



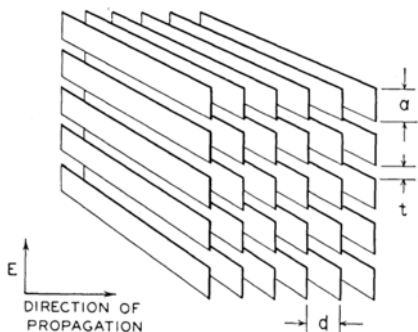
# **Fundamental Analyses and Discovery of Topological EM Materials**

**AFOSR EM Portfolio Review**

**Daniel Sievenpiper, UC San Diego  
January 8, 2025**

# From Artificial Dielectrics to Topological Metamaterials

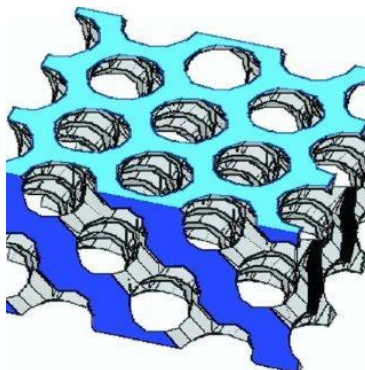
## Artificial Dielectrics



Sharpless  
1940s to 1960s

Metal strips, plates, or spheres in dielectric change effective permittivity

## Photonic Crystals



Yablonovitch et. al.  
1980s and 1990s

Specific lattices (periodicity) can create band gaps (omnidirectional frequency filters)

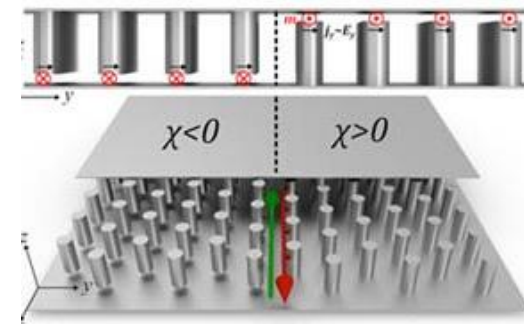
## Metamaterials



Smith et. al.  
2000s

Resonance can create any value of permittivity and/or permeability, including negative or near zero

## Photonic Topological



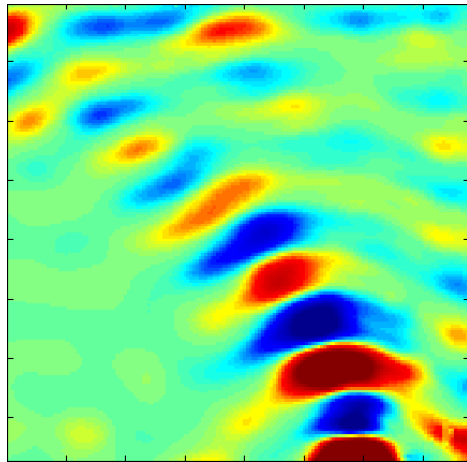
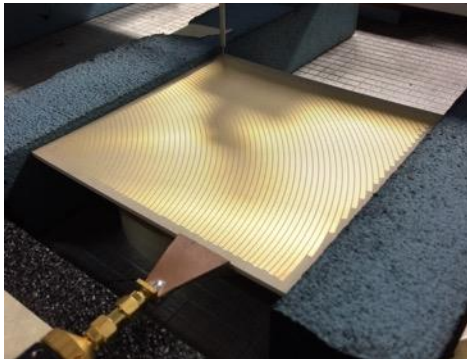
Shvets et. al.  
2010s

"Spin-orbit coupling" for unidirectional modes (two opposite polarizations propagate in opposite directions)

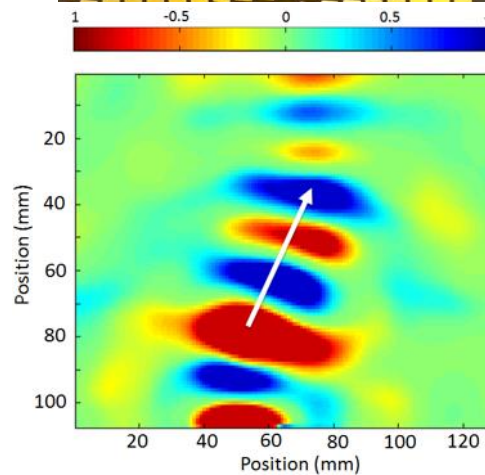
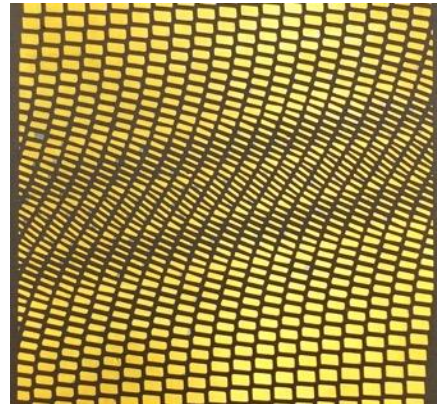
- **Topological electromagnetic structures represent the next advancement and new degree of freedom in artificial media or metamaterials**

# Examples of Patterned Metasurfaces for Various Functions

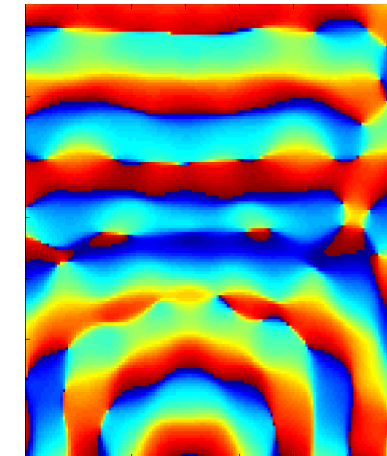
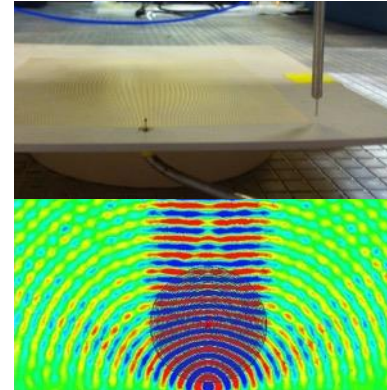
**Bend Around Curve**



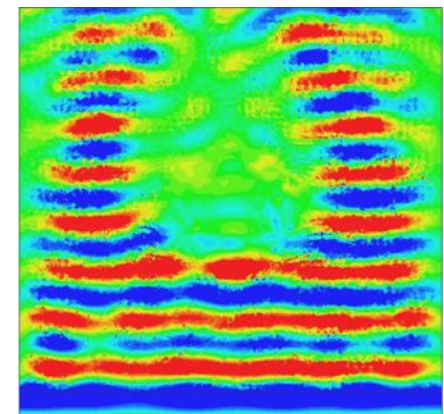
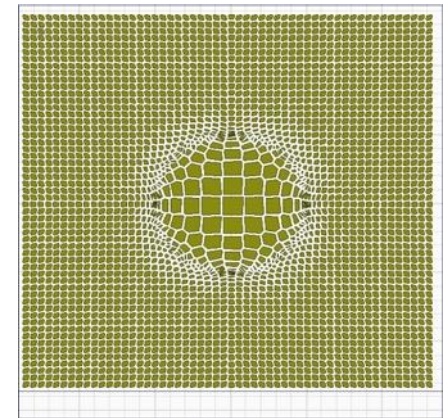
**Beam Shifter**



**Planar Lens**



**Avoiding Hole**

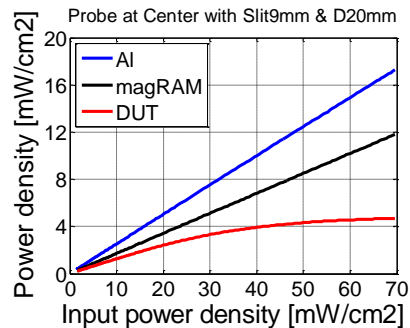
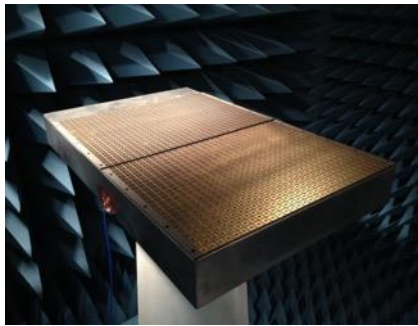
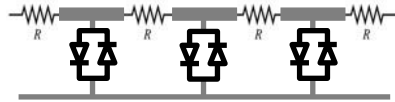


- Patterning method allows for wide range of impedance functions
- Enables control over surface wave propagation and radiation

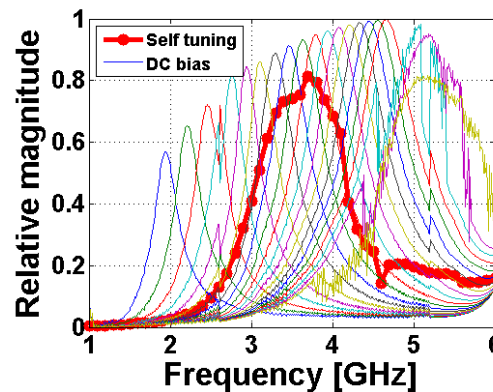
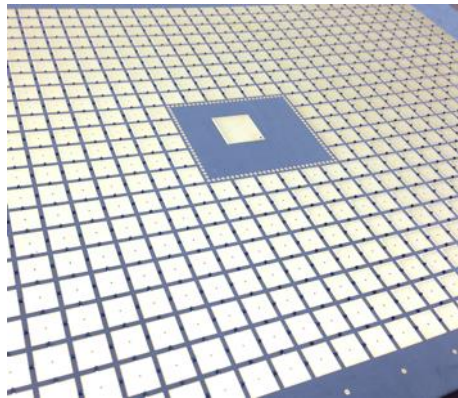


# Examples of Active Metasurfaces for Various Functions

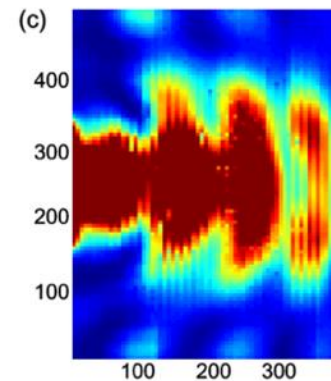
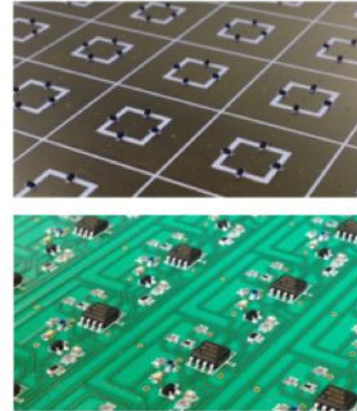
## Nonlinear Absorber



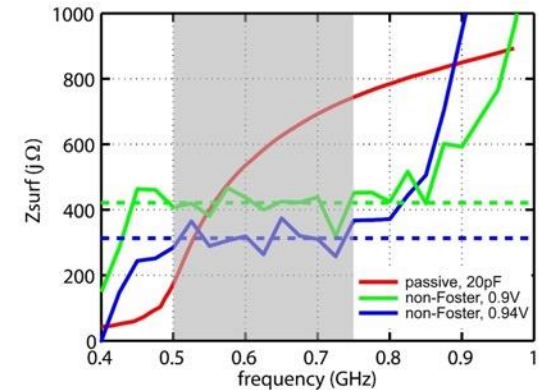
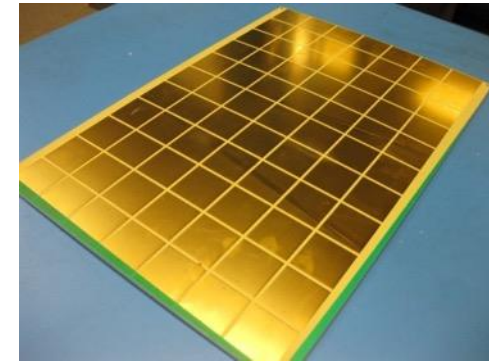
## Self-Tuning Surface



## Self-Focusing Surface



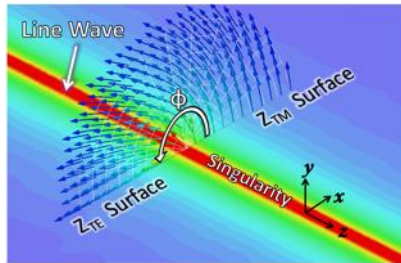
## Broadband Non-Foster Based Surface



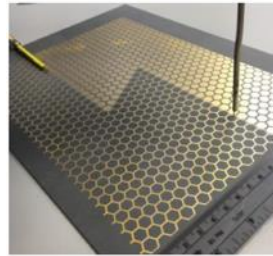
- A wide range of new surface properties are enabled by including nonlinear, active, feedback, or non-Foster circuits

# Overview of Topological and Chiral Structures Studied Recently

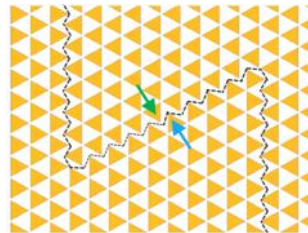
Line waves



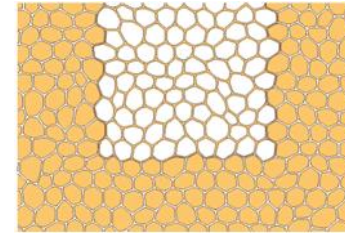
Metallic PTI (spin)



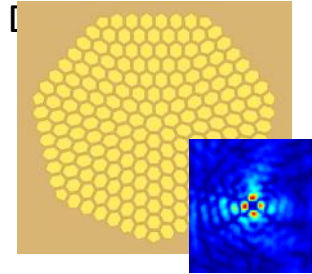
Metallic PTI (valley)



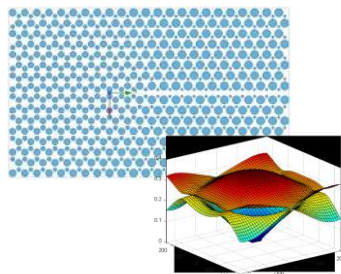
Effects of Disorder



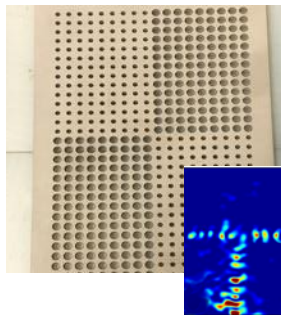
Real Space



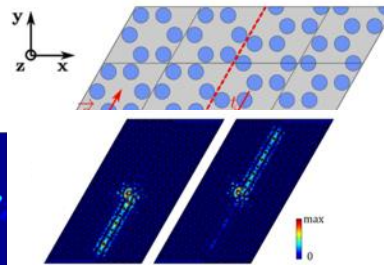
Dirac Point Propagation



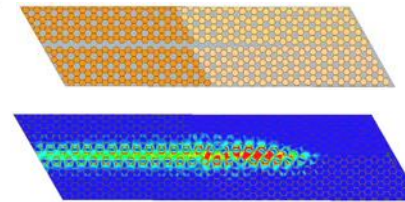
Zak Phase Structures



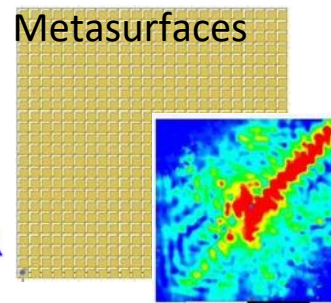
Antiphase Boundaries



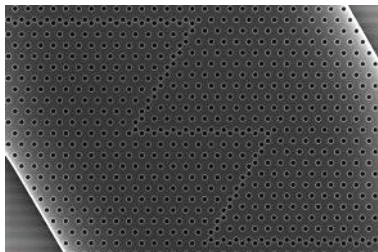
Vernier/Zipper Structures



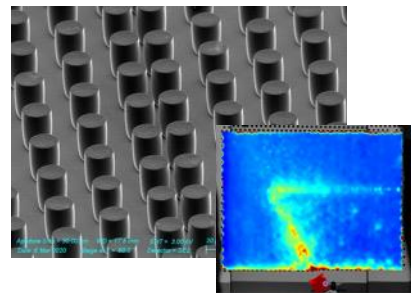
Chiral Metasurfaces



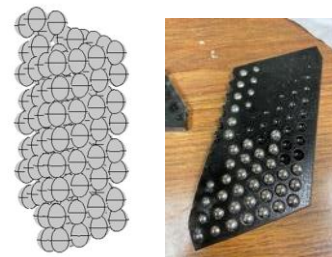
Triangular Lattice Boundaries (Photonic)



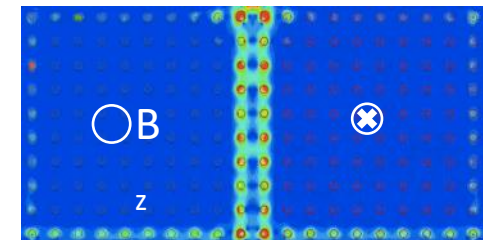
Triangular Lattice Boundaries (Phononic)



Acoustic & EM 3D Screw Discontinuities



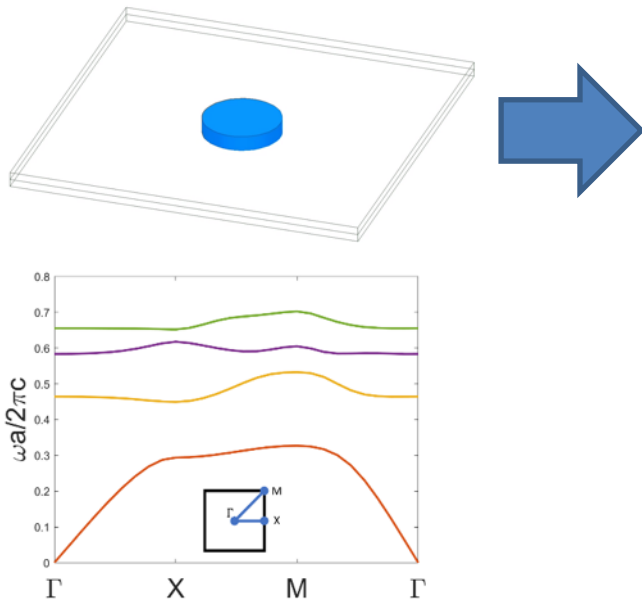
Topological Plasma Structures





# Characterizing Topological Modes Numerically

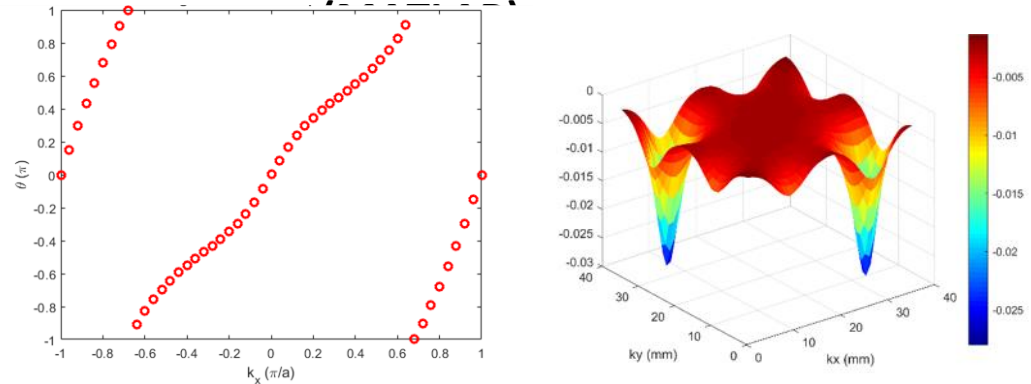
## Unit cell design (HFSS)



## Automation Tools

- Dispersion diagram generators
- Eigenfield extractors
- Lattice constructors

## Topological Analysis



## Analysis Packages

- Mode profile
- Berry curvature
- Chern number
- Spin Chern number
- Valley Chern number
- Wilson loops
- Real space invariants

$$C = \frac{1}{2\pi} \sum_{\text{BZ}} \phi_j = \frac{1}{2\pi} \sum_{\text{BZ}} \text{Im} \left[ \log \prod_i \langle u_{k_i} | u_{k_{i+1}} \rangle \right]$$

$$D_L \equiv \frac{1}{2\pi i} \left[ \sum_{k_\ell \in \partial \mathcal{B}^-} A_1(k_\ell) - \sum_{k_\ell \in \mathcal{B}^-} F_{12}(k_\ell) \right]$$

$$W(k_i) = -\text{Im} \left( \log \prod_{k_j} \langle u_{(k_i, k_j)}(\mathbf{r}) | u_{(k_i, k_{j+1})}(\mathbf{r}) \rangle \right)$$

- We have made all these codes freely available
- <https://github.com/Applied-Electromagnetics-Lab>



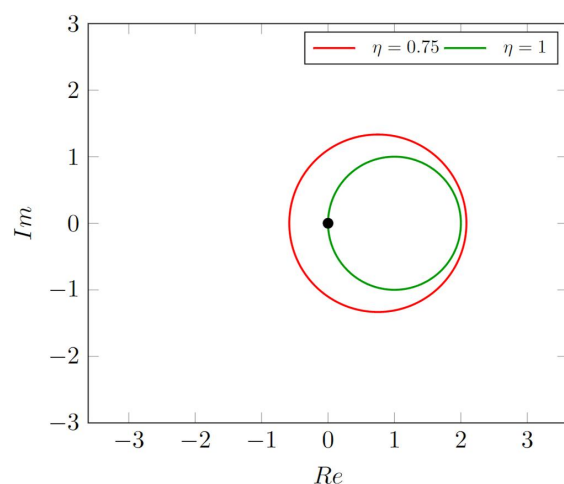
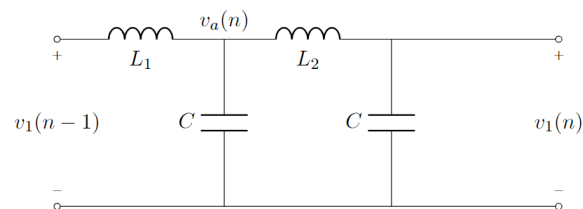
## Current Goals

- Conduct topological electromagnetic material discovery and optimization using random patterns
  - Which geometrical features cause which effects, and why?
- Create circuit models for topological effects to allow improved understanding and integration with circuit components
  - Can we explain these effects with something we already understand?
- Explore the possibility of unidirectional bulk surface waves or unidirectional bulk 3D wave propagation
  - Create structures that control scattering or thermal propagation
- Answer fundamental questions about the rules that govern various classes of topological materials
  - Which geometrical structures have which properties, and are they consistent?



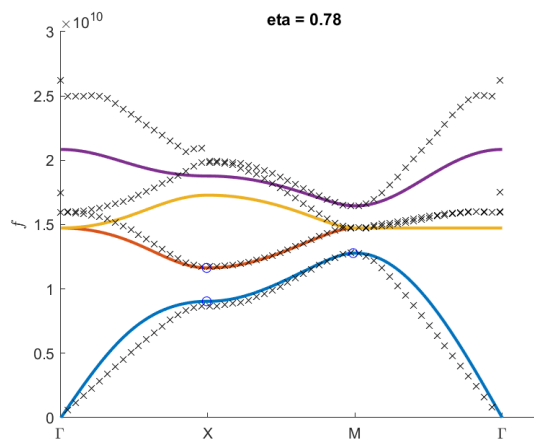
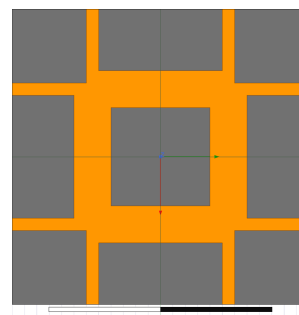
# Circuit Models of Topological EM Structures

## 1D Structures



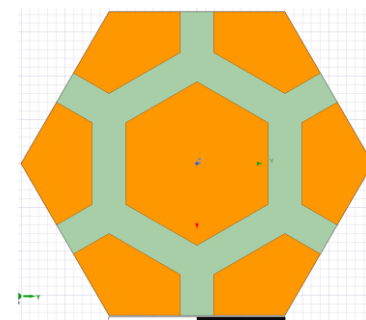
- Emergent Zak phase from circuit model

## 2D Structures | Square Lattice

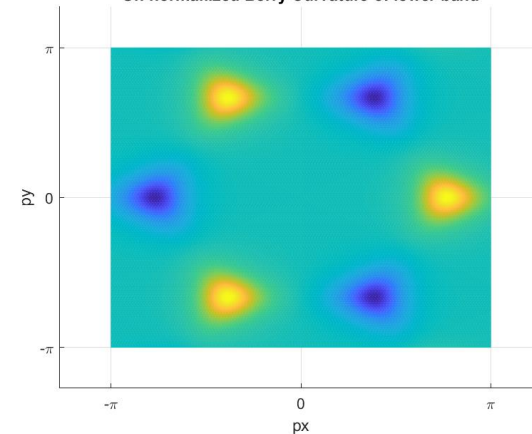


- Stepping stone for Hexagonal lattices

## 2D Structures | Hexagonal Lattice



Un-normalized Berry Curvature of lower band

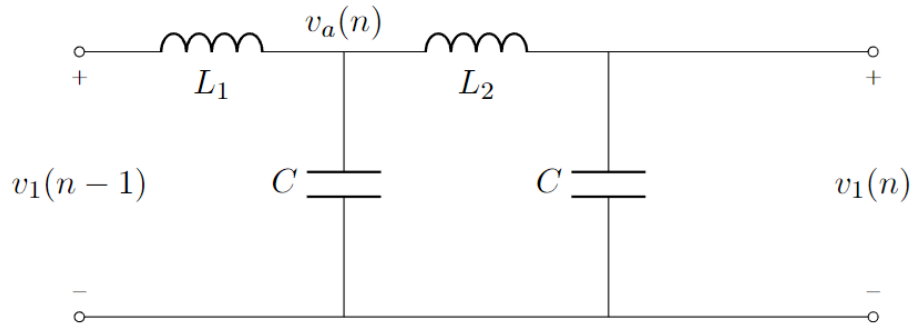


- Emergent winding/Chern numbers





# TxLine Modelling of PTIs | 1D

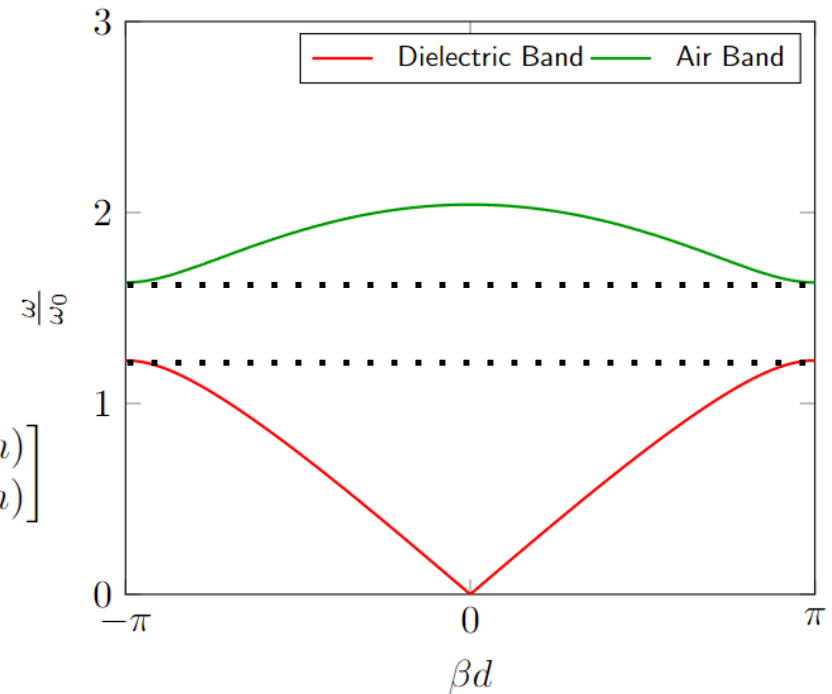


$$j\omega C \begin{bmatrix} v_a(n) \\ v_1(n) \end{bmatrix} = \frac{1}{j\omega} \begin{bmatrix} \left(\frac{1}{L_1} + \frac{1}{L_2}\right) & \left(\frac{e^{j\beta d}}{L_1} + \frac{1}{L_2}\right) \\ \left(\frac{1}{L_2} + \frac{e^{-j\beta d}}{L_1}\right) & \left(\frac{1}{L_1} + \frac{1}{L_2}\right) \end{bmatrix} \begin{bmatrix} v_a(n) \\ v_1(n) \end{bmatrix}$$

$$\frac{v}{Z} = \mathbf{Y}_p v$$

$$\eta = \sqrt{\frac{L_2}{L_1}}$$

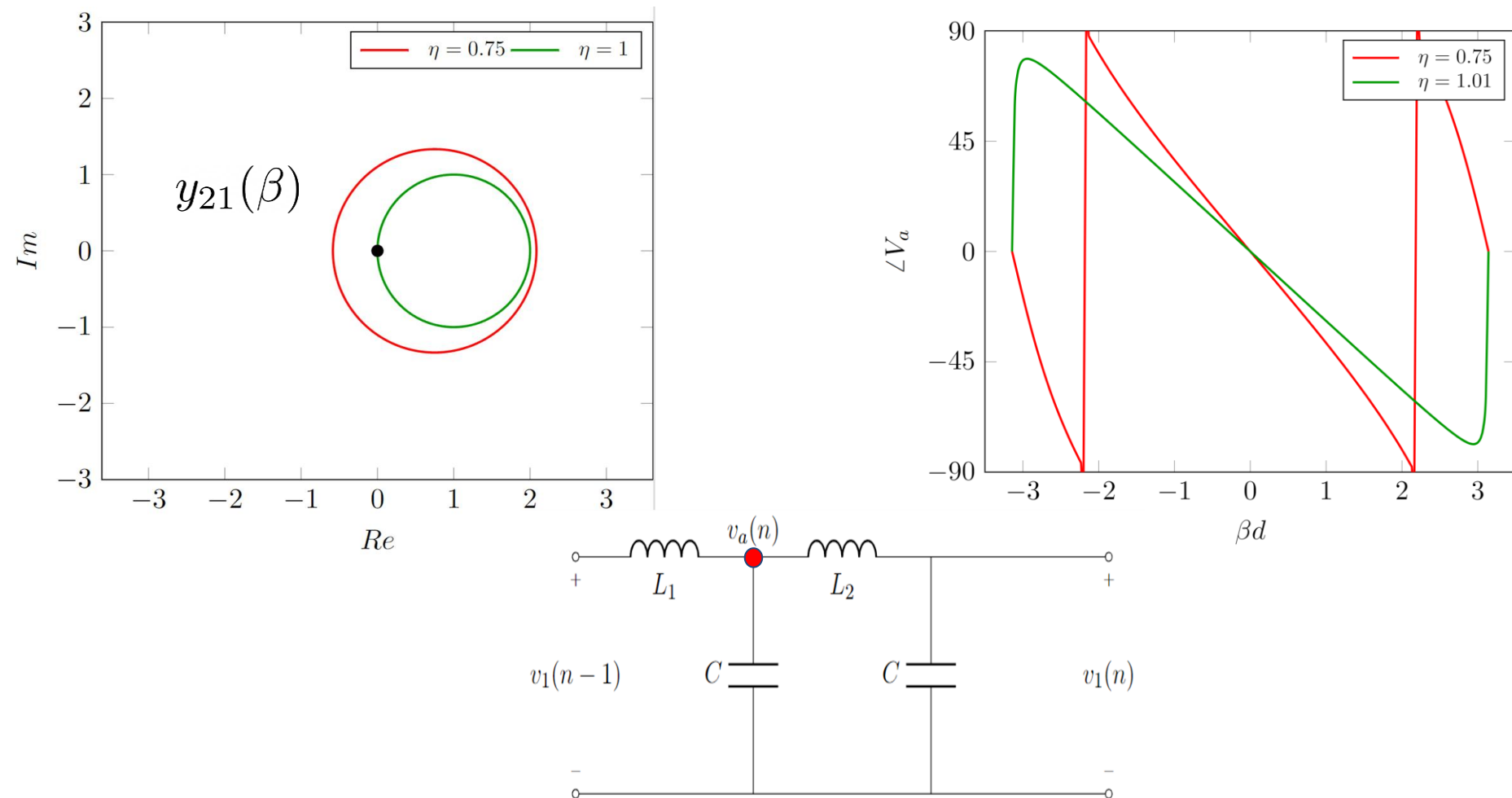
Band Diagram for  $\eta$  of 0.75



- Periodic circuit analysis can be easily carried out by writing KCL and enforcing periodic boundary conditions. Similar to Blochwave analysis with Y parameter matrices
- Alternating the inductors (or capacitors) opens up a band gap which is topologically non-trivial for  $\eta < 1$



# Circuit Models for Photonic Topological Insulators

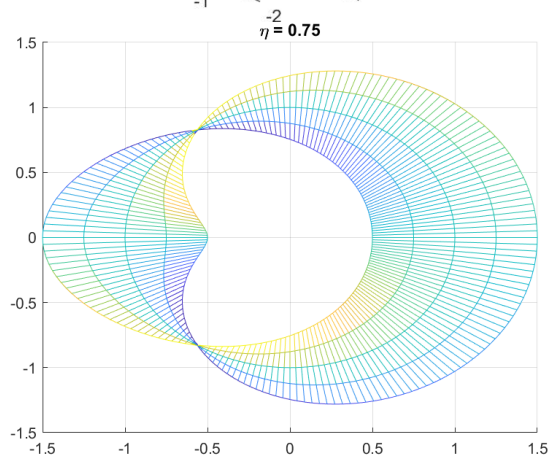
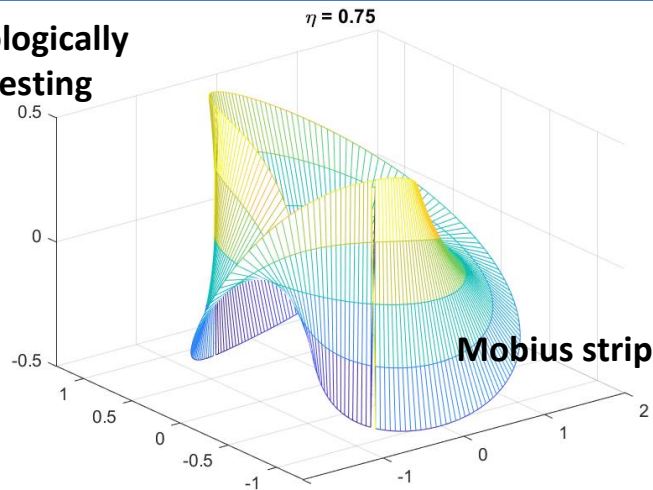


- $y_{21}$  encircling the origin is indicative of a pole (or zero) and is the Zak phase
- The phase of the voltage at the internal node has abrupt phase discontinuities which gives rise to topological protection

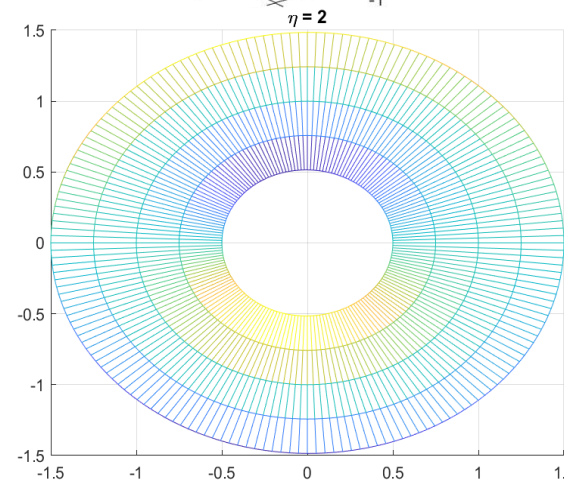
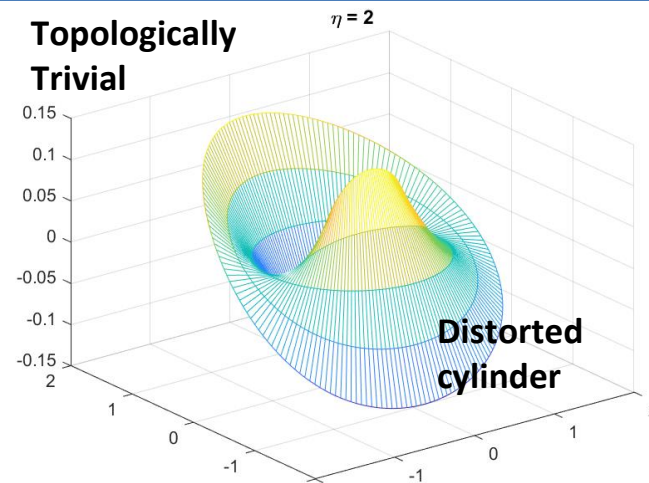


# Circuit Models for Photonic Topological Insulators

**Topologically  
Interesting**



**Topologically  
Trivial**



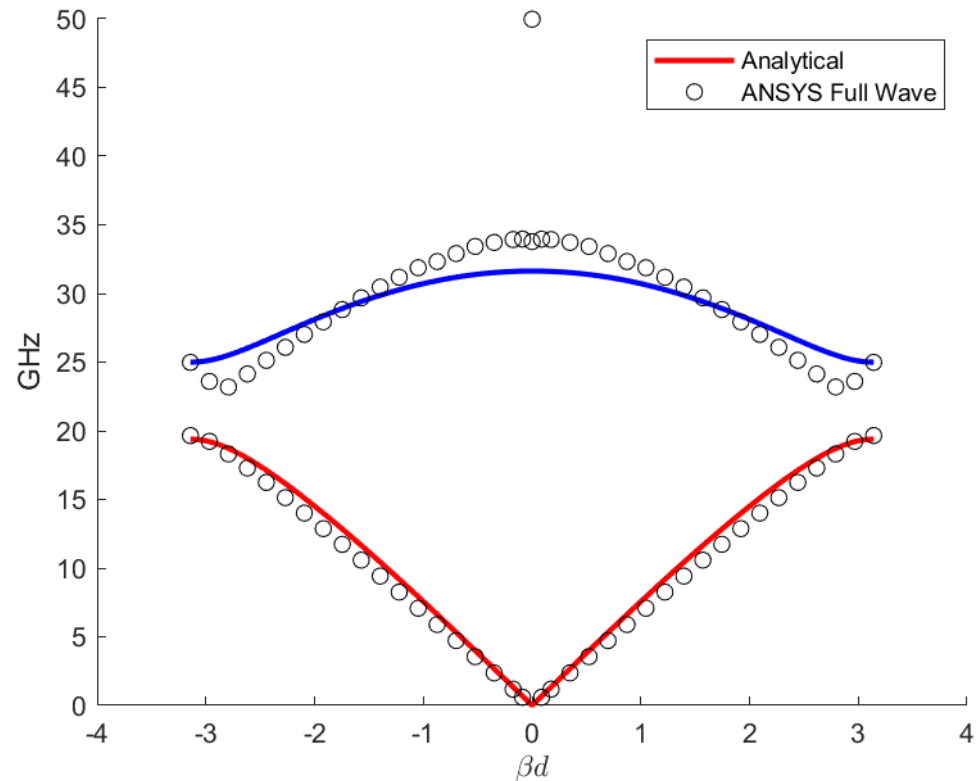
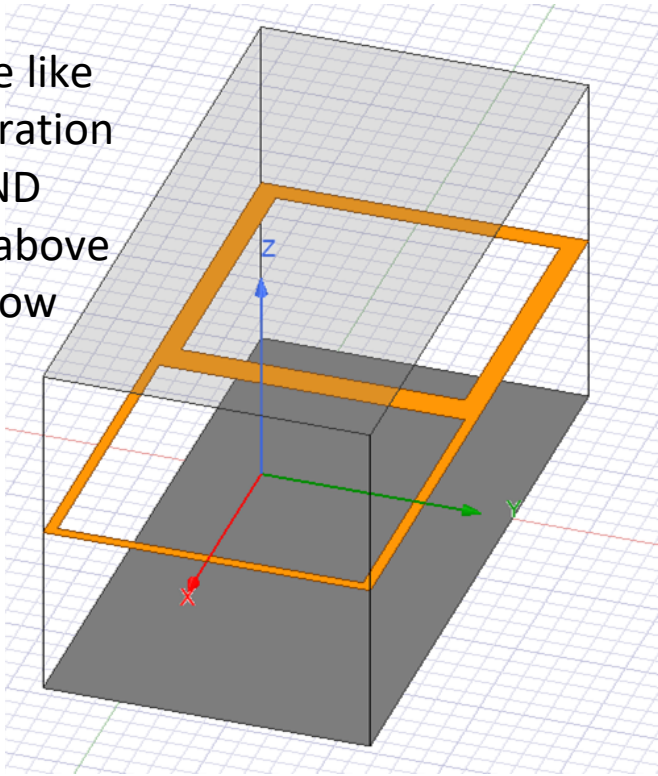
- Figure formed by plotting the phase of the internal node (a 3D vector in the xz and yz plane) over the full Brillouin Zone (a circle in the xy plane)
- Topologically nontrivial structures form a Mobius strip, while trivial structures form a cylindrical loop





# Simple EM Structure Implementation

Stripline like  
configuration  
with GND  
planes above  
and below

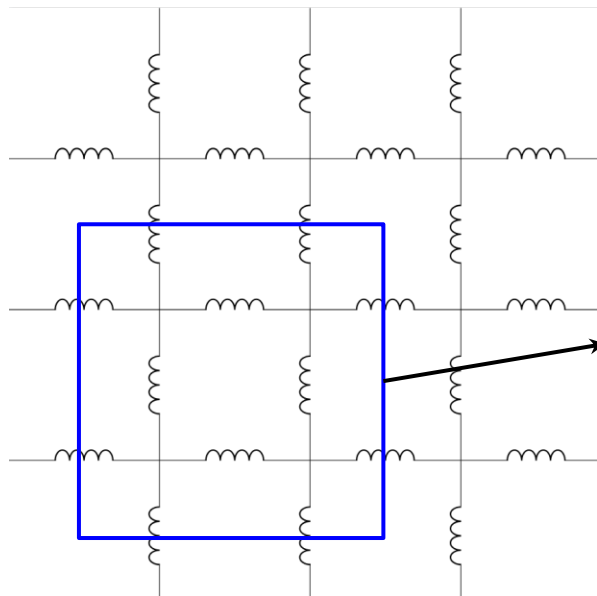


w1 (Ring 1 Thickness)	0.1mm
w2 (Ring 2 Thickness)	0.2mm
d (Cell Periodicity)	3mm

**Circuit Model is in good agreement with fullwave simulation conducted using Ansys HFSS**

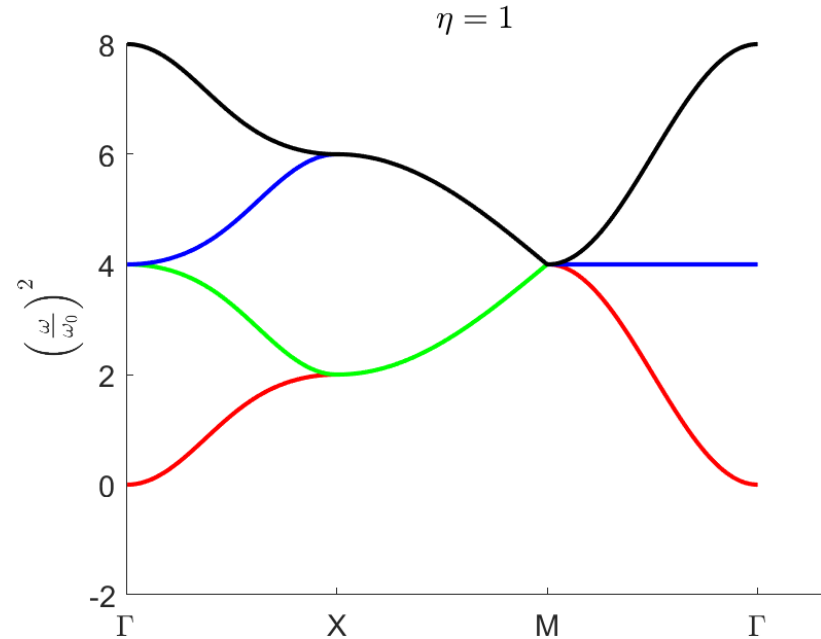


# 2D Square Lattice and Band Structure



Unitcell  
highlighted

Capacitors to  
ground not  
shown



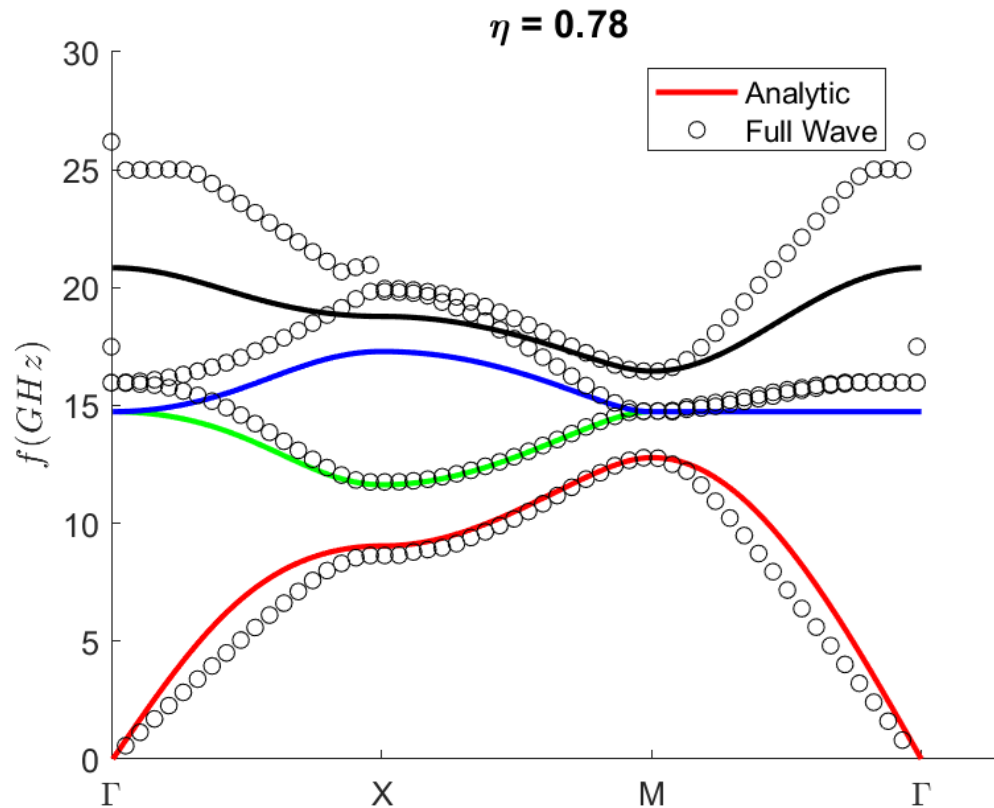
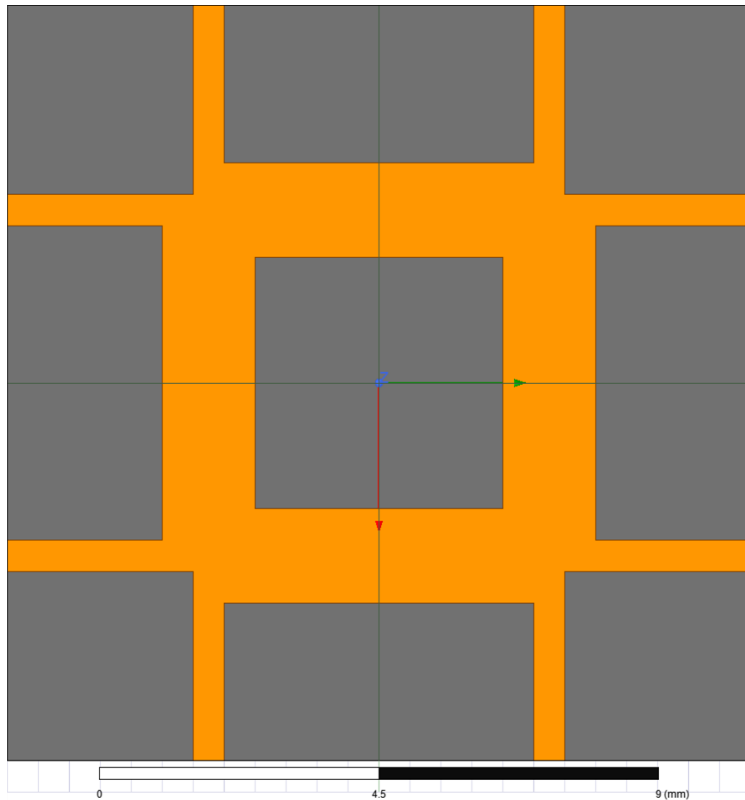
$$\begin{bmatrix} 2\left(\eta + \frac{1}{\eta}\right) & -\left(\eta e^{jk_y a_y} + \frac{1}{\eta}\right) & 0 & -\left(\eta e^{jk_x a_x} + \frac{1}{\eta}\right) \\ -\left(\eta e^{-jk_y a_y} + \frac{1}{\eta}\right) & 2\left(\eta + \frac{1}{\eta}\right) & -\left(\eta e^{-jk_x a_x} + \frac{1}{\eta}\right) & 0 \\ 0 & -\left(\eta e^{-jk_x a_x} + \frac{1}{\eta}\right) & 2\left(\eta + \frac{1}{\eta}\right) & -\left(\eta e^{-jk_y a_y} + \frac{1}{\eta}\right) \\ -\left(\eta e^{jk_x a_x} + \frac{1}{\eta}\right) & 0 & -\left(\eta e^{jk_y a_y} + \frac{1}{\eta}\right) & 2\left(\eta + \frac{1}{\eta}\right) \end{bmatrix} \begin{bmatrix} v_a \\ v_b \\ v_c \\ v_d \end{bmatrix} = \left(\frac{\omega}{\omega_0}\right)^2 \begin{bmatrix} v_a \\ v_b \\ v_c \\ v_d \end{bmatrix}$$

$$\eta = \sqrt{\frac{L_u}{L_c}} \quad \omega_0^2 = \frac{1}{C\sqrt{L_u L_c}}$$

- Similar to the 1D case, simple LC circuits can be written for 2D square lattices which accurately describes the dispersion relation.

## Comparing 2D Square Lattice Analytical Approximation with Full EM Simulation

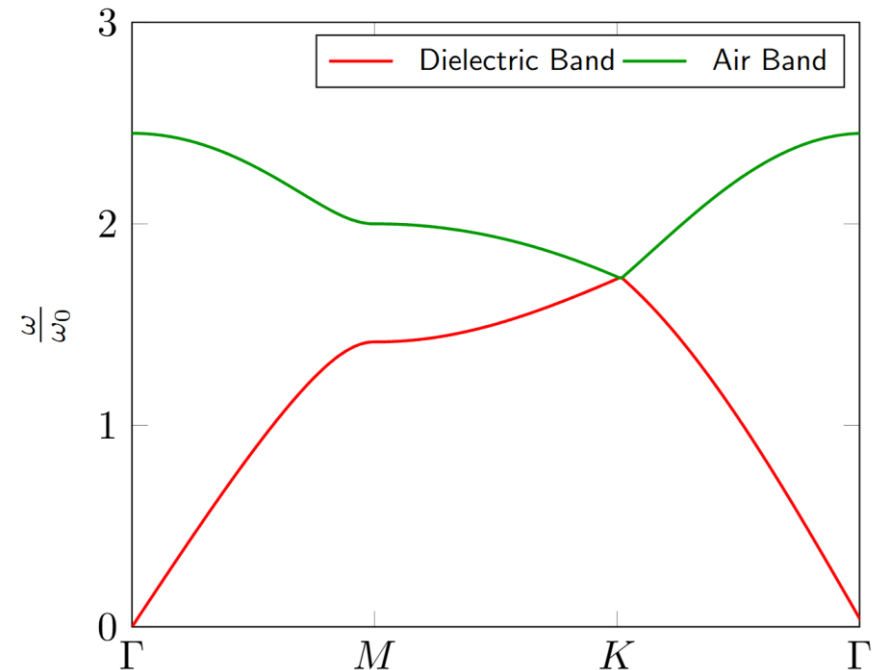
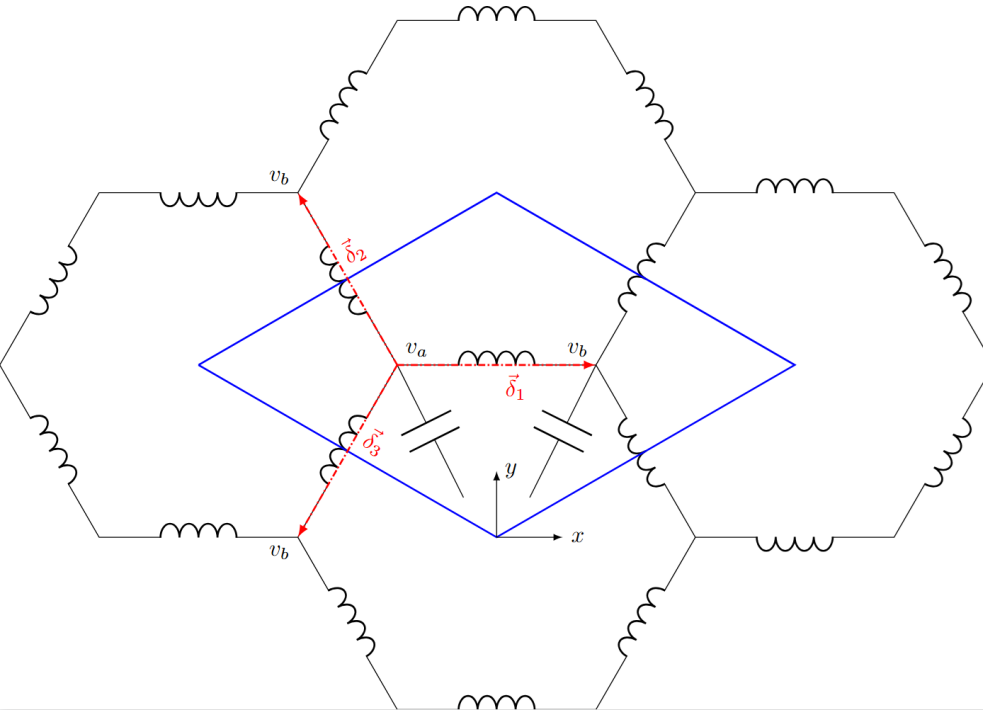
Microstrip like configuration with GND plane below



- Circuit Model is in good agreement with full wave simulation using Ansys HFSS for lower bands



# Simple 2D Hexagonal Lattice



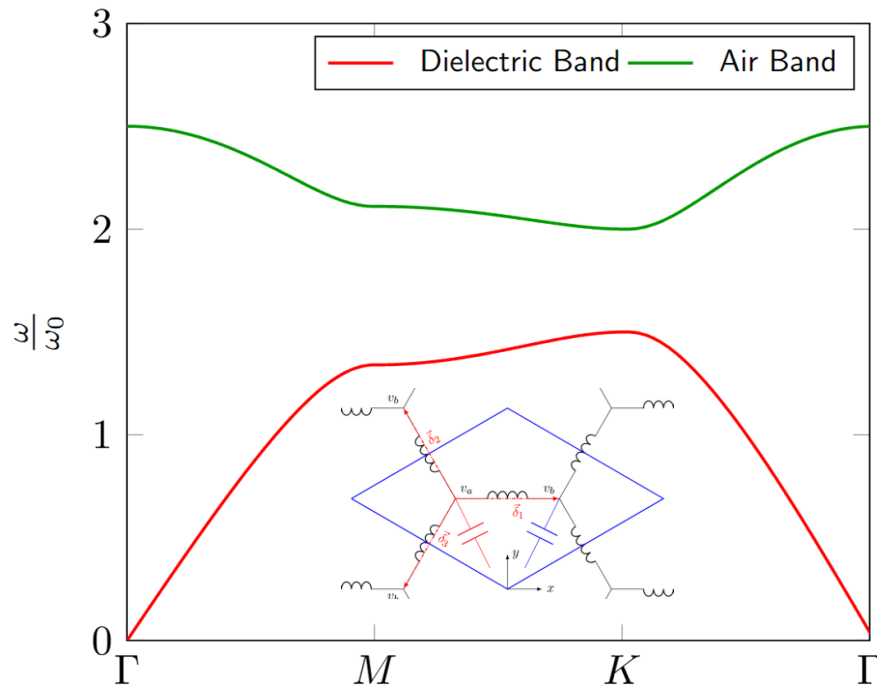
$$\begin{bmatrix} 3 & -\sum_i e^{-j\vec{k} \cdot \vec{\delta}_i} \\ -\sum_i e^{+j\vec{k} \cdot \vec{\delta}_i} & 3 \end{bmatrix} \begin{bmatrix} v_a \\ v_b \end{bmatrix} = \left( \frac{\omega}{\omega_0} \right)^2 \begin{bmatrix} v_a \\ v_b \end{bmatrix} \quad \omega_0 = \frac{1}{\sqrt{LC}}$$

- Simple 2D LC Circuits predict Dirac Cones at K(K') points which can be broken through varying C values
- Circuit model provides a path to engineer the dispersion relation near the K(K') points with different topological properties

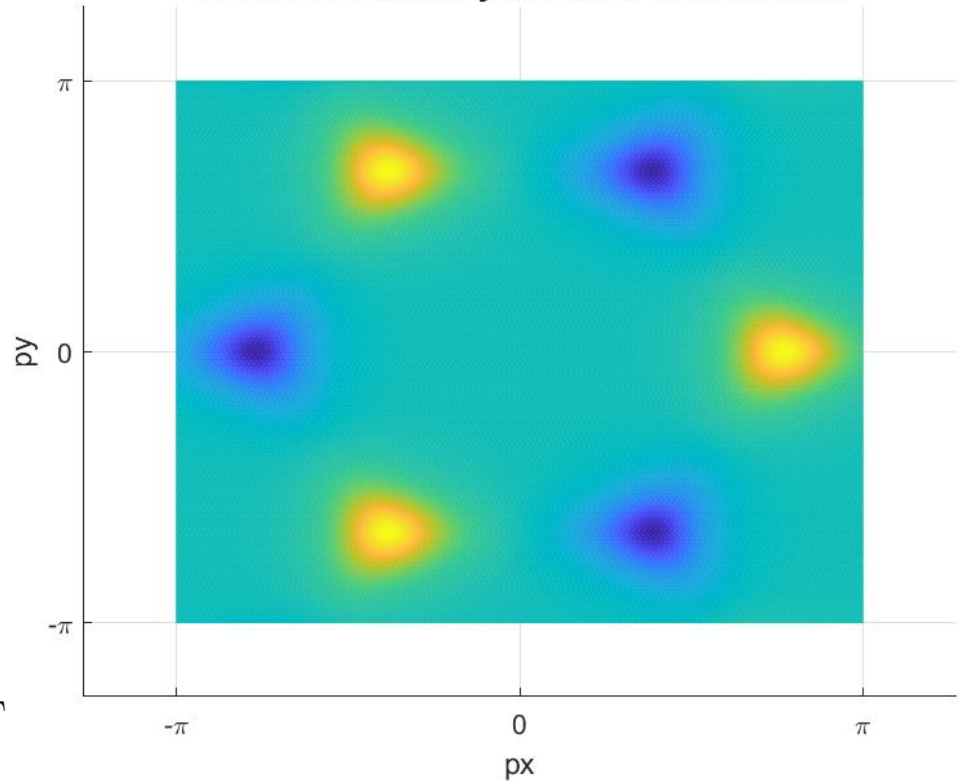


## 2D Hexagonal Lattice Band Structure and Berry Curvature Calculated from Circuit Model

Band Diagram for  $\eta$  of 0.75



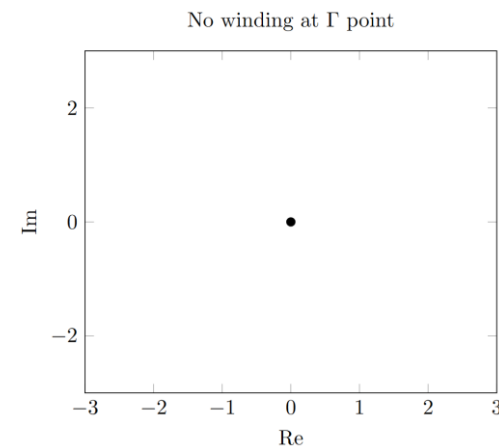
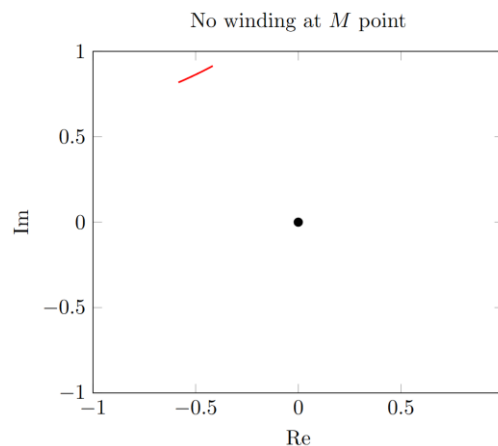
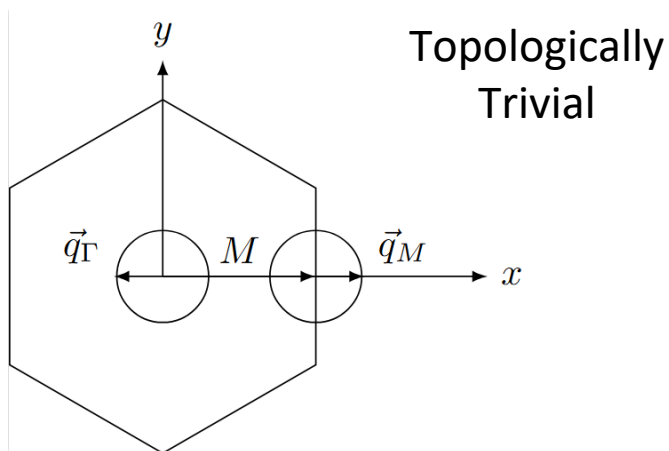
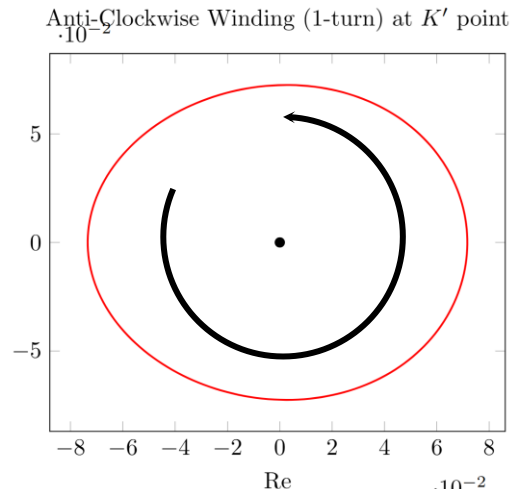
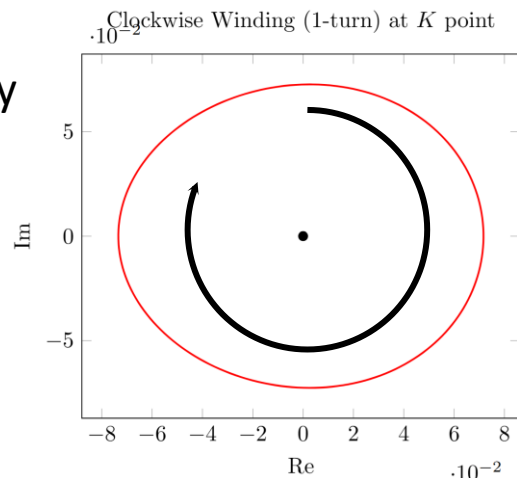
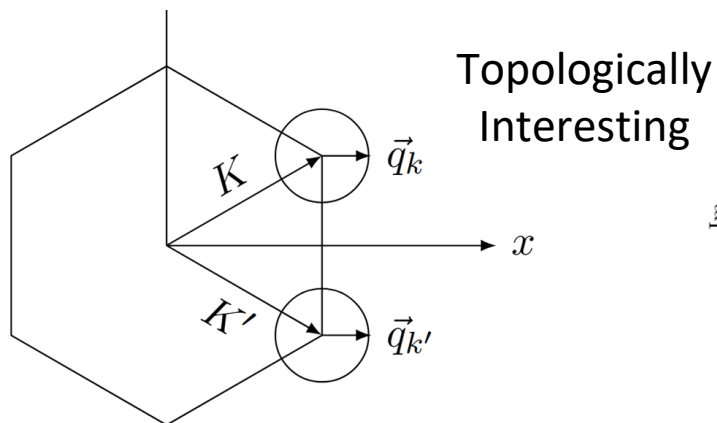
Un-normalized Berry Curvature of lower band



$$\begin{bmatrix} 3\eta & -\eta \sum_i e^{-j\vec{k} \cdot \vec{\delta}_i} \\ -\frac{1}{\eta} \sum_i e^{-j\vec{k} \cdot \vec{\delta}_i} & \frac{3}{\eta} \end{bmatrix} \begin{bmatrix} v_a \\ v_b \end{bmatrix} = \left( \frac{\omega}{\omega_0} \right)^2 \begin{bmatrix} v_a \\ v_b \end{bmatrix} \quad \eta = \sqrt{\frac{C_2}{C_1}} \text{ and } \omega_0^2 = \frac{1}{L\sqrt{C_1 C_2}}$$

- Circuit model predicts alternating Berry Curvature at K and K' points indicating topological protection at these High Symmetry Points.
- Width of curvature depends on band gap. If curvature at K and K' touch, they cancel each other

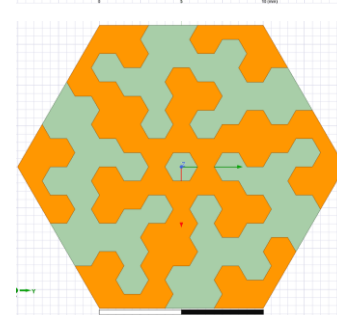
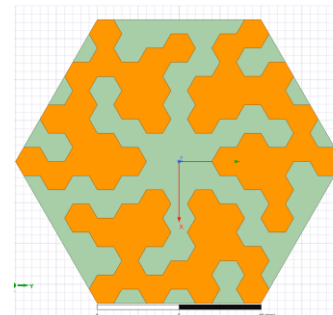
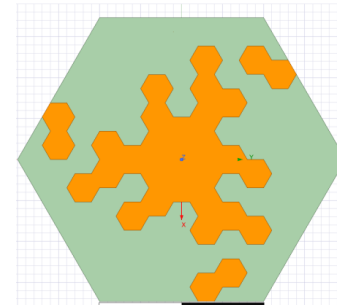
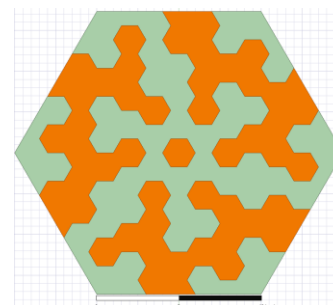
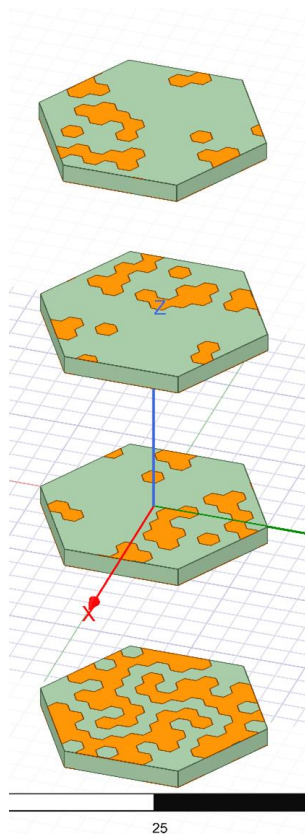
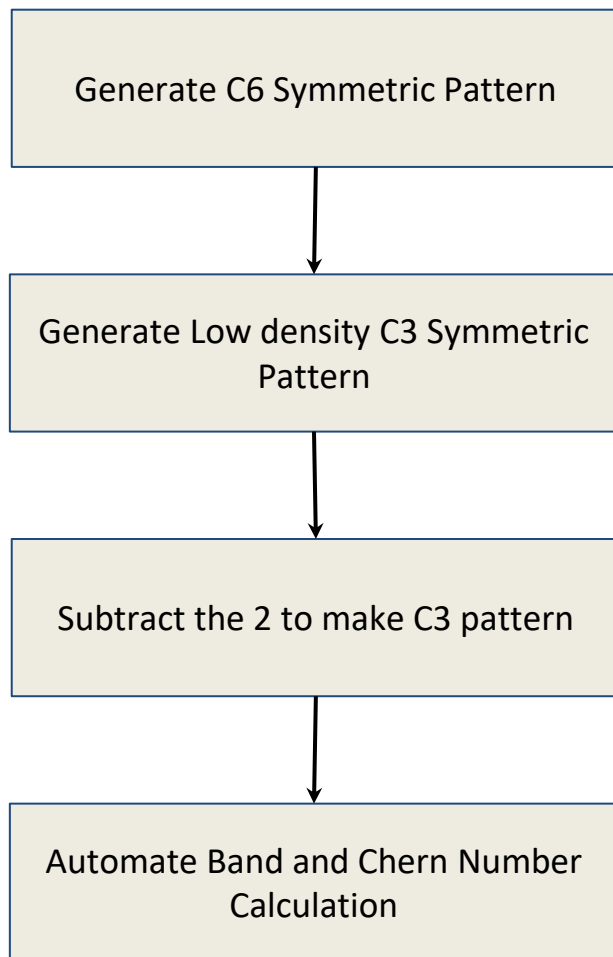
## 2D Hexagonal Lattice Analysis at K and K' Points



- In the vicinity of the K and K' points,  $y_{21}$  traces out a circle which encloses the origin indicating the presence of a pole (or zero). The phase acquired is  $\pm\pi \Rightarrow$  Valley Chern Number =  $\pm 0.5$
- Near the  $\Gamma$  and M point, there is no such winding



# Constructing Random Patterns of Hexagons with Desired Symmetry Properties

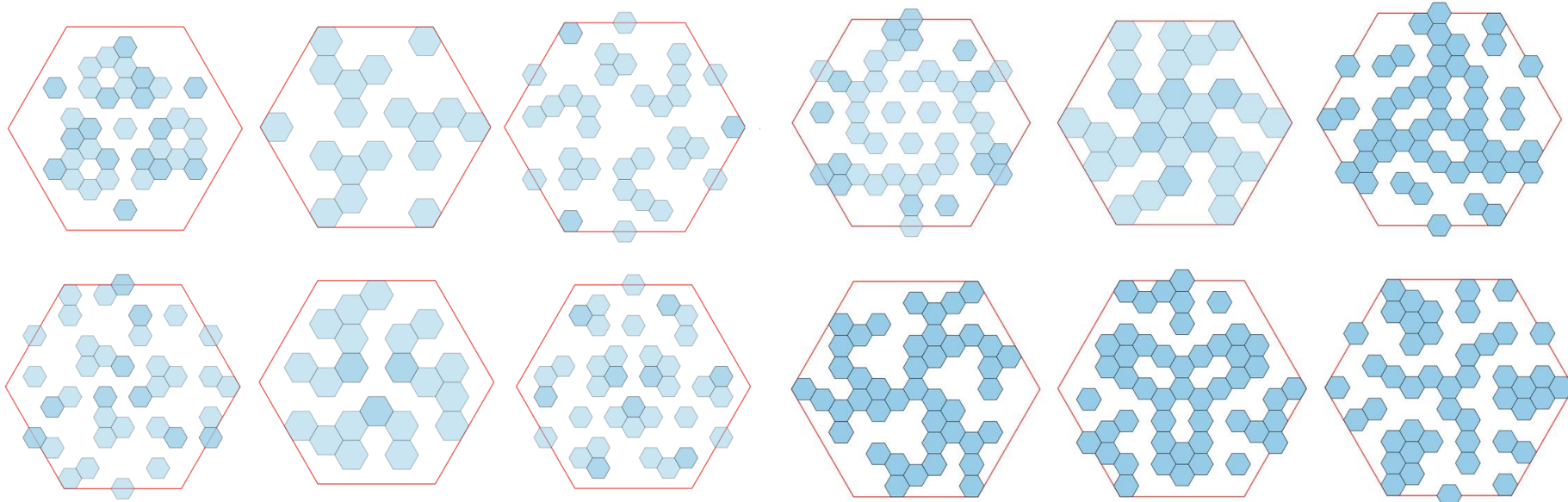




# Discovering Trivial vs. Nontrivial Structures Through Random Patterns

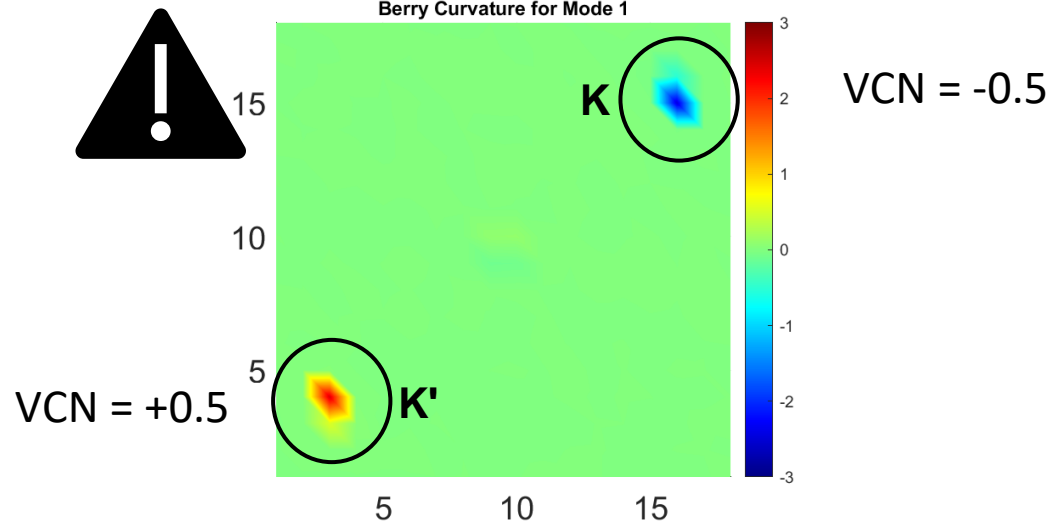
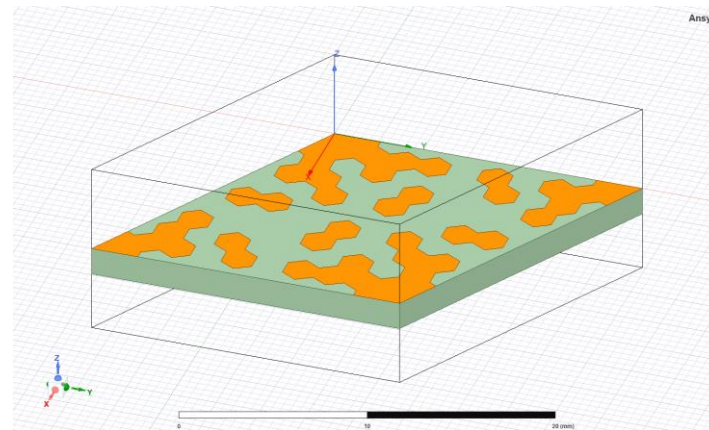
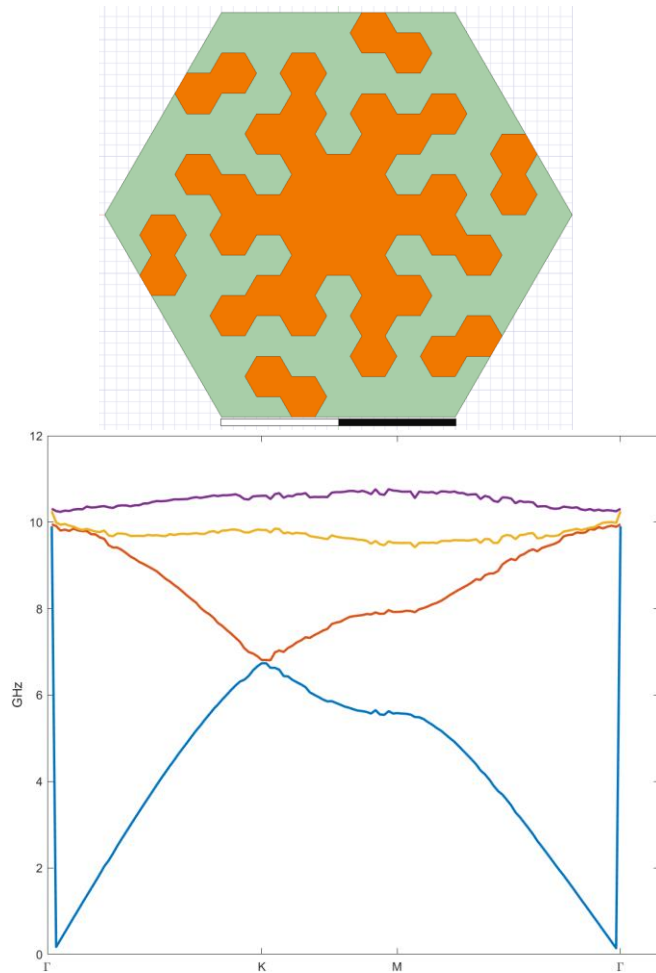
Trivial

Nontrivial



- **Most research today is based on certain geometries with no clear reason for why they were chosen**
  - Someone else studied this one in the past?
  - A grad student somehow happened to find a good one?
- **Automating simulation of random patterns will allow us to develop an understanding of which geometrical features lead to which properties, e.g. ...**
  - Wide band gap
  - Nontrivial topological behavior

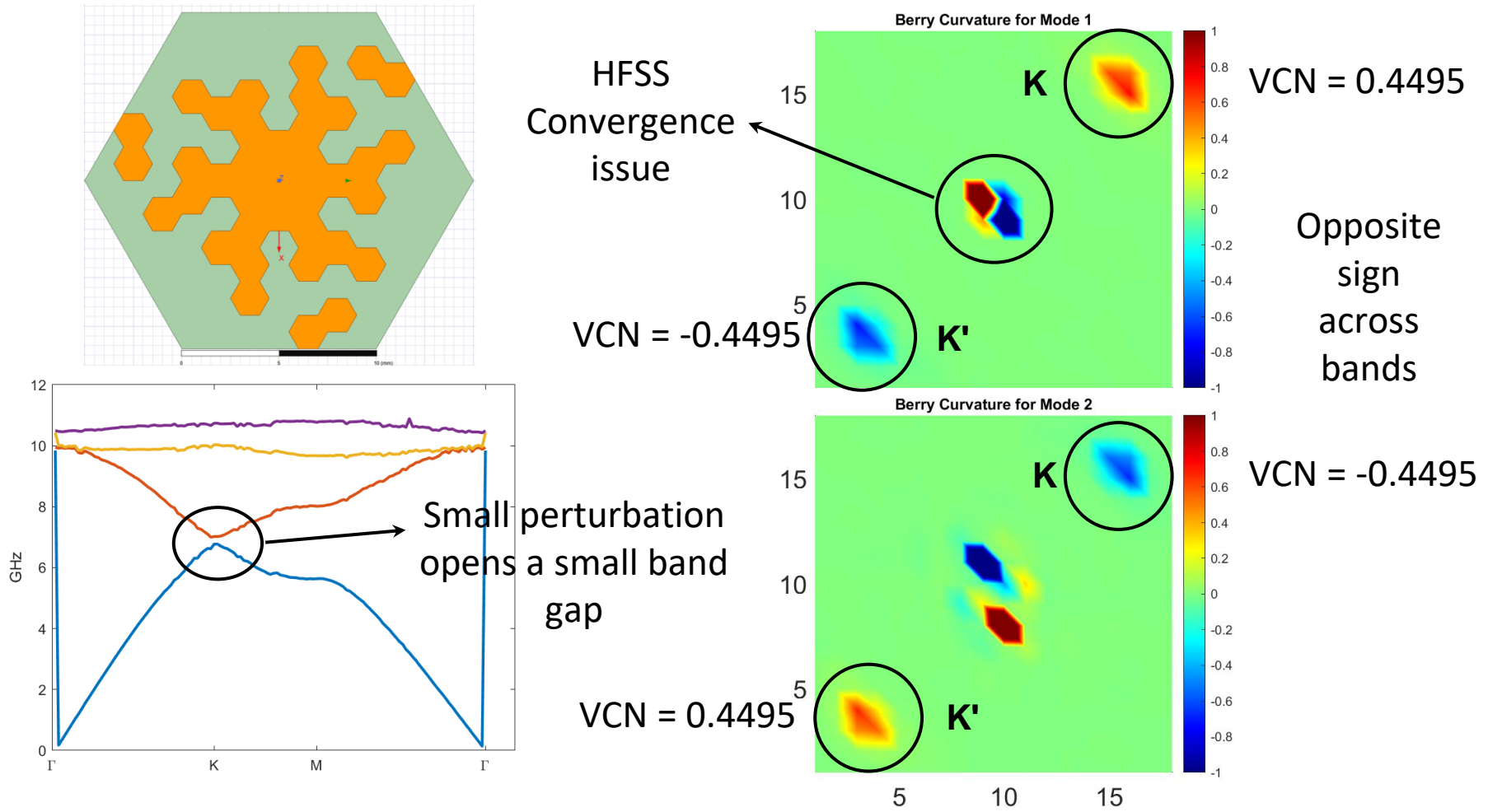
# Random Patterns on Hexagonal Lattice and Some Challenges of Identifying Nontrivial Structures



Such a C6 symmetric structure might show curvature at the K and K' points for a single band, but in reality as 2 bands become degenerate at the same point, you have to sum curvatures from both bands which will add to 0

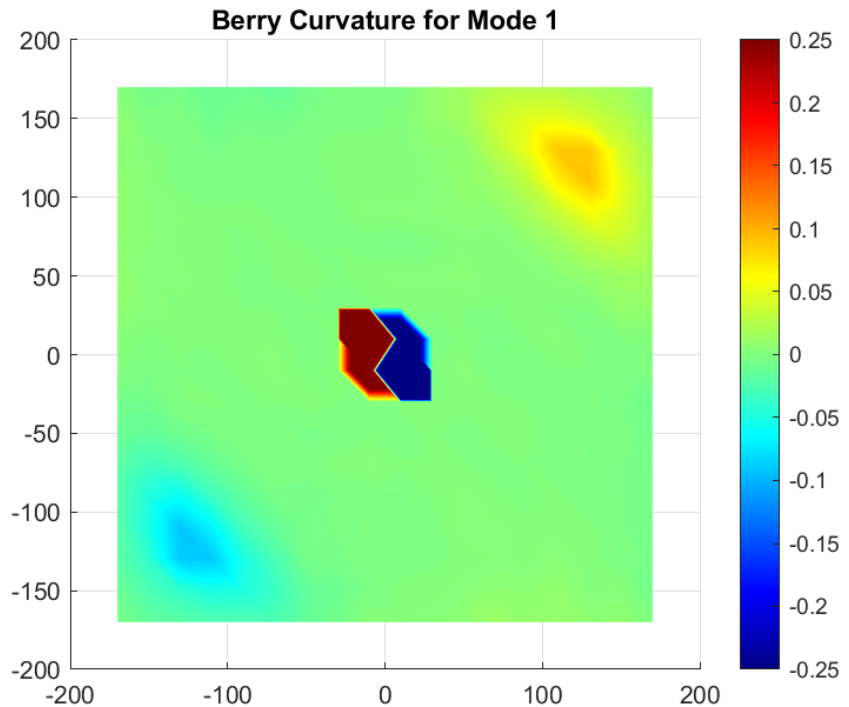
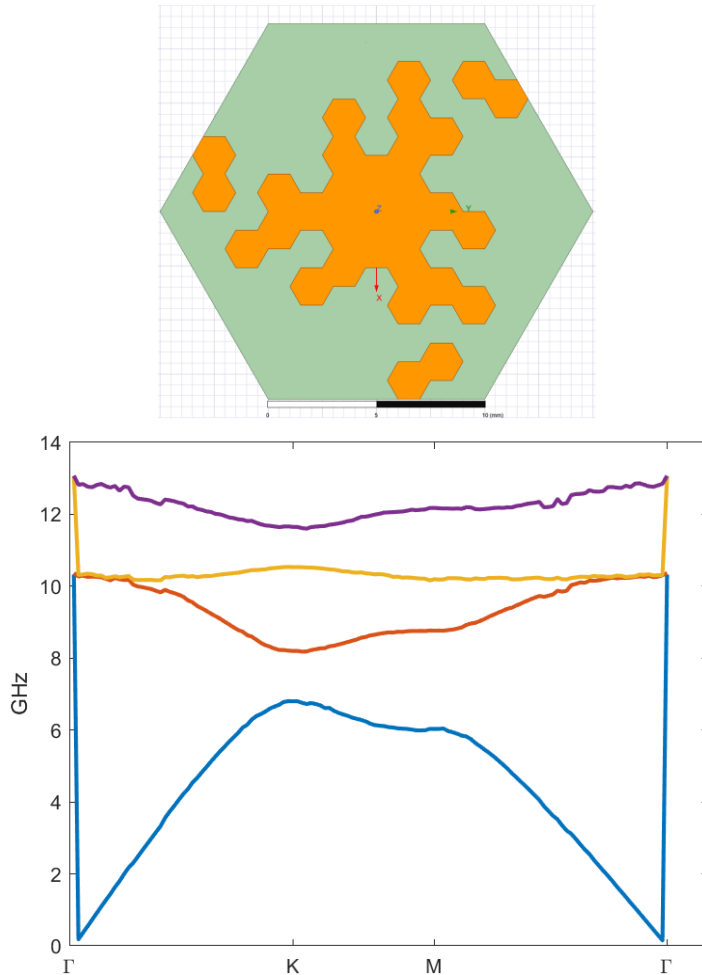


## Random Patterns on Hexagonal Lattice: C3 Structures with Gaps



- Generated Random Pattern is topologically interesting and can form a Valley type PTI
- A very narrow bandgap makes it difficult to excite only the edge mode. To increase the width of the band gap, degree of perturbation is increased

## Random Patterns on Hexagonal Lattice: Wider Band Gaps with Greater Perturbation

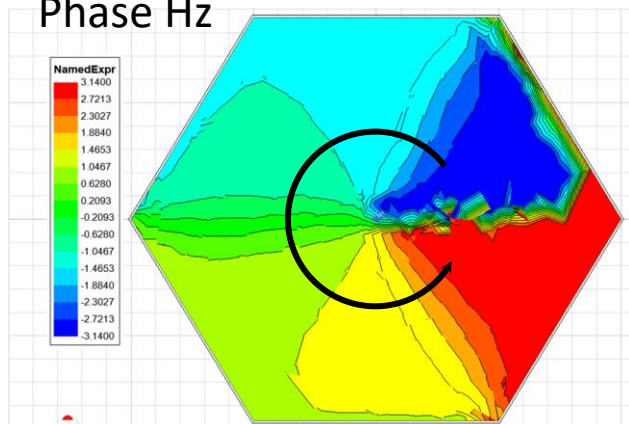


- As strength of perturbation is increased, the band gap increases as predicted by the model
- As the degree of perturbation is increased, the region of Berry curvature widens
- Can be used to construct a Valley type PTI with a unidirectional edge mode

## Unit Cell Phase Profile at K(K') point

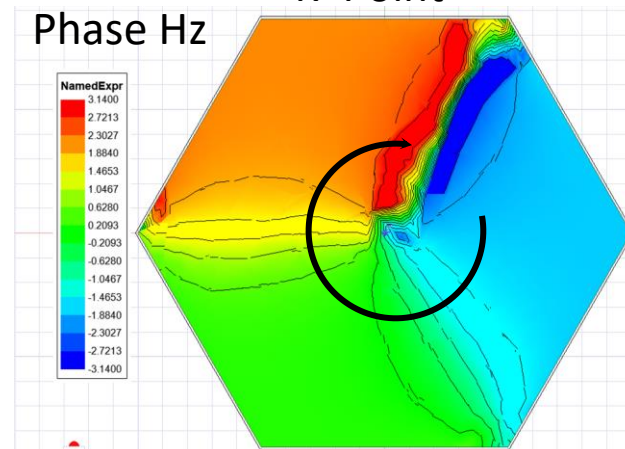
K Point

Phase Hz

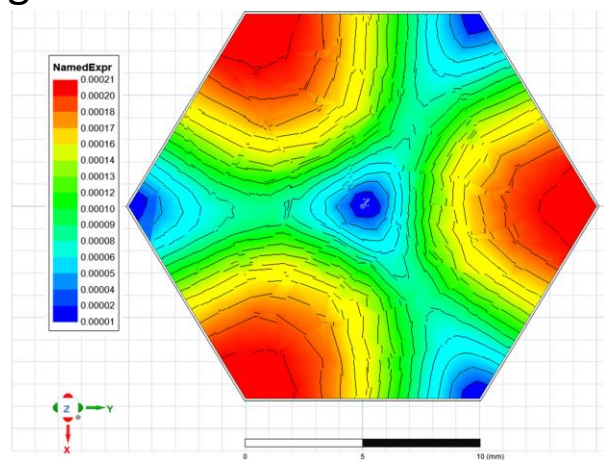


K' Point

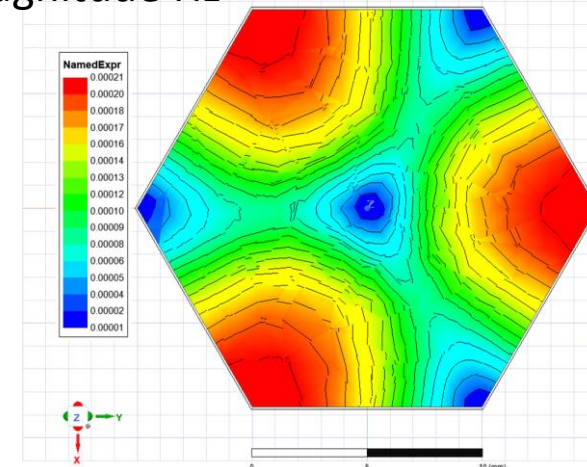
Phase Hz



Magnitude Hz



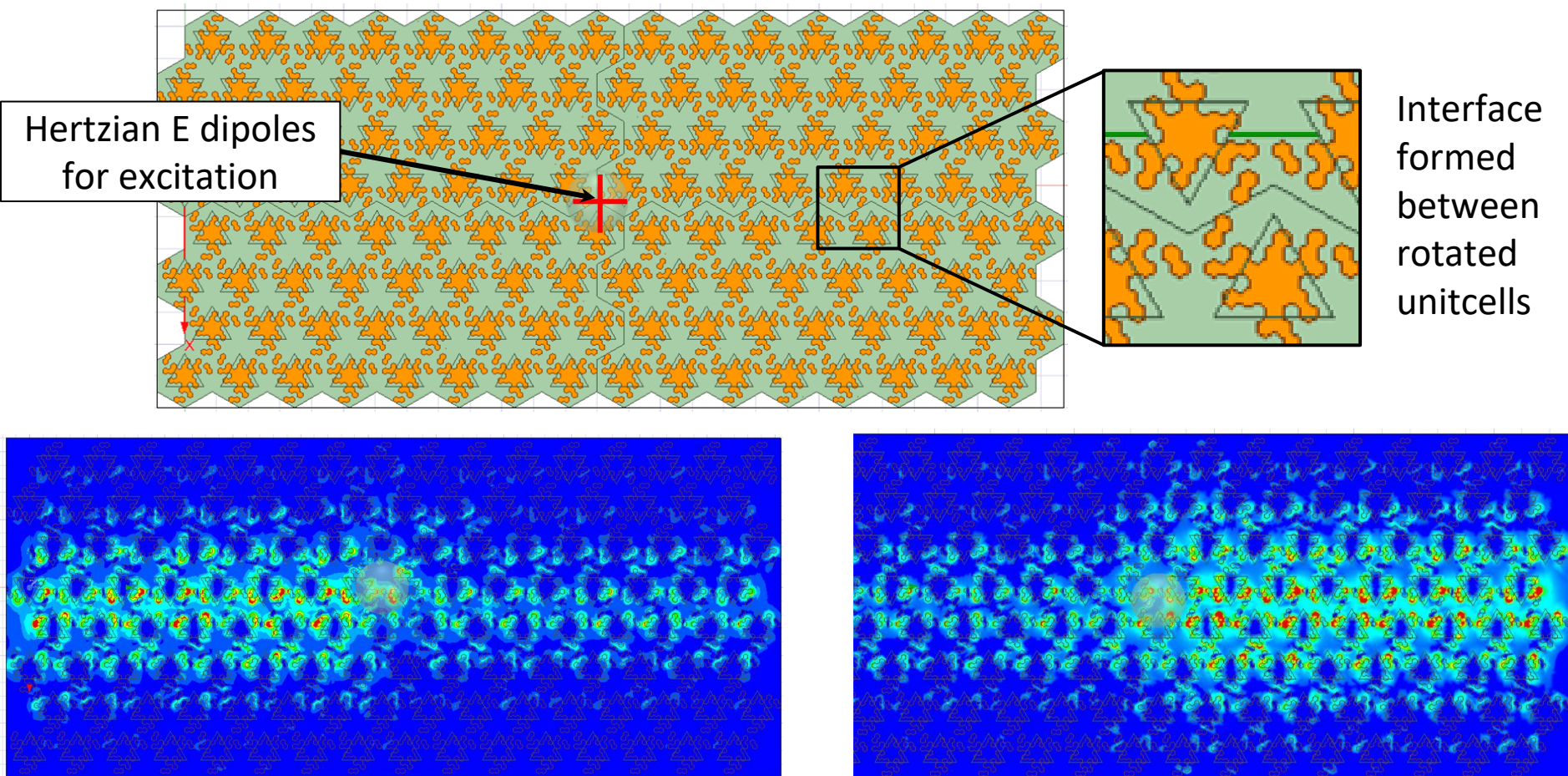
Magnitude Hz



- Counter rotating phase profiles on the unitcell at K and K' points as predicted
- The fields at K and K' points are indistinguishable



## Unidirectional Edge Mode in a Lattice of Random C3 Cells



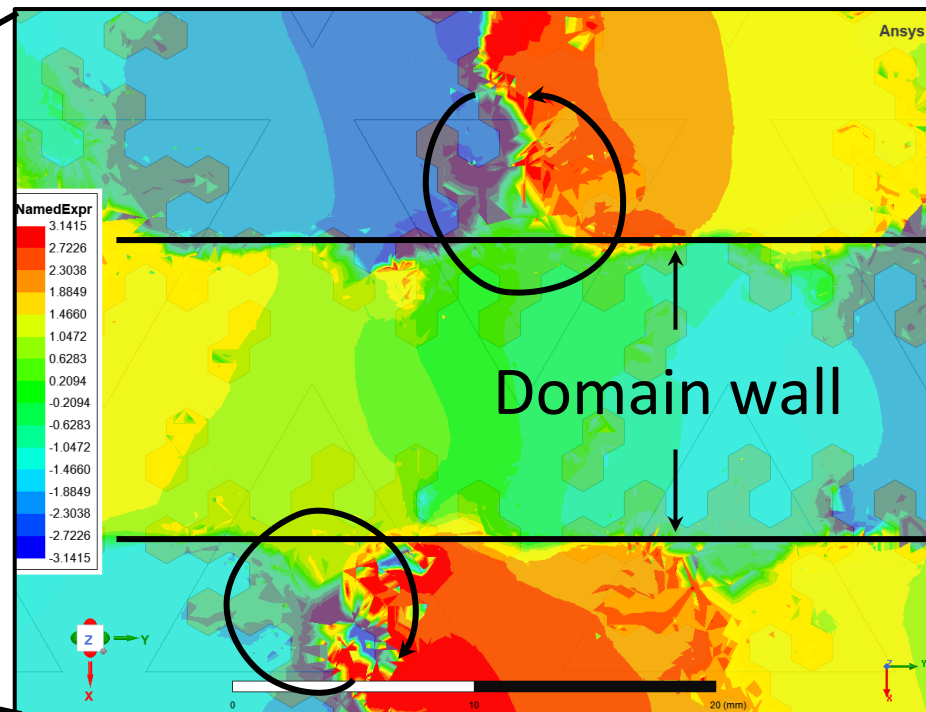
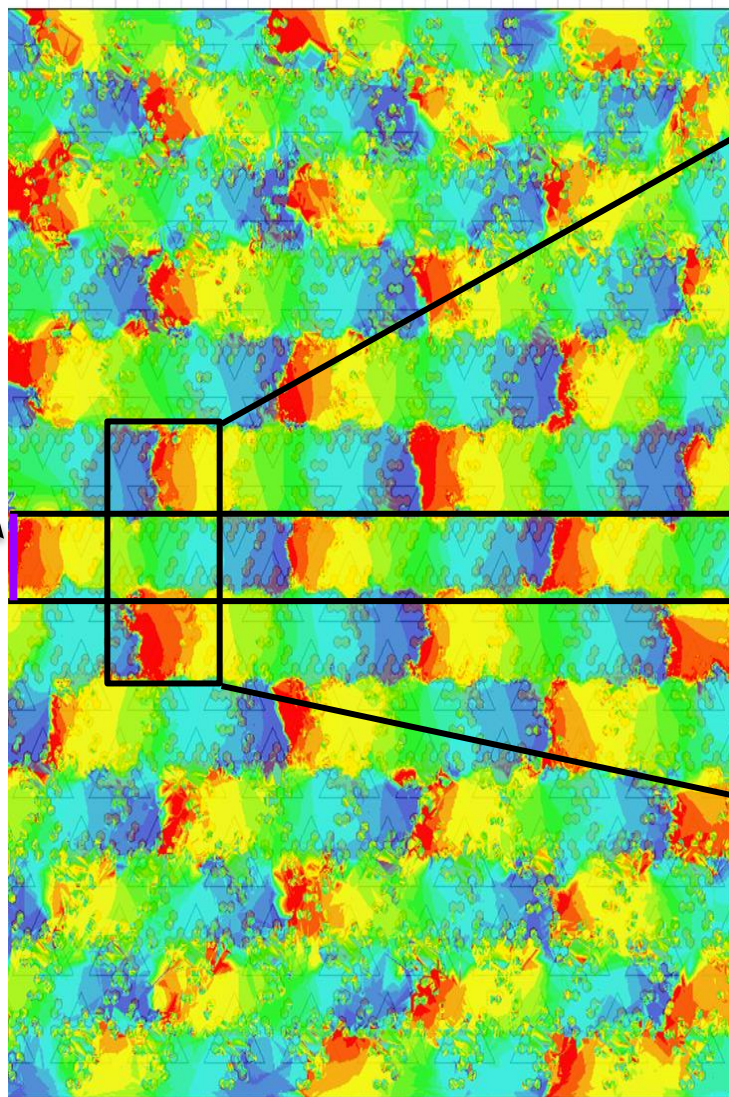
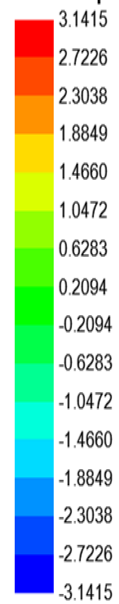
- Unidirectional edge mode is excited using 2 inplane orthogonal electric dipoles
- LHCP and RHCP excitation selectively couples into forward and backward modes



# Unidirectional Edge Mode: Phase of transverse field

Waveport

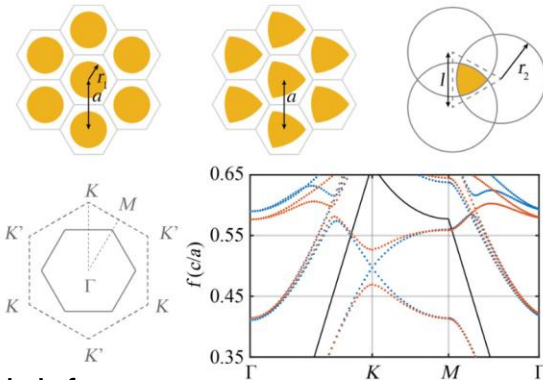
NamedExpr



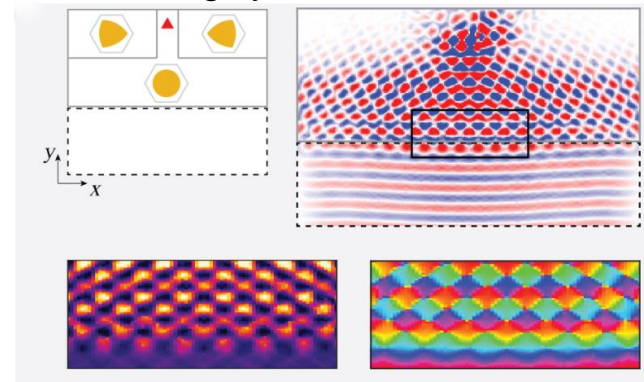
Counter rotating phase profiles across the domain wall which is responsible for unidirectional propagation

# Recent Work in Topology and Metasurfaces with Applications in Novel Energy Transport Mechanisms

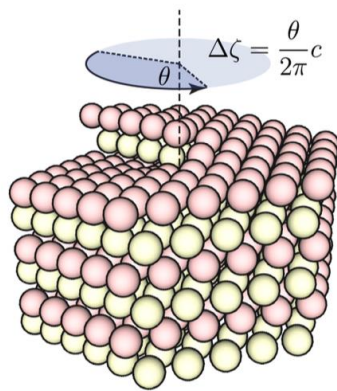
Band structure of grounded metal patch arrays showing Dirac point



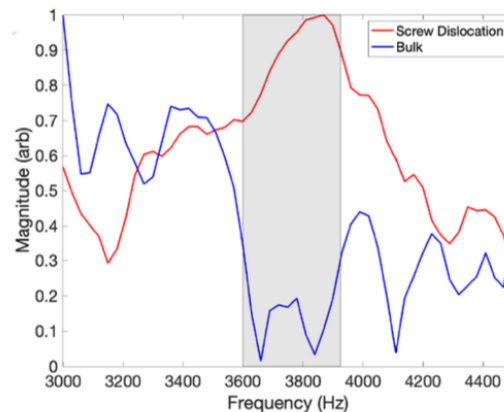
Pseudodiffusive transport with broad, highly coherent wavefront



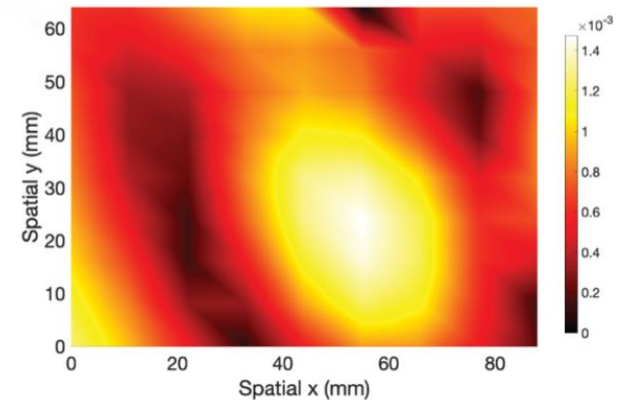
Helical defect (screw discontinuity) in 3D phononic crystal



Measured transport within bulk band gap



Measured hot spot at screw discontinuity

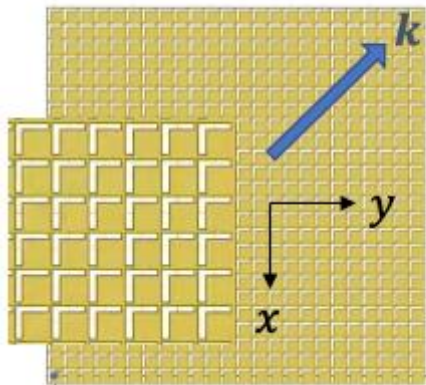


- Unique transport properties such as coherent phase locking in a broad beam
- Unidirectional propagation in 3D structures – bulk unidirectional waves?

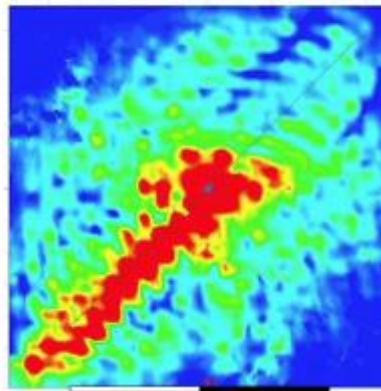


# Chiral Materials Effect on Scattering and Electrically Controlled Impedance Surfaces

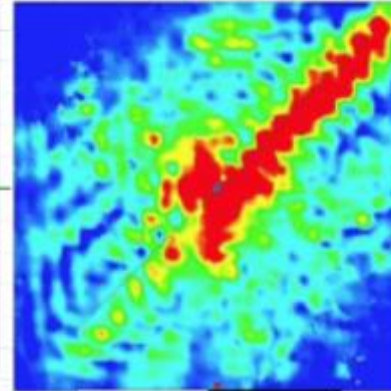
Chiral Metasurface



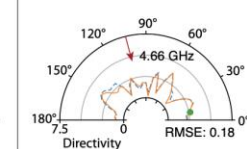
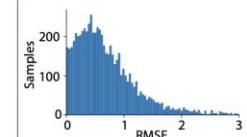
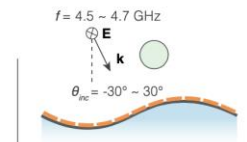
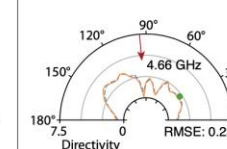
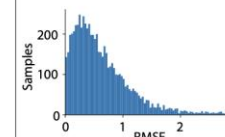
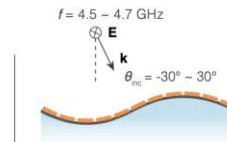
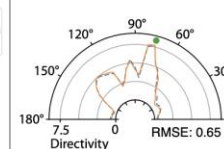
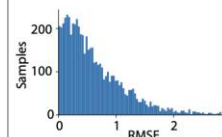
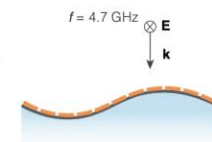
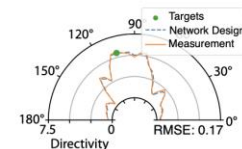
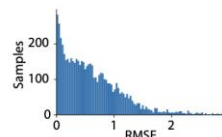
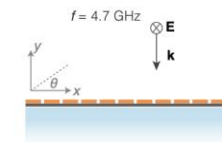
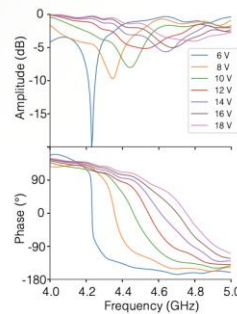
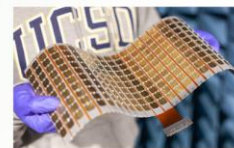
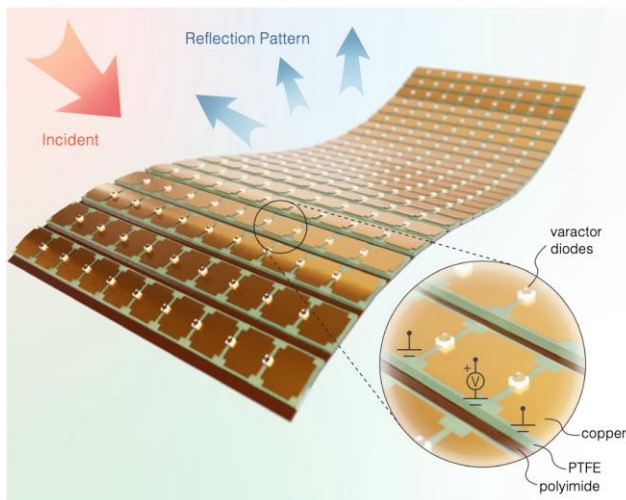
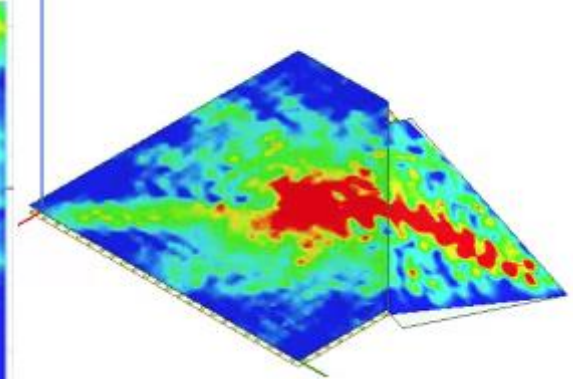
RCP Excitation



LCP Excitation

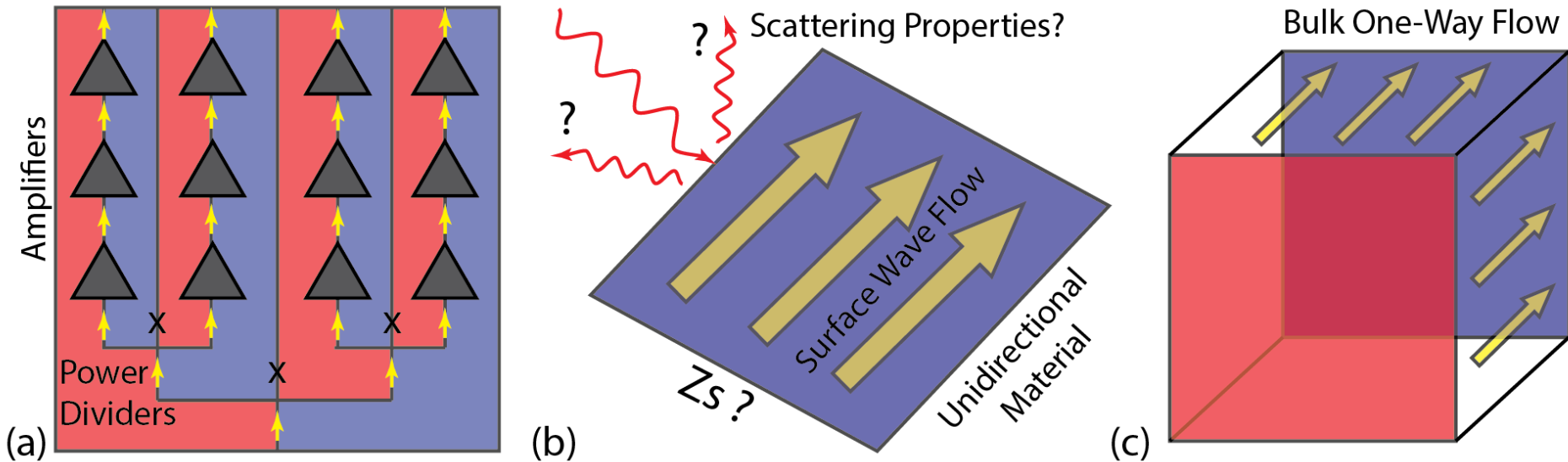


Propagation over Bend



- Potential applications in scattering control using chiral surfaces
- Combine with electronic control of surface properties

# Other Potential Applications of Unidirectional Transport in Topological Materials



- Combine with circuit components for compact amplifiers with reduced feedback based on one-way propagation along interfaces
- New surfaces with unique scattering properties based on unidirectional surface wave flow (free-space scattering is linked to surface wave properties)
- One-way bulk propagation in 2D or 3D materials for EM or thermal transport