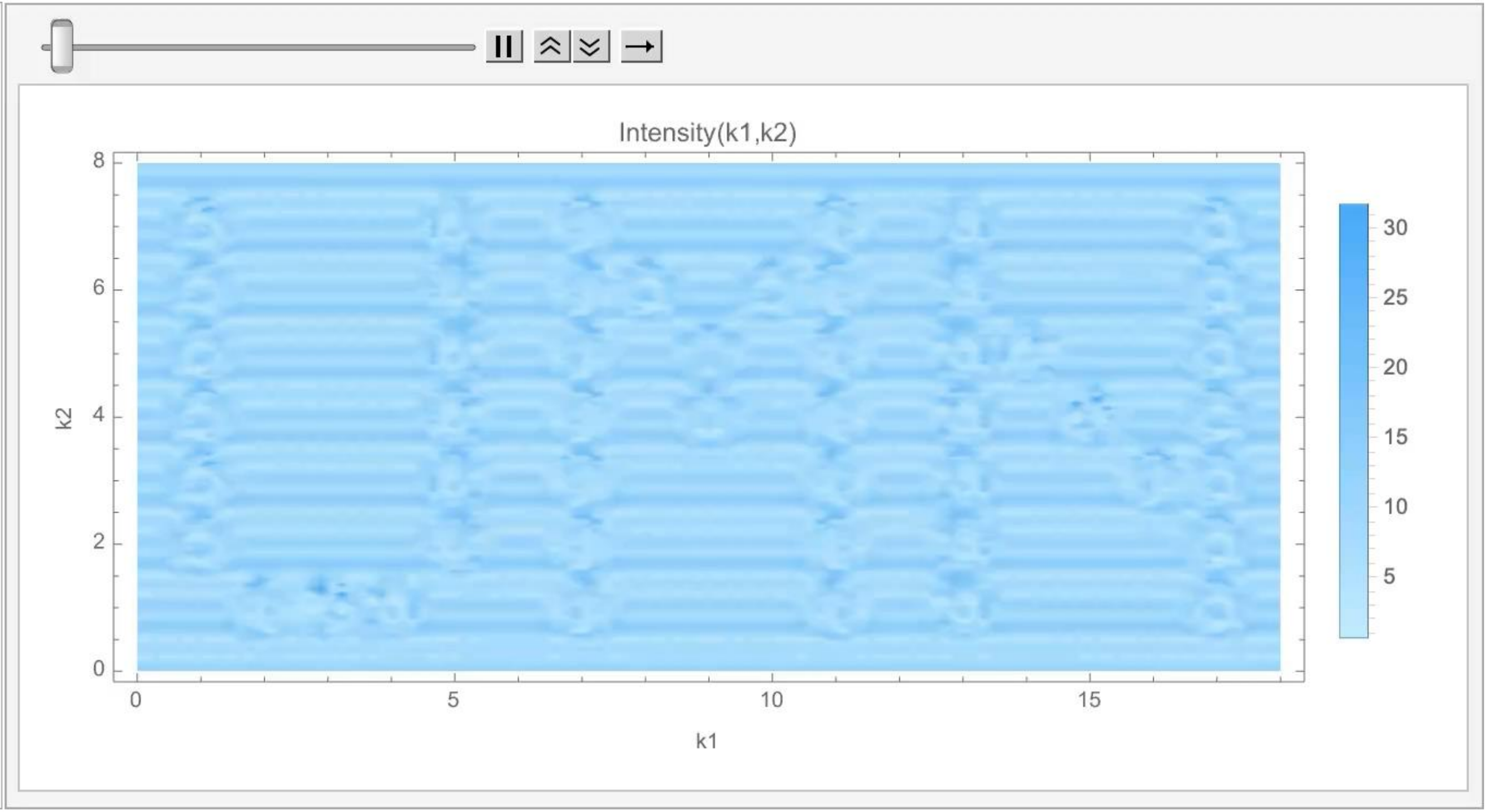
Application 3 : Display in parameter space

Observation point is along a direction perpendicular to screen in far field





When the structure is completely folded into helical shape, we can see the letters “UMN” within interference pattern in parameter space.

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Back to Maxwell's equation, but in free space

$$\mathbf{E}(\mathbf{x},t)=\frac{1}{2\pi}e^{-i\omega t}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\mathbf{T}_\omega(k_x,k_y)e^{i(k_x x+k_y y+\gamma z)}dk_x dk_y$$

Constraints :

- $k_x^2+k_y^2+\gamma^2=\omega^2/c^2$ (Monochromatic solution)
- $\mathbf{T}_\omega\cdot(k_x,k_y,\gamma)=0$

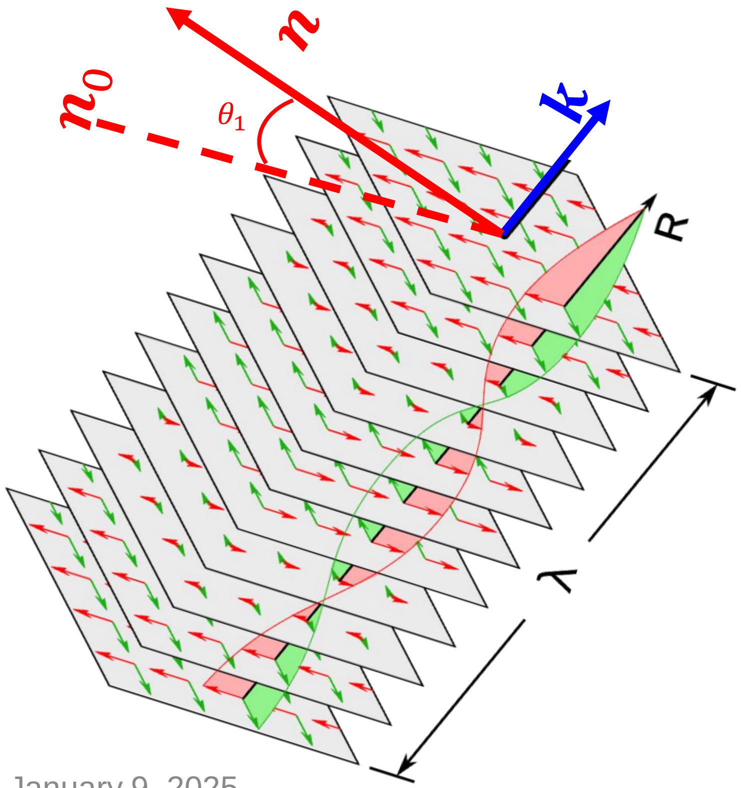
Plane-wave theory of time-domain field, Thorkild B. Hansen and Arthur D. Yaghjian

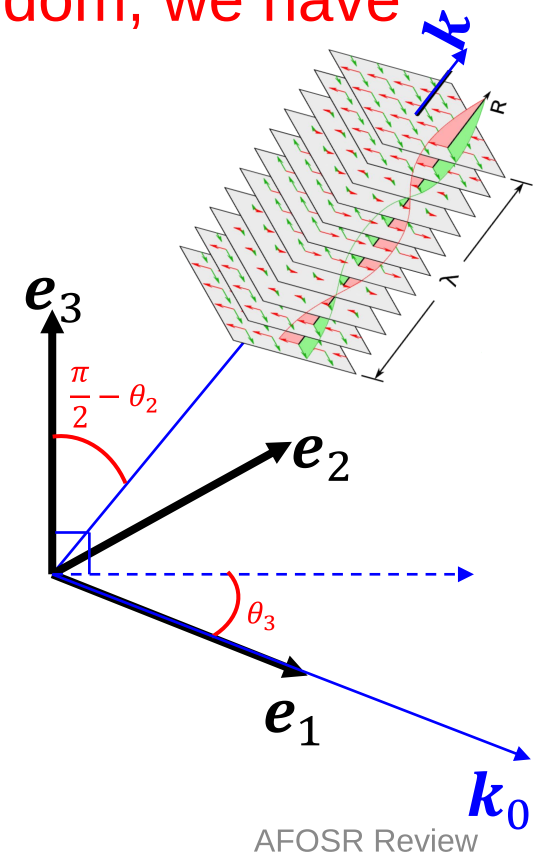
Another approach…

$$\mathbf{E}(\mathbf{x},t) = e^{-i\omega t}\int_{-\pi}^{\pi}\int_{-\pi/2}^{\pi/2}\int_{-\pi}^{\pi} A(\theta_1,\theta_2,\theta_3)\mathbf{R}_3(\theta_3)\mathbf{R}_2(\theta_2)\mathbf{R}_1(\theta_1)\mathbf{n}_0 e^{i(\mathbf{R}_3(\theta_3)\mathbf{R}_2(\theta_2)\mathbf{k}_0\cdot\mathbf{x})}d\theta_1 d\theta_2 d\theta_3$$

Freedom, we have

$\mathbf{R}_i(\theta_i)$ is rotation about $\mathbf{e}_i$ by θ_i.





General idea :

- Start with $\mathbf{E}_0(\mathbf{x}) = \mathbf{n}_0 e^{i\mathbf{k}_0\cdot\mathbf{x}}$ such that $|\mathbf{k}_0|^2 = \omega^2/c^2$.
- Rotate this in all possible direction and superimpose

Kalpesh Jaykar, Richard D. James (2025), General monochromatic solution of Maxwell's equation in free space and its application(preprint)

Approximation by trapezoidal rule

$$\mathbf{E}(\mathbf{x},t) \approx e^{-i\omega t} \sum_l \sum_m \sum_n A_{lmn}(\theta_{1_l}, \theta_{2_m}, \theta_{3_n}) \underbrace{\mathbf{R}_3(\theta_{3_n})\mathbf{R}_2(\theta_{2_m})\mathbf{R}_1(\theta_{1_l})\mathbf{n}_0 e^{i(\mathbf{R}_3(\theta_{3_n})\mathbf{R}_2(\theta_{2_m})\mathbf{k}_0\cdot\mathbf{x})}}$$

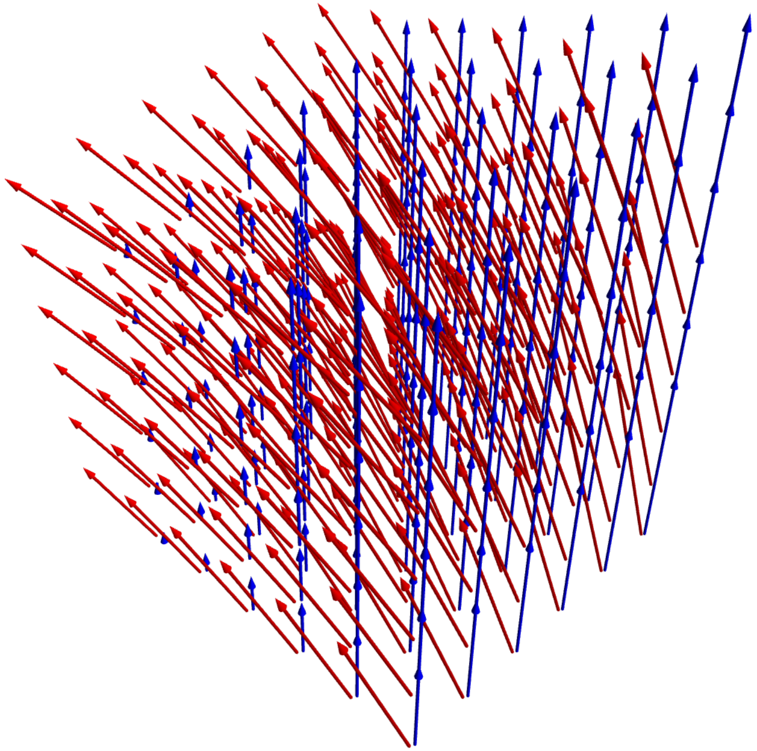
Maxwell’s equations are satisfied at the discrete level as well.

Choose different amplitudes $A_{lmn}(\theta_{1_l}, \theta_{2_m}, \theta_{3_n})$ in approximate formula or $A(\theta_1, \theta_2, \theta_3)$ in exact solution to design different radiations

Kalpesh Jaykar, Richard D. James (2025), General monochromatic solution of Maxwell’s equation in free space and its application(preprint)

Fast convergence of trapezoidal rule (Illustrative example)

Red – exact solution
Blue – approximate solution



- Number of terms 16
- Maximum normalized error 0.32

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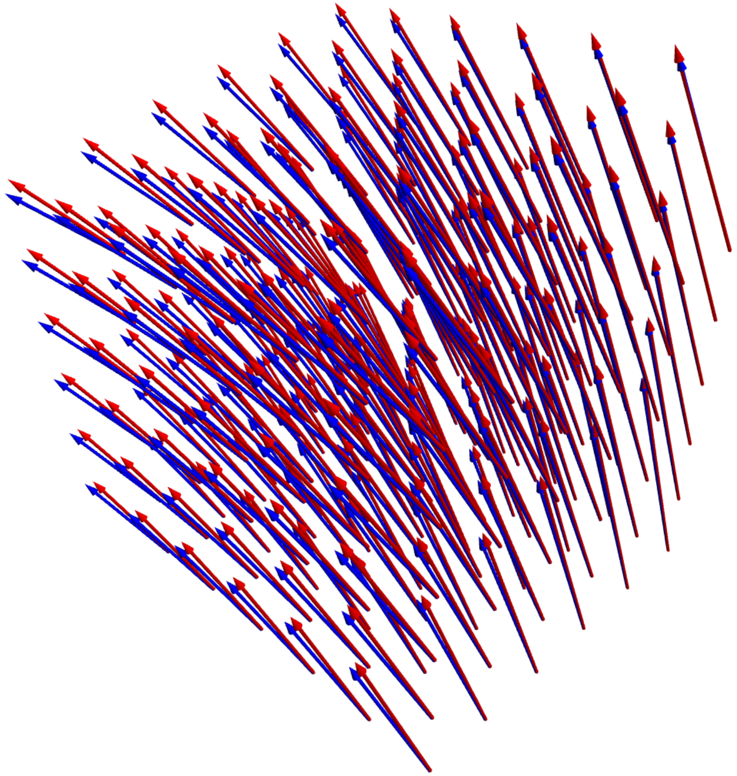
Fast convergence of trapezoidal rule (Illustrative example)



Red – exact solution
Blue – approximate solution

- ❑ Number of terms 36
- ❑ Maximum normalized error 0.11

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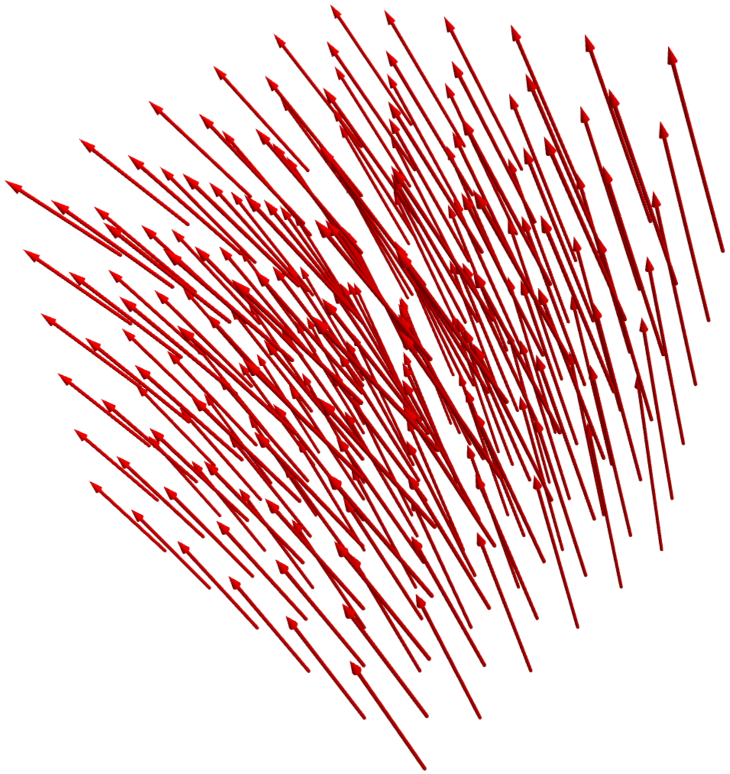
Fast convergence of trapezoidal rule (Illustrative example)

Red – exact solution
Blue – approximate solution



- Number of terms: 64
- Maximum normalized error: 6.26 x 10-6

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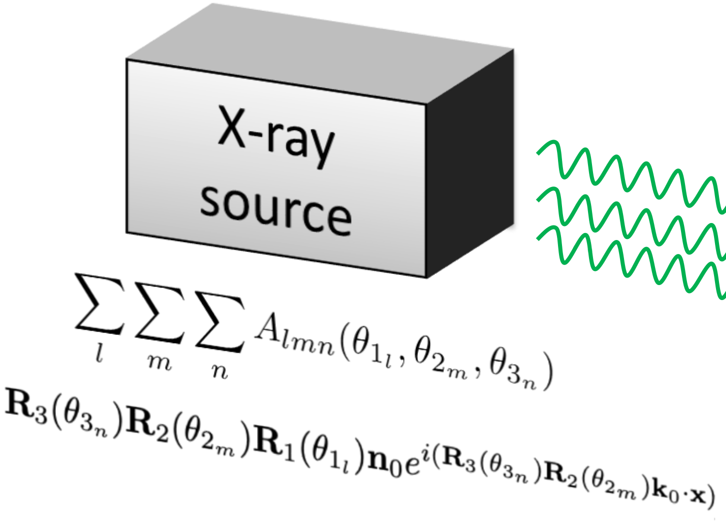
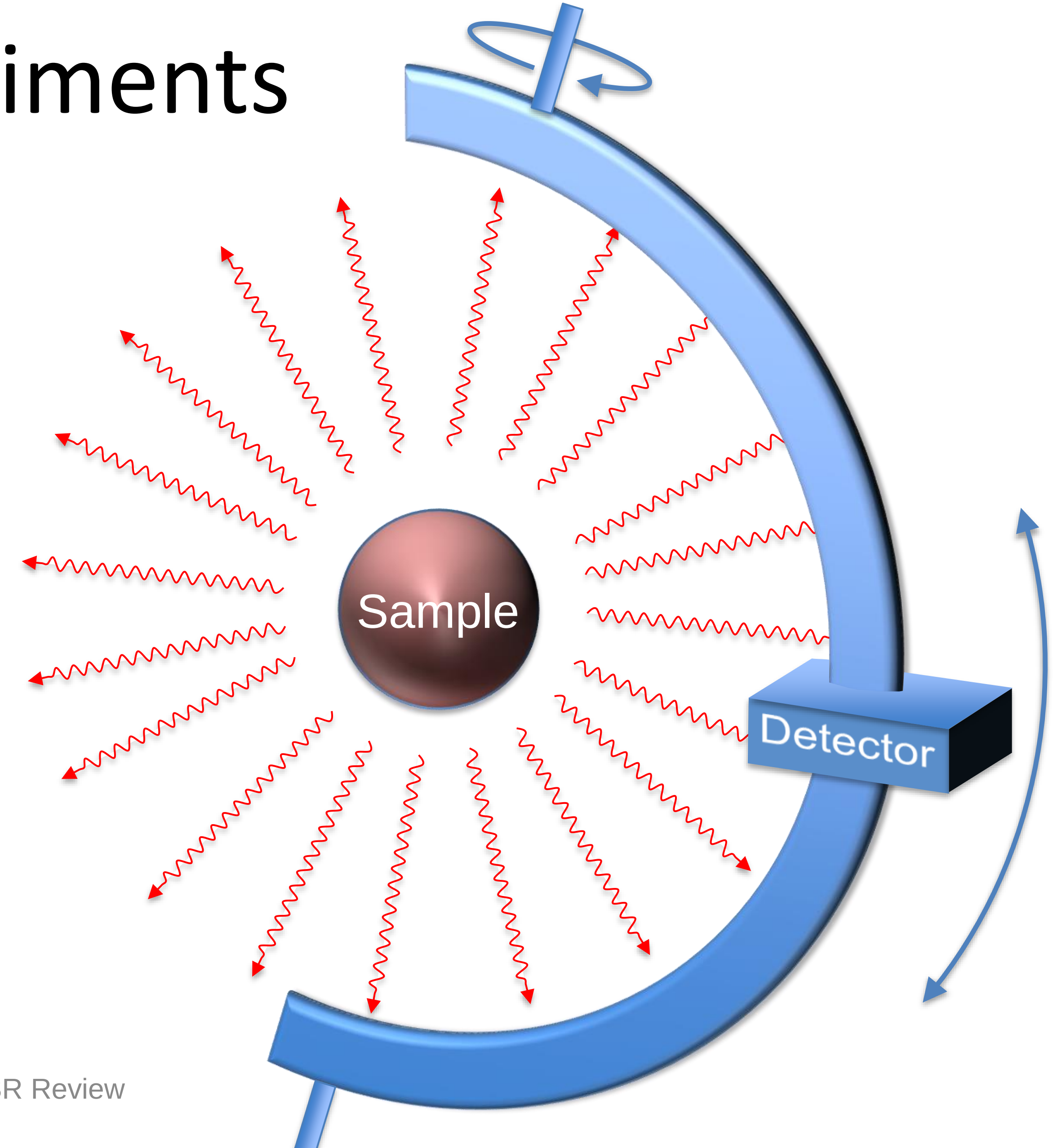
Why this solution is interesting?

$$\mathbf{E}(\mathbf{x},t) \approx e^{-i\omega t} \sum_{l} \sum_{m} \sum_{n} A_{lmn}(\theta_{1_l}, \theta_{2_m}, \theta_{3_n}) \mathbf{R}_3(\theta_{3_n}) \mathbf{R}_2(\theta_{2_m}) \mathbf{R}_1(\theta_{1_l}) \mathbf{n}_0 e^{i(\mathbf{R}_3(\theta_{3_n})\mathbf{R}_2(\theta_{2_m})\mathbf{k}_0 \cdot \mathbf{x})}$$

- Exact solution of the time harmonic Maxwell’s equations
- Extremely fast convergence of approximation by the Trapezoidal Rule
- The terms in the approximation are synchronized “dipole antennas”
- Time harmonic Maxwell’s equations are satisfied at both the discrete level and for the limit

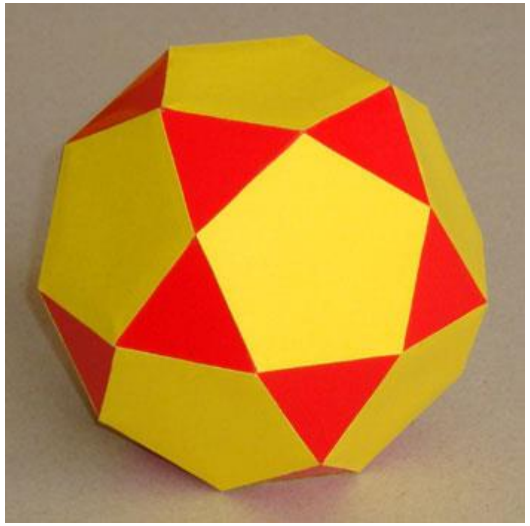
Schematic for structure determination experiments



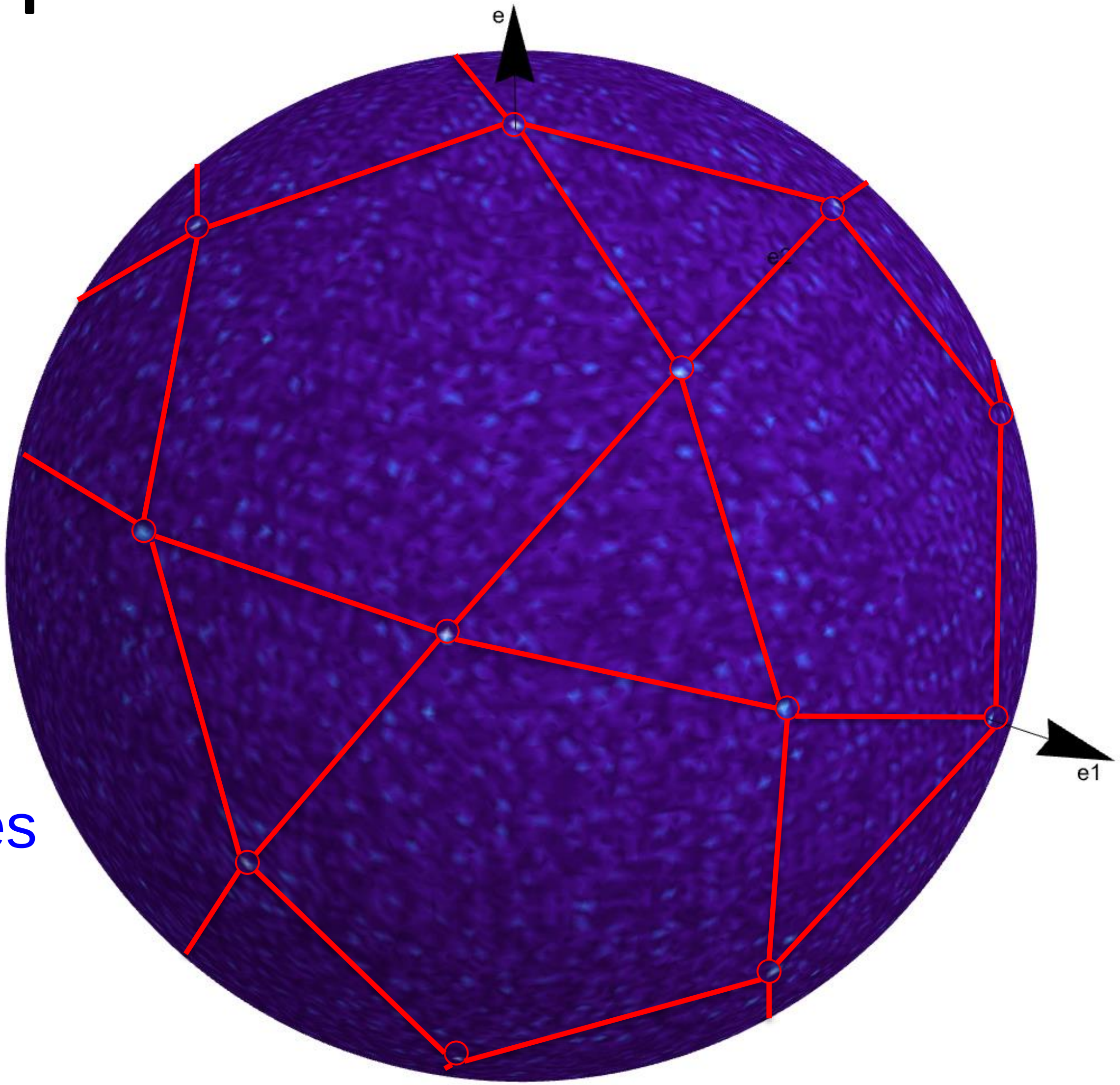


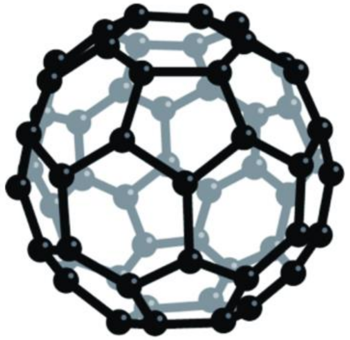
Design A_{lmn} to maximize the intensity in given direction

Intensity pattern with molecules like C_{60}



icosidodecahedron





C_{60} molecule

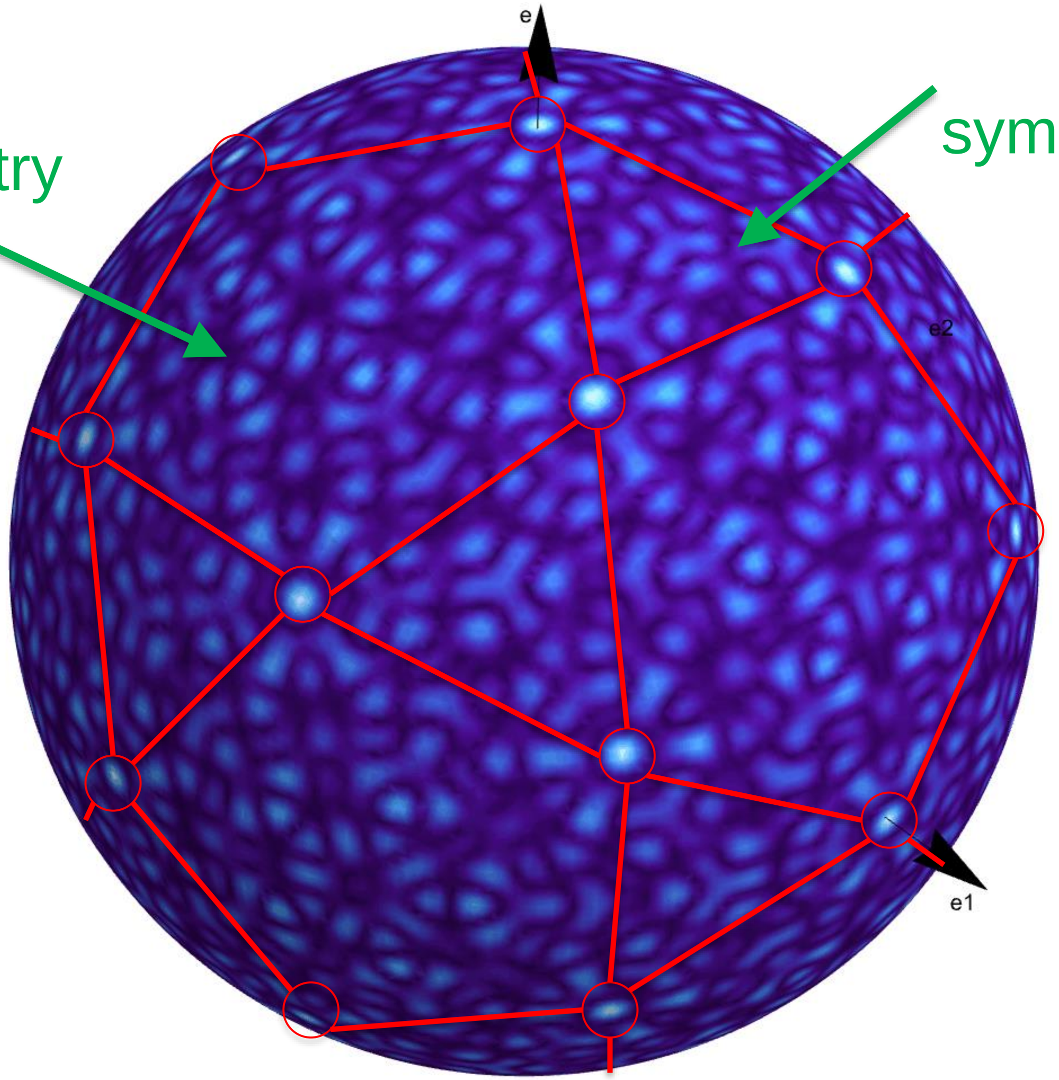
Vertices of peak intensities forms icosidodecahedron

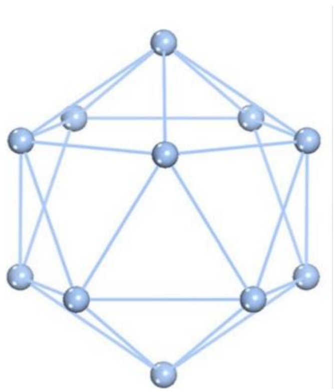
Image courtesy : Olivier Berné et al. (2015), 30 years of cosmic fullerenes.

Intensity pattern with molecule like Ag_{12}

5-fold symmetry

3-fold symmetry





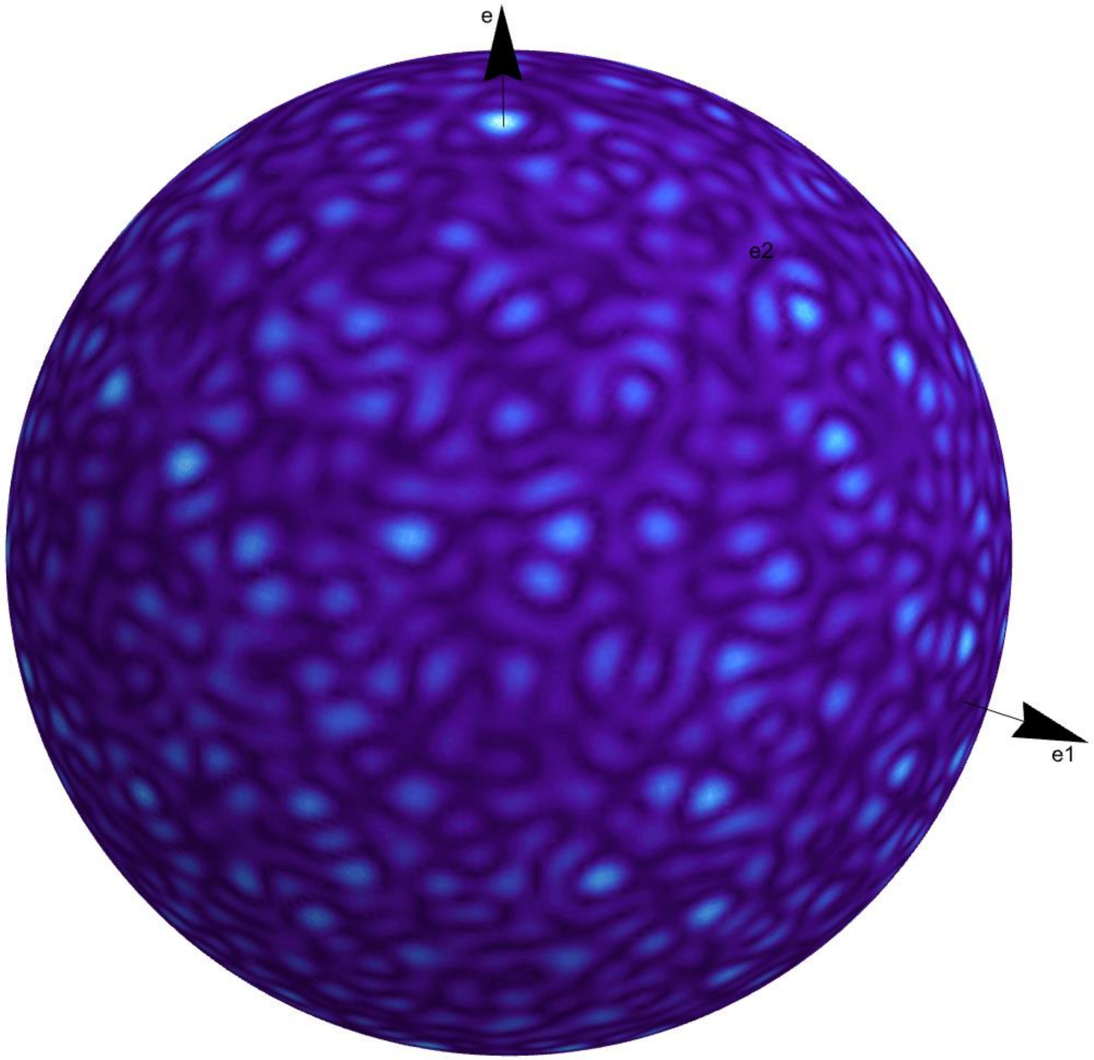
Ag_{12} molecule

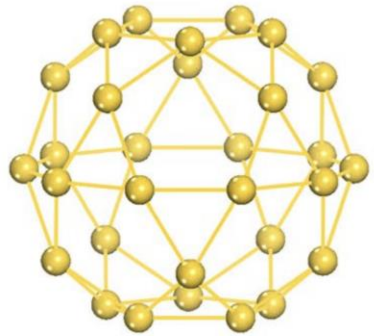
Again, vertices of peak intensities forms icosidodecahedron

Image courtesy : Zhaohua Chen, Zun Xie (2022), *J Mol Model.*

Intensity pattern with molecule like Au_{30}

Need to solve the inverse problems for these materials





Au_{30} molecule

Image courtesy : Zhaohua Chen, Zun Xie (2022), *J Mol Model.*

Thank you

Questions?

Applications in radar technology