

# **Efficient Implementation of Discrete Linear Unitary Operations with Photonic Circuits**

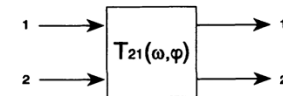
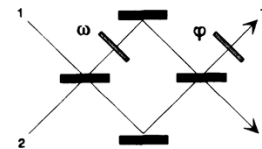
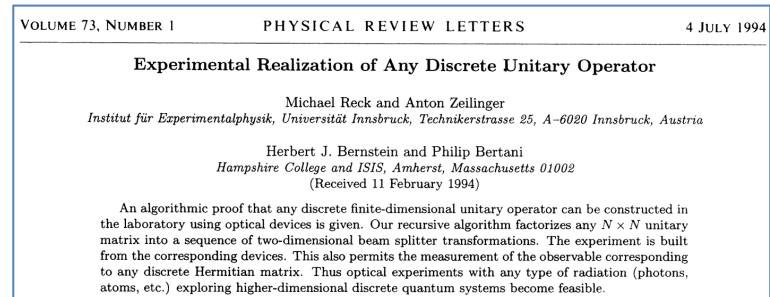
Mohammad-Ali Miri

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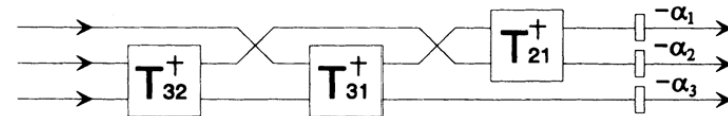
Queens College of the City University of New York  
The Graduate Center of the City University of New York

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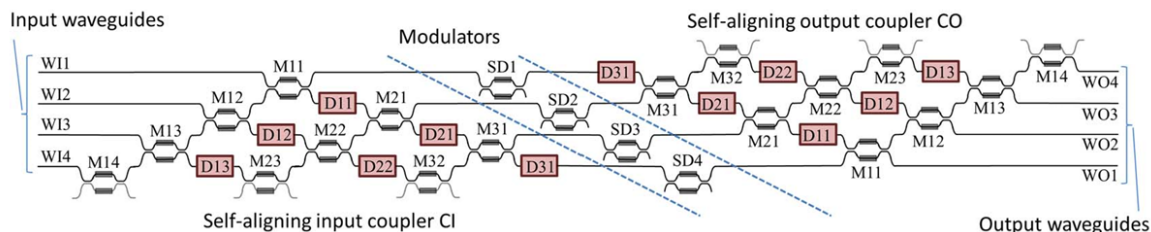
- Unitary matrix :  $N^2$  parameters
- Parameterization in a recursive fashion and in a feedforward triangular mesh architecture of beam splitters and phase shifters
- $U(N)$ 
  - $N^2$  phase shifters
  - $N(N - 1)/2$  beam splitters



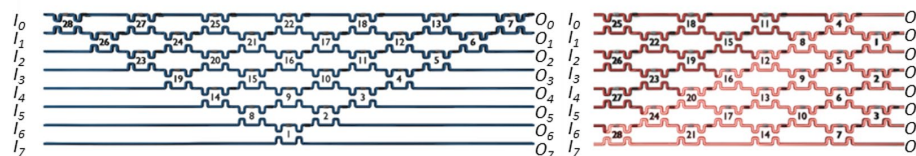
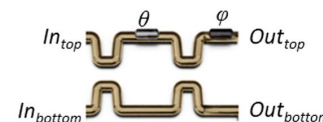
$$U(N) = (T_{N,N-1} \cdot T_{N,N-2} \cdots T_{2,1} \cdot D)^{-1}$$



- Integrated photonics
- Waveguide meshes containing Mach-Zehnder interferometers and phase shifters
- In theory,  $N^2$  phase shifters and  $N(N-1)/2$  MZIs
- In practice, needs additional phase shifters to compensate imbalance
- Several architectures

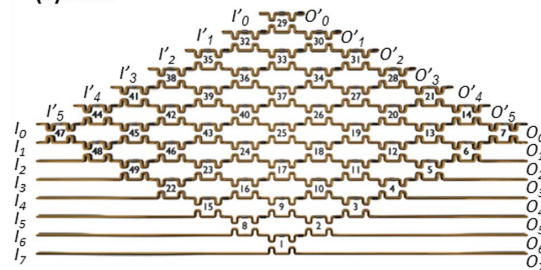


(a) 2x2 Building Block

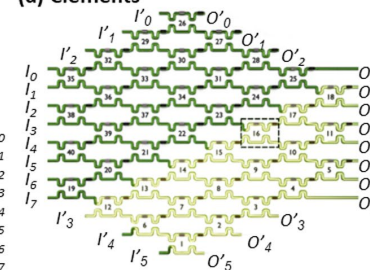


(b) Reck

(d) Clements



(c) Diamond



(e) Bokun

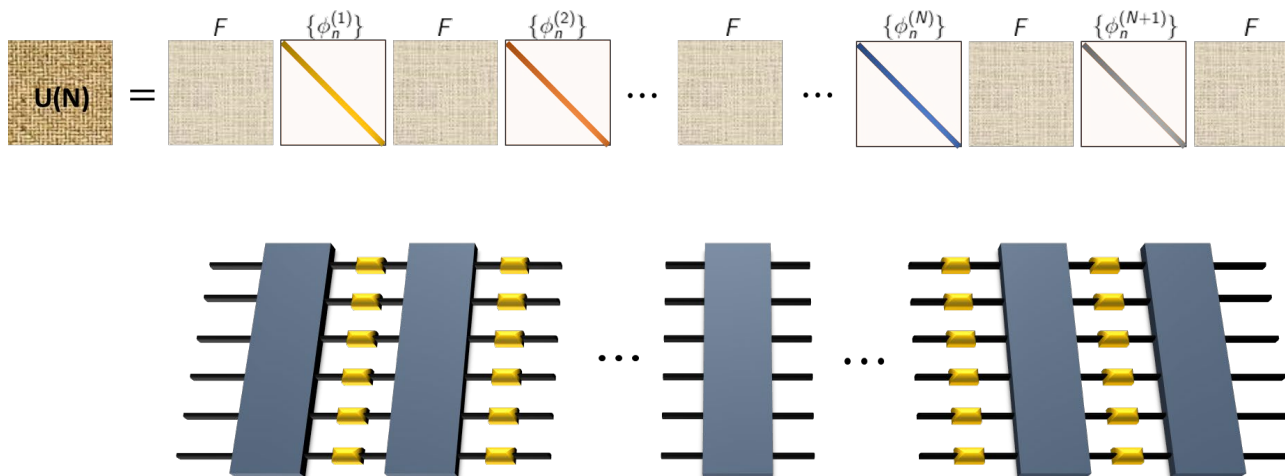
D. A. Miller, *Self-configuring universal linear optical component*, *Photonics Research* 1, 1 (2013).

K. Rahbardar Mojaver, B. Zhao, E. Leung, S. Safaee, and O. Liboiron-Ladouceur, "Addressing the programming challenges of practical interferometric mesh based optical processors," *Opt. Express* **31**, 23851–23866 (2023).

- Can we parametrize unitaries in form of interlacing of diagonal phases and a fixed matrix?

$$U(N) = F P_M F P_{M-1} \cdots F \cdots P_2 F P_1 F$$

- A solution exists:
  - for certain choices of  $F$
  - for  $M = N + 1$  layers



- Factorizing the group of  $N \times N$  unitary matrices  $U(N)$ :

$$U(N) = \mathbf{F} \mathbf{P}_M \mathbf{F} \mathbf{P}_{M-1} \cdots \mathbf{F} \cdots \mathbf{P}_2 \mathbf{F} \mathbf{P}_1 \mathbf{F}$$

$N$ -point Discrete Fractional  
Fourier Transform (DFrFT)

$\mathbf{F}$

$$F = \exp\left(i \frac{\pi}{2} H\right)$$

$$H = \begin{bmatrix} 0 & \kappa_{1,2} & \cdots & 0 & 0 \\ \kappa_{1,2} & 0 & & 0 & 0 \\ \vdots & & \ddots & \vdots & \\ 0 & 0 & & 0 & \kappa_{N-1,N} \\ 0 & 0 & \cdots & \kappa_{N-1,N} & 0 \end{bmatrix}$$

$$\kappa_{n,n+1} = \frac{1}{2} \sqrt{(N-n)n}; n = 1, \dots, N-1$$

$N$ -parameter diagonal matrices

$\mathbf{P}_m$

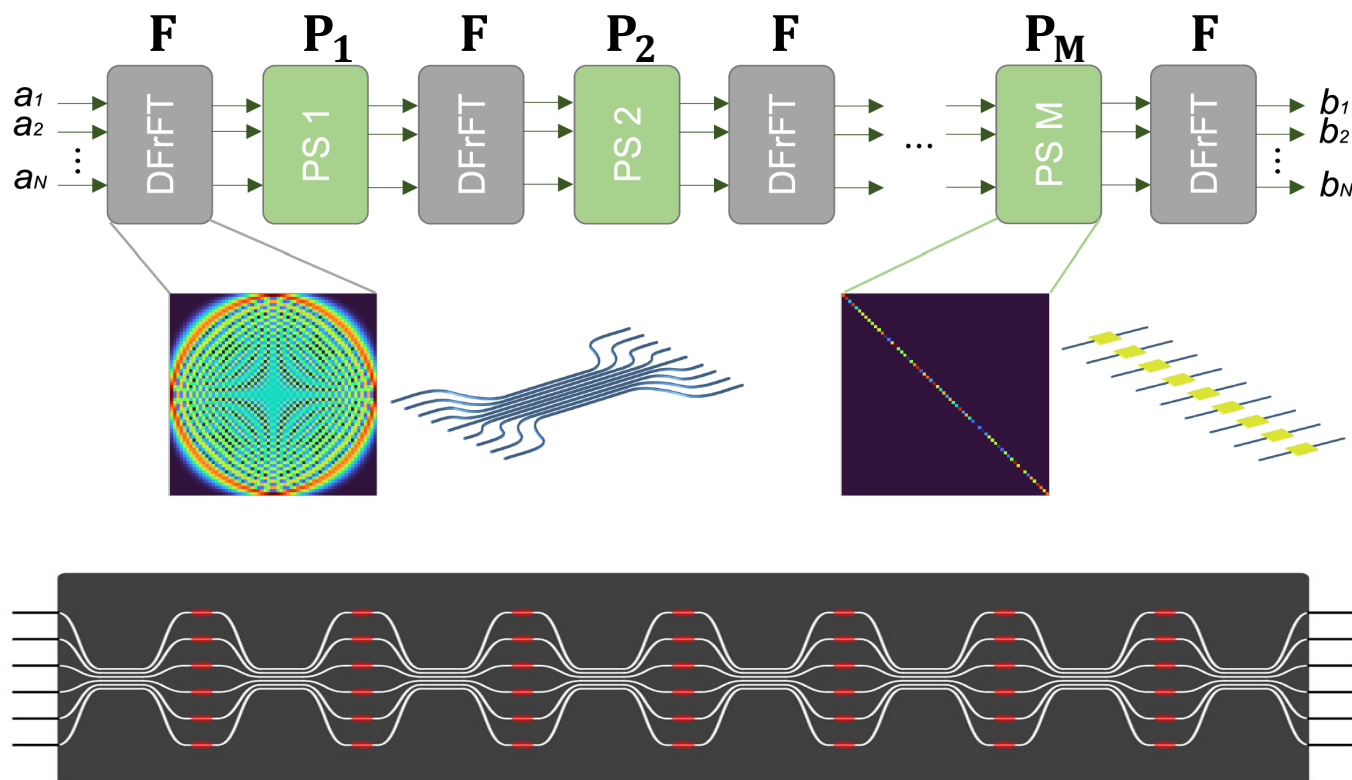
$$P_m = \begin{bmatrix} e^{i\phi_1^m} & 0 & \cdots & 0 & 0 \\ 0 & e^{i\phi_2^m} & & 0 & 0 \\ \vdots & & \ddots & \vdots & \\ 0 & 0 & \cdots & e^{i\phi_{N-1}^m} & 0 \\ 0 & 0 & & 0 & e^{i\phi_N^m} \end{bmatrix}$$

M. Markowitz, M.-A. Miri, *arXiv:2307.07101* (2023).

A. Perez-Leija, R. Keil, A. Kay, et al., *Phys. Rev. A*, 87(1), 012309, (2013).

A. Perez-Leija, R. Keil, H. Moya-Cessa, A. Szameit, and D. N. Christodoulides, *Phys. Rev. A*, 87, 022303, (2013).

S. Weimann, A. Perez-Leija, M. Lebugle, et al, *Nat. Commun.*, 7, 1–8, (2016).



M. Markowitz, M.-A. Miri, arXiv:2307.07101 (2023).

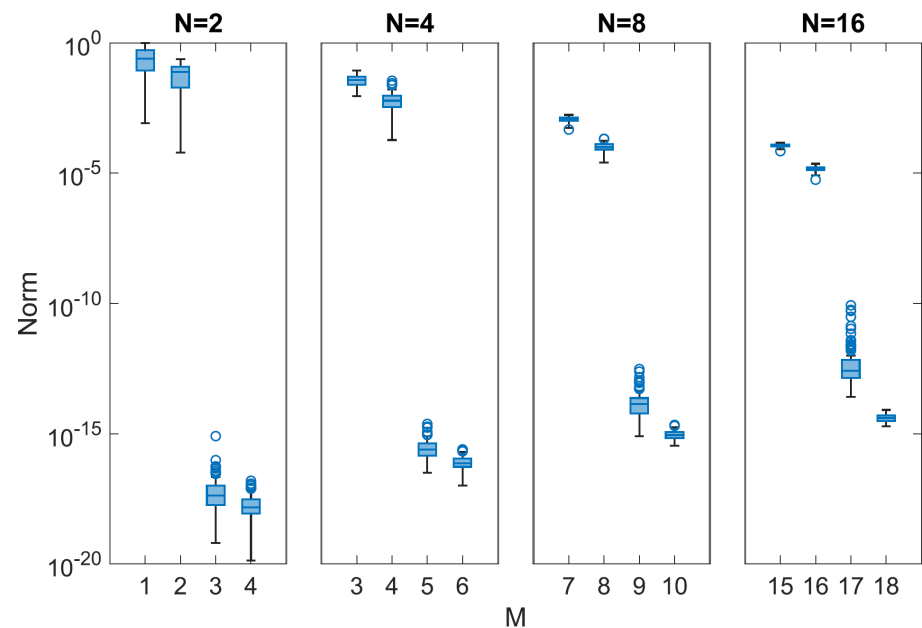
K. Zelaya, M. Markowitz, and M.-A. Miri, "The Goldilocks Principle of Learning Unitaries by Interlacing Fixed Operators with Programmable Phase Shifters on a Photonic Chip," Scientific Reports (2023).

- Numerical results for universal representation of  $U(N)$ , for cases of  $N = 2, 4, 8$  and  $16$ .
- A set of 100 target unitary matrices ( $U_t$ ) are generated at random. Then the phase parameters in our factorization ansatz:

$$\tilde{U} = \mathbf{F}\mathbf{P}_M\mathbf{F} \cdots \mathbf{P}_2\mathbf{F}\mathbf{P}_1\mathbf{F}$$

are optimized so the norm of the representation error  $\|\tilde{U} - U_t\|$  approaches zero.

- In all cases, we observe a phase transition from  $M = N$  to  $M = N + 1$ .



# Extension to Non-unitary Operations

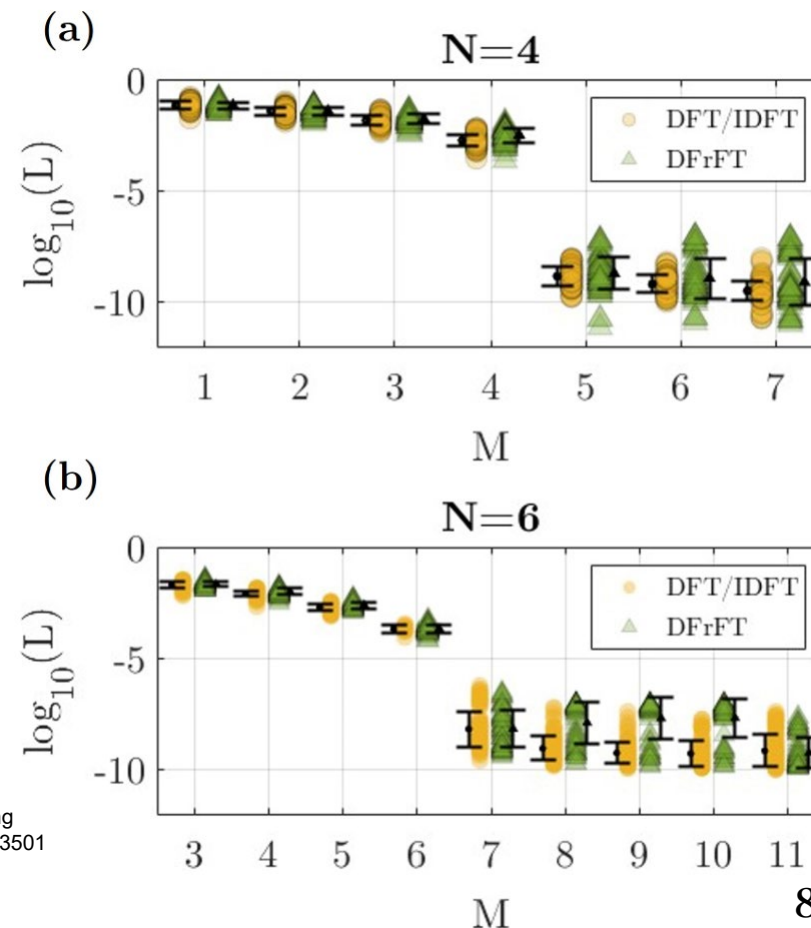
- Factoring arbitrary  $N \times N$  matrices in terms of  $2N - 1$  diagonal and circulant matrices:

- $$\mathbf{A} = \mathbf{D}^{(2N-1)} \mathbb{F} \mathbf{D}^{(2N-2)} \mathbb{F}^{-1} \mathbf{D}^{(2N-3)} \dots \mathbf{D}^{(3)} \mathbb{F} \mathbf{D}^{(2)} \mathbb{F}^{-1} \mathbf{D}^{(1)}$$

- Proposed factorization:

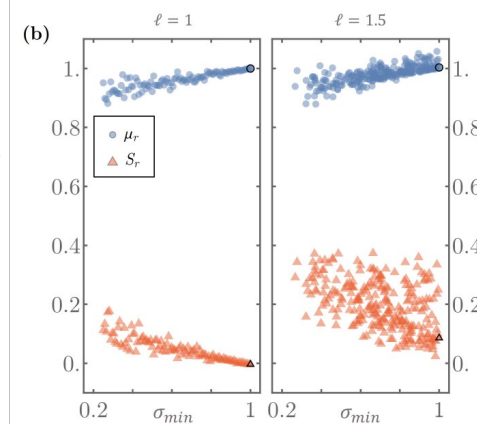
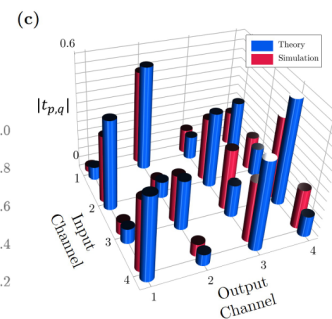
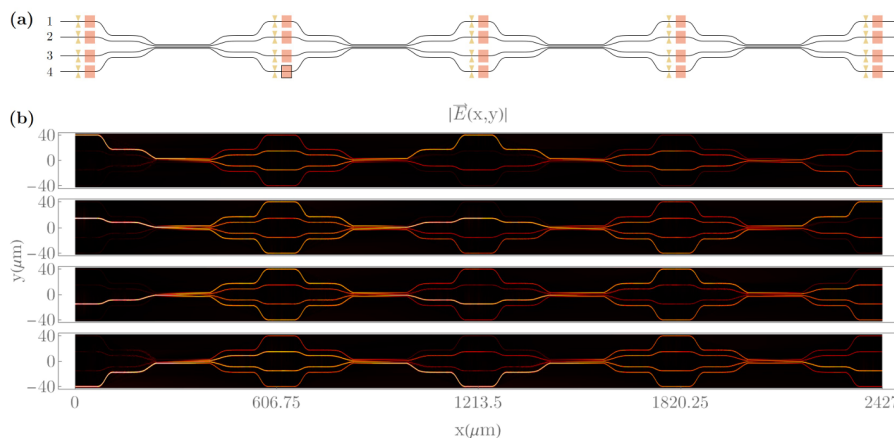
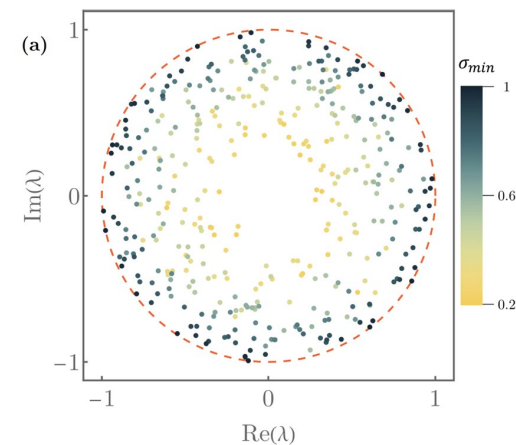
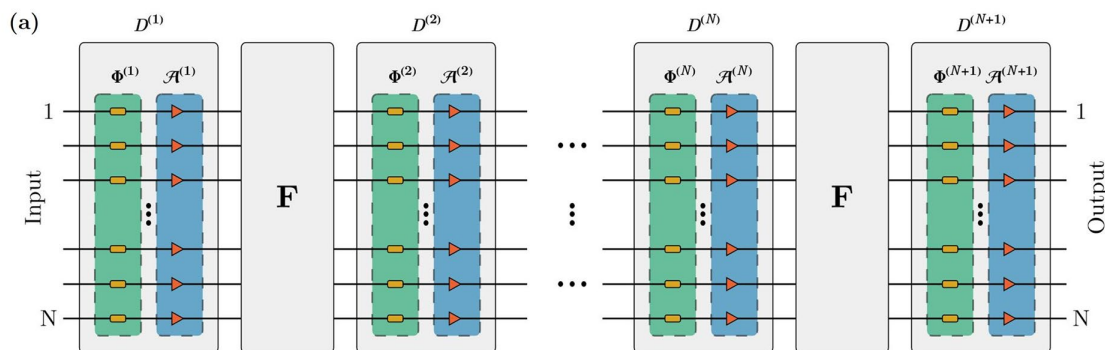
- $$\mathbf{A} = \mathbf{D}^{(M)} \mathbb{F} \mathbf{D}^{(M-1)} \dots \mathbf{D}^{(3)} \mathbb{F} \mathbf{D}^{(2)} \mathbb{F} \mathbf{D}^{(1)}$$

*R. M. Gray, Toeplitz and circulant matrices : A review, Foundations and Trends in Communications and Information Theory 2,155(2006).*

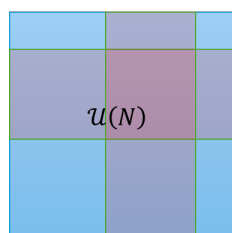
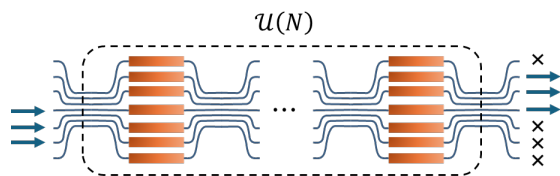




# From Unitaries to Non-Unitaries



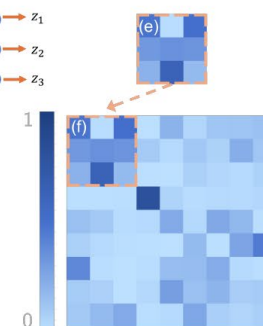
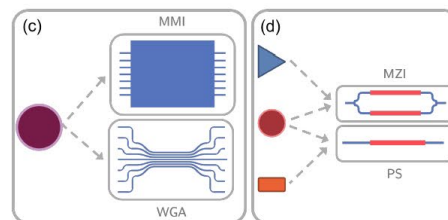
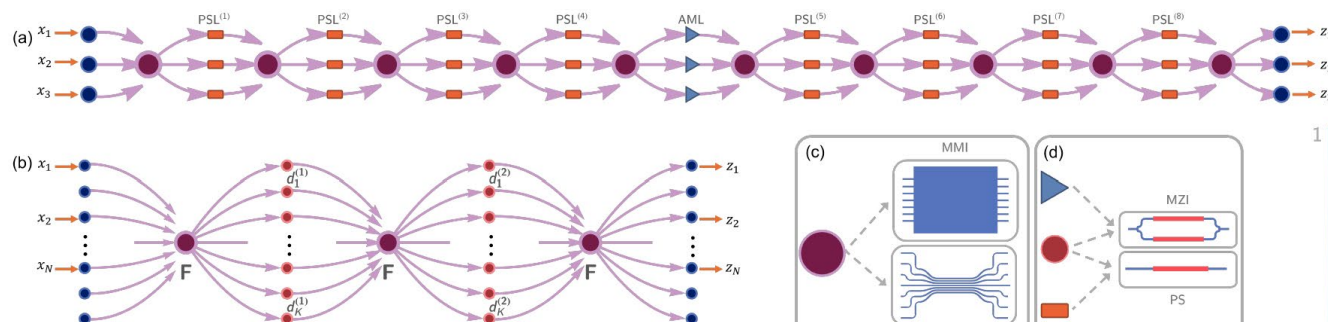
# Reshaping the Architecture



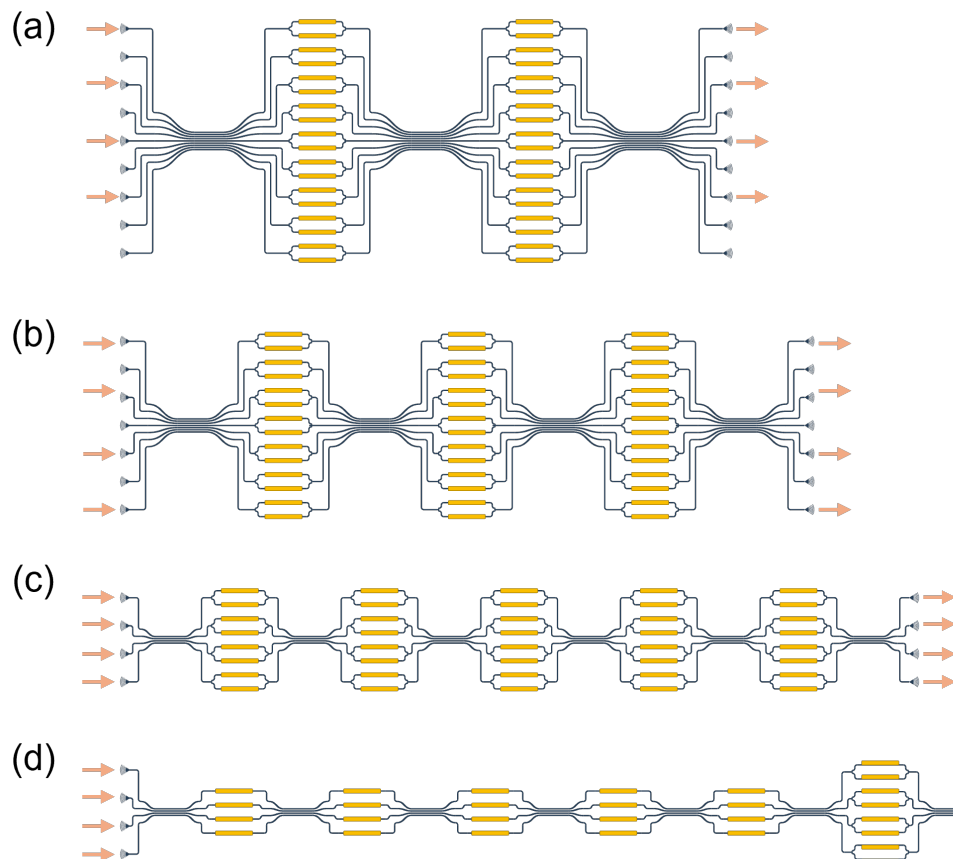
$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$



$$\begin{bmatrix} a & b & c & d \\ d & a & b & c \\ c & d & a & b \\ b & c & d & a \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} y_1 \\ d.c. \\ y_2 \\ d.c. \end{bmatrix}$$



# Reshaping the Architecture



N-port device  
M layers  
target: K-port

non-unitary

$$K = \left\lfloor \frac{2N^2 - 2}{M} + 1 \right\rfloor \approx 2 \frac{N^2}{M}$$

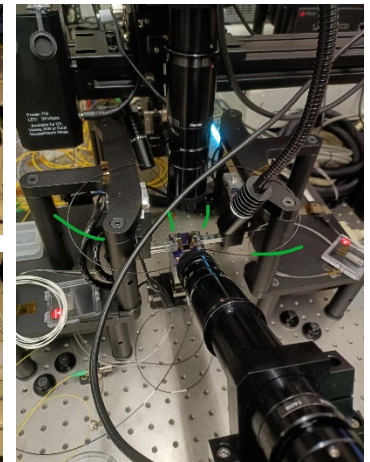
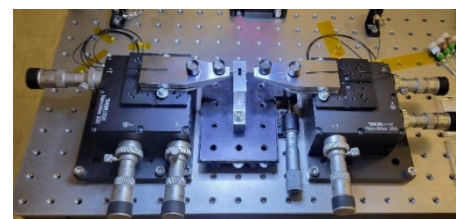
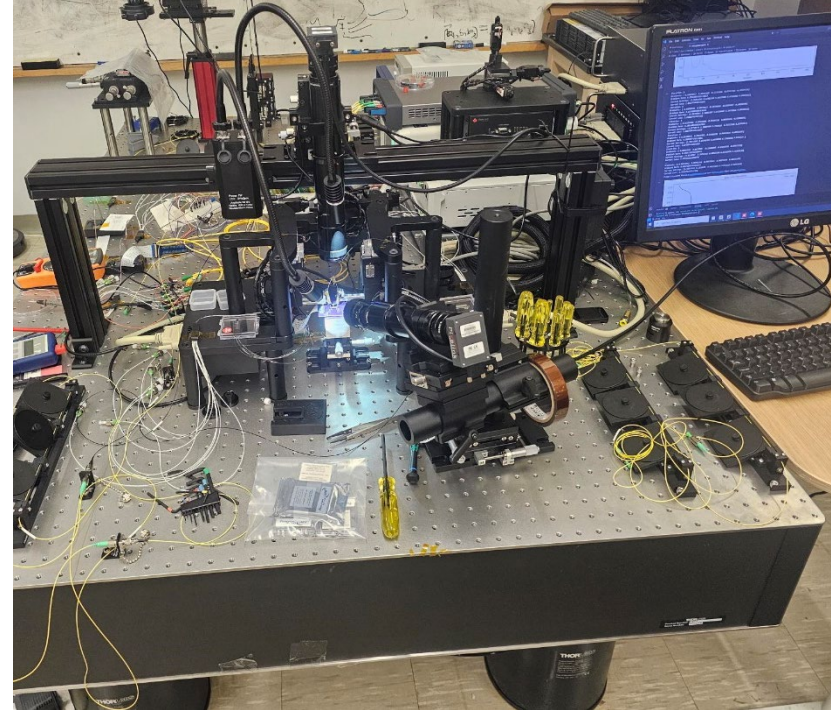
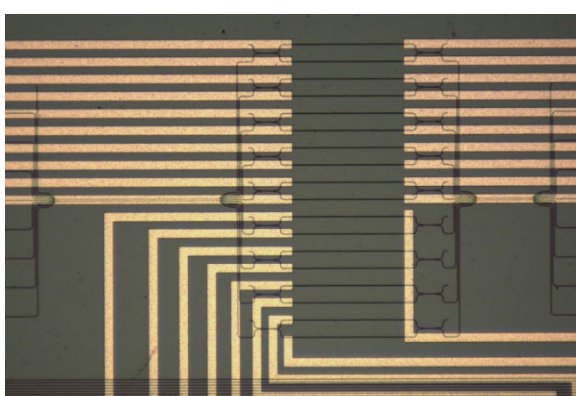
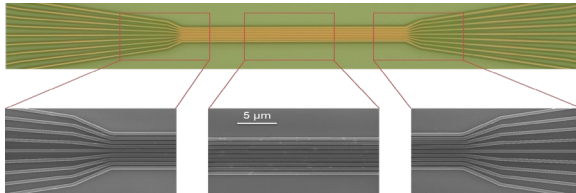
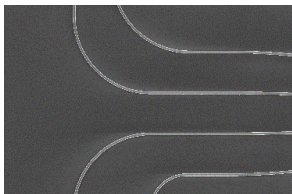
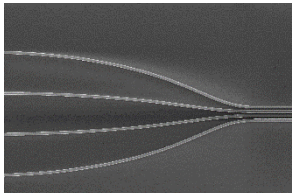
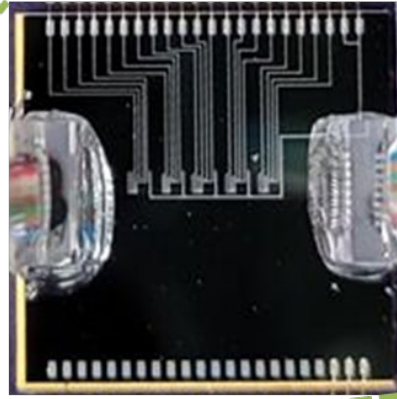
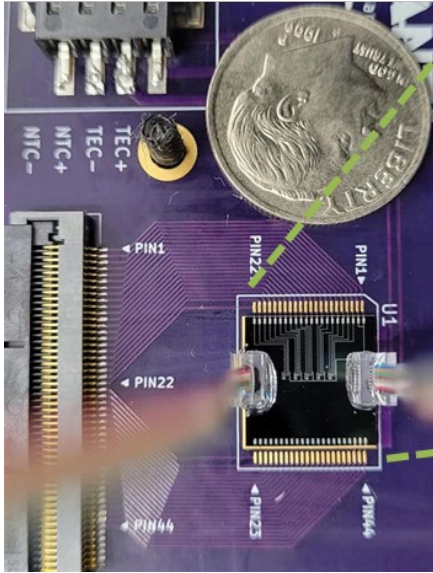
unitary

$$K = \left\lfloor \frac{2N^2 - 3}{2M} + 1 \right\rfloor \approx \frac{N^2}{M}$$

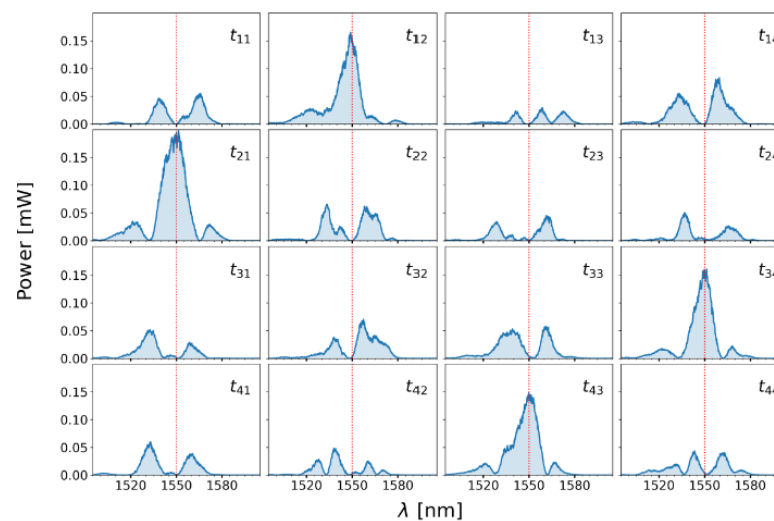
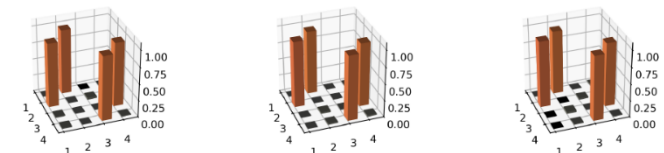
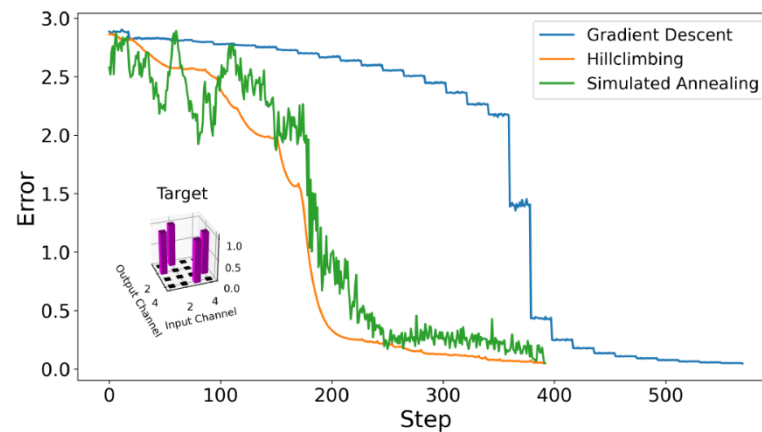
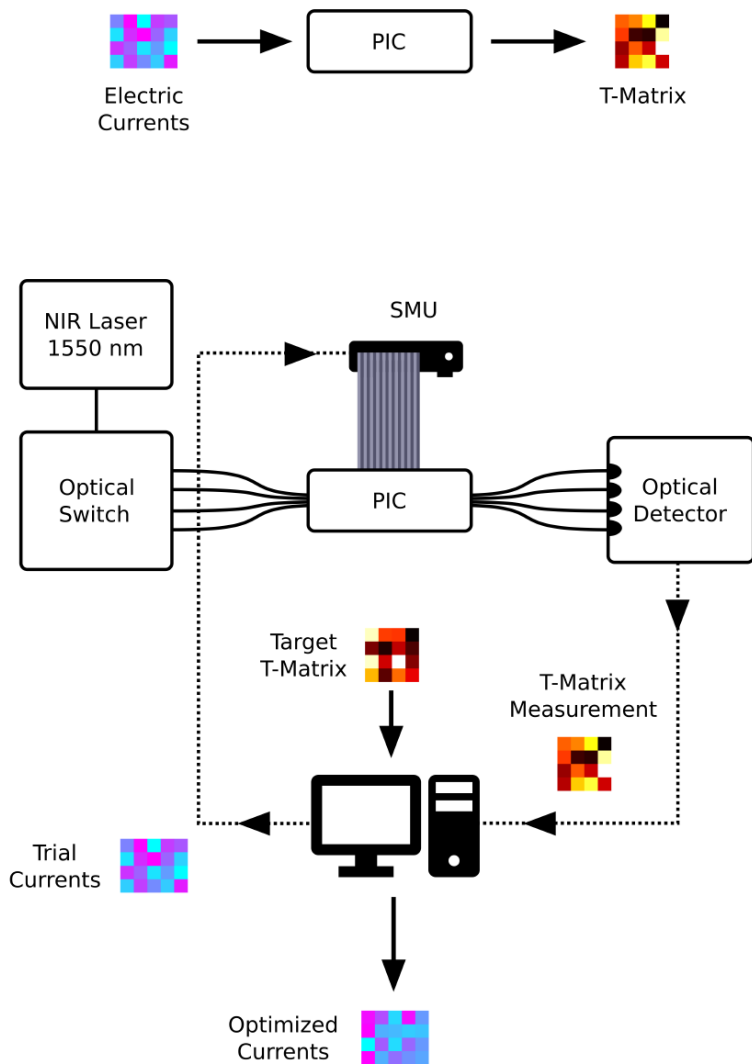
# Experimental Demonstration



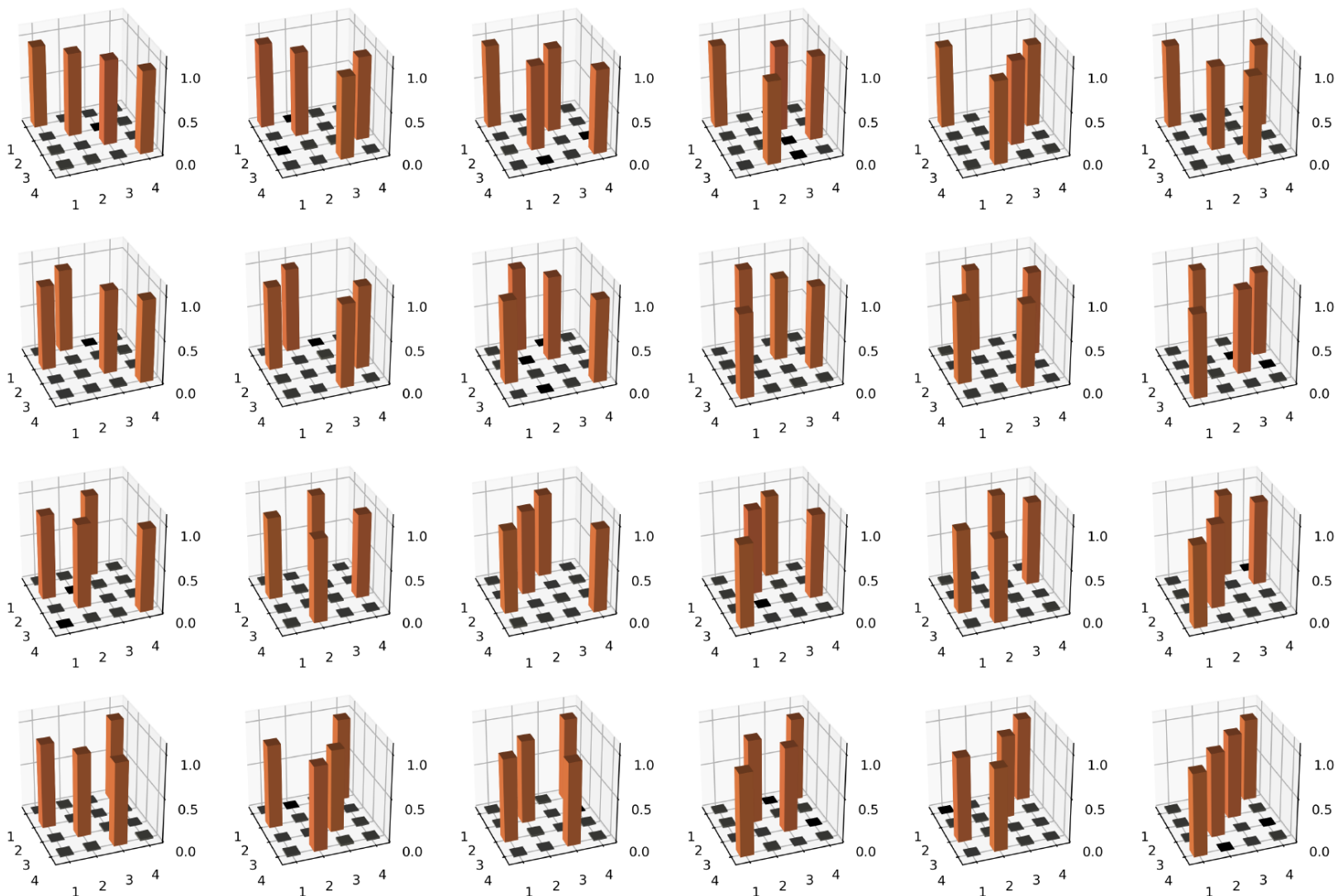
# Experimental Demonstration



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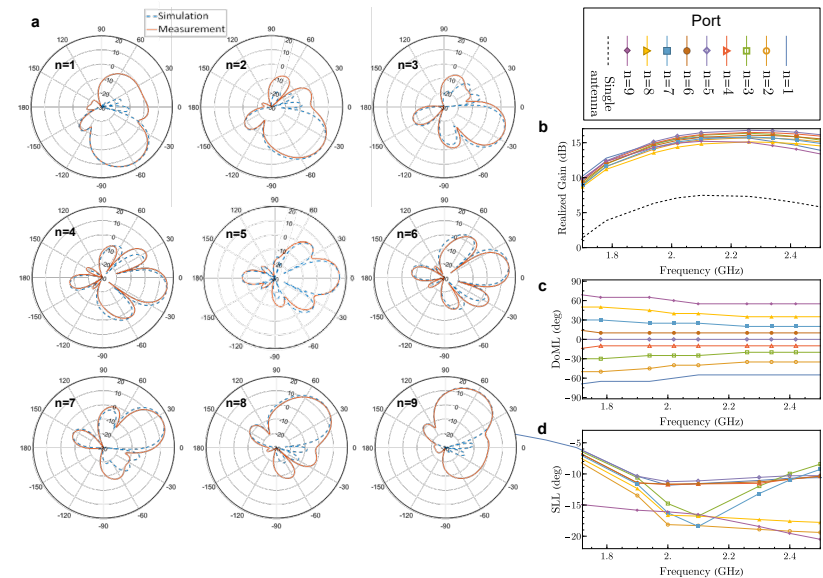
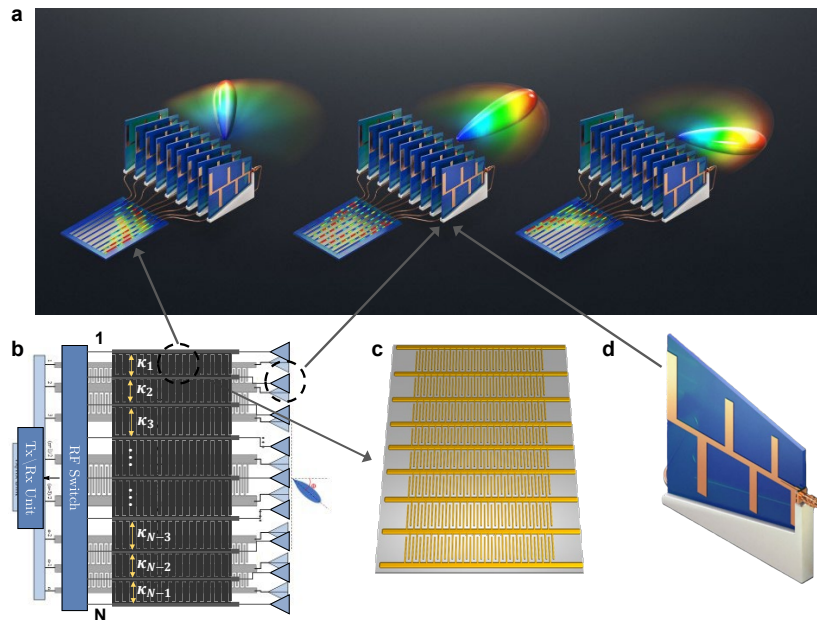
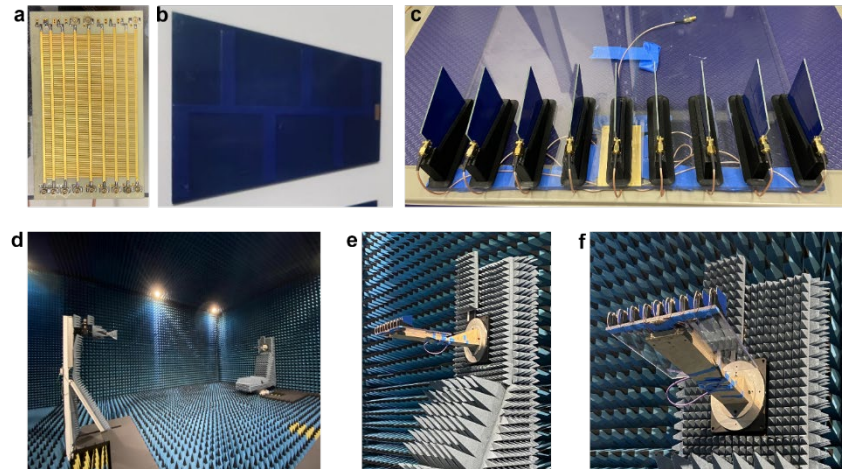
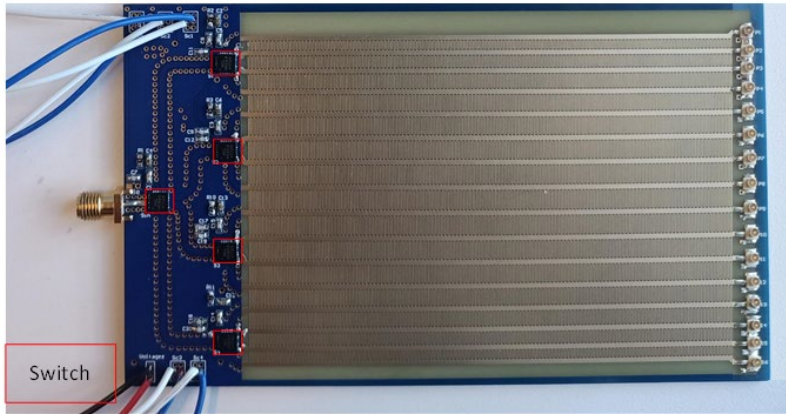


# Experimental Demonstration

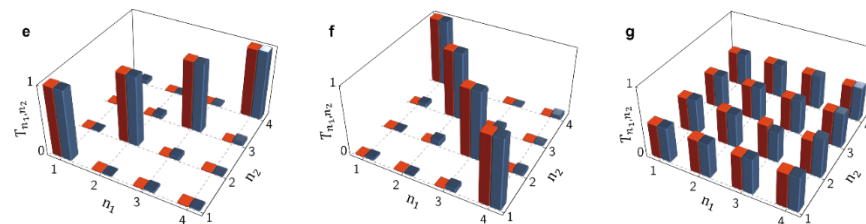
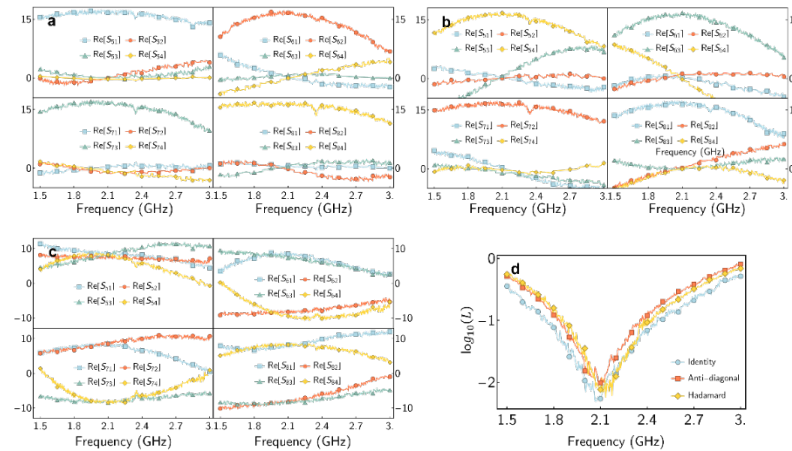
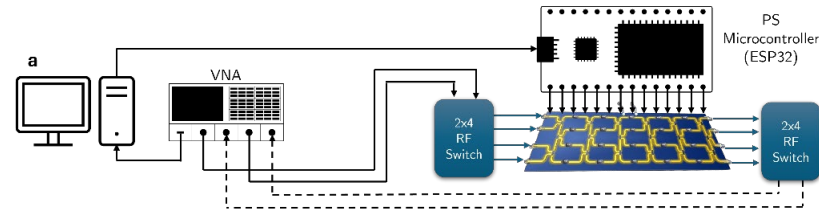




# RF/Microwave Realization

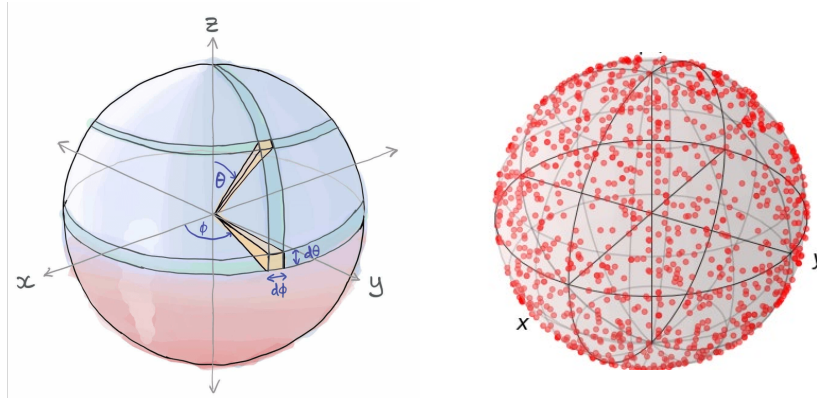




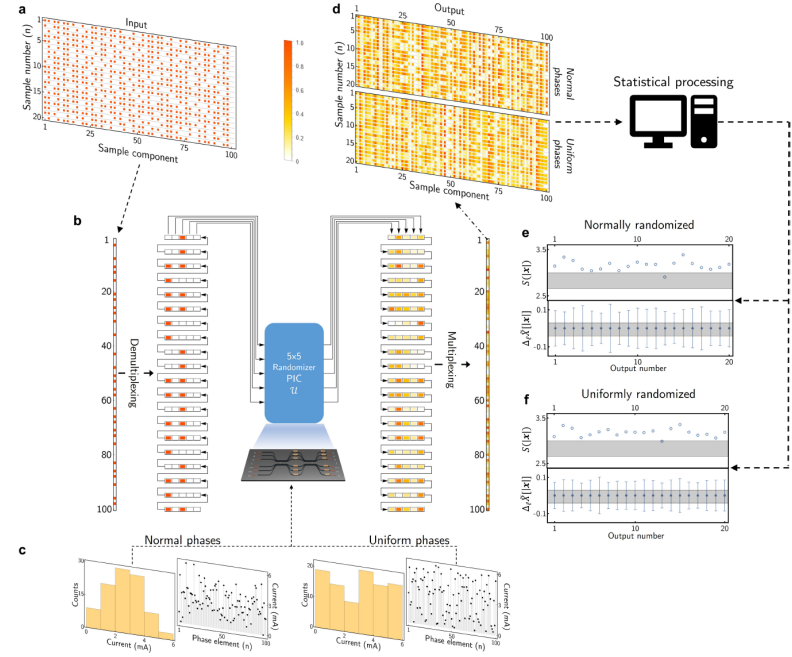
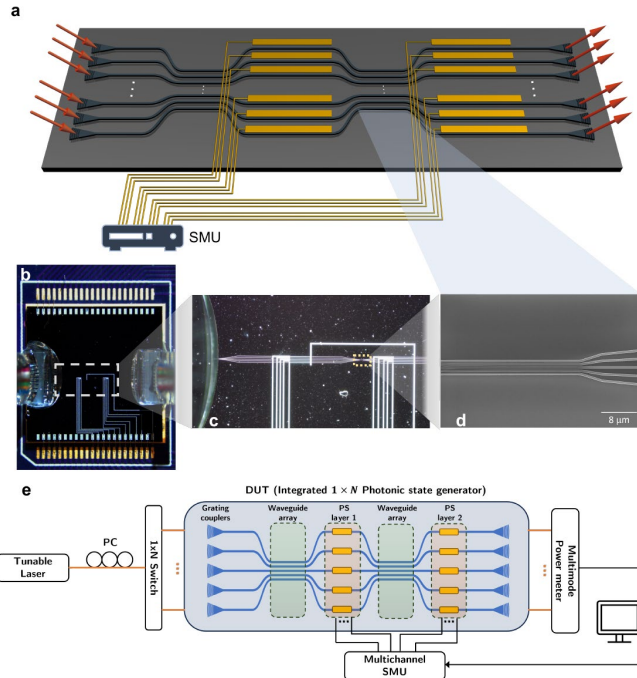
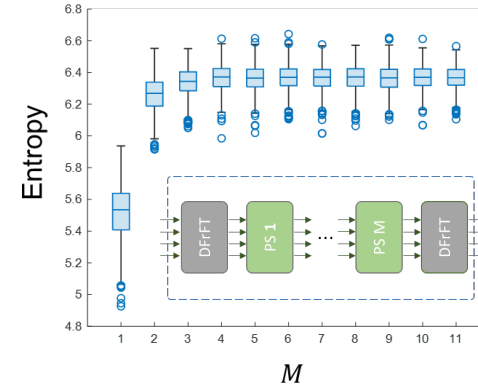


# Device Applications

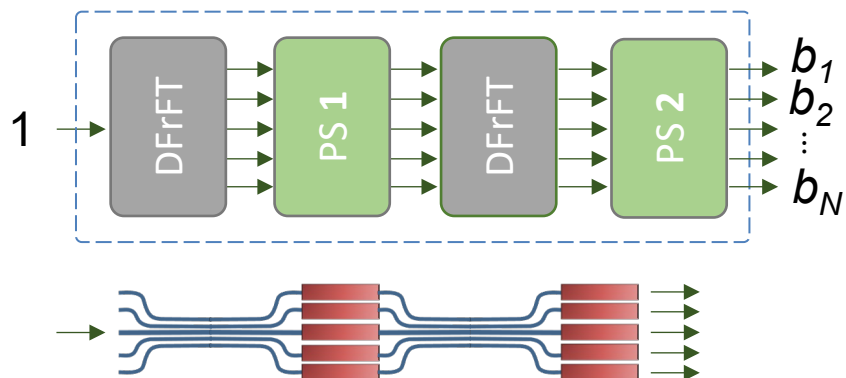
# Random Matrix Generation



[https://pennylane.ai/qml/demos/tutorial\\_haar\\_measure/](https://pennylane.ai/qml/demos/tutorial_haar_measure/)

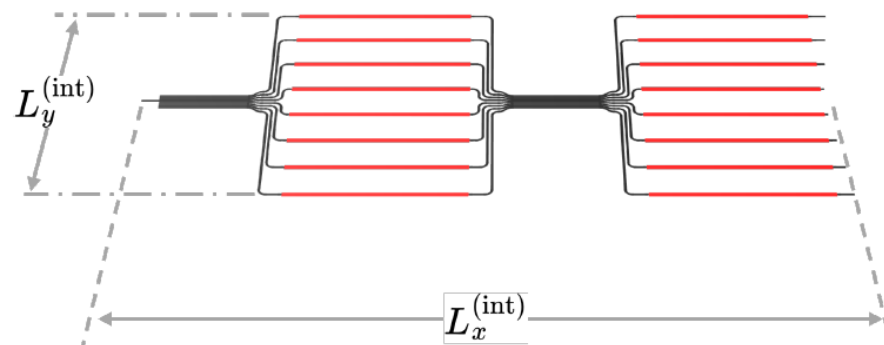
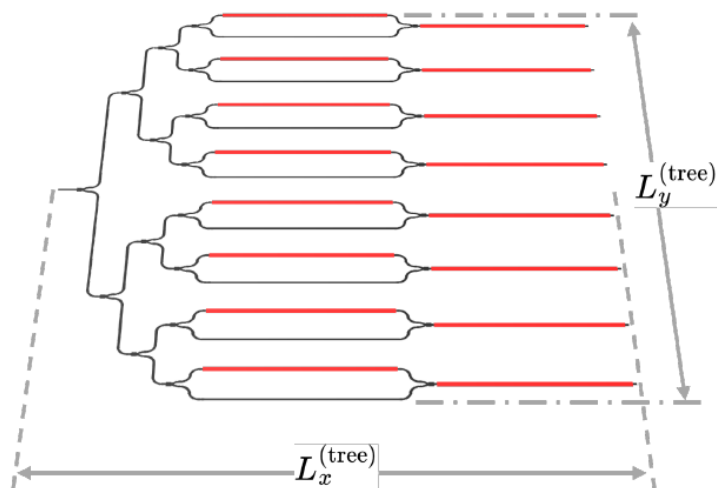


# Unitary Preparation of Arbitrary States

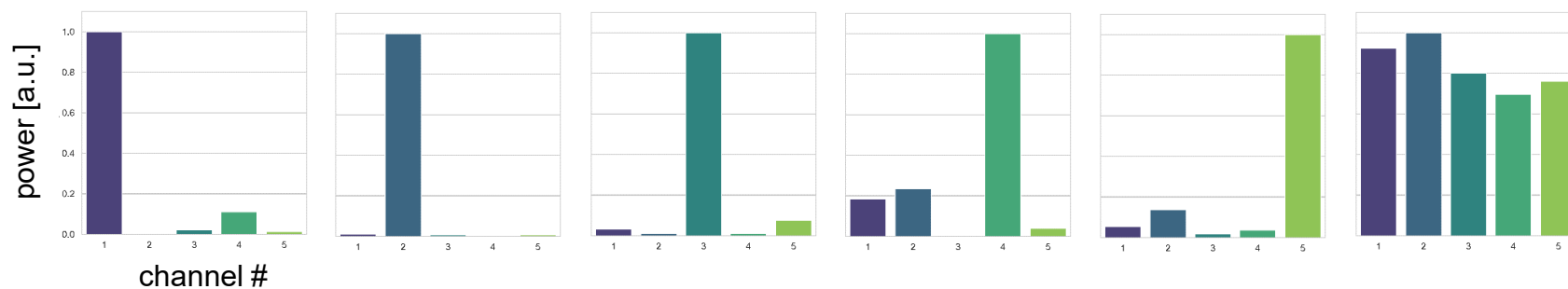
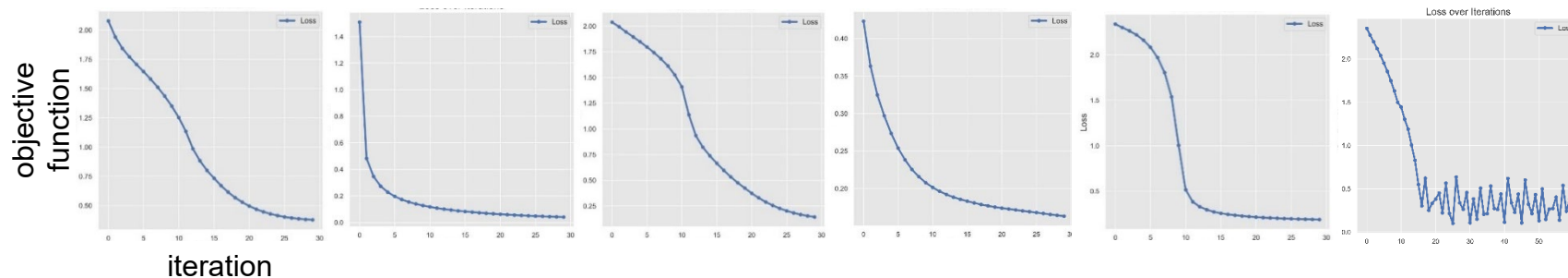
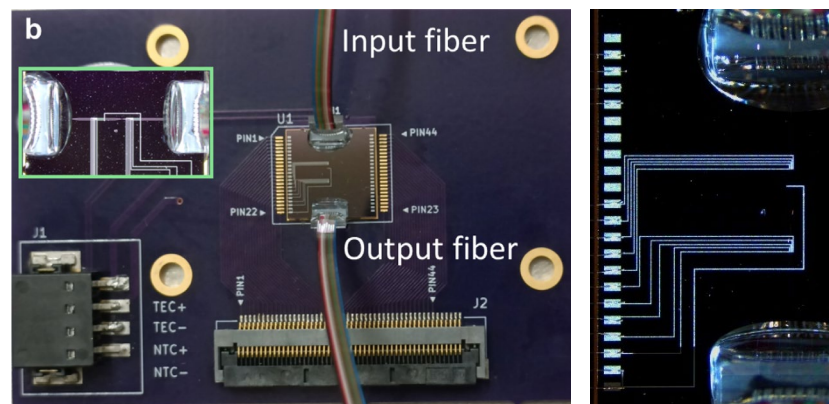
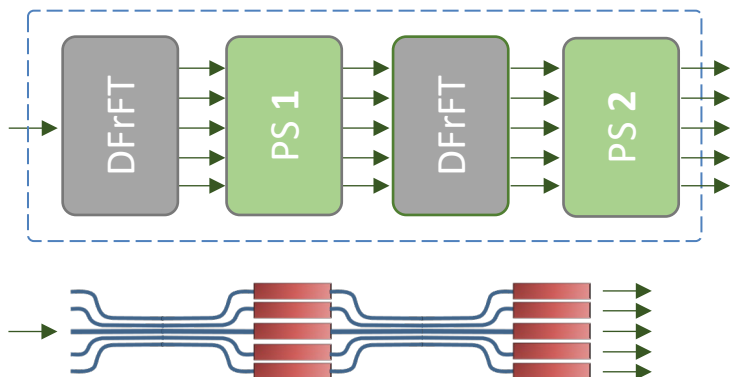


amplitude  $\rightarrow$  1-layer

amplitude & phase  $\rightarrow$  2-layers

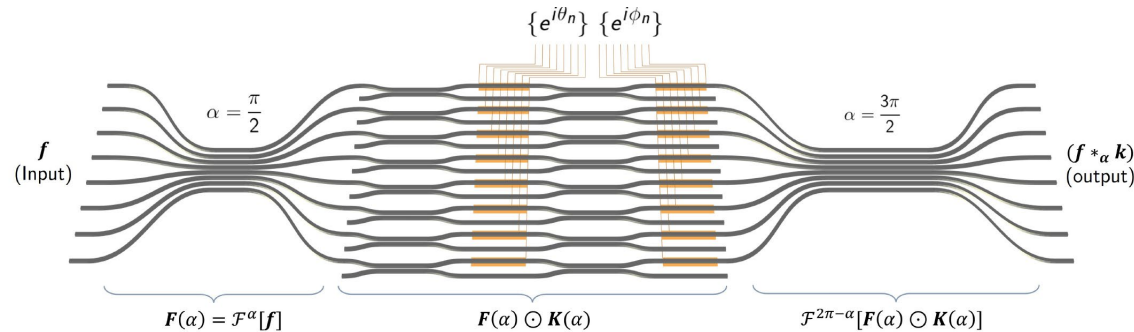
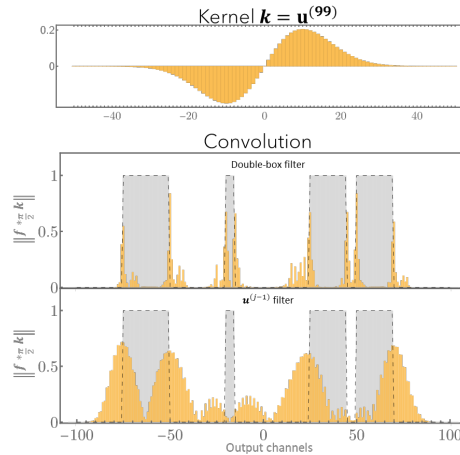
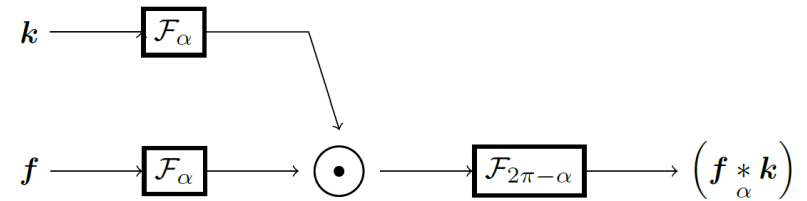


# Unitary Preparation of Arbitrary States



- Convolution can be cast as matrix-vector multiplication
- Or more conveniently it can be viewed in the Fourier domain
- We define a modified convolution based on DFrFT and a layer of amplitude modulation

$$\sqrt{N} \left( f *_{\alpha} k \right)_q = \mathcal{F}^{2\pi-\alpha} [F(\alpha) \odot K(\alpha)]_q$$



- M. Markowitz, K. Zelaya, and **M.-A. Miri**, "Learning Arbitrary Complex Matrices by Interlacing Amplitude and Phase Masks with Fixed Unitary Operations," *Physical Review A*, 110(3), 033501 (2024).
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- J. Friedman, K. K. Mandal, K. Zelaya, M. Honari-Latifpour, and M.-A. Miri, "In-situ training of programmable photonic unitaries" *in preparation* (2025).
- R. Keshavarz, K. Zelaya, N Shariati, **M.-A. Miri**, "Programmable Microwave Integrated Circuits for Performing Analog Matrix Operations in Real-Time," *in preparation* (2025).

