

Super-Resolution SAR Moving Target Imaging

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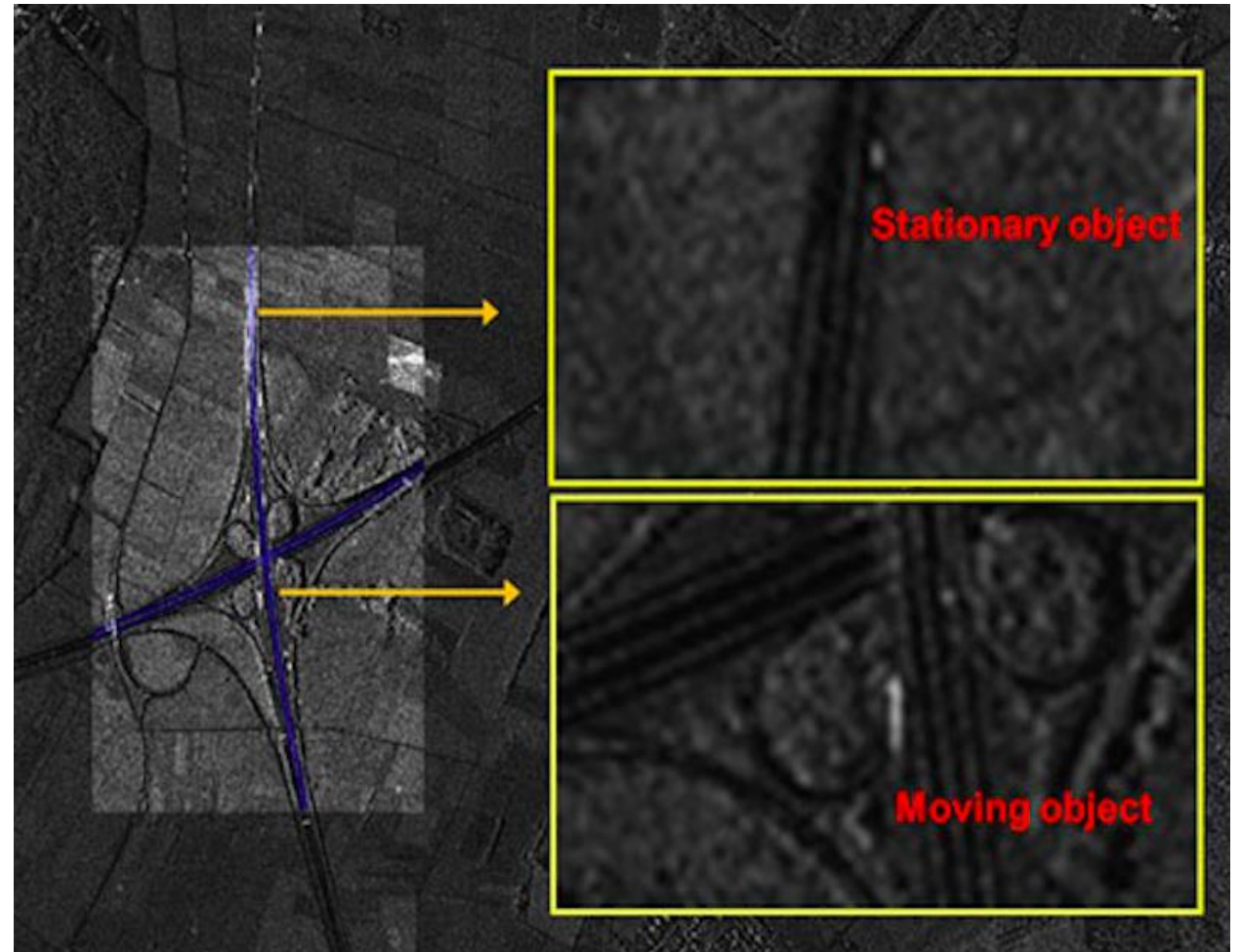
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Engineering



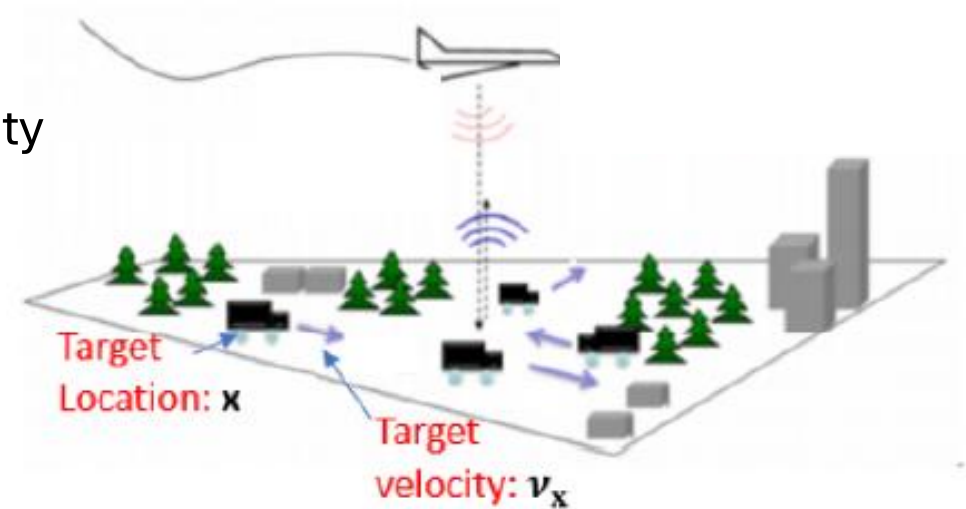
SAR-Ground Moving Target Imaging

- Challenge for SAR image reconstruction
 - Classical SAR: optimal for stationary targets
 - Moving targets appear smeared & displaced
- ***SAR-GMTI: reconstruct as a focused image***, estimate the target velocity



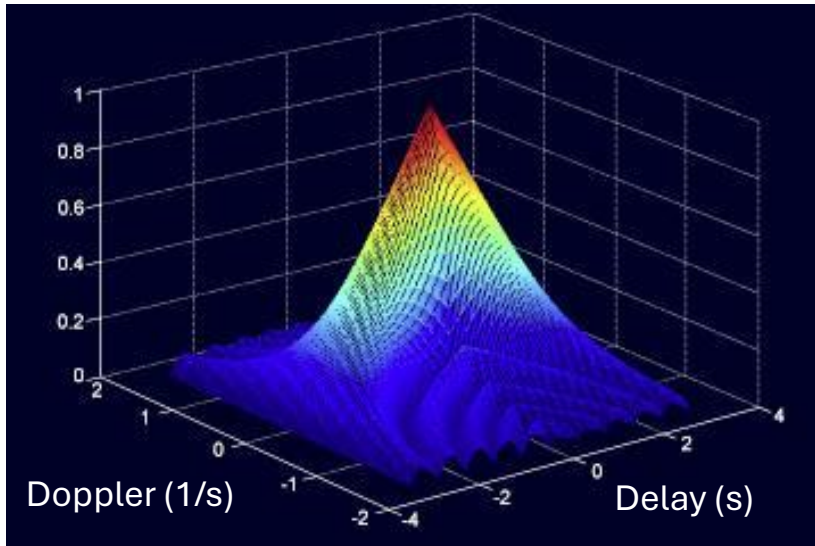
SAR-GMTI State-of-the-Art

- Prior-Art on SAR-GMTI
 - Multiple phase centers: ATI & DPCA
 - Space-time adaptive processing
 - Single antenna: time-frequency analysis for velocity estimation
 - Low rank + sparse data decompositions
- Single antenna SAR-GMTI via optimization
 - Focus moving targets & 2D velocity estimation
 - Use the structure of the phase-space reflectivity of the scene



Motivation & Objectives

- **Super-resolution** for single antenna SAR-GMTI: simultaneous recovery & focusing of stationary extended targets and moving point-targets
- *Why super-resolution capability for moving target imaging?*
 - *Range resolution vs. Doppler resolution trade-off in RD processing*



$$\Delta v = \frac{c_0}{2f_0 T}$$

Velocity resolution

$$\Delta x = \frac{c_0}{2B}$$

Range resolution

Fundamental
trade-off for SAR
systems

$B \uparrow == T \downarrow \Rightarrow$ Increased range-resolution
Decreased velocity-resolution


Super-Resolution for SAR-GMTI

- *Why super-resolution for SAR-GMTI? → range resolution vs. Doppler resolution trade-off in RD processing*
- *Increased accuracy in localization & tracking with SAR systems*
 - Signals of opportunity, S & L-band SARs → low bandwidth, long pulse duration systems
 - **Super-resolution in range: estimate location to precision $\Delta x < \frac{c_0}{2B}$**
 - X-band SAR → high bandwidth, short pulse duration systems
 - **Super-resolution in velocity: estimate velocities to precision $\Delta v < \frac{c_0}{2f_0 T}$**

Our Approach & Its Advantages

- Part 1: moving target & stationary background decomposition via convex optimization
 - *Use structure in the phase-space reflectivity*
 - *Capability in dense moving target environments*
- Part 2: super-resolution imaging and velocity estimation for moving targets via atomic norm minimization
 - *Sparsity of moving targets and structure in the received signal*
 - *Localization and velocity estimation with resolution beyond Fourier techniques*

SAR-GMTI Received Signal Model

- Linear target motion model : $\mathbf{z}(s) = \mathbf{x} + \mathbf{v}_x s$  s : slow – time variable, $s \in [0, \text{CPI}]$
 \mathbf{x} : target position at $s = 0$
 \mathbf{v}_x : constant target velocity

SAR-GMTI Received Signal Model

- Linear target motion model : $\mathbf{z}(s) = \mathbf{x} + \mathbf{v}_x s \longrightarrow$ s : slow – time variable, $s \in [0, \text{CPI}]$
 \mathbf{x} : target position at $s = 0$
 \mathbf{v}_x : constant target velocity
- Under the Born, and the start-stop approximations:

$$d(s, t) = \int e^{-i\omega(t - R(s, \mathbf{x}, \mathbf{v}_x))/c_0} A(\omega, s, \mathbf{x}, \mathbf{v}_x) \rho(\mathbf{x}) d\mathbf{x} d\omega$$

Received signal

t : fast – time variable

Range to the target

$$R(s, \mathbf{x}, \mathbf{v}_x) = 2|\gamma(s) - \mathbf{x} - \mathbf{v}_x s|$$

Scene reflectivity function

SAR-GMTI Received Signal Model

- Linear target motion model : $\mathbf{z}(s) = \mathbf{x} + \mathbf{v}_x s$

- Under the Born, and the start-stop approximations:

$$d(s, t) = \int e^{-i\omega(t-R(s, \mathbf{x}, \mathbf{v}_x))/c_0} A(\omega, s, \mathbf{x}, \mathbf{v}_x) \rho(\mathbf{x}) d\mathbf{x} d\omega$$

- *Lifting* via velocity integration \rightarrow a linear map over the *phase-space reflectivity (PSR)*

$$d(s, t) = \int e^{-i\omega(t-R(s, \mathbf{x}, \mathbf{v})/c_0} A(\omega, s, \mathbf{x}, \mathbf{v}) \rho(\mathbf{x}) \delta(\mathbf{v} - \mathbf{v}_x) d\omega d\mathbf{x} d\mathbf{v}$$

PSR:

$$q(\mathbf{x}, \mathbf{v}) = \rho(\mathbf{x}) \delta(\mathbf{v} - \mathbf{v}_x)$$



Structure of the PSR Function

- *The phase-space reflectivity (PSR) structure*

Discretized PSR:
N-sized location grid &
M-sized velocity grid



$$q(\mathbf{x}, \mathbf{v}) = \rho(\mathbf{x})\delta(\mathbf{v} - \mathbf{v}_{\mathbf{x}})$$

$$\mathbf{Q} = \begin{bmatrix} \rho(\mathbf{x}_1)\delta(\mathbf{v}_1 - \mathbf{v}_{\mathbf{x}_1}) & \cdots & \rho(\mathbf{x}_N)\delta(\mathbf{v}_1 - \mathbf{v}_{\mathbf{x}_N}) \\ \rho(\mathbf{x}_1)\delta(\mathbf{v}_2 - \mathbf{v}_{\mathbf{x}_1}) & \cdots & \rho(\mathbf{x}_N)\delta(\mathbf{v}_2 - \mathbf{v}_{\mathbf{x}_N}) \\ \vdots & \vdots & \vdots \\ \rho(\mathbf{x}_1)\delta(\mathbf{v}_M - \mathbf{v}_{\mathbf{x}_1}) & \cdots & \rho(\mathbf{x}_N)\delta(\mathbf{v}_M - \mathbf{v}_{\mathbf{x}_N}) \end{bmatrix} = \mathbf{Q}_s + \mathbf{Q}_\nu$$

Structure of the PSR Function

- The phase-space reflectivity (PSR) structure

Discretized PSR:
N-sized location grid &
M-sized velocity grid



$$q(\mathbf{x}, \mathbf{v}) = \rho(\mathbf{x})\delta(\mathbf{v} - \mathbf{v}_x)$$

$$\mathbf{Q} = \begin{bmatrix} \rho(\mathbf{x}_1)\delta(\mathbf{v}_1 - \mathbf{v}_{x_1}) & \cdots & \rho(\mathbf{x}_N)\delta(\mathbf{v}_1 - \mathbf{v}_{x_N}) \\ \rho(\mathbf{x}_1)\delta(\mathbf{v}_2 - \mathbf{v}_{x_1}) & \cdots & \rho(\mathbf{x}_N)\delta(\mathbf{v}_2 - \mathbf{v}_{x_N}) \\ \vdots & \vdots & \vdots \\ \rho(\mathbf{x}_1)\delta(\mathbf{v}_M - \mathbf{v}_{x_1}) & \cdots & \rho(\mathbf{x}_N)\delta(\mathbf{v}_M - \mathbf{v}_{x_N}) \end{bmatrix} = \mathbf{Q}_s + \mathbf{Q}_\nu$$

An example with two moving point-targets, and a dense stationary background

$$\mathbf{Q}_s = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ \rho(\mathbf{x}_1) & 0 & \rho(\mathbf{x}_3) & \cdots & \rho(\mathbf{x}_{k-1}) & 0 & \rho(\mathbf{x}_{k+1}) & \cdots & \rho(\mathbf{x}_N) \\ 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \end{bmatrix} \quad \mathbf{Q}_\nu = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 & \rho(\mathbf{x}_k) & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ 0 & \rho(\mathbf{x}_2) & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \end{bmatrix}$$


Discretized Signal Model

- *The phase-space reflectivity (PSR) structure*

$$\mathbf{Q} = \begin{bmatrix} \rho(\mathbf{x}_1)\delta(\mathbf{v}_1 - \mathbf{v}_{\mathbf{x}_1}) & \cdots & \rho(\mathbf{x}_N)\delta(\mathbf{v}_1 - \mathbf{v}_{\mathbf{x}_N}) \\ \rho(\mathbf{x}_1)\delta(\mathbf{v}_2 - \mathbf{v}_{\mathbf{x}_1}) & \cdots & \rho(\mathbf{x}_N)\delta(\mathbf{v}_2 - \mathbf{v}_{\mathbf{x}_N}) \\ \vdots & \vdots & \vdots \\ \rho(\mathbf{x}_1)\delta(\mathbf{v}_M - \mathbf{v}_{\mathbf{x}_1}) & \cdots & \rho(\mathbf{x}_N)\delta(\mathbf{v}_M - \mathbf{v}_{\mathbf{x}_N}) \end{bmatrix} = \mathbf{Q}_s + \mathbf{Q}_\nu$$

$$d(s_n, t_m) \approx \sum_{l=1}^L \sum_{k=1}^N \sum_{k'=1}^M F(\omega_l, s_n, t_m, \mathbf{x}_k, \mathbf{v}_{k'}) \mathbf{Q}(\mathbf{x}_k, \mathbf{v}_{k'})$$

The discretized
signal model



Lifted linear forward map over the PSR space

$$d(s_n, t_m) := \langle \mathbf{F}(s_n, t_m), \mathbf{Q} \rangle_F \quad \rightarrow \quad \mathbf{d} := \mathcal{F}(\mathbf{Q})$$

SAR-GMTI Inverse Problem

- Simultaneous recovery of a sparse moving target component and a rank-1 stationary background component:

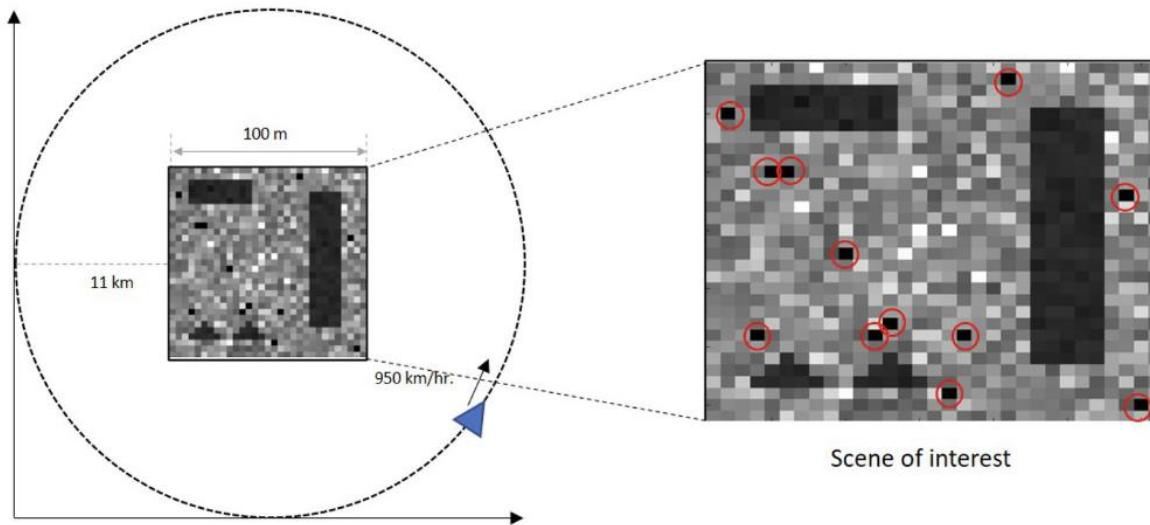
$$\begin{aligned} \min_{\mathbf{Q}_s, \mathbf{Q}_\nu \in \mathbb{R}^+} \quad & \|\mathbf{Q}_\nu\|_1 \\ \text{s.t.} \quad & \|\mathbf{d} - \mathcal{F}(\mathbf{Q}_s + \mathbf{Q}_\nu)\|_F \leq \delta \\ & \mathbf{Q}_s \in C_1, \mathbf{Q}_\nu \in C_1^c, \end{aligned}$$

- Solve via ADMM \rightarrow *convergence is guaranteed in residual & functional value via convexity*

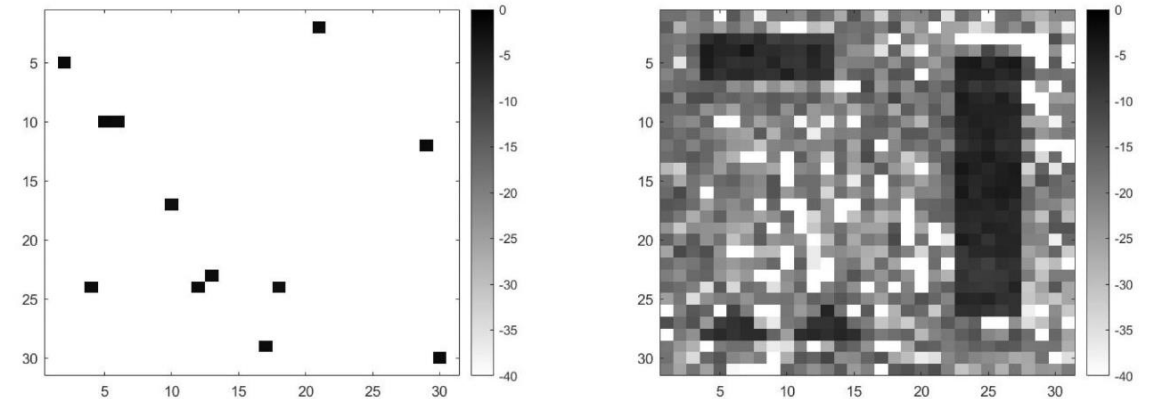
$$\begin{aligned} \min \quad & \lambda \|\mathbf{Q}_\nu\|_1 + \frac{1}{2} \|\mathcal{F}(\mathbf{Q}) - \mathbf{d}\|_2^2 + \mathcal{I}_{C_1}(\mathbf{Q}_s) + \mathcal{I}_{C_1^c}(\mathbf{Q}_\nu) \\ \text{s.t.} \quad & \mathbf{Q} = \mathbf{Q}_s + \mathbf{Q}_\nu. \end{aligned}$$

Simple convex projections
& soft-thresholding

Numerical Simulations: Part-1

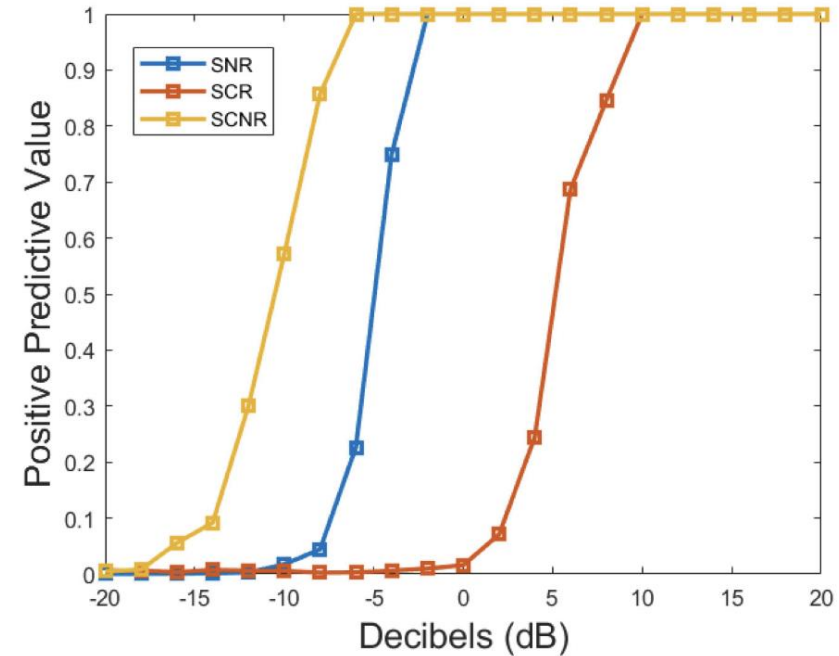
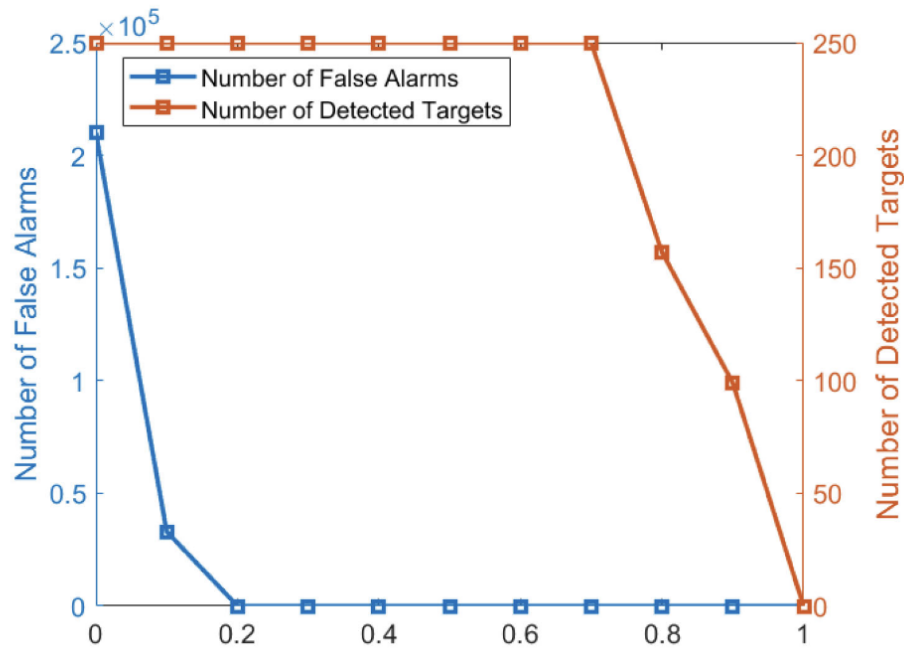


Reconstructed moving & stationary components



- Moving target focused & superposed on the stationary background buried in statistical clutter & noise (0 dB SCR)
- Velocity cell = 2 m/s x 2 m/s → **super-resolution in velocity, imaged at range-resolution limit**
- Conventional limit: 8ms chirp pulse at 50 MHz & 9 GHz carrier → time-BW product of 400000

Numerical Simulations: Part-1



- Consistent detection & false alarm characteristics in dense moving target environments
- Graceful degradation with decreasing SNR, SCR, SCNR:
$$PPV = \frac{\text{true positives}}{\text{true positives} + \text{false positives}}$$

Optimization-Based SAR-GMTI

- Single antenna SAR-GMTI
- Applicability to arbitrary flight trajectories & imaging scenarios
- Capable of recovering the stationary background simultaneously to focusing the moving foreground
- ***Super-resolution capability***
- ***Off-grid localization and velocity estimation → atomic norm minimization (ANM)***

Atomic Norm Minimization for SAR-GMTI

- ANM for SAR-GMTI: line spectral estimation per slow-time

$$\rho(\boldsymbol{x}) = \sum_{k=1}^K \rho_k \delta(\boldsymbol{x} - \boldsymbol{x}_k), \quad \longrightarrow \quad \mathbf{f}_n(\mathcal{A}) = \sum_{k=1}^K \rho_k e^{-i \frac{2\pi f_0}{c_0} R(s_n, \boldsymbol{x}_k, \boldsymbol{v}_k)} \mathbf{a}(\phi(s_n, \boldsymbol{x}_k, \boldsymbol{v}_k)).$$

$$\mathcal{A} = \{\mathbf{a}(\phi) \in \mathbb{C}^M \mid \phi \in [0, 1), a_m = e^{-i2\pi\phi m}, m = 1, \dots, M\}$$

Atomic Norm Minimization for SAR-GMTI

- ANM for SAR-GMTI: line spectral estimation per slow-time

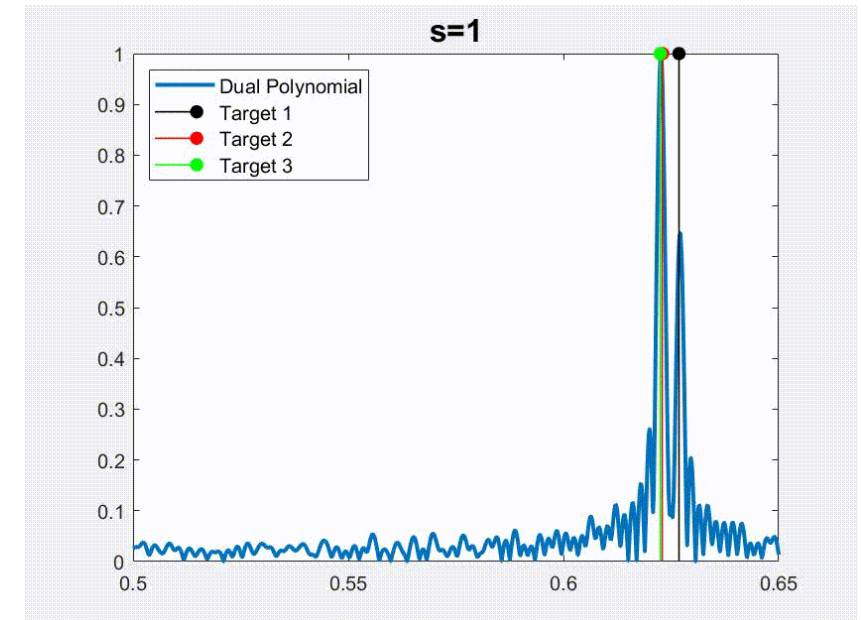
$$\rho(x) = \sum_{k=1}^K \rho_k \delta(x - x_k), \quad \longrightarrow \quad \mathbf{f}_n(\mathcal{A}) = \sum_{k=1}^K \rho_k e^{-i \frac{2\pi f_0}{c_0} R(s_n, \mathbf{x}_k, \mathbf{v}_k)} \mathbf{a}(\phi(s_n, \mathbf{x}_k, \mathbf{v}_k)).$$

$$\mathcal{A} = \{\mathbf{a}(\phi) \in \mathbb{C}^M \mid \phi \in [0, 1), a_m = e^{-i2\pi\phi m}, m = 1, \dots, M\}$$

- Tomographic imaging \rightarrow invert the forward map
- SAR-GMTI via ANM \rightarrow **decompose the received signal to the echo of each target**

$$\hat{\theta}_k(s_n) = \phi(s_n, \mathbf{x}_k, \mathbf{v}_k)$$

- Estimate location & velocity from the recovered phases per target



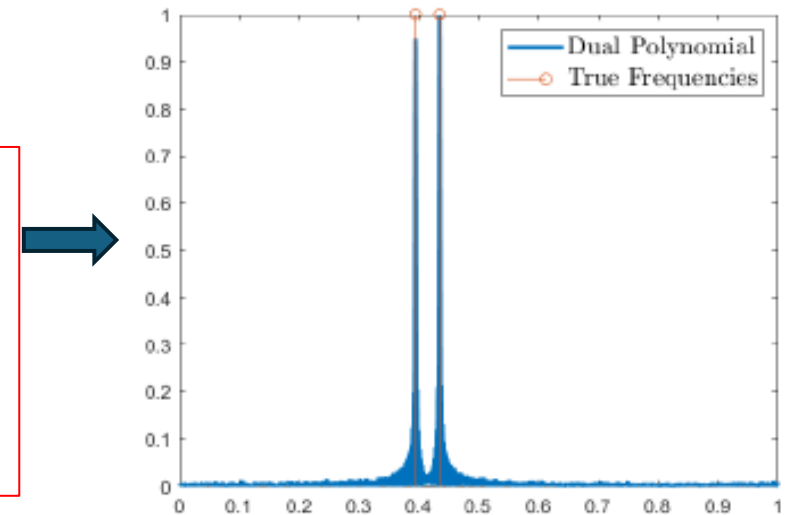
ANM Optimality for Line Spectral Estimation

- A computational tool for sparse recovery without a pre-fixed grid

$$\mathcal{A} = \{\mathbf{a}(\phi) \in \mathbb{C}^M \mid \phi \in [0, 1), a_m = e^{-i2\pi\phi m}, m = 1, \dots, M\}$$
$$\mathbf{f}_n(\mathcal{A}) = \sum_{k=1}^K \rho_k e^{-i\frac{2\pi f_0}{c_0} R(s_n, \mathbf{x}_k, \mathbf{v}_k)} \mathbf{a}(\phi(s_n, \mathbf{x}_k, \mathbf{v}_k)).$$

Atomic norm denoising: $\min_{\mathbf{f}} \frac{1}{2} \|\mathbf{f} - \mathbf{d}_n\|_2^2 + \lambda \|\mathbf{f}\|_{\mathcal{A}}$

$$\frac{1}{M} \|\mathbf{f}_n - \hat{\mathbf{f}}\|_2^2 = \mathcal{O} \left(\sigma \sqrt{\frac{\log M}{M}} \sum_{k=1}^K |\rho_k| \right)$$



MSE optimality for line spectral estimation [Bhaskar-Tang-Recht]
Exact recovery of phases (noise free) [Granda-Candes]

Localization and Velocity Estimation

- *Objective → use super-resolved phases for precise localization & velocity estimation per target*

$$\frac{c_0}{\Delta f} \hat{\theta}_k(s_n) = 2 \|\gamma(s) - \mathbf{x}_k - \mathbf{v}_k s_n\|, \quad n = 0, \dots, N-1$$

Localization and Velocity Estimation

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Flat topography

A system of quadratic equations over $\mathbf{p} = [x_1 \ x_2, v_1, v_2]^T$

$$\mathbf{b}_k = \mathbf{M}\boldsymbol{\beta}(\mathbf{p}), \quad \boldsymbol{\beta}(\mathbf{p}) = [x_1, x_2, v_1, v_2, x_1^2, x_2^2, v_1^2, v_2^2, x_1 v_1, x_2 v_2]^T$$

$$\mathbf{M}^T = \begin{bmatrix} -2\gamma_1(s_0) & \cdots & -2\gamma_1(s_{N-1}) \\ -2\gamma_2(s_0) & \cdots & -2\gamma_2(s_{N-1}) \\ -2s_0\gamma_1(s_0) & \cdots & -2s_{N-1}\gamma_1(s_{N-1}) \\ -2s_0\gamma_2(s_0) & \cdots & -2s_{N-1}\gamma_2(s_{N-1}) \\ 1 & \cdots & 1 \\ 1 & \cdots & 1 \\ s_0^2 & \cdots & s_{N-1}^2 \\ s_0^2 & \cdots & s_{N-1}^2 \\ 2s_0 & \cdots & 2s_{N-1} \\ 2s_0 & \cdots & 2s_{N-1} \end{bmatrix}.$$

Determined by imaging geometry, PRF, aperture

Localization and Velocity Estimation

- *Objective → use super-resolved phases for precise localization & velocity estimation per target*

$$\frac{c_0}{\Delta f} \hat{\theta}_k(s_n) = 2 \|\gamma(s) - \mathbf{x}_k - \mathbf{v}_k s_n\|, \quad n = 0, \dots, N-1$$

A system of quadratic equations over $\mathbf{p} = [x_1 \ x_2, v_1, v_2]^T$

- *Solve for the GMTI parameters (2D location and 2D velocity) as a non-linear least squares problem*

$$\mathbf{p}_k^* = \arg \min_{\mathbf{p} \in \mathcal{D} \times \mathcal{V}} \ell(\mathbf{p}) := \|\mathbf{b}_k - \mathbf{M}\boldsymbol{\beta}(\mathbf{p})\|^2.$$

$$\mathbf{p}^{l+1} = \mathbf{p}^l + (\mathbf{J}(\mathbf{p}^l)^T \mathbf{J}(\mathbf{p}^l))^{-1} \mathbf{J}(\mathbf{p}^l)^T (\mathbf{b} - \mathbf{M}\boldsymbol{\beta}(\mathbf{p}^l))$$

$$\mathbf{J}(\mathbf{p}^l) := \mathbf{M} \frac{\partial \boldsymbol{\beta}^T}{\partial \mathbf{p}} \Big|_{\mathbf{p}=\mathbf{p}^l}$$

Gauss-Newton
method

Recovery Guarantees via ANM & NNLS

- Convergence of GN: circular trajectory, polynomials satisfying $\gamma_1(s) \in P(r_1), \gamma_2(s) \in P(r_2)$, with $\max(r_1, r_2) \geq \min(r_1, r_2) + 2$
- ANM recovery guarantees:

$$\min_{k \neq l} |\phi(s_n, \mathbf{x}_k, \mathbf{v}_k) - \phi(s_n, \mathbf{x}_l, \mathbf{v}_l)| \geq \frac{2}{M} \Rightarrow \|\mathbf{z}_k(s_n) - \mathbf{z}_l(s_n)\| \geq c_0/B.$$

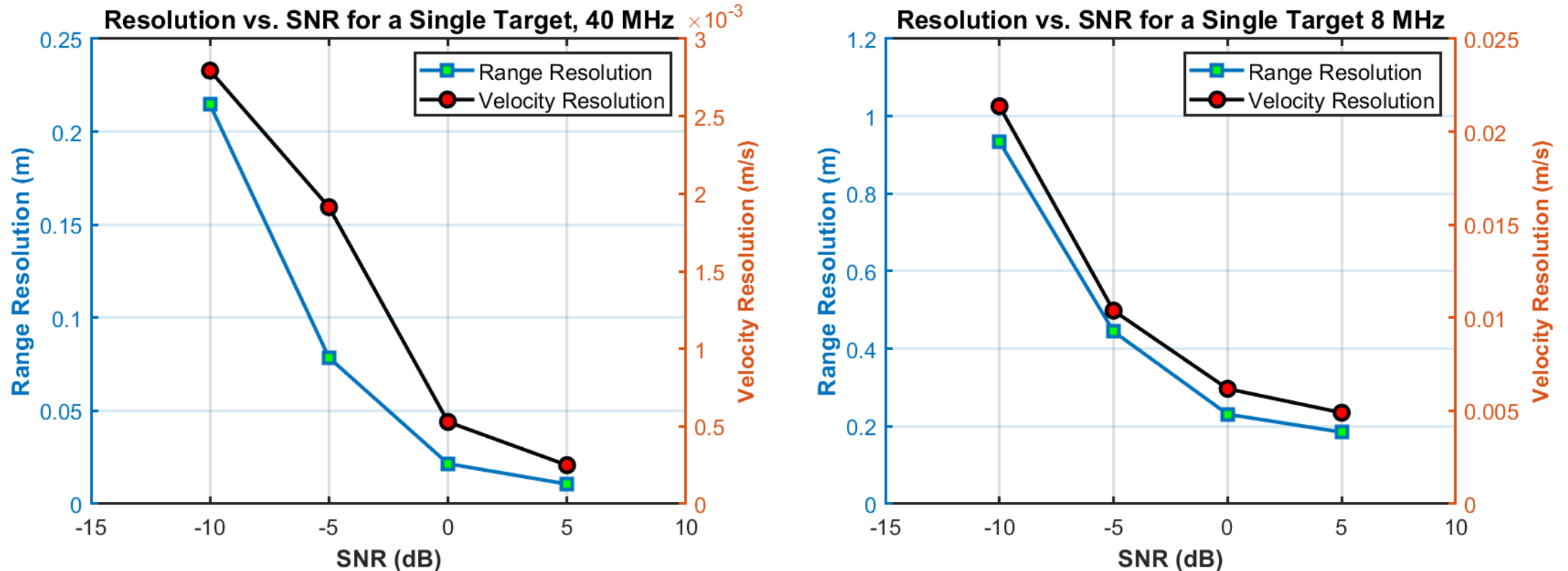
Minimal separation condition in phase

Minimal separation in space

- Sample complexity for K-number of moving targets:

$$K = o(\sqrt{M/\log M}) \quad N = \mathcal{O}(K^2).$$

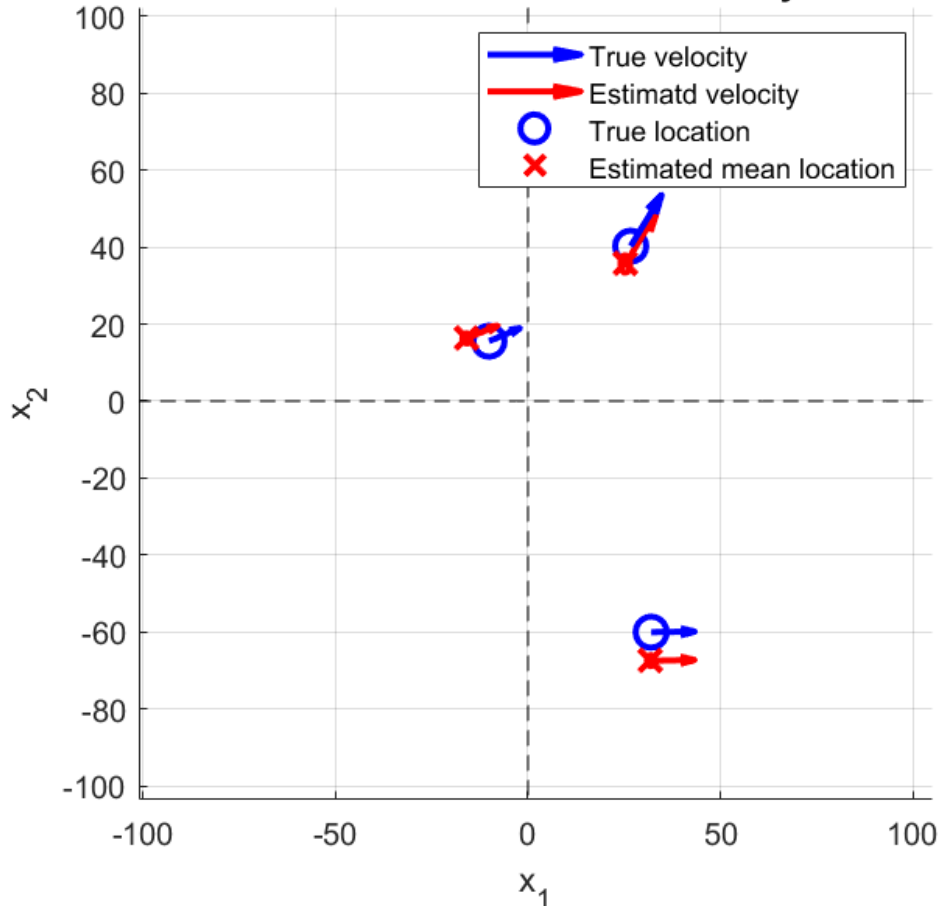
Monte-Carlo Evaluations: Single Target



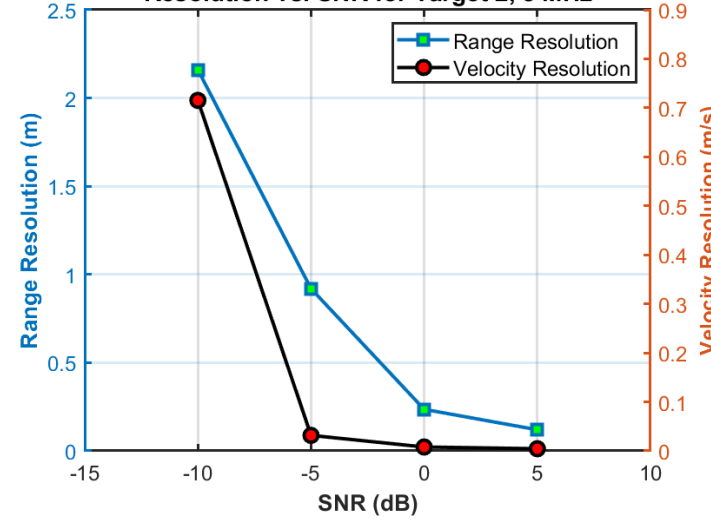
- Standard deviation of numerical location & velocity estimates, using 10 instances of noisy reconstructions (range resolution limit of 3.75 meters & 18.75 meters respectively)

Preliminary Multi-Target Evaluations

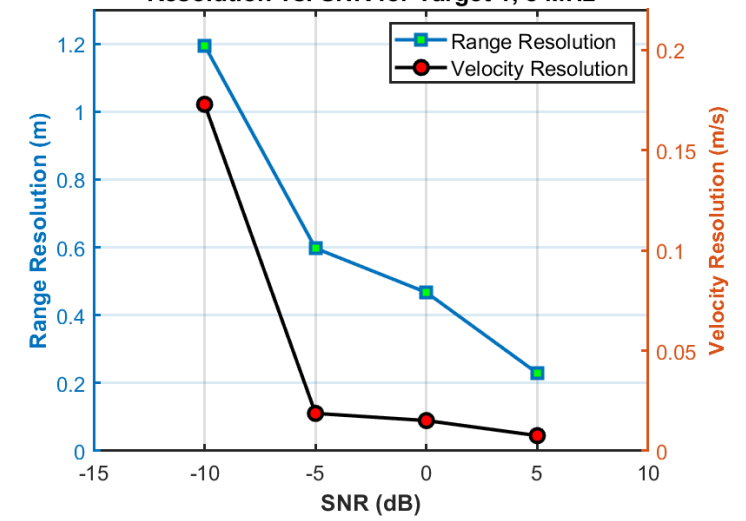
Estimated Location and Velocity



Resolution vs. SNR for Target 2, 8 MHz



Resolution vs. SNR for Target 1, 8 MHz



- Localization within a resolution-cell
- Velocity estimation without a specified grid
- Graceful degradation with respect to SNR

Conclusion & Future Work

- A method capable of focusing moving targets **off-grid with super-resolution**
 - Velocity super-resolution in high bandwidths, range super-resolution in low bandwidths
- **Novel approach to detection & tracking of moving targets**
 - Separate processing of each target for location & velocity estimates
- *Exact recovery guarantees for SAR-GMTI using ANM & NLLS*
 - General conditions for antenna trajectories & aperture length
 - Increased noise floor in phase estimates
 - Theoretical study of bias and variance of the ANM method