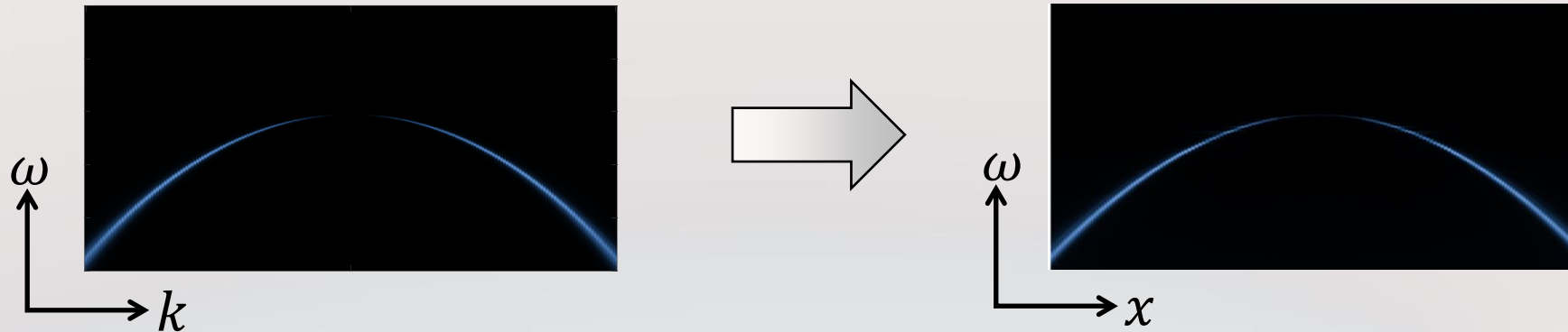


# Nonlocal metasurfaces for control of light beyond the Generalized Snell's Law



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EM Portfolio Review, Dr. Arje Nachman

January 7, 2025

# Field: Metasurfaces

'Meta-' era of optics

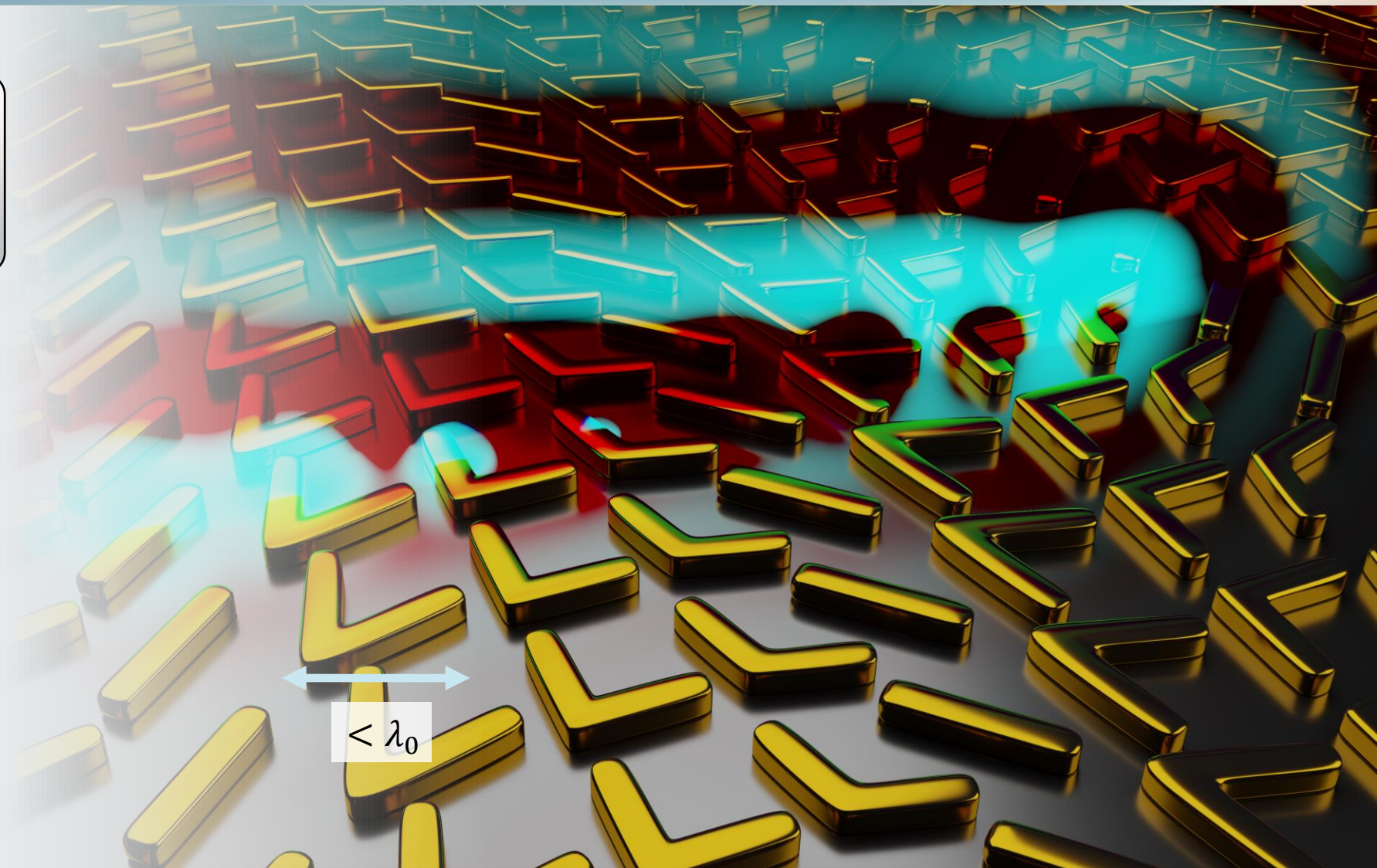
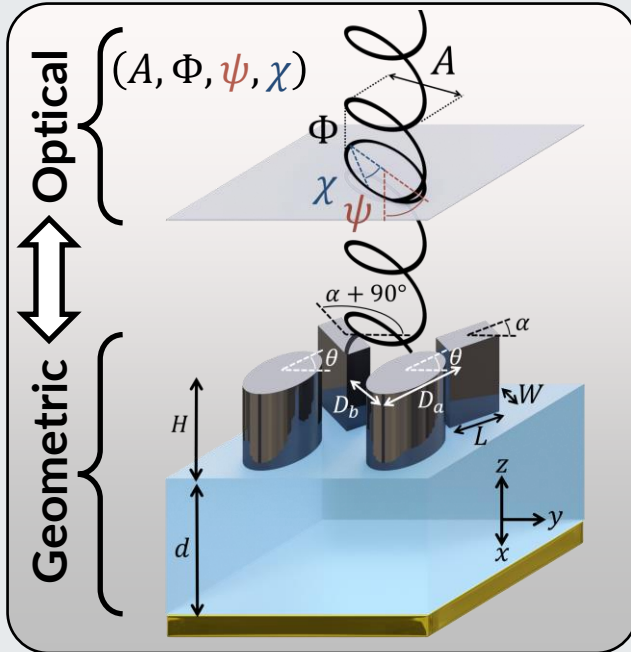
**Less:**

"how does light behave?"

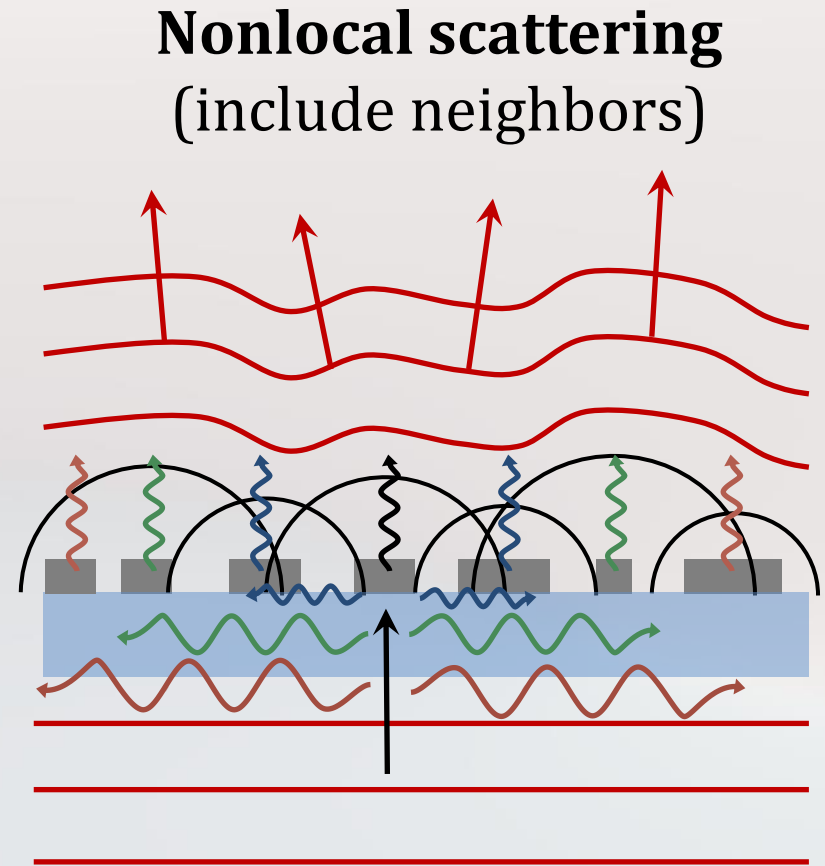
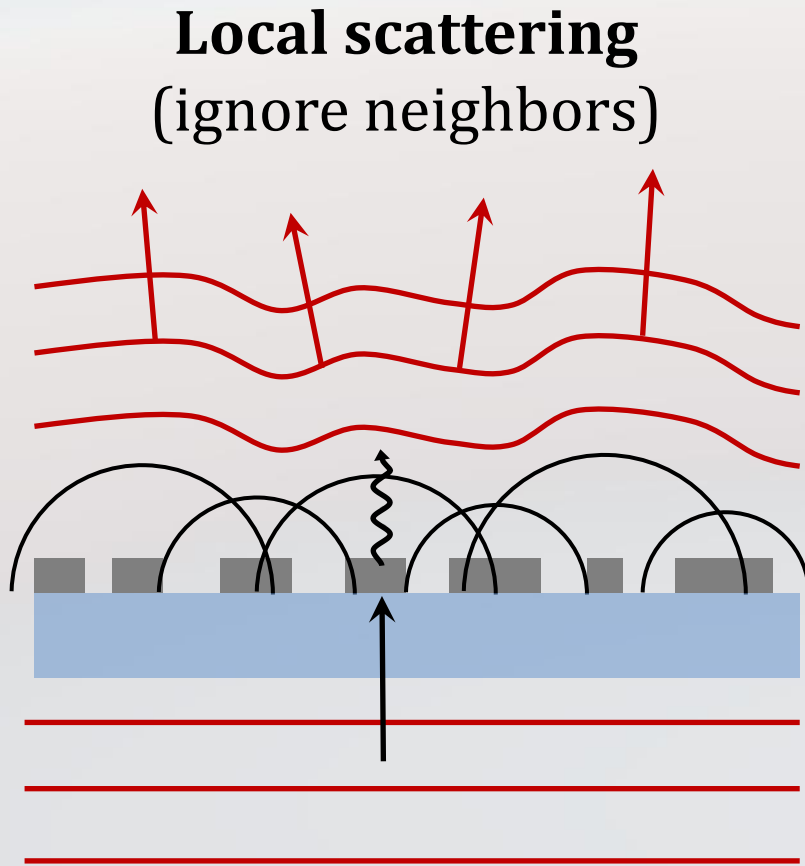
**More:**

"how do we *make* light behave?"

Degrees of Freedom

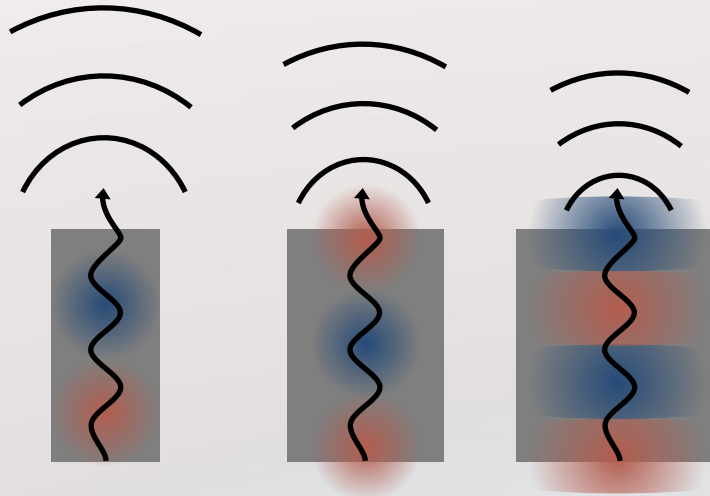


# Trend: From local to nonlocal



# Challenge: Local control of nonlocal modes

## Metasurfaces

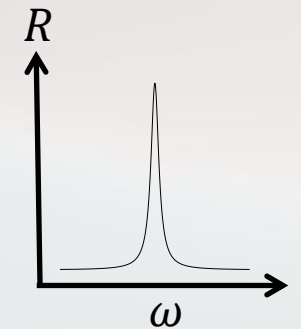


Control of	
Local scattering	Yes
Inter-element coupling	No

## Photonic crystal slabs



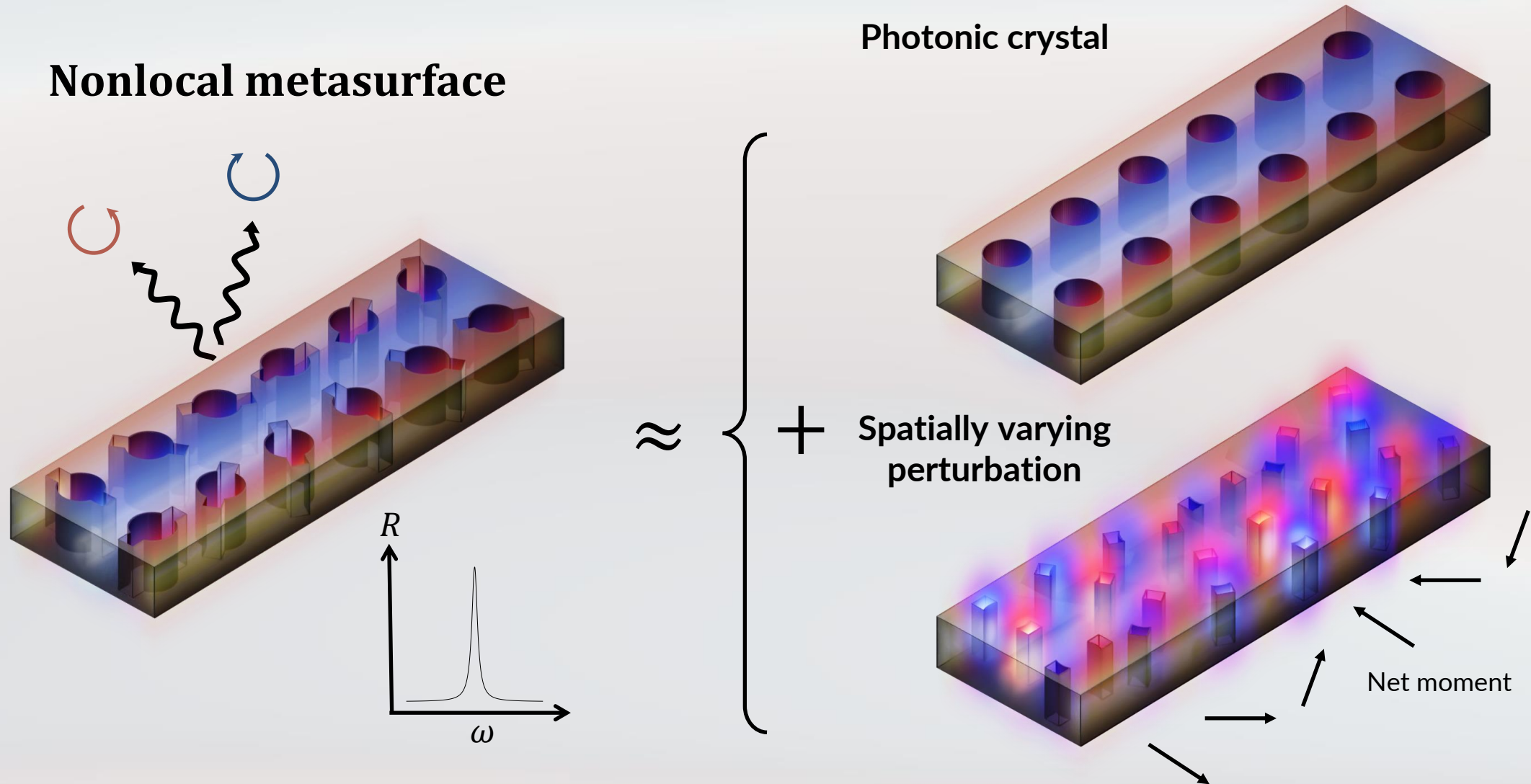
Control of	
Local scattering	No
Inter-element coupling	Yes



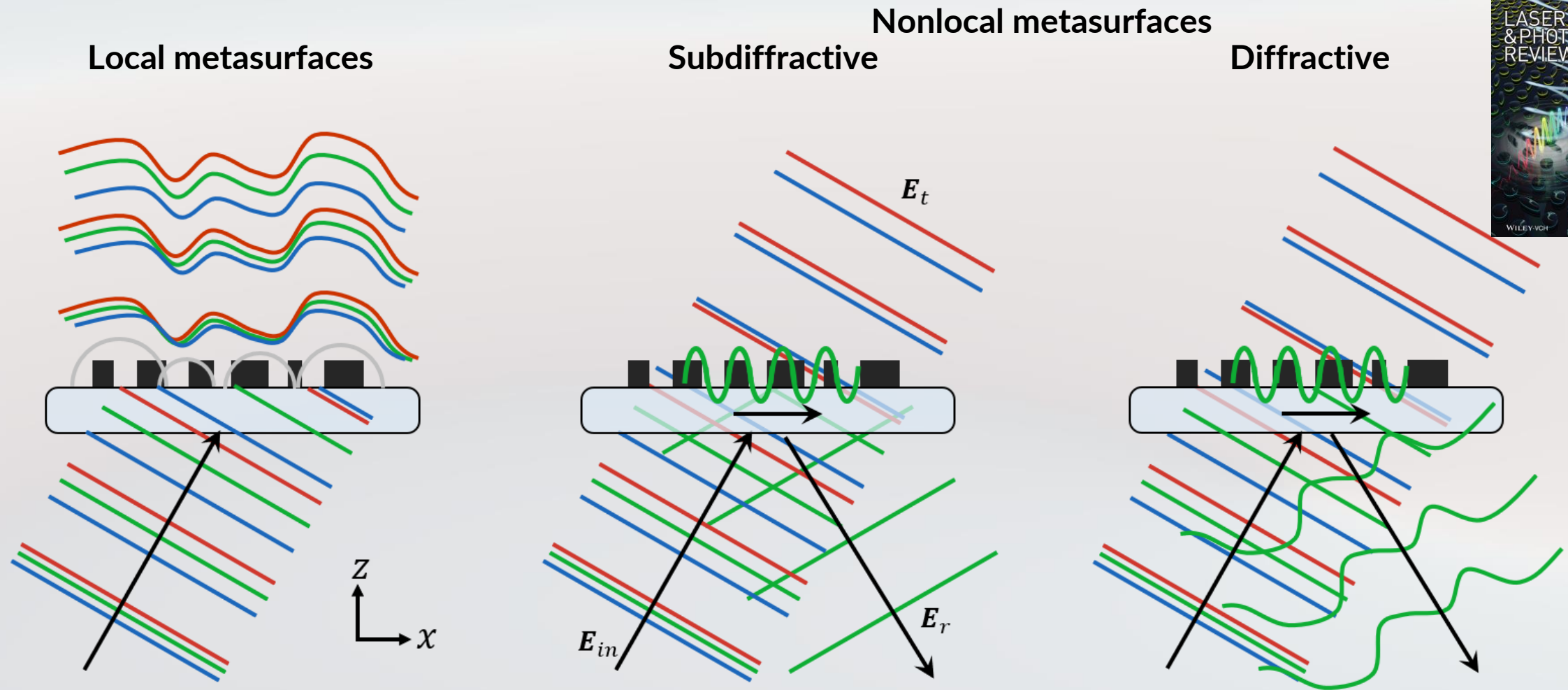
*How do we turn a 'bug' into a 'feature'?*



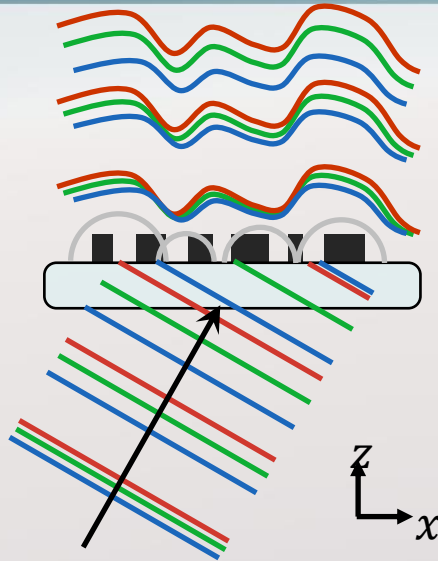
# Solution: Small perturbations



# Novelty: New class of device



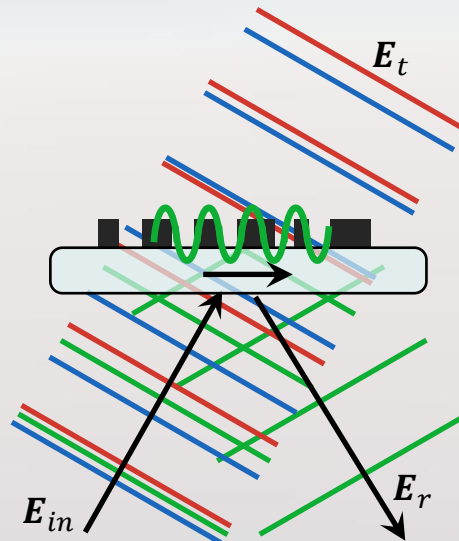
# Significance: General scattering behavior



**Space-frequency description:**

$$E_r(x, \omega) = r(x, \omega) E_{in}(x, \omega)$$

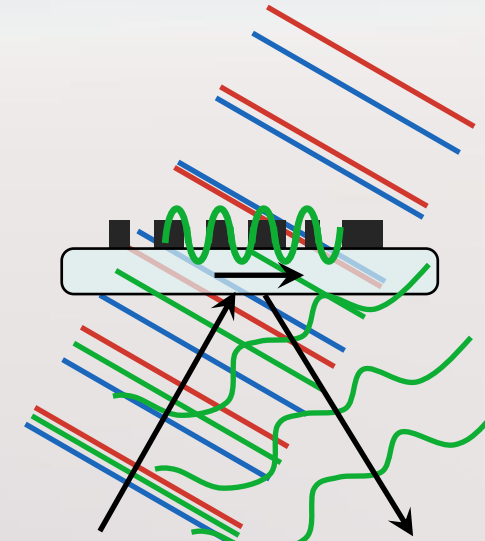
$$E_t(x, \omega) = t(x, \omega) E_{in}(x, \omega)$$



**Momentum-frequency description:**

$$E_r(k, \omega) = r(k, \omega) E_{in}(k, \omega)$$

$$E_t(k, \omega) = t(k, \omega) E_{in}(k, \omega)$$



**Space-frequency description:**

$$E_r(x, \omega) = \int dx' \rho(x, x', \omega) E_{in}(x', \omega)$$

$$E_t(x, \omega) = \int dx' \tau(x, x', \omega) E_{in}(x', \omega)$$

**Locality:**

$$\begin{aligned} \rho(x, x', \omega) &= r(x, \omega) \delta(x - x') \\ \tau(x, x', \omega) &= t(x, \omega) \delta(x - x') \end{aligned}$$

**Homogeneity:**

$$\begin{aligned} \rho(x, x', \omega) &= r(x - x', \omega) \\ \tau(x, x', \omega) &= t(x - x', \omega) \end{aligned}$$

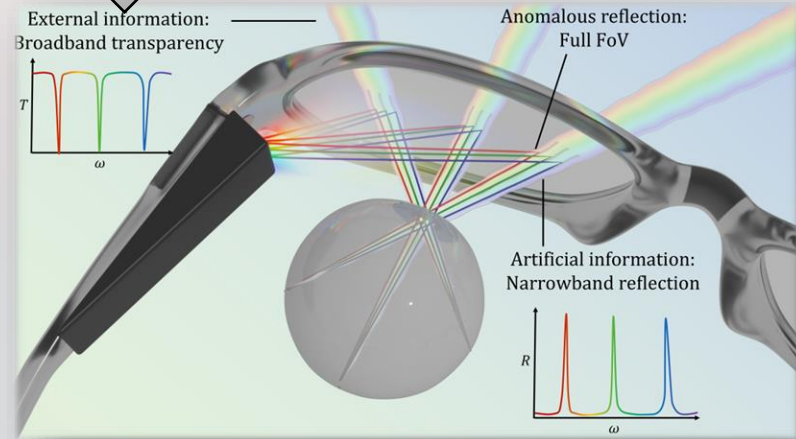


# Applications: Controlling open systems with symmetries

Free space

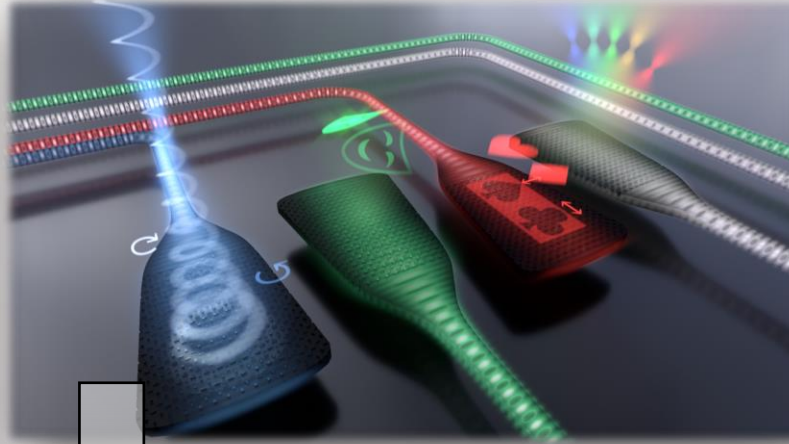


Augmented reality

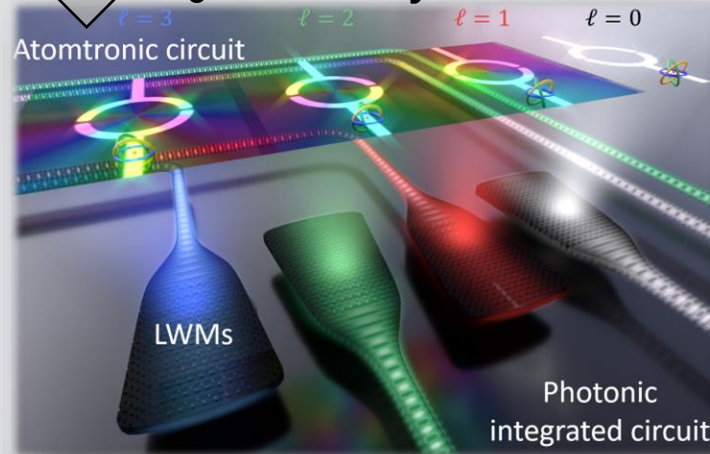


*Light: Sci. & App.* **11**, 246 (2022)

Integrated Photonics

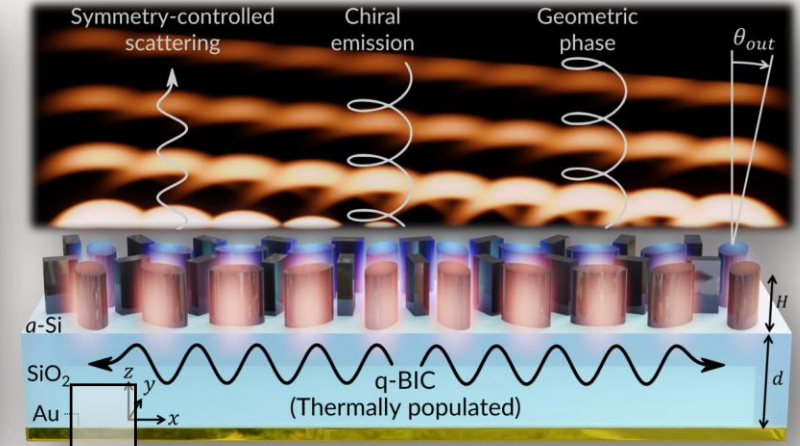


Quantum systems

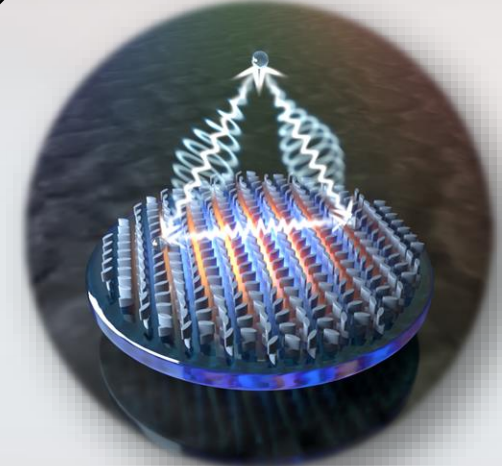


*Nat. Nanotechnol.* **18**, 580-588 (2023)

Thermal Metasurfaces



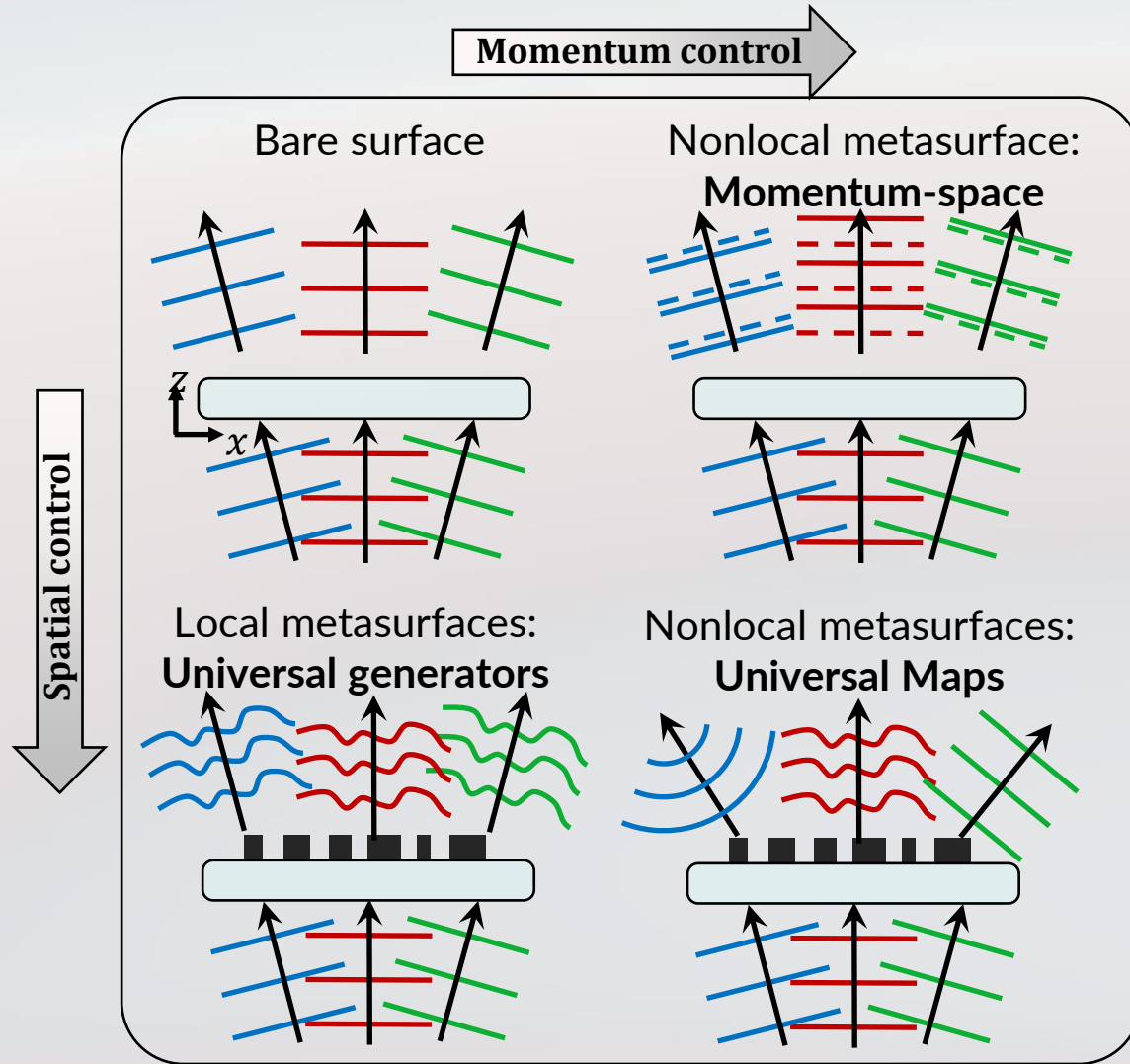
Custom sources



*PRX* **18**, 021050 (2021)  
*Nat. Nanotechnol.* (2024)



# Opportunity: Beyond the Generalized Snell's Law



Periodic thin film media:

$$S_{ij}(k_i, k'_j) = \sum_m a_m(k_i, k'_j) \delta[f_m(k_i, k'_j)]$$

Conservation of quasi-momentum:

$$f_m(k_i, k'_j) = k_i - k'_j - m \frac{2\pi}{P}$$

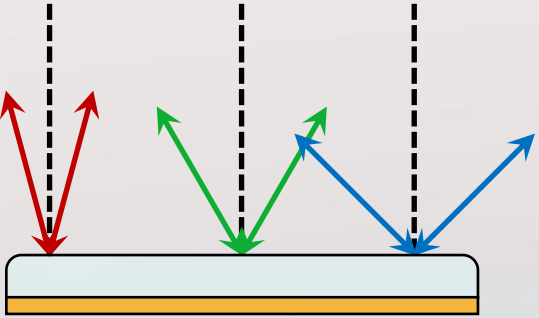
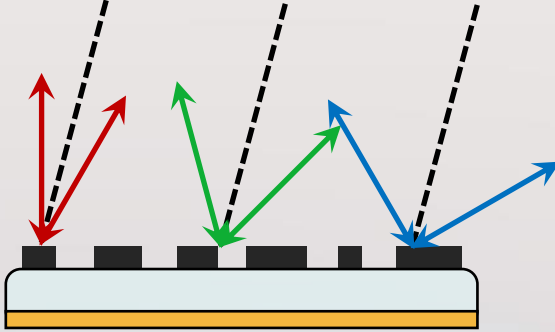
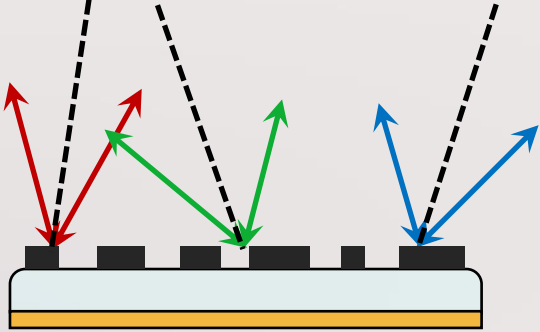
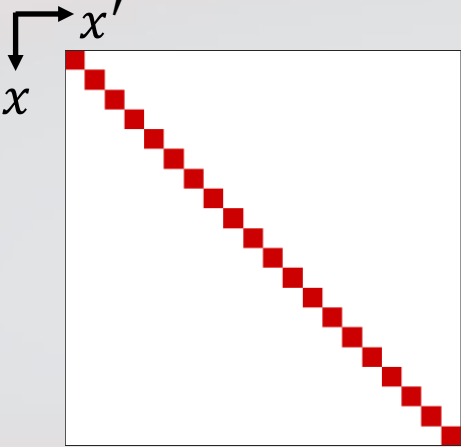
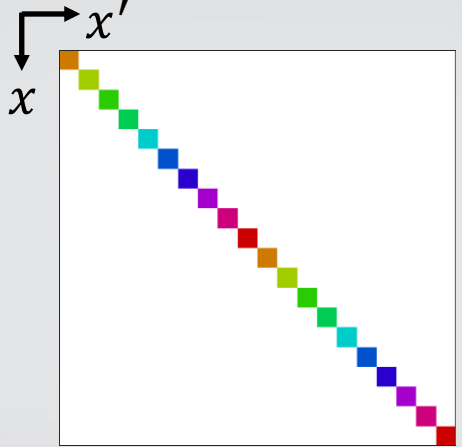
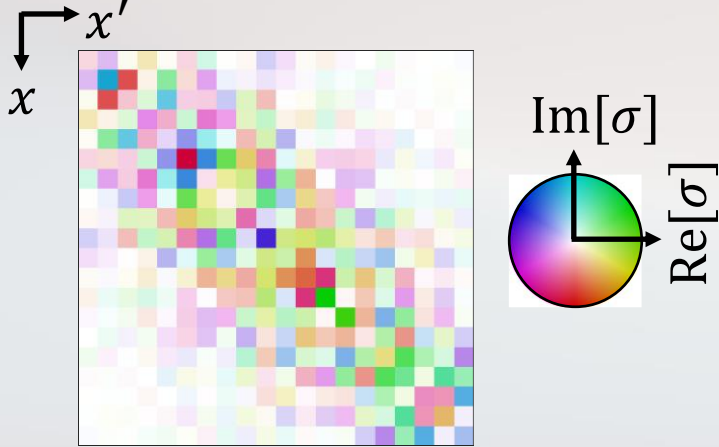
Case	$f(k_i, k'_j)$
Snell's Law	$k_i - k'_j$
Generalized Snell's Law	$k_i - k'_j - \nabla\Phi$
Nonlocal Generalized Snell's Law	$k_i - k'_j - \nabla\Phi_{ij}(k'_j)$

Reciprocity:

$$S_{ij}(k_a, k_b) = S_{ji}(-k_b, -k_a)$$

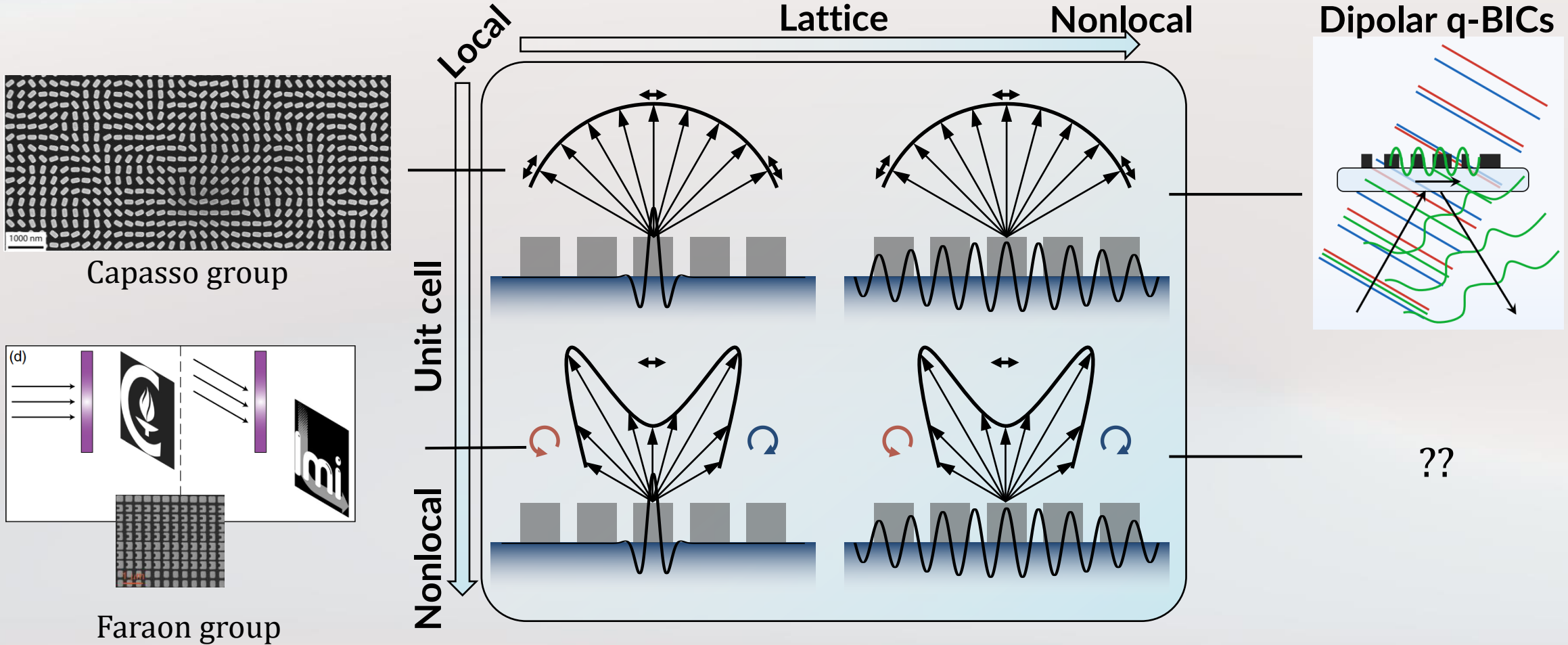
$$\nabla\Phi_{ij}(k_b) = \nabla\Phi_{ji}(-k_a)$$

# Opportunity: information density

	Bare interface	Generalized Snell's Law	Nonlocal Generalized Snell's Law
Law	$\sin(\theta_{out}) = \sin(\theta_{in})$	$\sin(\theta_{out}) = \sin(\theta_{in}) + \frac{1}{k_0} \frac{d\Phi}{dx}$	$\sin(\theta_{out}) = \sin(\theta_{in}) + \frac{1}{k_0} \frac{d\Phi}{dx}(\theta_{in})$
Functionality			
Information density			

# Unexplored regime

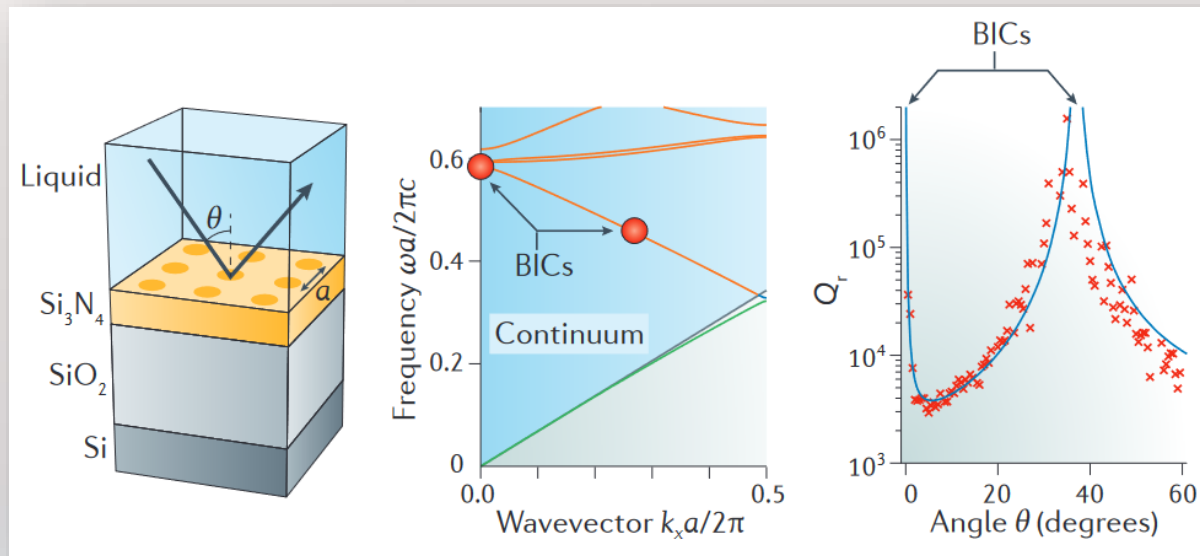
## Nonlocality in spatially varying metasurfaces





# Hot topic: Bound states in the continuum

## Diverging Q-factors

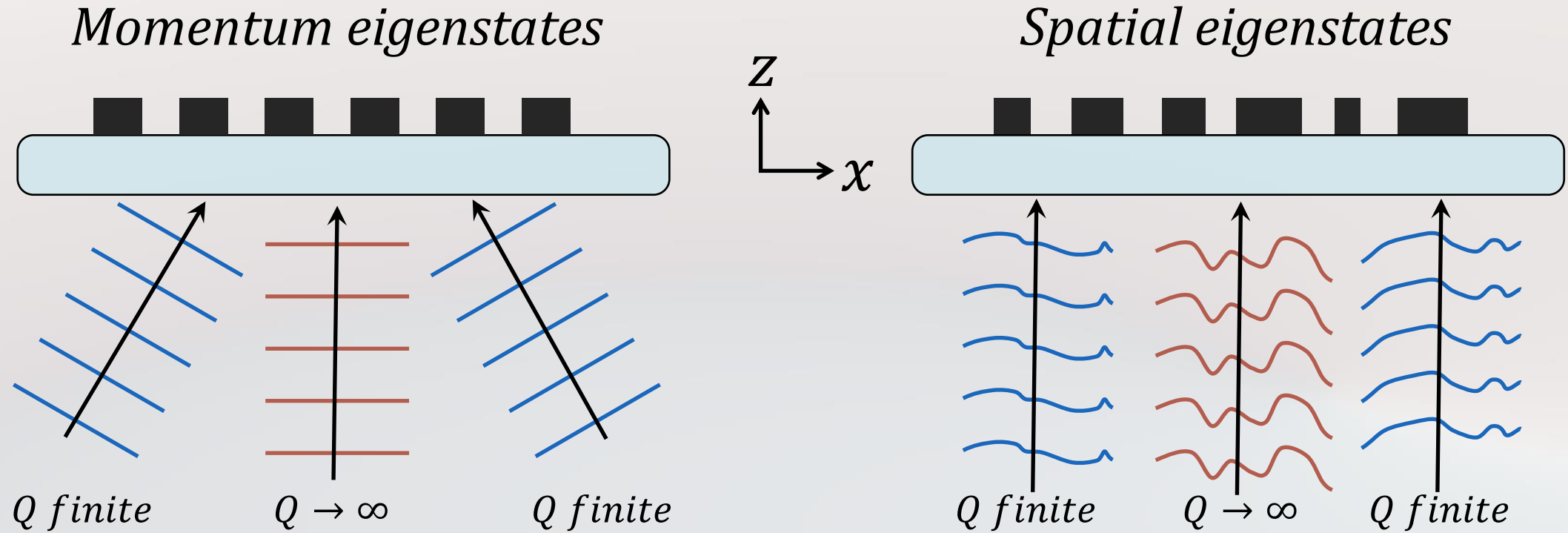


C. Hsu, et al. *Nat. Rev. Mater.* **1**, 16048 (2016).

### Types:

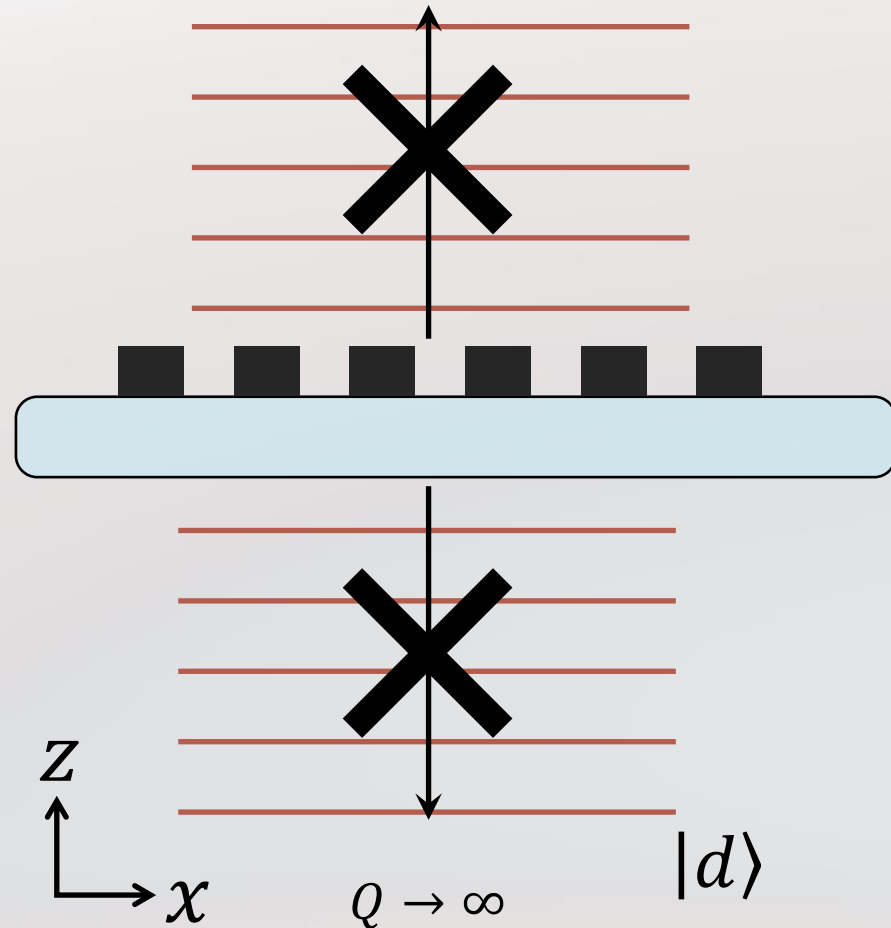
- **Symmetry-protected BICs**
- **Accidental BICs**
- **Friedrich-Wintgen BICs**
- **Unidirectional Guided Resonances**

# New idea: Spatial eigenstates in the continuum



# Spatial eigenstates in the continuum

## Momentum eigenstate



### Hamiltonian:

$$Ha = \omega a$$

$$H = \omega_0 - \left( \frac{b - i\gamma_q}{2} \right) \frac{\partial^2}{\partial x^2}$$

### Outgoing waves:

$$|s_-(x)\rangle = |d(x)\rangle \frac{\partial a(x)}{\partial x}$$

### Solutions:

$$a = e^{ikx}$$

$$\omega_k = \omega_0 + \frac{b - i\gamma_q}{2} k^2$$

As

$$a \rightarrow 1$$

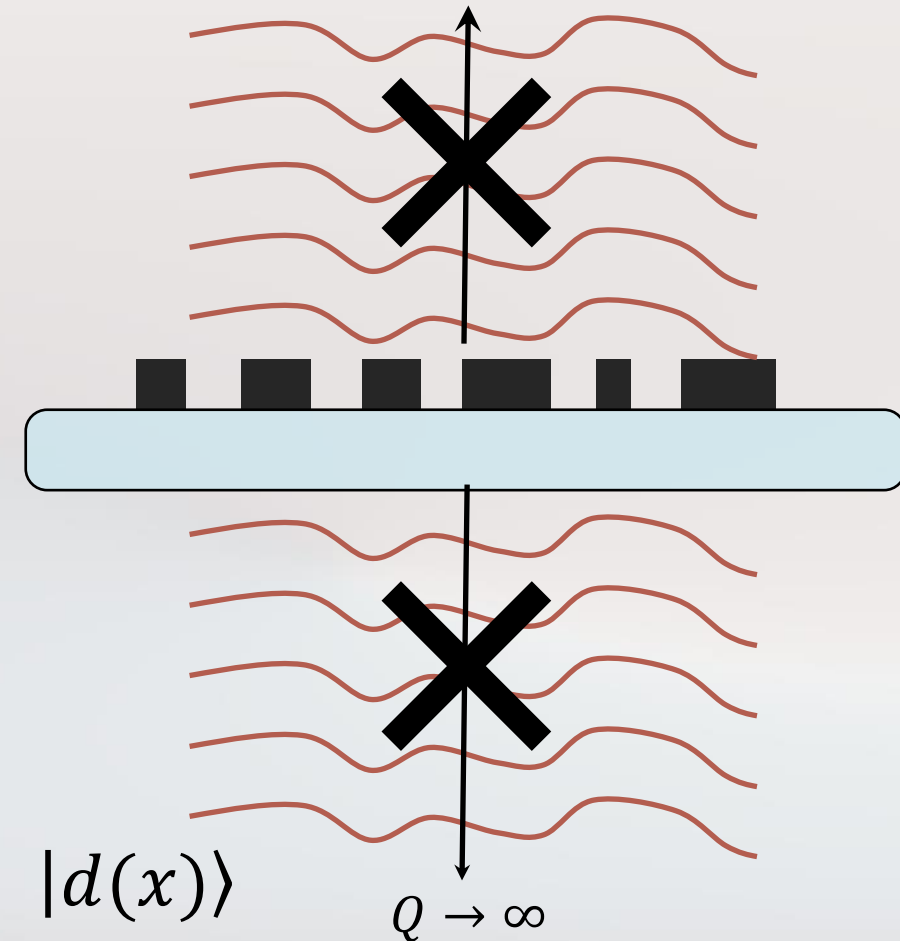
We have

$$Q \rightarrow \infty$$

For the wave

$$|s_-(x)\rangle \rightarrow 0 \times |d(x)\rangle$$

## Spatial eigenstate





# Conventional monomer lattices

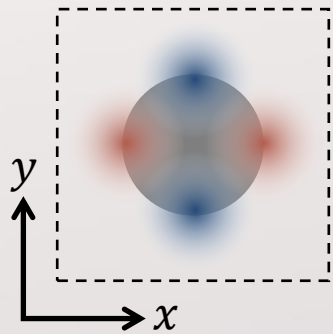
Geometry

Reflection

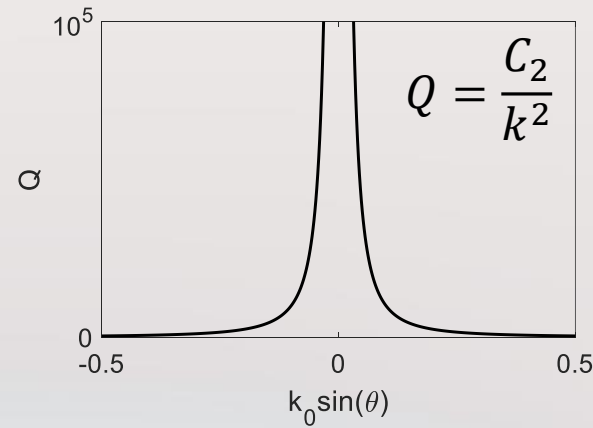
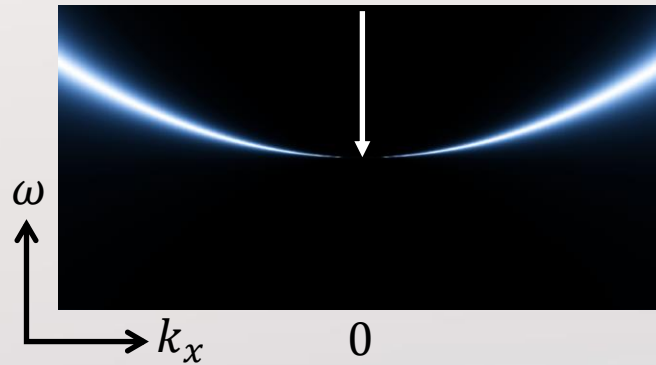
Q-factor

Scattering profile

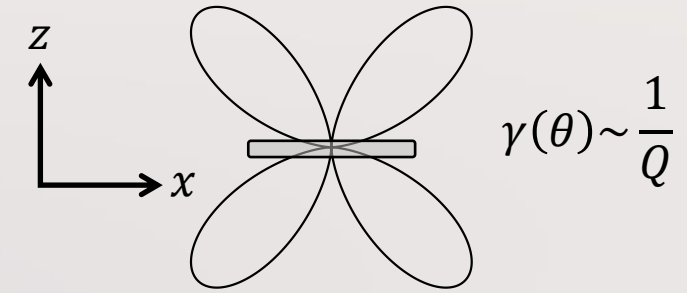
Unperturbed



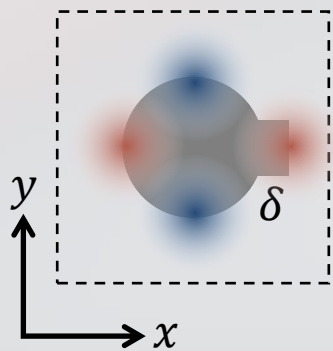
BIC



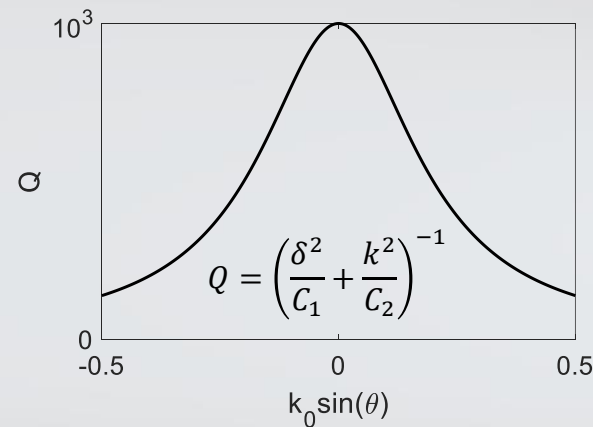
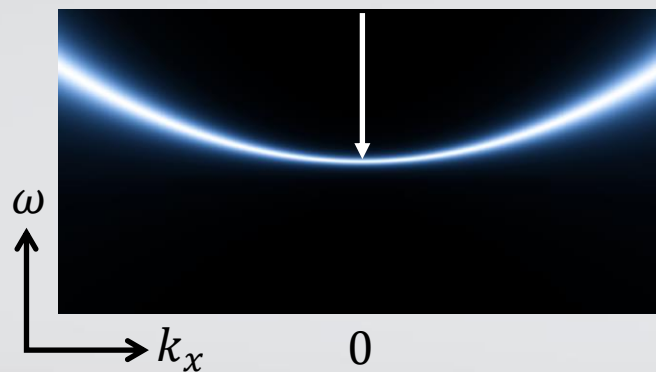
Quadrupole  
(uncontrolled)



Perturbed

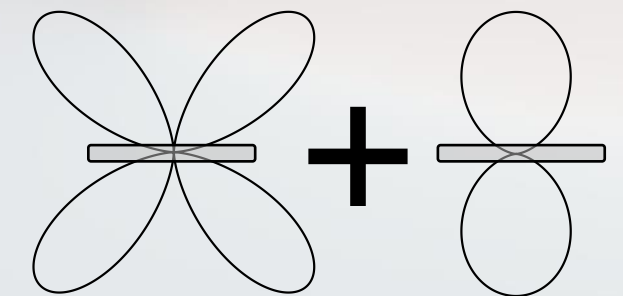


q-BIC



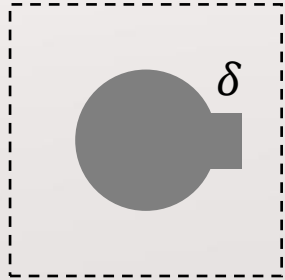
Quadrupole  
(uncontrolled)

Dipole  
(controlled)

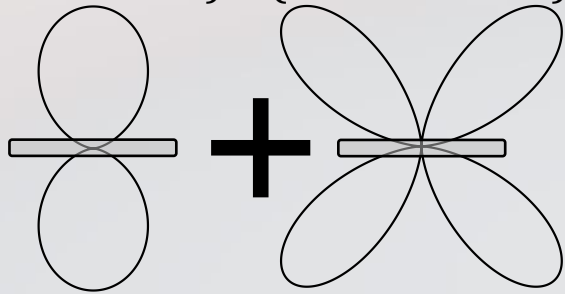


# BICs and q-BICs: perturbation approaches

## Monomer



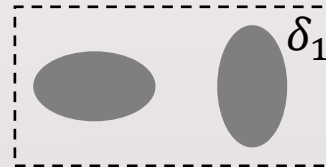
Dipole  
(controlled)      Quadrupole  
(uncontrolled)



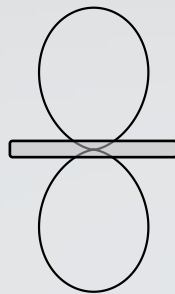
$$Q \sim \left( \frac{\delta^2}{C_1} + \frac{k^2}{C_2} \right)^{-1}$$

## Dimer

### Dipolar

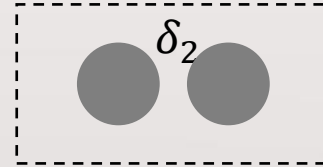


Dipole  
(controlled)

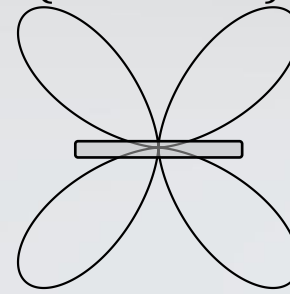


$$Q \sim \frac{C_1}{\delta_1^2}$$

### Quadrupolar

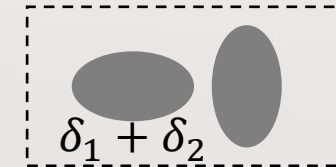


Quadrupole  
(controlled)

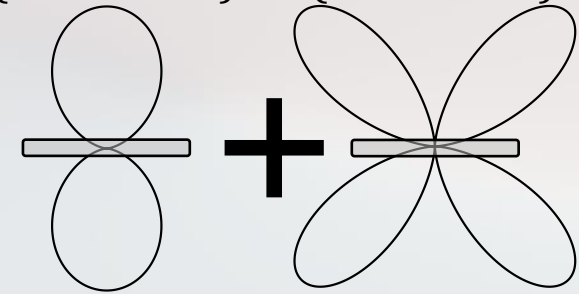


$$Q \sim \frac{C_2}{\delta_2^2} \frac{1}{k^2}$$

### Both



Dipole  
(controlled)      Quadrupole  
(controlled)



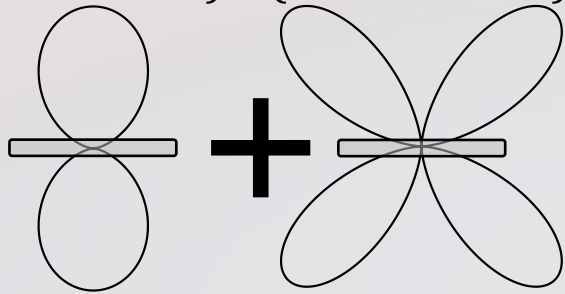
$$Q \sim \left( \frac{\delta_1^2}{C_1} + \frac{\delta_2^2 k^2}{C_2} \right)^{-1}$$

# BICs and q-BICs: perturbation approaches

## Monomer

Common approach  
in the literature

Dipole  
(controlled)      Quadrupole  
(uncontrolled)



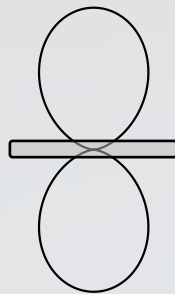
$$Q \sim \left( \frac{\delta^2}{C_1} + \frac{k^2}{C_2} \right)^{-1}$$

## Dimer

### Dipolar

Past  
work

Dipole  
(controlled)

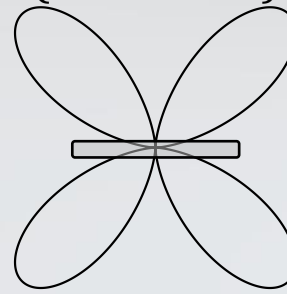


$$Q \sim \frac{C_1}{\delta_1^2}$$

### Quadrupolar

Today's  
main  
topic

Quadrupole  
(controlled)

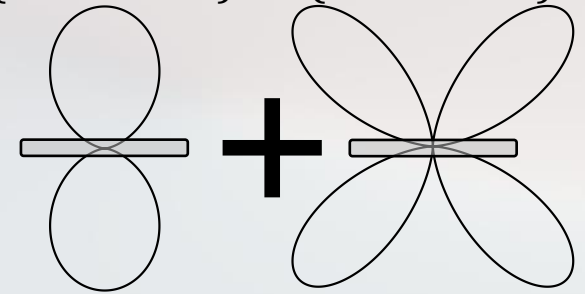


$$Q \sim \frac{C_2}{\delta_2^2} \frac{1}{k^2}$$

### Both

Ongoing  
work

Dipole  
(controlled)      Quadrupole  
(controlled)

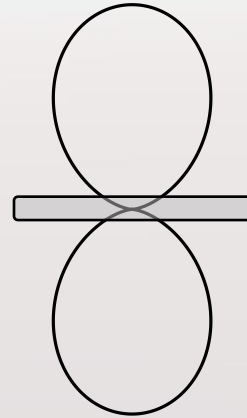
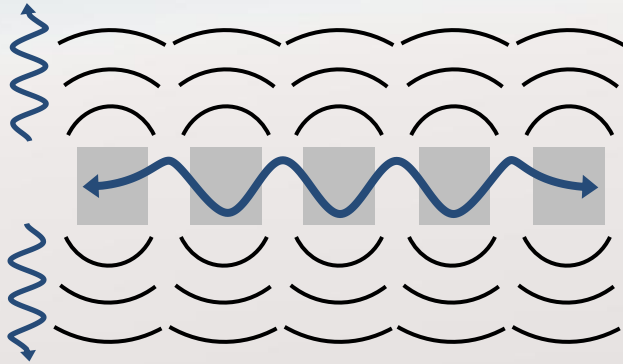


$$Q \sim \left( \frac{\delta_1^2}{C_1} + \frac{\delta_2^2 k^2}{C_2} \right)^{-1}$$

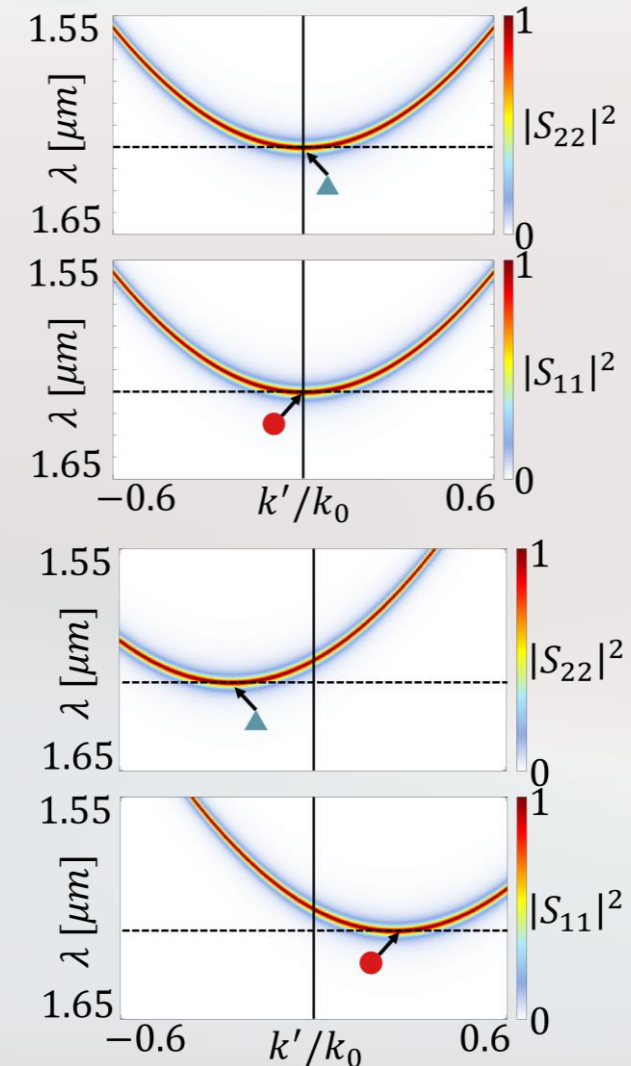
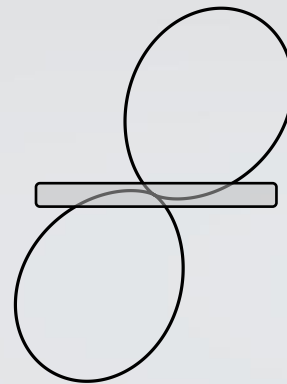
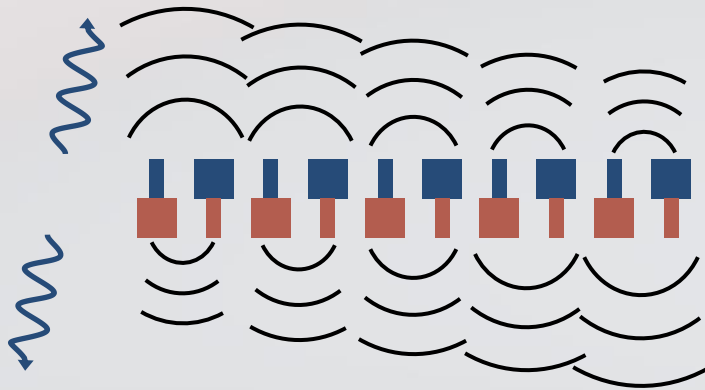


# Past work: Dipolar nonlocal phase gradients

## Periodic

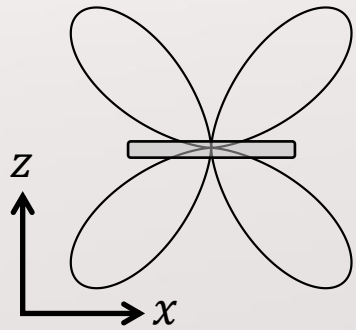


## Nonlocal metasurface

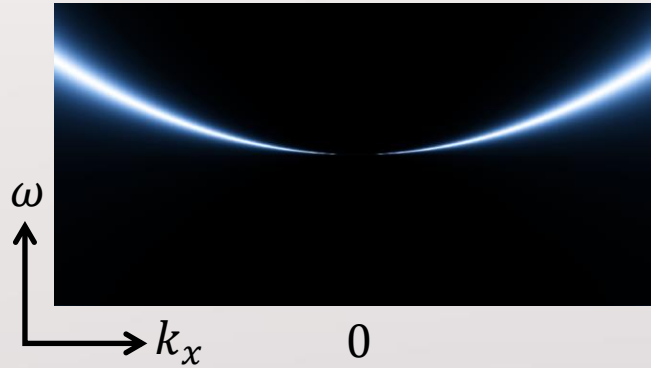


# Now apply this to quadrupolar states

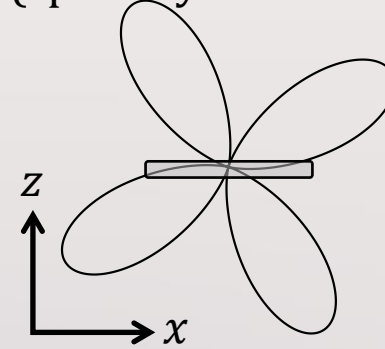
Quadrupole



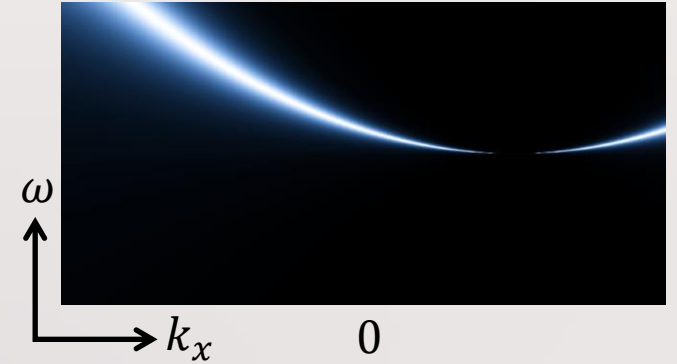
Quadrupolar q-BIC



Quadrupole  
(spatially controlled)



Quadrupolar q-BIC +  
phase gradient



**Momentum BICs:**

$$|d\rangle = \begin{bmatrix} d_1 \\ d_2 \end{bmatrix}$$

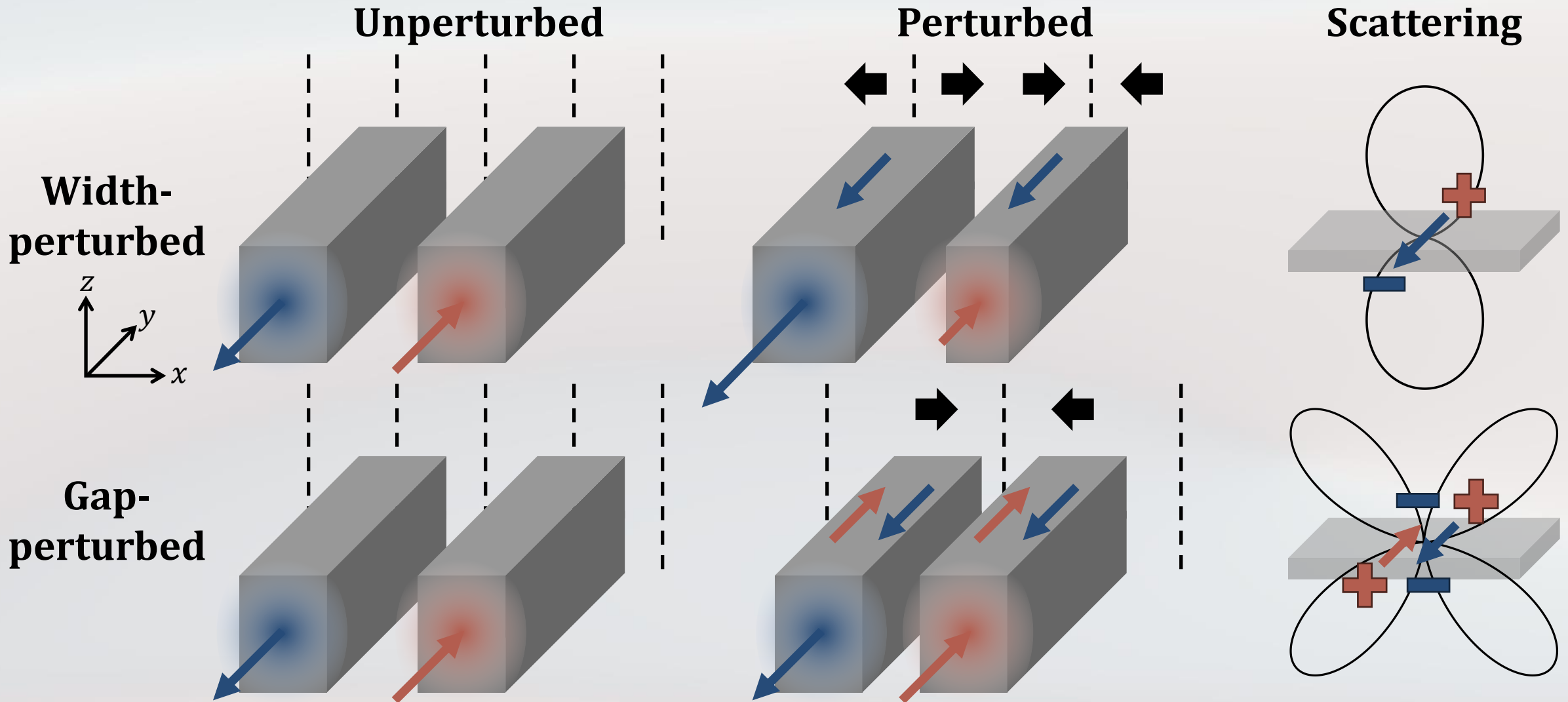
= Planewaves

**Spatial BICs:**

$$|d\rangle = \sqrt{\frac{\gamma_q}{2}} \begin{bmatrix} \exp(-i\Phi(x)) \\ -i \exp(i\Phi(x)) \end{bmatrix}$$

= Custom waves

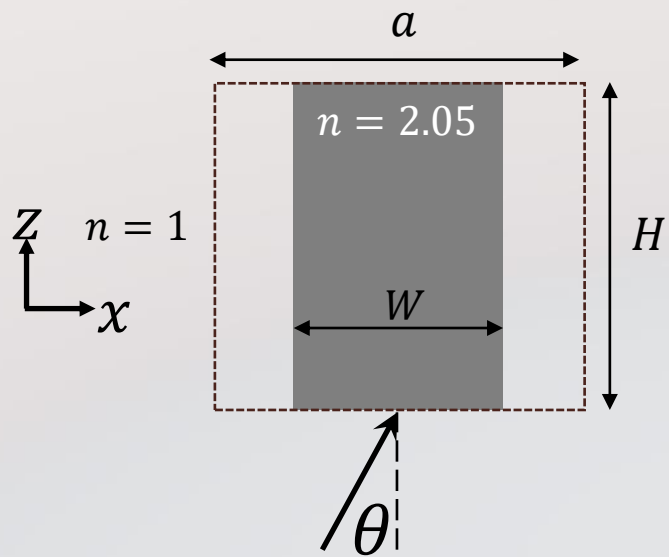
# Perturbations for fundamental TE mode



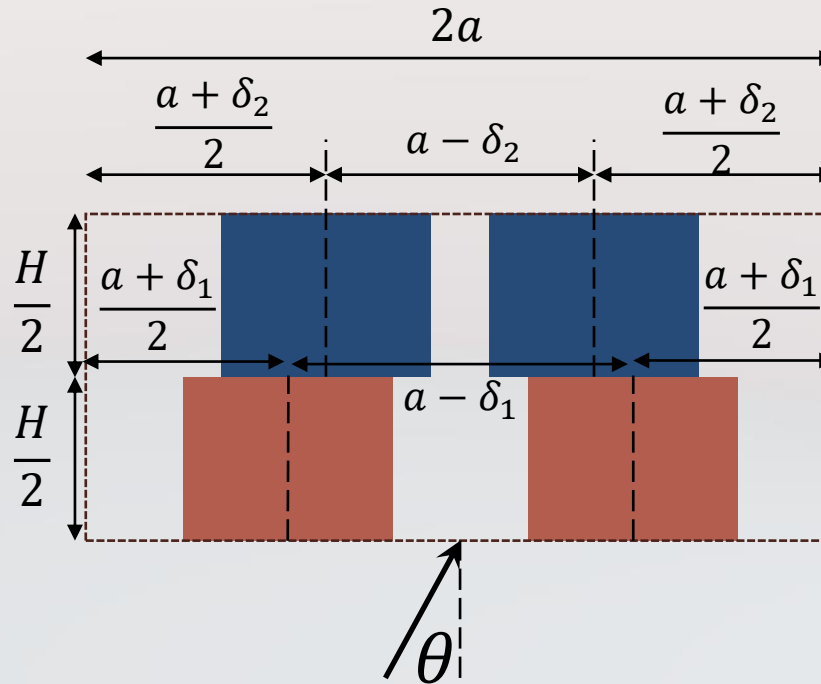


# Perturbation scheme: dual-layer gap-perturbed

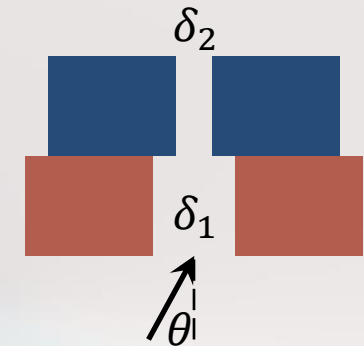
Unperturbed



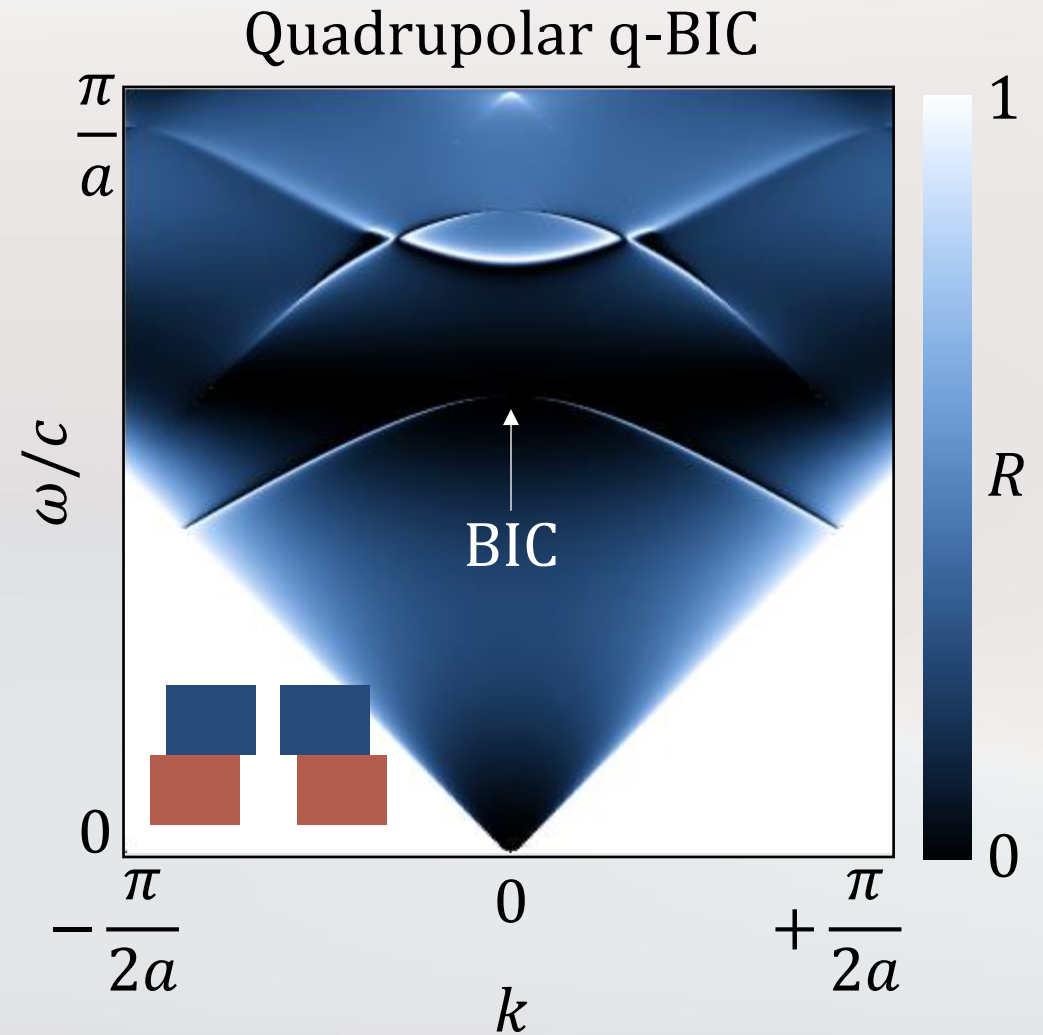
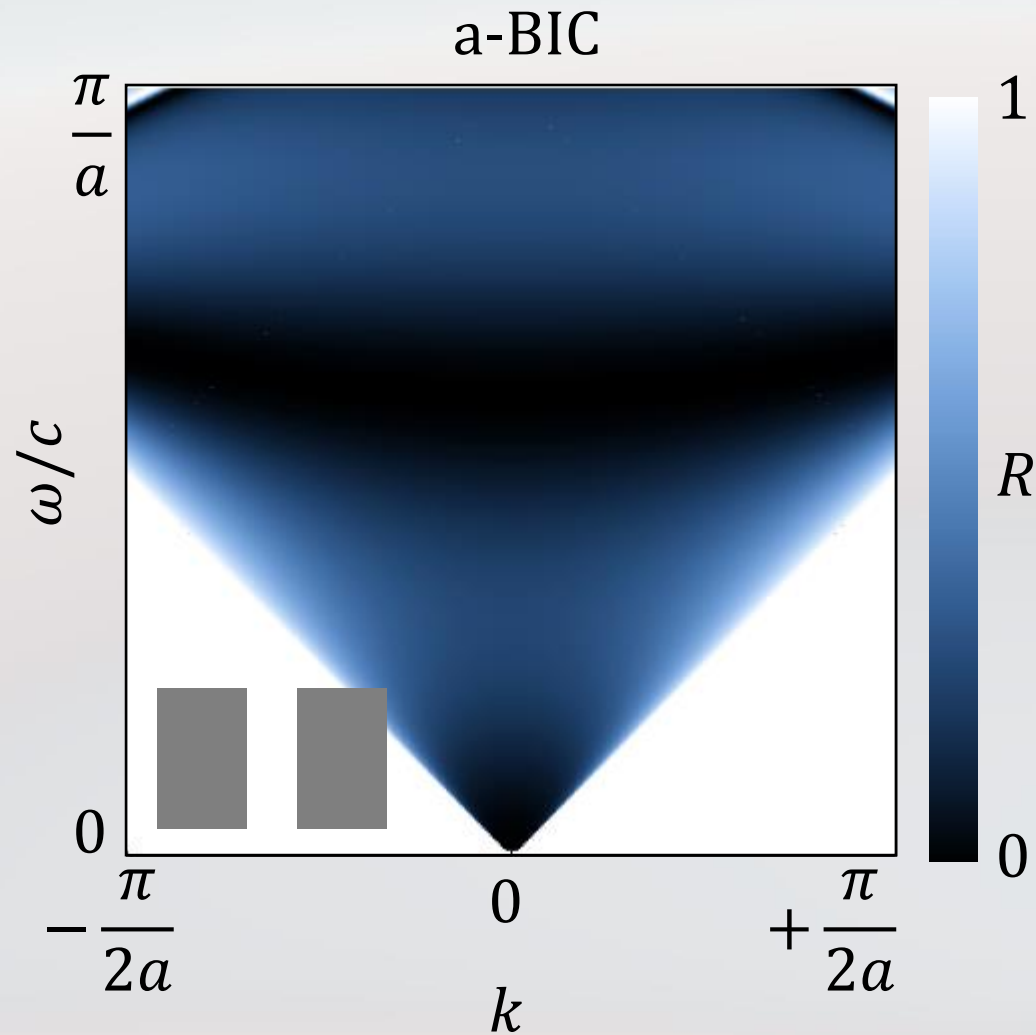
Perturbed



Simplified schematic:



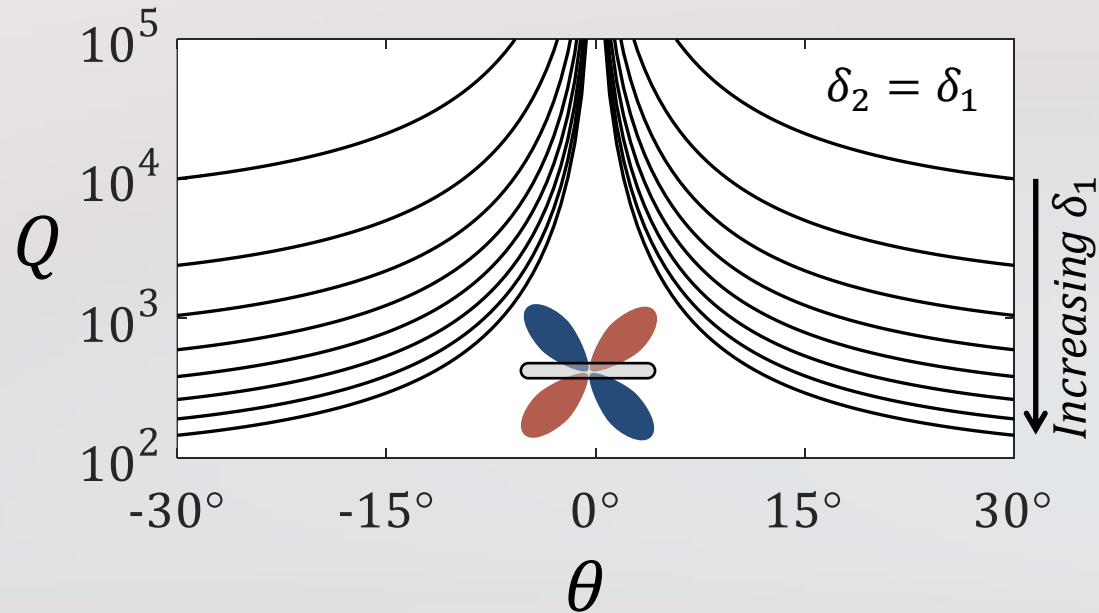
# Zone-folded, symmetry-protected BIC



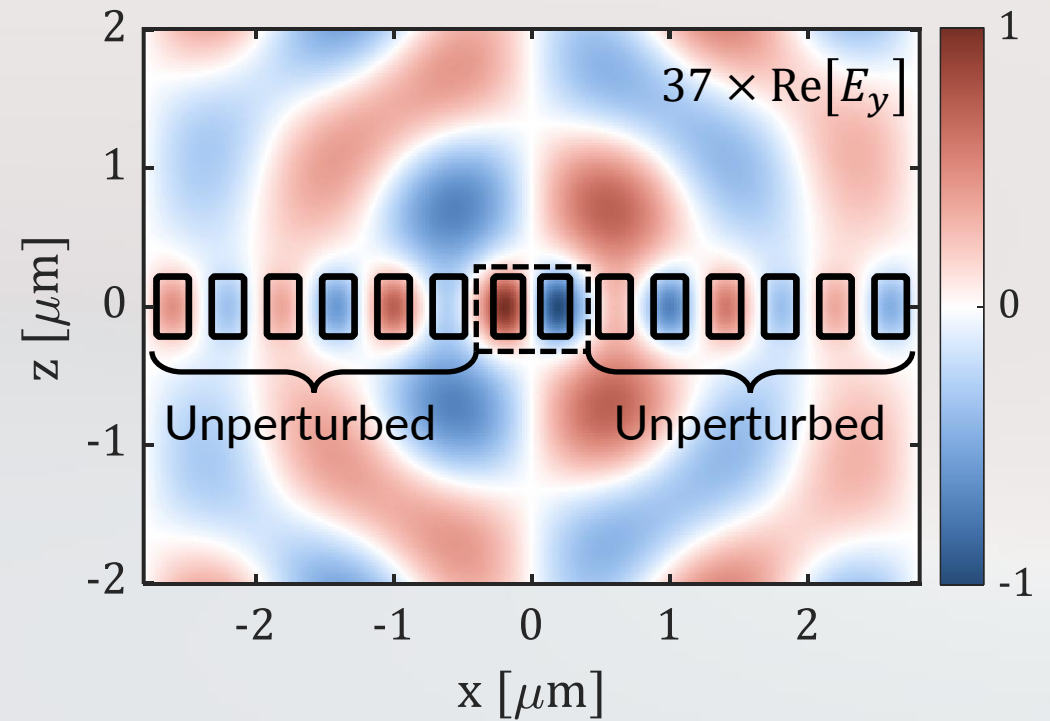
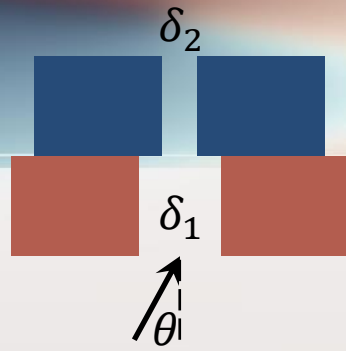
# Quadrupolar scattering

**Controllable Q-factor  
(nonlocal property)**

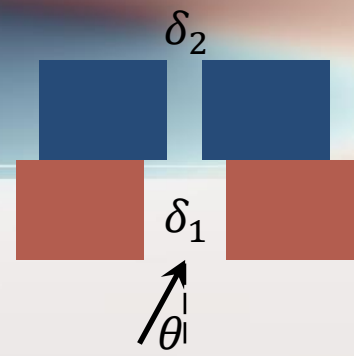
$$Q \propto \frac{1}{k^2} \frac{1}{\delta_1^2 + \delta_2^2}$$



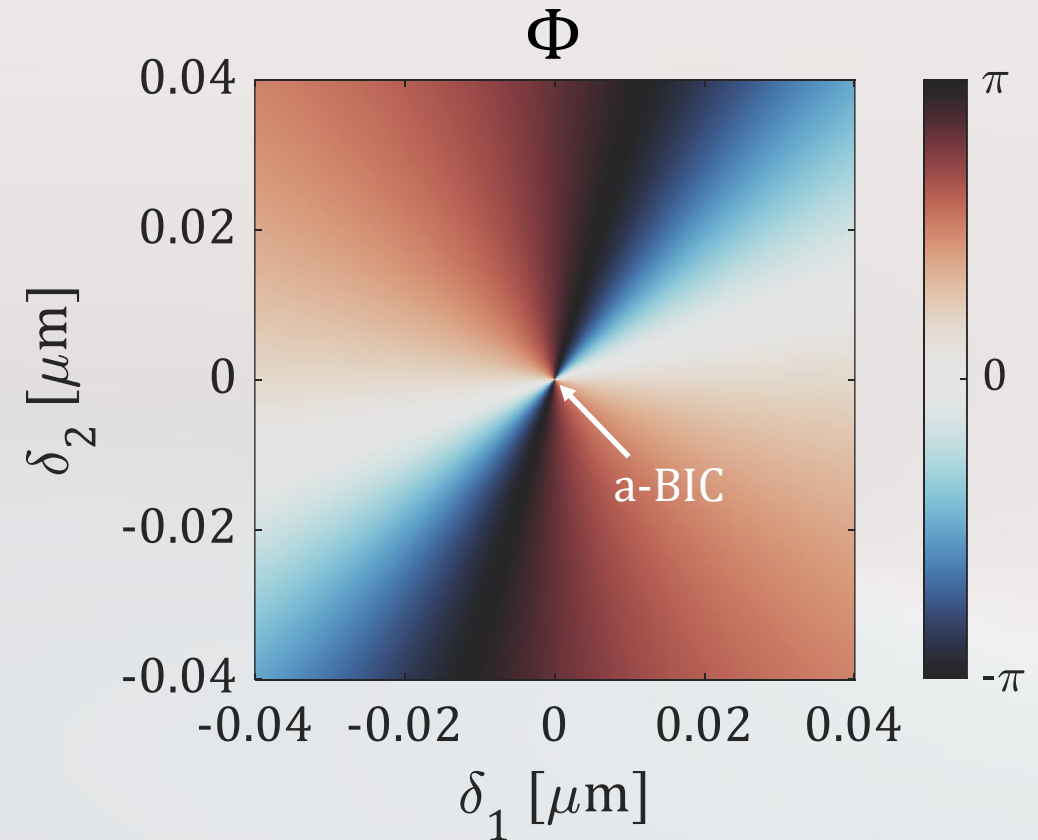
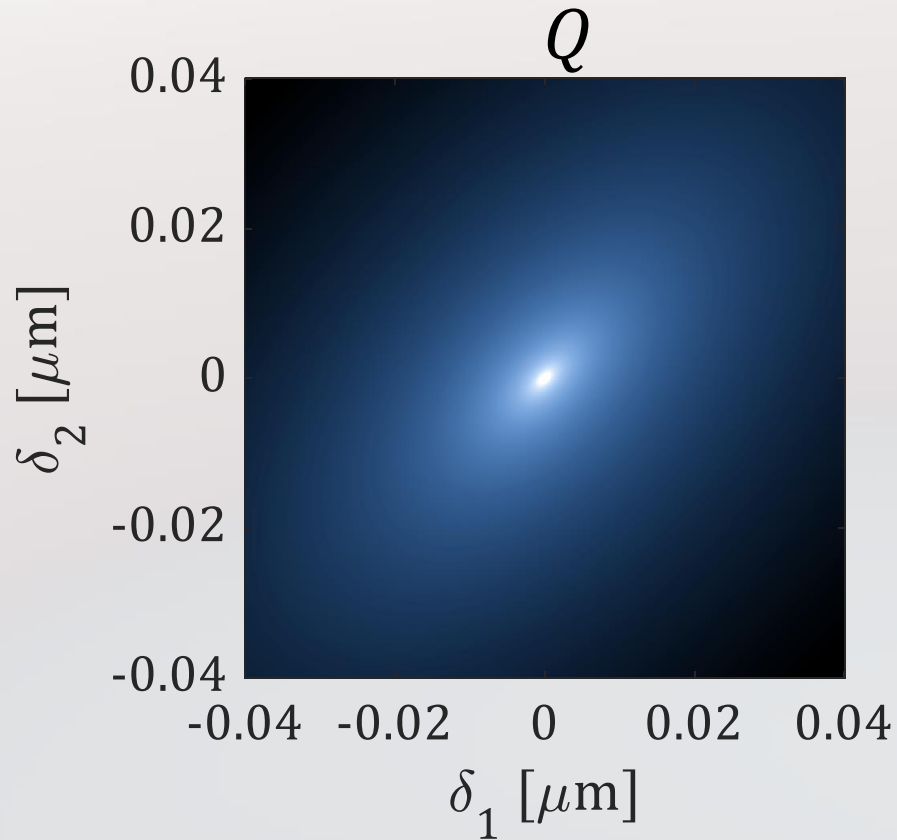
**Local leakage  
(local property)**



# Arbitrary Q-factor and phase

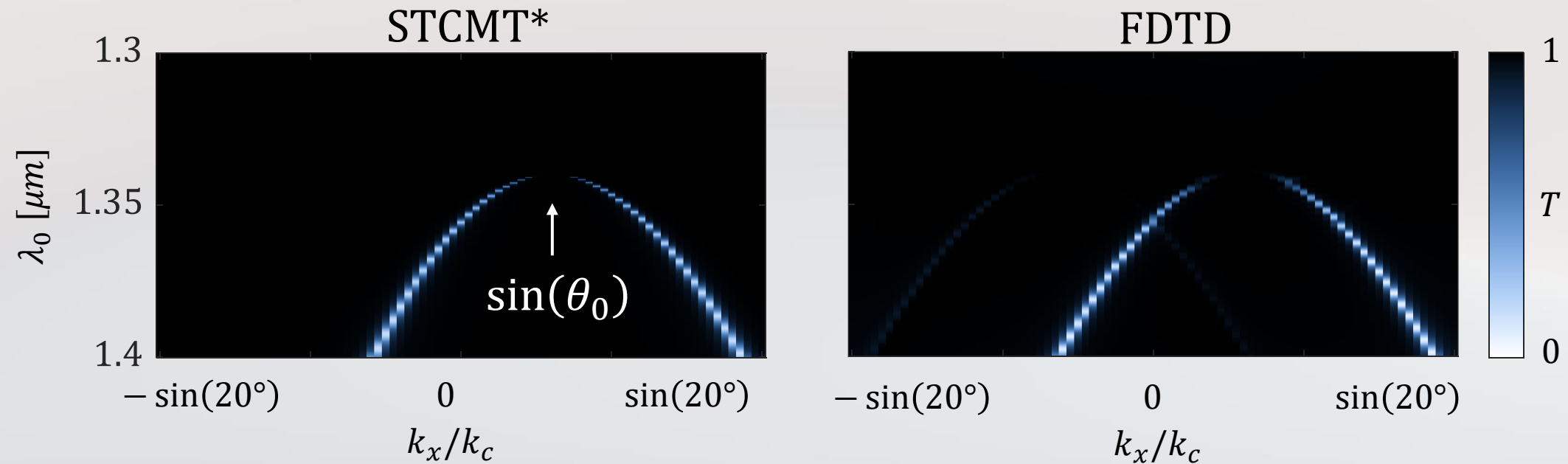
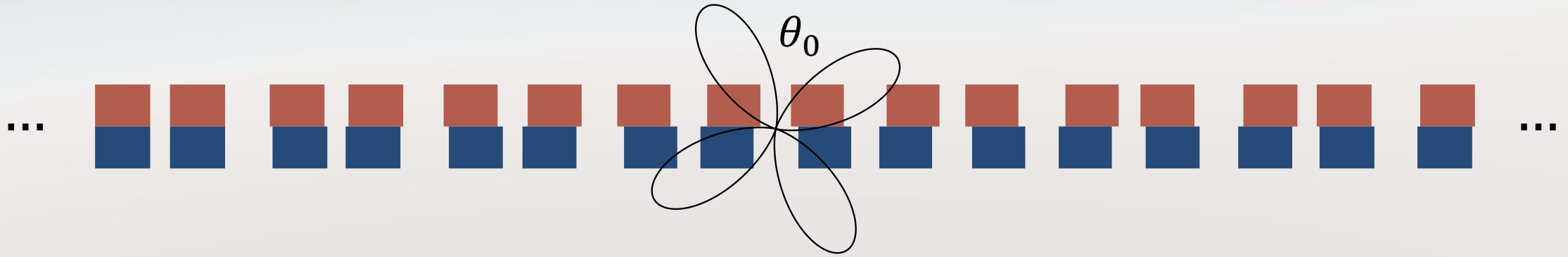


$$\theta = 2^\circ$$





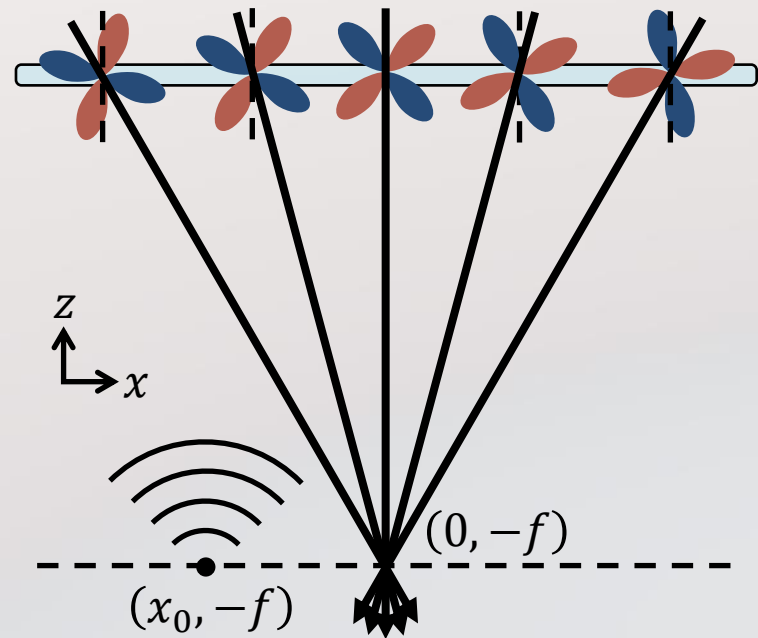
# Quadrupolar phase gradient



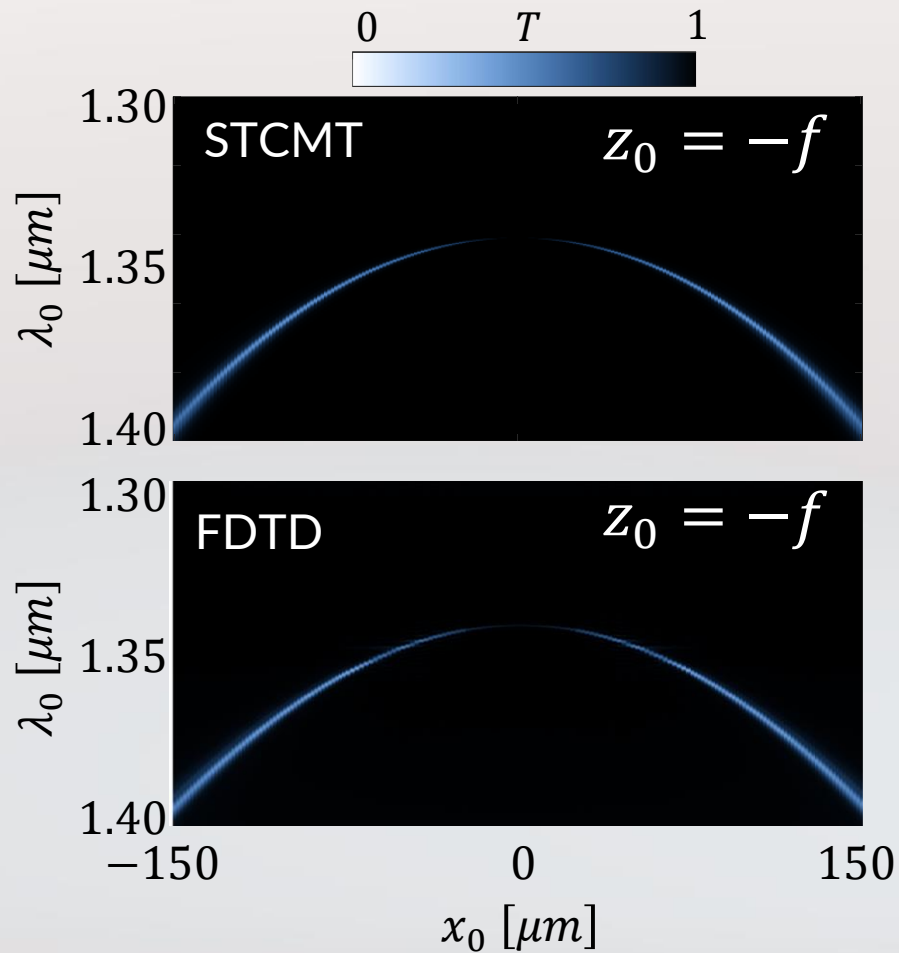
\*A. Overvig, S. Mann, and A. Alù, "Spatiotemporal Coupled Mode Theory", *Light: Science & Applications*, **13**, 28 (2024).

# BIC in the *spatial domain*

Scan the focal plane



Response

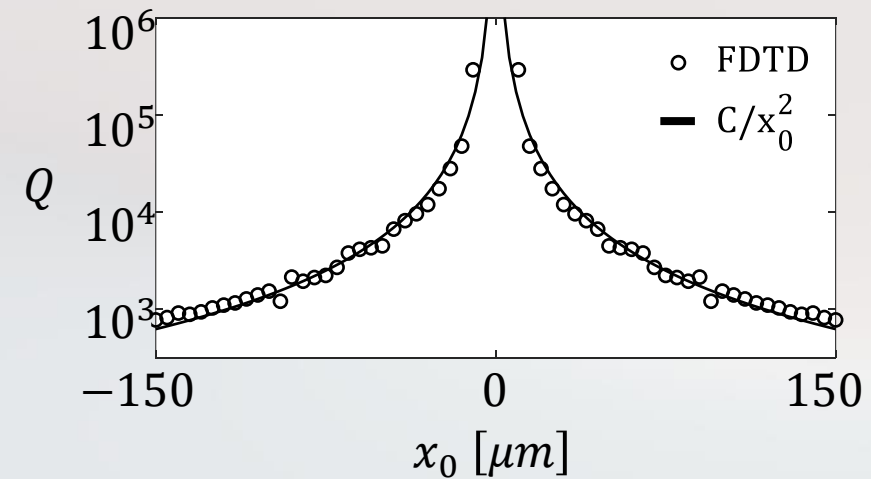


Momentum space

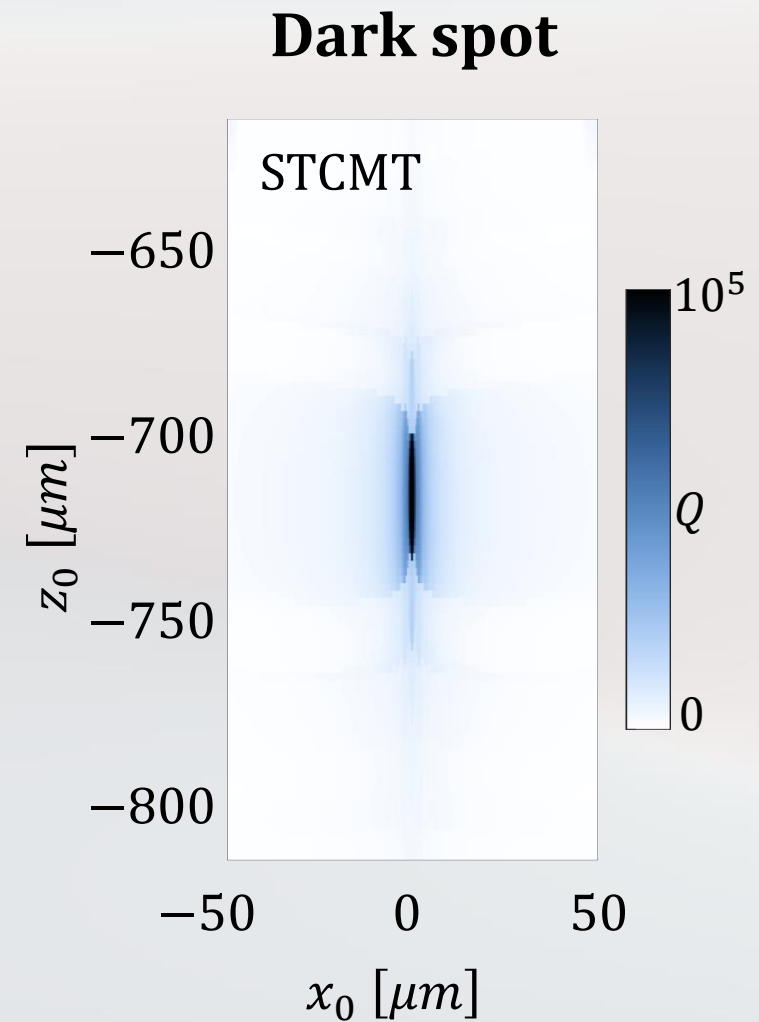
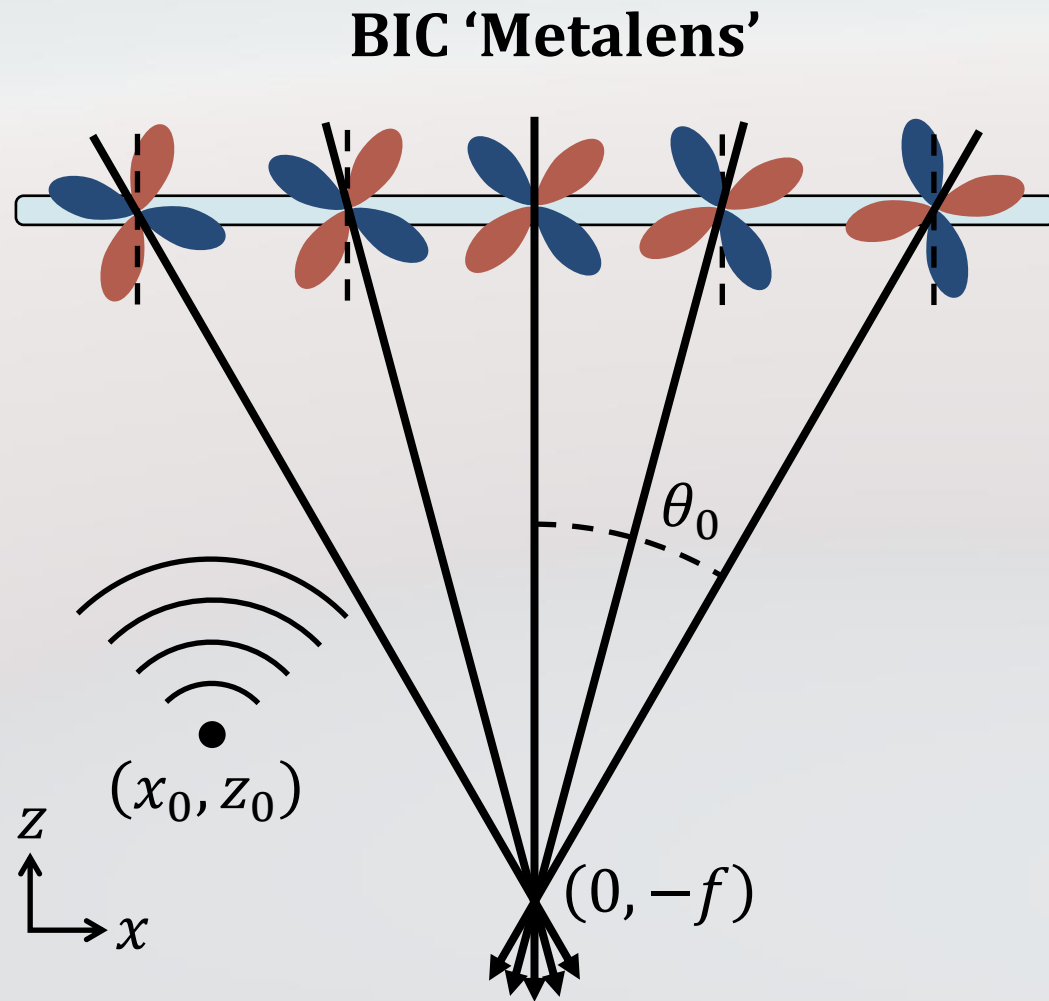


Real space

$$Q \sim \frac{1}{k^2} \rightarrow Q \sim \frac{1}{x_0^2}$$

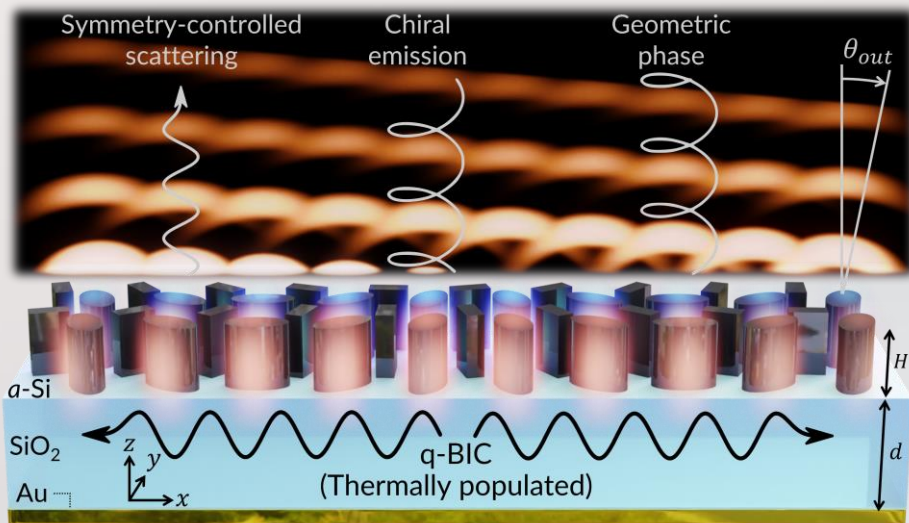


# Away from the focal plane

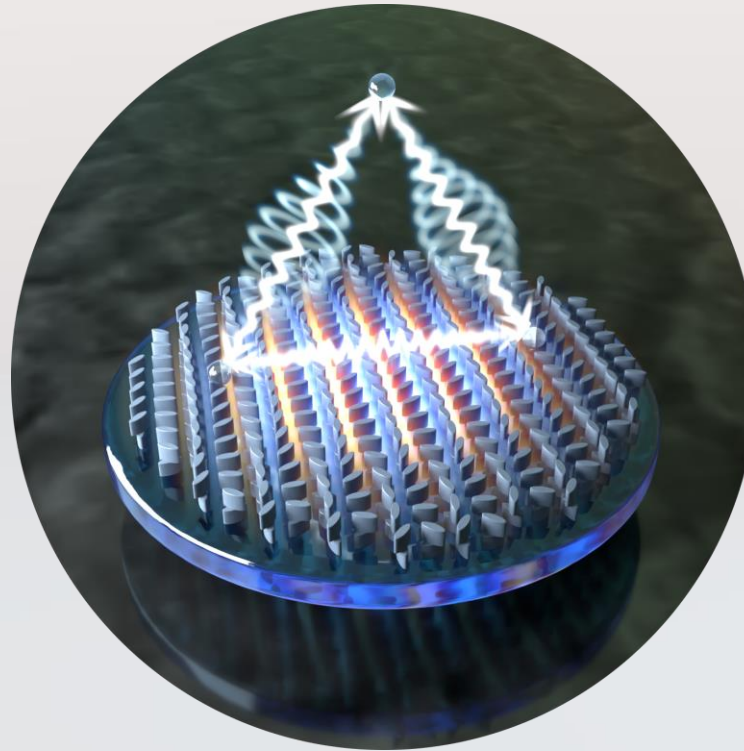


# Potential applications: custom sources

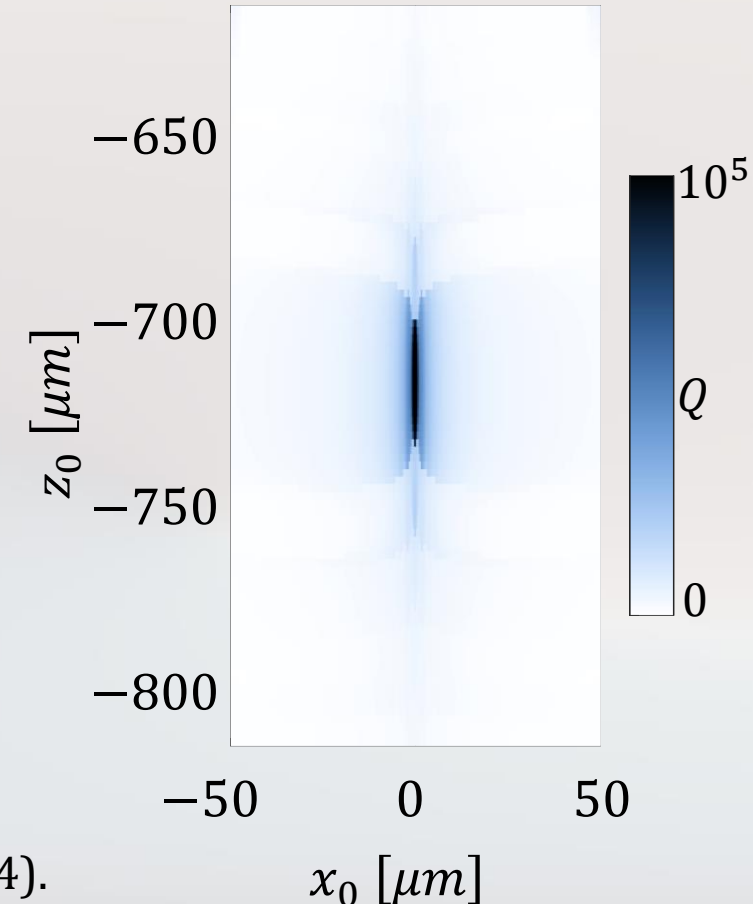
## Thermal emission



## Laser emission



## ...focused



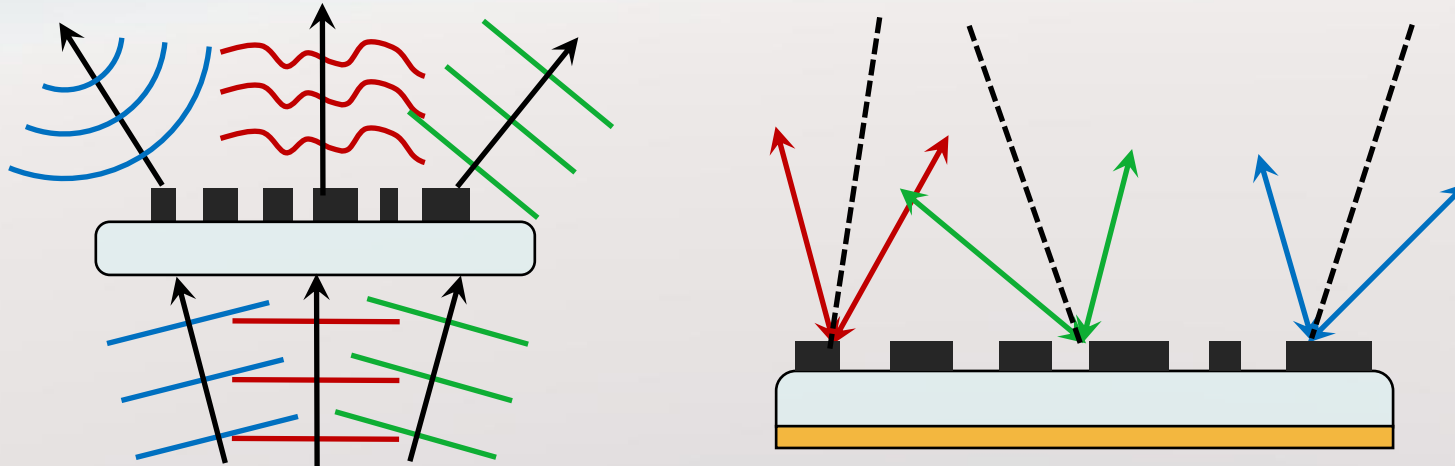
A. Overvig, S. Mann, A. Alu, *PRX* **18**, 021050 (2021)

J. Nolen, A. Overvig, M. Cotrufo, A. Alu, *Nature Nanotechnology* **19**, 1627-1634 (2024).

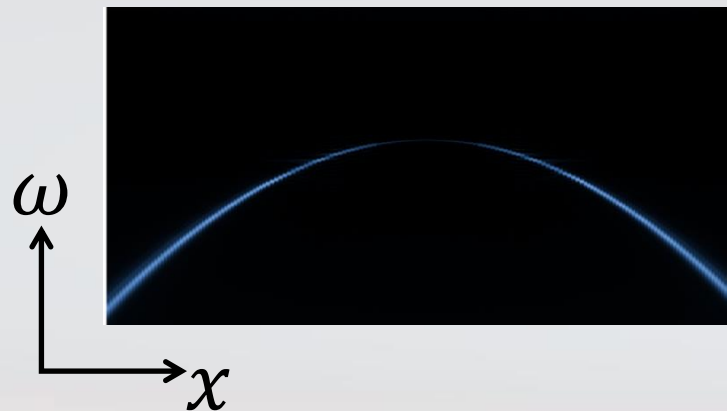
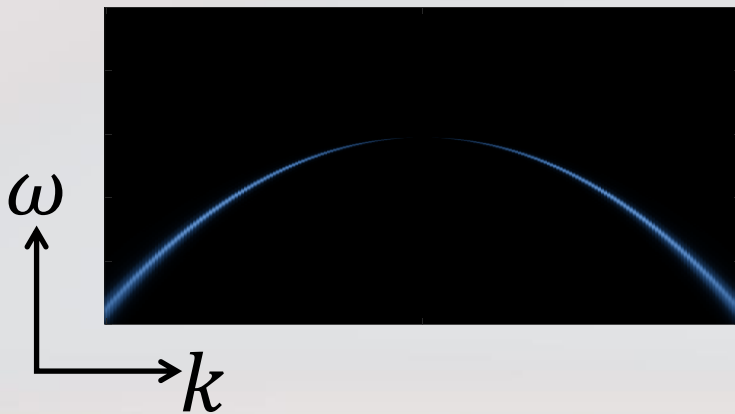


# Summary

Nonlocal metasurfaces = opportunities beyond Generalized Snell's Law



Generalizes BICs from the *momentum* eigenstates to *spatial* eigenstates



## Publications, 2024

- [1] A. Overvig, S. Mann, and A. Alù, "Spatiotemporal Coupled Mode Theory", *Light: Science & Applications*, **13**, 28 (2024).
- [2] J. Nolen, A. Overvig, M. Cotrufo, A. Alu, *Nature Nanotechnology* **19**, 1627-1634 (2024).
- [3] A. Overvig, F. Monticone, in preparation
- [4] A. Overvig, in preparation

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