Deterministic Singularities in Partially Coherent Vortex Beams

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UNC CHARLOTTE

AFOSR Electromagnetics Review,
January 2025

Advantages of structured light in FSOC

- Light propagating in atmospheric turbulence suffers from distortions that can adversely affect sensing and communications
- Partially coherent (PC) beams are resistant to turbulence
- Vortex structures can be used to encode and multiplex information, and are (in a sense) resistant to atmospheric distortions
- Can we mix PC and vortex properties to improve communications? Problem: vortex is phase structure, PC implies randomized phase
- Phase vortices imposed on a PC beam typically “decompose” into correlation vortices

Deterministic vortices in PC fields

We can now identify many ways in which a partially coherent vortex beam can manifest a “deterministic” vortex on propagation:

- Beams with a deterministic vortex always present: “separable phase” or “circular coherence” beams
- Beams that evolve a deterministic vortex in far zone: “Rankine vortex beams”
- Beams that introduce a deterministic vortex at designed distance: “Deterministic vortex beams” **(RELATIVELY NEW)**
- Beams that start with a deterministic vortex and end with a deterministic vortex “Elliptical basis beams” **(NEW)**
- Beams that periodically revive a deterministic vortex: “Deterministic vortex revivals” **(NEW)**

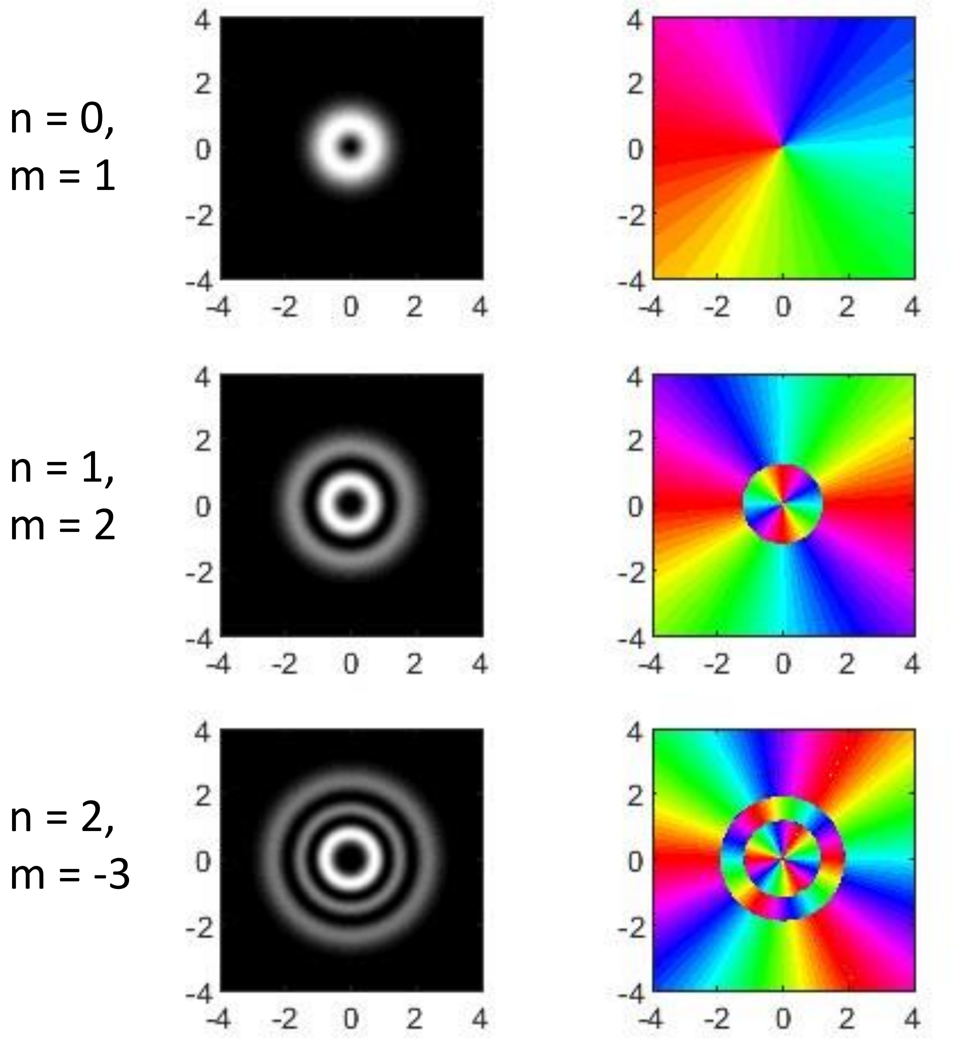
Phase vortices in coherent fields

- **In transverse plane of a monochromatic beam, points of zero intensity are singularities of phase, and the phase has a discrete “twist” around the zero that is a discrete multiple of** 2π: the “topological charge”

- The topological charge is discrete and stable under perturbations and multiple vortex orders can be optically multiplexed and demultiplexed

- These properties have led to vortex beams being considered for optical communications

- A phase vortex that circles a zero of intensity we will call a deterministic vortex



Basics concepts of optical coherence theory

All light waves possess random fluctuations in space and time. Single mode lasers have low fluctuations, thermal sources have high fluctuations; between these cases we talk of "partial coherence"

We look at fields that are (relatively) narrowband, quasi-monochromatic, with fluctuations in space: *spatially partially coherent*. These fields can be characterized by the cross-spectral density:

$$\boxed{W(\mathbf{r}_1, \mathbf{r}_2, \omega) = \langle U^*(\mathbf{r}_1, \omega) U(\mathbf{r}_2, \omega) \rangle_\omega}$$

The angle brackets represent an average over a monochromatic ensemble of realizations

Can always write the cross-spectral density in terms of fields/correlations (or intensities/correlations):

$$W(\mathbf{r}_1, \mathbf{r}_2) = U^*(\mathbf{r}_1) U(\mathbf{r}_2) \mu(\mathbf{r}_1, \mathbf{r}_2)$$

Vortices and partial coherence

As a function of $\mathbf{r}_1$ or $\mathbf{r}_2$, the cross-spectral density satisfies a Helmholtz equation like a coherent monochromatic field:

$$\left[\nabla_1^2 + k^2\right] W(\mathbf{r}_1, \mathbf{r}_2) = 0 \qquad \left[\nabla_2^2 + k^2\right] W(\mathbf{r}_1, \mathbf{r}_2) = 0$$

$$k = \frac{\omega}{c}$$

If we fix $\mathbf{r}_1$ or $\mathbf{r}_2$, we expect to normally see vortices of the cross-spectral density with respect to the other variable; we call these correlation vortices.

These vortices are vortices of the two-point correlation function: they are not related to the intensity, and they depend on choice of observation point!

Deterministic vortices in partially coherent fields?

Under what condition do we typically expect to see deterministic vortices in partially coherent fields? Every cross-spectral density can be written in a *coherent mode representation*:

$$W(\mathbf{r}_1, \mathbf{r}_2) = \sum_{n=1}^{N} \lambda_n \phi_n^*(\mathbf{r}_1)\phi_n(\mathbf{r}_2)$$

Where $\lambda_n > 0$, ϕ_n are orthonormal, and *N* is total nonzero modes. Intensity of light can be found by setting $\mathbf{r}_1 = \mathbf{r}_2$:

$$I(\mathbf{r}) = \sum_{n=1}^{N} \lambda_n |\phi_n(\mathbf{r})|^2$$

E. Wolf, "New Theory of Partial Coherence in the Space-frequency Domain: Part I: Spectra and Cross-spectra of Steady-state Sources", *J. Opt. Soc. Amer.* **72**, 343–351 (1982).

Deterministic vortices in partially coherent fields?

$$I(\mathbf{r}) = \sum_{n=1}^{N} \lambda_n |\phi_n(\mathbf{r})|^2$$

Let us look for points of zero intensity in a transverse *x, y* plane. We must have $\phi_n(\mathbf{r}) = 0$ for each mode in the same location; this means we must satisfy $2N$ equations with two degrees of freedom: *x, y*. The only time that this will typically happen is if $N = 1$. This is the fully coherent case!

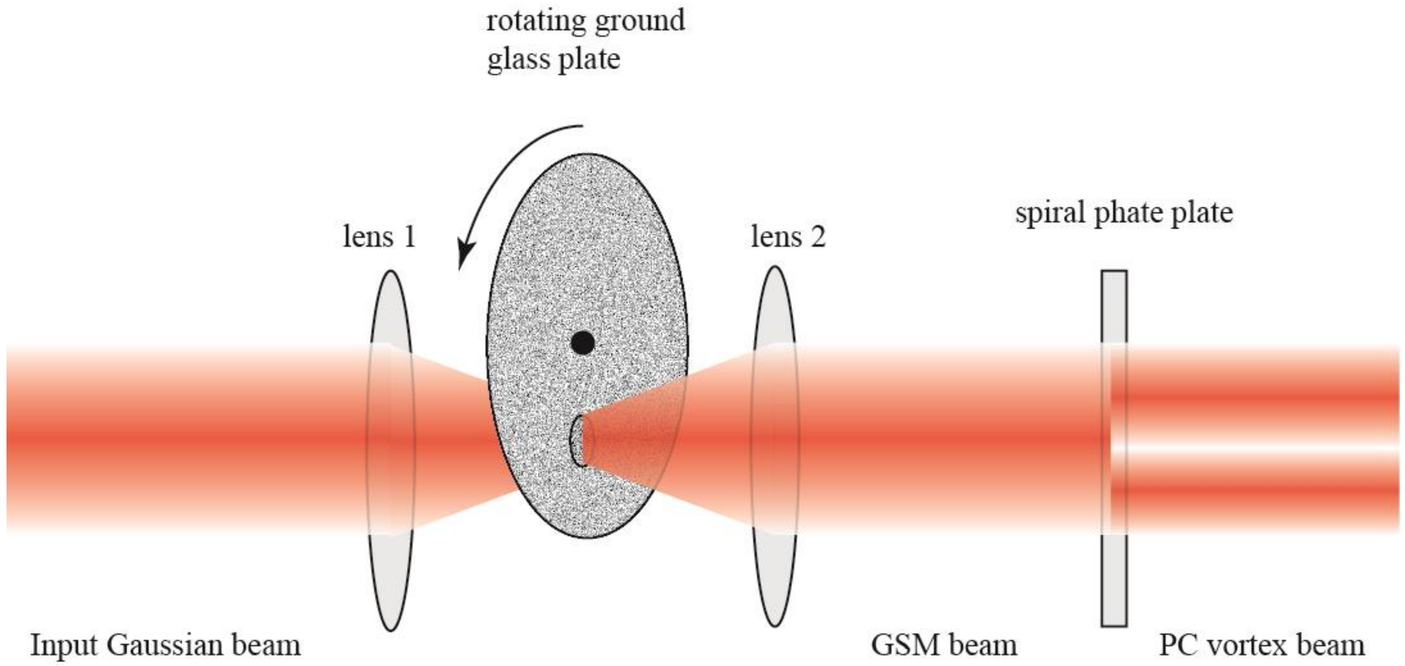
We conclude: zeros of intensity, and deterministic vortices, are not typical features of partially coherent fields. Under what special conditions can they arise?

G. Gbur, T.D. Visser and E. Wolf, "'Hidden' singularities in partially coherent wavefields", J. Opt. A 6 (2004), S239.

Example: Gaussian Schell-model vortex beam

Consider a Gaussian Schell-model vortex beam, of mathematical form:

$$W(\mathbf{r}_1, \mathbf{r}_2) = I_0 \underbrace{e^{-r_1^2/2\sigma_S^2} e^{-r_2^2/2\sigma_S^2}}_{\text{Gaussian beam}} \underbrace{e^{-|\mathbf{r}_1 - \mathbf{r}_2|^2/2\sigma_g^2}}_{\text{Schell-model}} \underbrace{e^{-im\phi_1} e^{im\phi_2}}_{\text{Imposed vortex}}$$



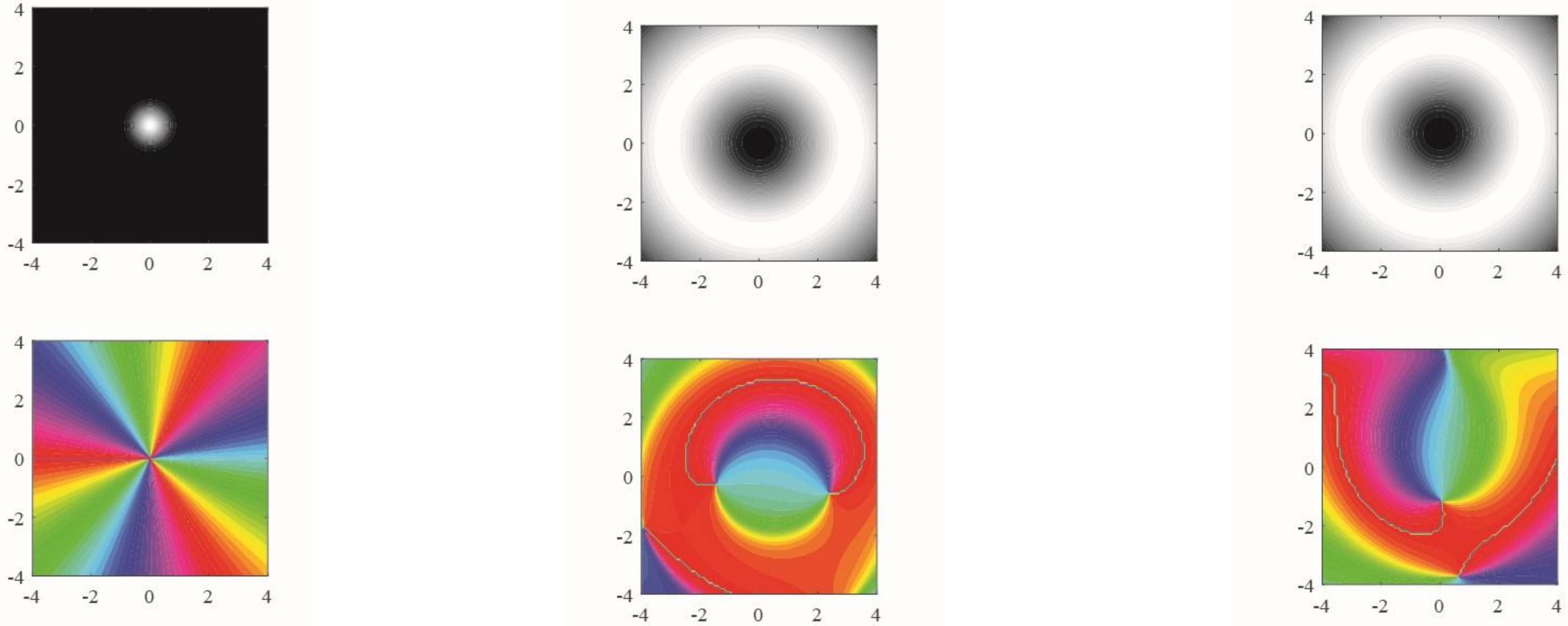
GSMv beam on propagation

On propagation, the deterministic vortex at the core immediately decomposes into a set of correlation vortices that are not associated with zeros of intensity!

Intensity and CSD phase at source

Intensity and CSD phase for one choice of fixed point

Intensity and CSD phase for different choice of fixed point



On propagation, GSMv beam possesses no deterministic vortex structure!

Separable phase beams

How can we make a beam with a deterministic vortex that “sticks”? We can use a coherent mode representation where each mode has the same azimuthal order:

$$W(\mathbf{r}_1, \mathbf{r}_2) = \sum_{n=1}^{N} \lambda_n \phi^*_{nm}(\mathbf{r}_1)\phi_{nm}(\mathbf{r}_2)$$

where each mode is for example a Laguerre-Gauss mode with the same azimuthal order *m* but a different radial order *n*. The combined beam will always have a zero on axis!

It is unclear how to easily make such a beam…

Galina V. Bogatyryova, Christina V. Fel’de, Peter V. Polyanskii, Sergey A. Ponomarenko, Marat S. Soskin, and Emil Wolf, "Partially coherent vortex beams with a separable phase," Opt. Lett. 28, 878-880 (2003)

Circular coherence beams

In 2017, Santarsiero et al. introduced a class of beams that are fully coherent in the azimuthal direction; we can view them as a random ensemble of beams with different focal lengths.

$$W_0(\mathbf{r}_1, \mathbf{r}_2) = \int \sqcap(\mathbf{v})\, \psi^*(\mathbf{r}_1) e^{-i\mathbf{v}\frac{r_1^2}{\delta_u^2}} \psi(\mathbf{r}_2) e^{i\mathbf{v}\frac{r_2^2}{\delta_u^2}} d\mathbf{v}$$

Here, **v** is the wavefront curvature of a member of the ensemble. We can also impose a vortex in the beam's center, and it will remain propagation invariant:

$$W_0(\mathbf{r}_1, \mathbf{r}_2) = \int \sqcap(\mathbf{v})\, \psi_m^*(\mathbf{r}_1) e^{-i\mathbf{v}\frac{r_1^2}{\delta_u^2}} \psi_m(\mathbf{r}_2) e^{i\mathbf{v}\frac{r_2^2}{\delta_u^2}} d\mathbf{v}$$

M. Santarsiero, R. Martínez-Herrero, D. Maluenda, J. C. G. de Sande, G. Piquero, and F. Gori, "Partially coherent sources with circular coherence," Opt. Lett. 42, 1512-1515 (2017).

Circular coherence vortex beams

Some results: on free space propagation, the circular coherence vortex beams maintain a deterministic vortex on propagation, but beam behavior is marred by self-focusing effects and non-trivial structure.

Can we do better, and make a "pure" deterministic vortex at a specified distance?

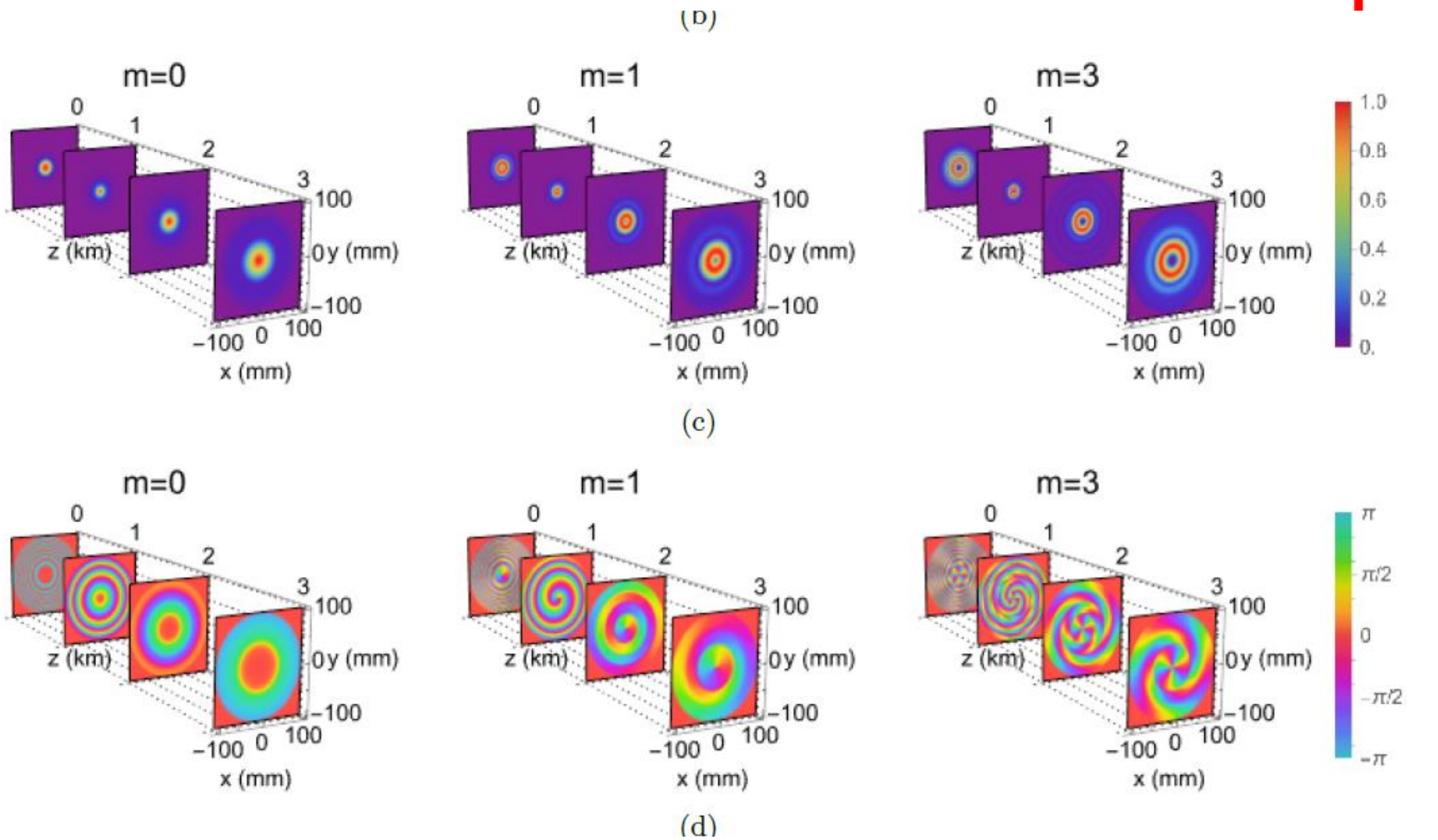
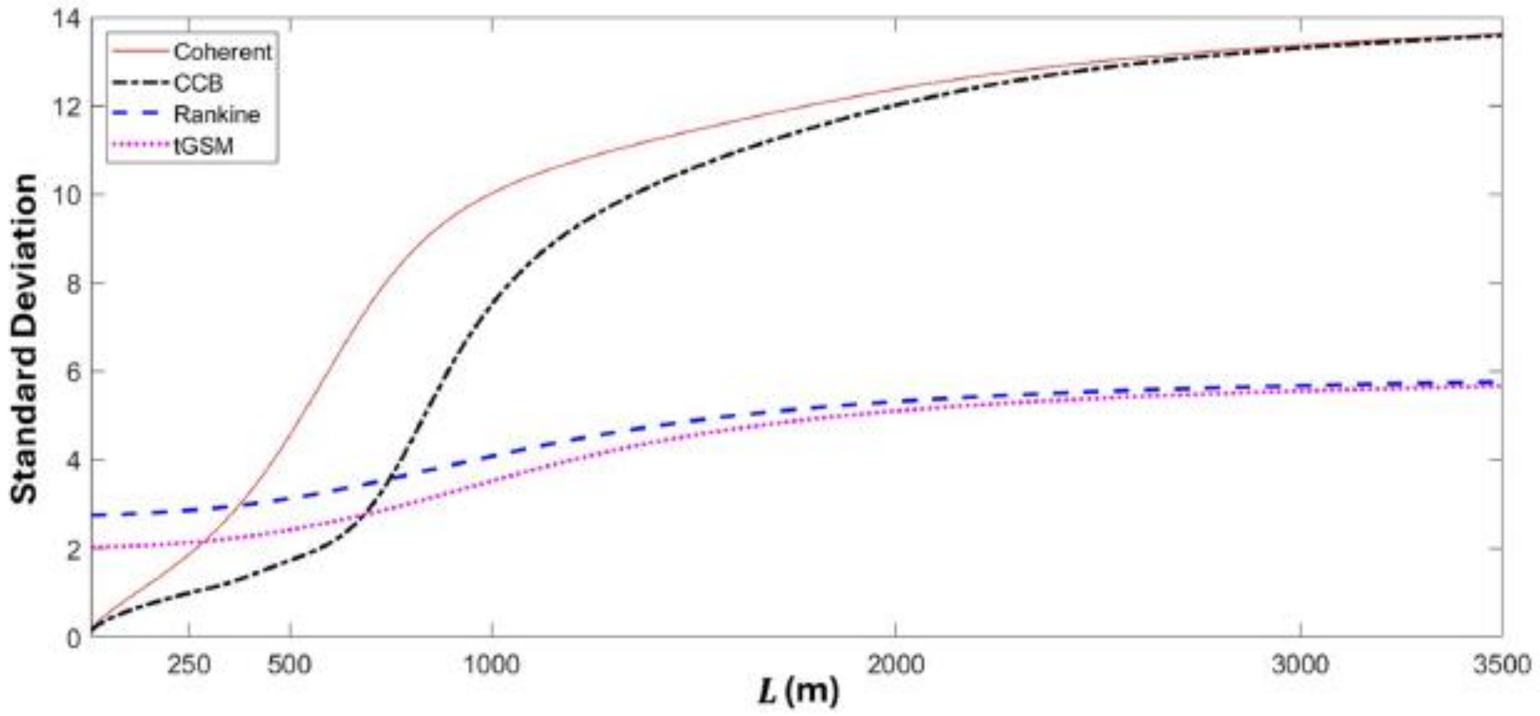


FIG. 4: Second-order coherence properties of circularly coherent LG beams on propagation with an azimuthal order of m=0, m=1, and m=3: (a) CSDs, (b) spectral degree of coherence, (c) amplitude and, (d) phase of CSDs under a fixed observing point, r_1, with $\sigma_s = 0.02\,\mathrm{m}$, $\delta_u = 0.044\,\mathrm{m}$, and $\lambda = 632.8\,\mathrm{nm}$ at the source plane, 1 km, 2 km, and 3 km, respectively. Observing points are $r_1 = 0.18\,\mathrm{mm}$, 0.27 mm, and 0.44 mm, respectively.

Rui Qi, Arash Shiri, and Greg Gbur, "Circularly coherent vortex beams with coherence singularities in free-space propagation," J. Opt. Soc. Am. A 42, 59-66 (2025).

Different OAM beam classes in turbulence

Early in 2024, we published a comparison of the OAM spectrum standard deviation for a number of OAM beam classes: the circularly coherent class beat the coherent class over short propagation distances.



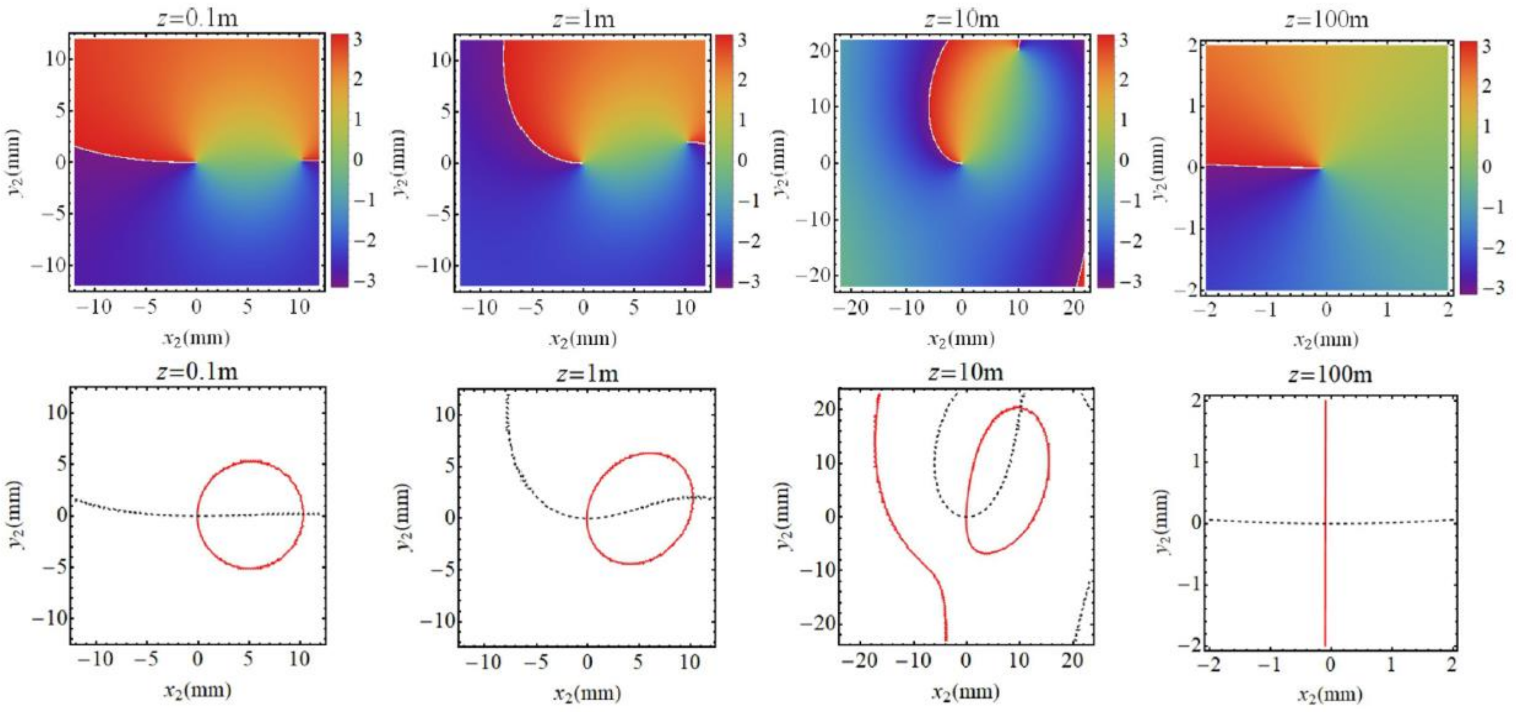
Arash Shiri and Greg Gbur, "Orbital angular momentum spectrum of model partially coherent beams in turbulence," Opt. Express 32, 18175-18192 (2024).

Deterministic vortex beams – a history

Since 2004, I have used a “beam wander model” to construct partially coherent vortex beams:

$$W(\mathbf{r}_1, \mathbf{r}_2, \omega) = \int P(\mathbf{r}_0) U^*(\mathbf{r}_1 - \mathbf{r}_0, \omega) U(\mathbf{r}_2 - \mathbf{r}_0, \omega) d^2 r_0$$

In 2020, with Zhang and Cai, looked at propagation of “beam wander” beams –
And the vortex appears to become more visible as the beam propagates to the far zone!



Y. Zhang, Y. Cai, and G. Gbur, “Partially coherent vortex beams of arbitrary radial order and a van Cittert–Zernike theorem for vortices,” Phys. Rev. A 101 (2020), 043812.

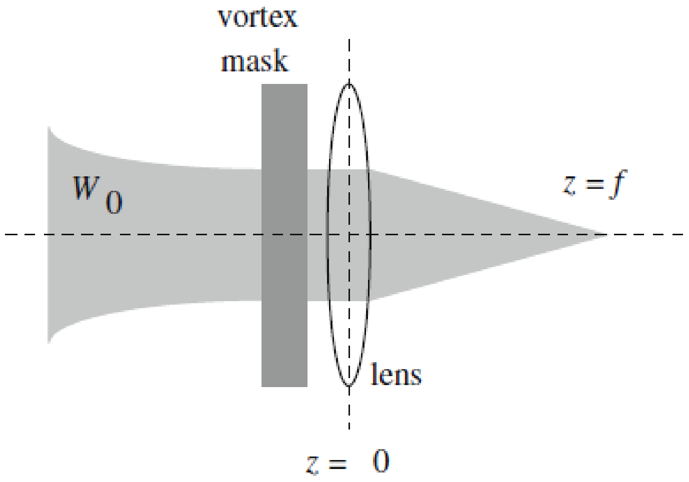
An explanation and a new approach

Explanation: a “beam wander” (Rankine) vortex beam is the Fourier transform of a Schell-model vortex beam:

$$W(\mathbf{r}_1,\mathbf{r}_2) = U_0^*(\mathbf{r}_1)U_0(\mathbf{r}_2)\mu_0(\mathbf{r}_2-\mathbf{r}_1)$$

$$\mu_0(\mathbf{r}_2-\mathbf{r}_1) = \int \tilde{\mu}_0(\mathbf{K})e^{-i\mathbf{K}\cdot(\mathbf{r}_2-\mathbf{r}_1)}d^2K$$

Schell-model beam is a series of tilts; the Fourier transform of this is a series of shifts! Far-zone of “beam wander” beam is Schell-model beam.



Y. Gu and G. Gbur, “Topological reactions of correlation vortices,” Opt. Commun. 282 (2009), 709.

Deterministic vortex beams

Fractional Fourier transforms roughly “mimic” diffraction theory. By use of a source which is a Fractional Fourier transform of a “fundamental” state, we can create beams that manifest a “pure” vortex at any propagation distance desired!

Start with:

$$W(\mathbf{r}_1, \mathbf{r}_2) = U_0^*(\mathbf{r}_1)U_0(\mathbf{r}_2)\mu_0(\mathbf{r}_2 - \mathbf{r}_1)$$

Expand as series of tilts:

$$\mu_0(\mathbf{r}_2 - \mathbf{r}_1) = \int \tilde{\mu}_0(\mathbf{K})e^{-i\mathbf{K}\cdot(\mathbf{r}_2 - \mathbf{r}_1)}d^2K$$

Use FracFT kernel:

$$K_\alpha(\mathbf{r}, \mathbf{r}') = \frac{ie^{-i\alpha}}{2\pi\sigma^2 \sin\alpha} e^{-\frac{i}{2\sigma^2}\cot\alpha r^2} e^{\frac{i}{\sigma^2 \sin\alpha}\mathbf{r}\cdot\mathbf{r}'} e^{-\frac{i}{2\sigma^2}\cot\alpha r'^2}$$

Introduce modified source “field”:

$$U_\alpha(\mathbf{r}) = \int K_\alpha(\mathbf{r}, \mathbf{r}')U_0(\mathbf{r}')d^2r'$$

Wenrui Miao, Yongtao Zhang, and Greg Gbur, "Deterministic vortices evolving from partially coherent fields," Optica 10, 1173-1176 (2023)

Deterministic vortex beams (II)

Consider free space propagation of FracFT field:

$$U_\alpha(\mathbf{r}, z) = \int G(\mathbf{r}, \mathbf{r}')U_\alpha(\mathbf{r}')d^2r'$$

Combine FracFT and propagation steps:

$$H_\alpha(\mathbf{r}, \mathbf{r}'') = \int G(\mathbf{r}, \mathbf{r}')K_\alpha(\mathbf{r}', \mathbf{r}'')d^2r'$$

$$H_\alpha(\mathbf{r}, \mathbf{r}'') = \frac{e^{ikz}e^{-i\alpha}}{2\pi i\beta^2} e^{\frac{ik}{2z}r^2} e^{-\frac{i}{2\sigma^2}\cot\alpha r''^2} e^{\frac{i\gamma}{2}(\mathbf{r}''-\mathbf{r}/\gamma)^2/\beta^2}$$

$$\beta^2 \equiv \sigma^2\sin\alpha - \frac{z}{k}\cos\alpha \qquad \gamma \equiv \frac{z}{k\sigma^2\sin\alpha}$$

For a given α, the field we be restored to GSM “pure” vortex state at z distance:

$$\boxed{z = k\sigma^2\tan\alpha}$$

Free-space propagation results – short range

Full simulation results show that the deterministic vortex appears exactly where it is predicted:

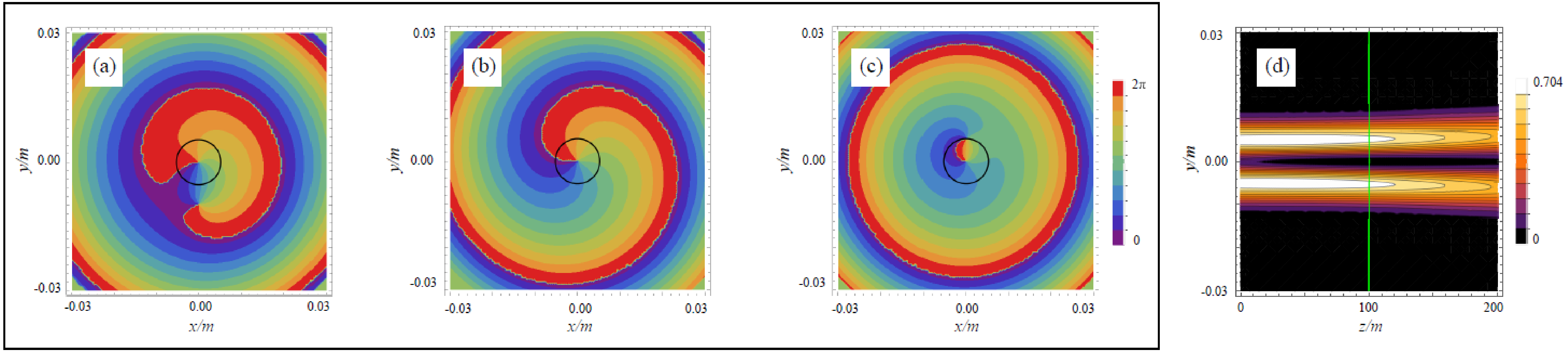


FIG. 1: Creating a Gaussian Schell-model vortex beam at $z_0 = 100m$. (a) z=77m, (b) z=100m, (c) z=121m; (d) intensity distribution along propagation. Here, in (a)-(c),the observation point r_1 is located at $(0.1mm, 0mm)$, $\alpha = 0.308, \sigma = 5 \times 10^{-3}m, \delta = 0.01m$. The black circle represents the range of beam width; in (d) the green line indicates the position where the deterministic vortex locates.

At other propagation distances, the vortex is a two-point correlation vortex, and appears with a singularity of opposite charge nearby

Self-focusing, depth of field

For very low starting spatial coherence, there is a "self-focusing" effect that becomes prominent around the position of the deterministic vortex.

The "depth of field" – the range over which the vortex appears effectively deterministic – decreases as the spatial coherence decreases.

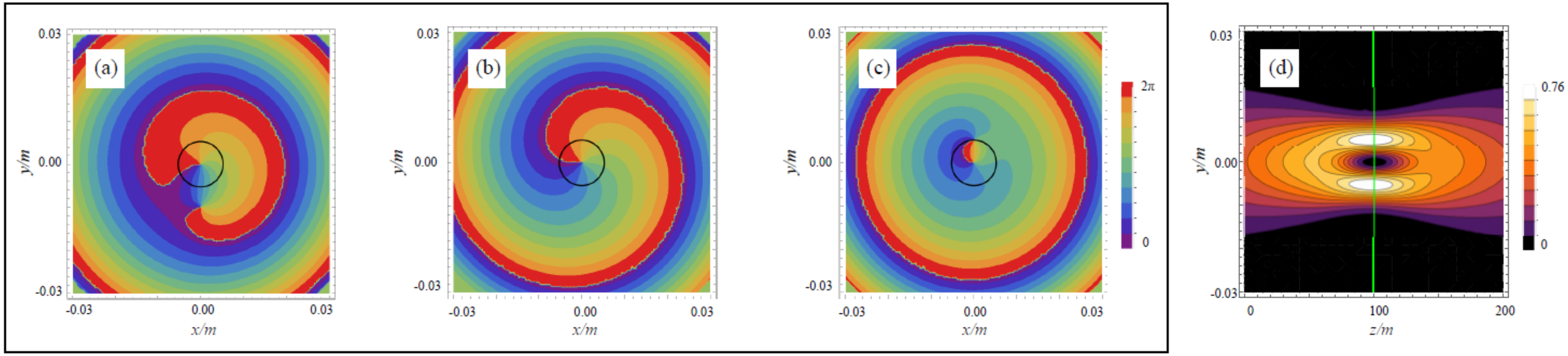


FIG. 3: Decrease the transverse correlation length δ to $0.001m$. (a) z=99.8m, (b) z=100m, (c) z=100.2m; (d) intensity distribution along propagation. Here, $\alpha = 0.308, \sigma = 5 \times 10^{-3}m$

Currently working on the properties of such beams when propagating in atmospheric turbulence.

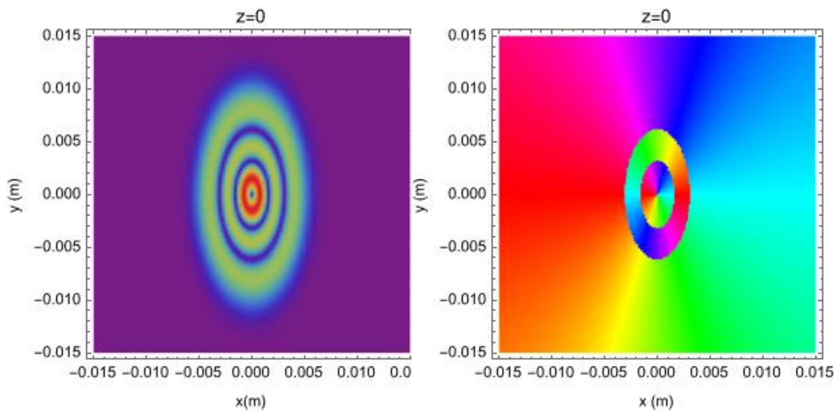
Elliptical basis beams

Can we make a beam that **starts** with a deterministic vortex and **ends** with a deterministic vortex?

- Elliptical LG beam

$$\psi_{nm}(x,y,a) = \left(\frac{\sqrt{2(x^2+a^2y^2)}}{\sigma_s}\right)^{|m|} L_n^{|m|}\left(\frac{2\,x^2+2\,a^2y^2}{\sigma_s^2}\right)$$
$$e^{-(x^2+a^2y^2)/\sigma_s^2}\, e^{im\tan^{-1}\left(\frac{ay}{x}\right)}$$

Elliptical LG_{21} $\sigma_s = 2\mathrm{mm}, a = 0.5$



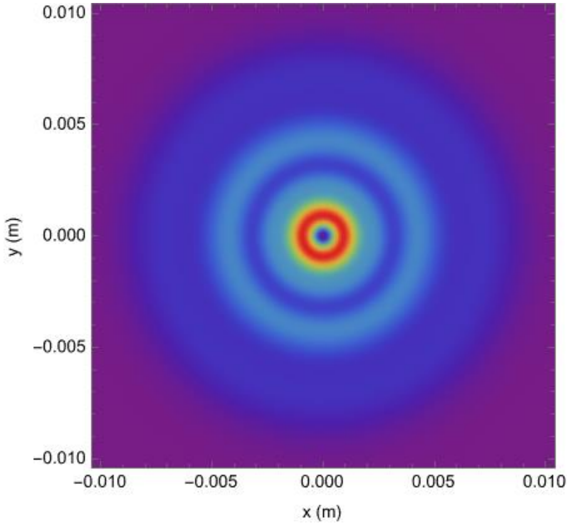
An elliptical Laguerre-Gauss beam will start in the source plane with a vortex at the center, and will evolve a complicated structure, and will eventually become an elliptical Laguerre-Gauss beam in the far zone that is rotated 90 degrees!

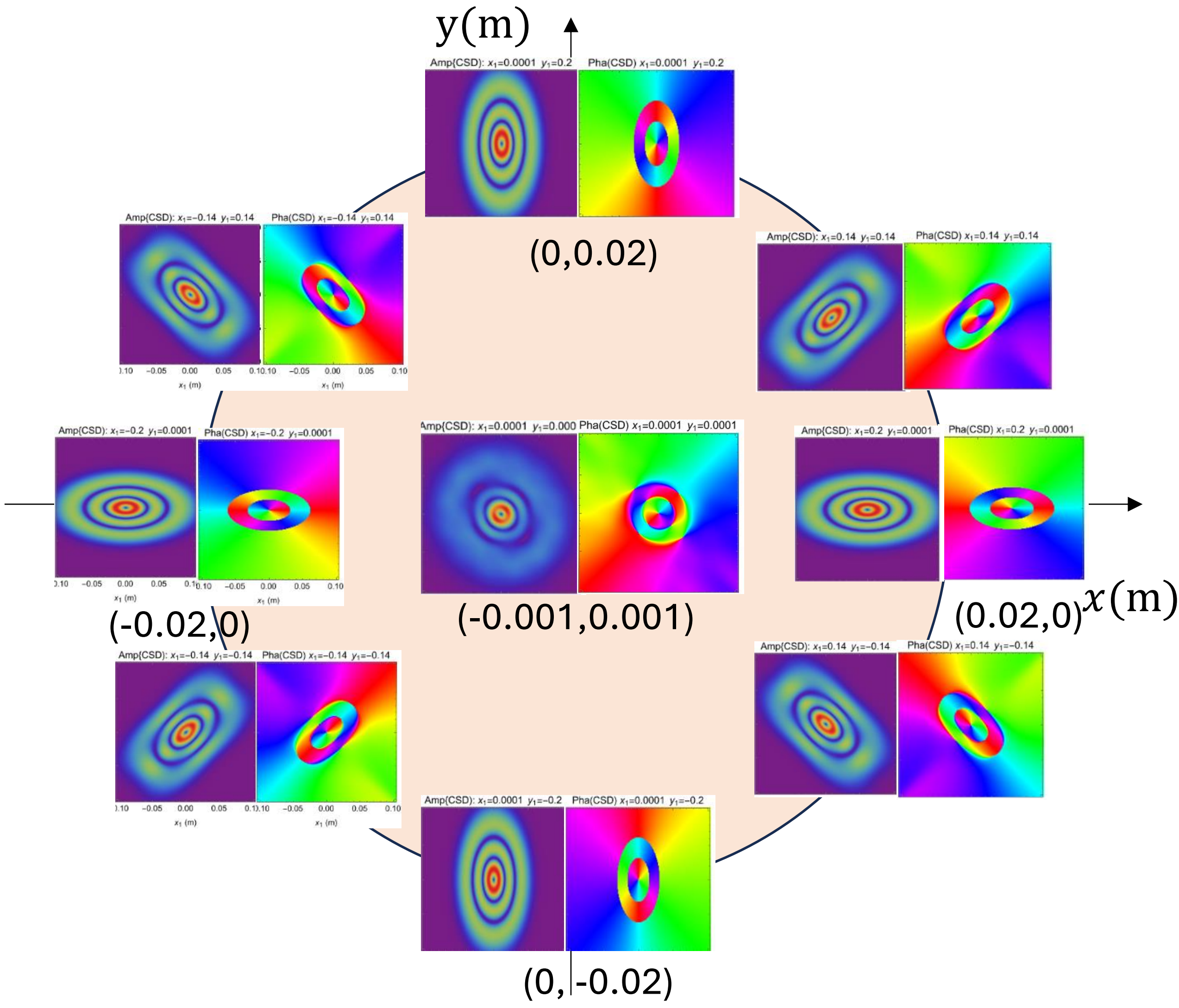
Construct a deterministic vortex beam from an incoherent superposition of all orientations!

Victor V. Kotlyar, Svetlana N. Khonina, Anton A. Almazov, Victor A. Soifer, Konstantins Jefimovs, and Jari Turunen, "Elliptic Laguerre-Gaussian beams," J. Opt. Soc. Am. A 23, 43-56 (2006)

Cross-spectral density at source: LG_{21}

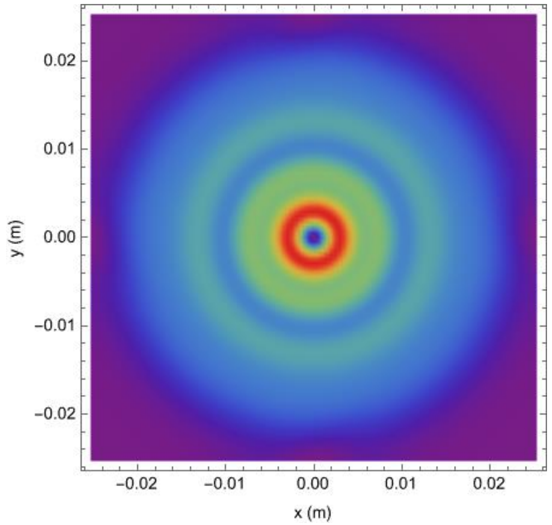
- $z = 0$ m
- Spectral density
  - Rotationally symmetric.
- CSD under fixed observing points
  - Vortex structure is intact under all observing points.

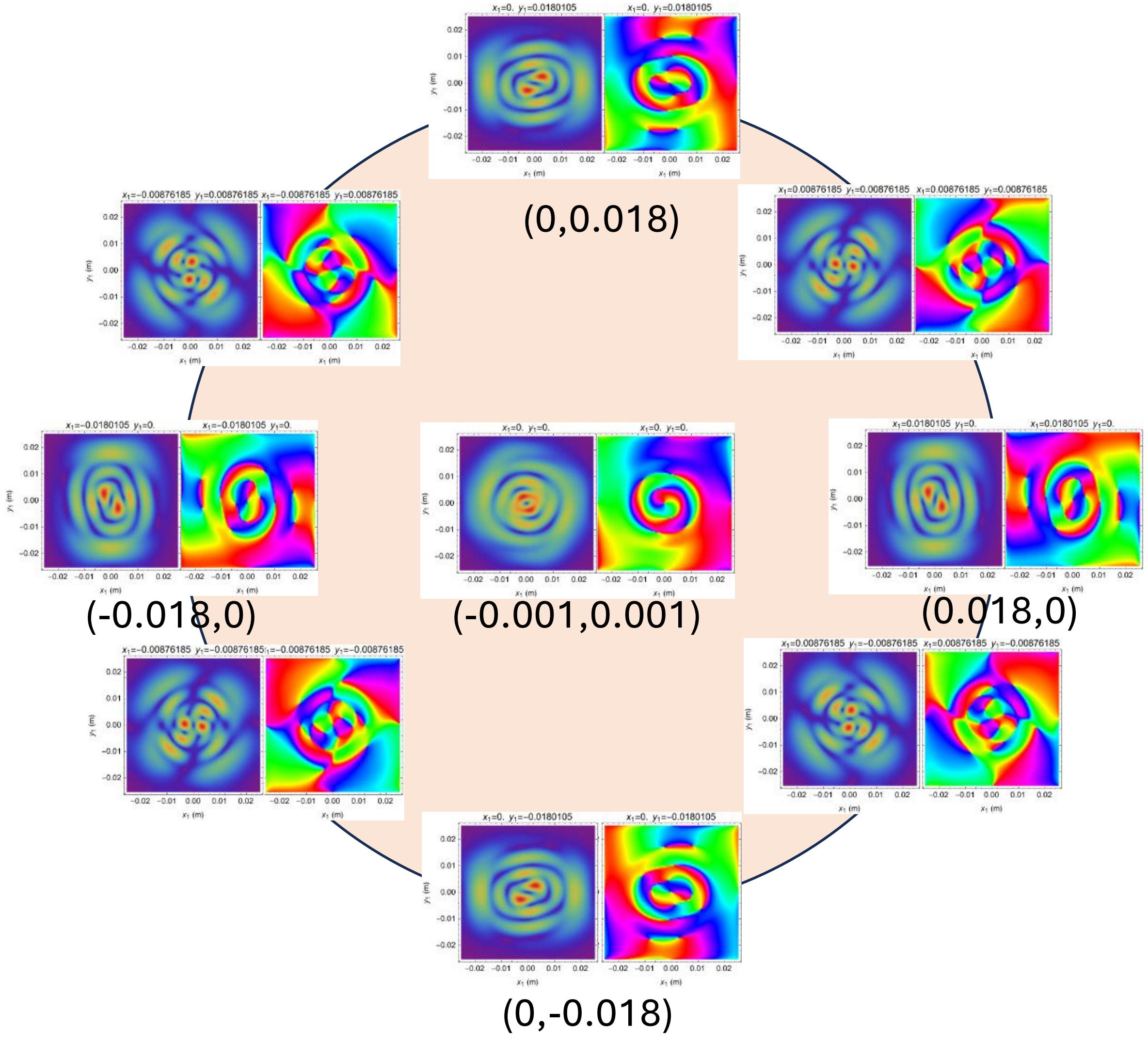




Cross-spectral density on propagation: LG_{21}

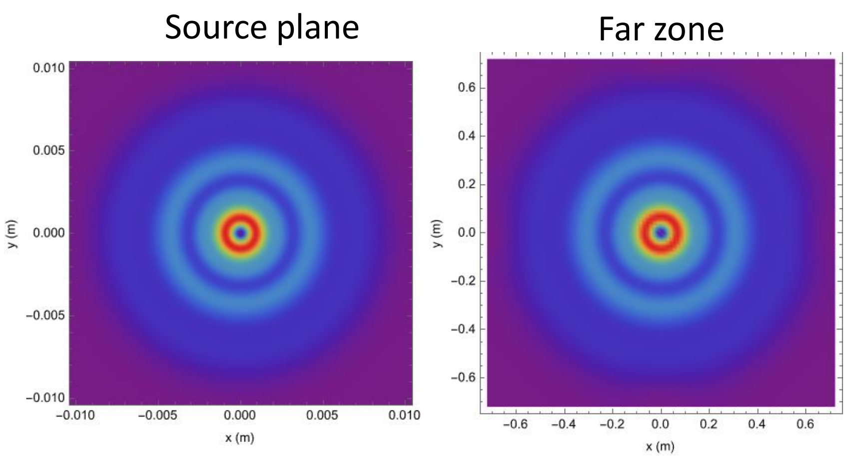
- $z = 100\ \mathrm{m}$
- Spectral density
  - Smoother distribution in radial direction.
- CSD under fixed observing points
  - Central vortex splits and shows random structure.

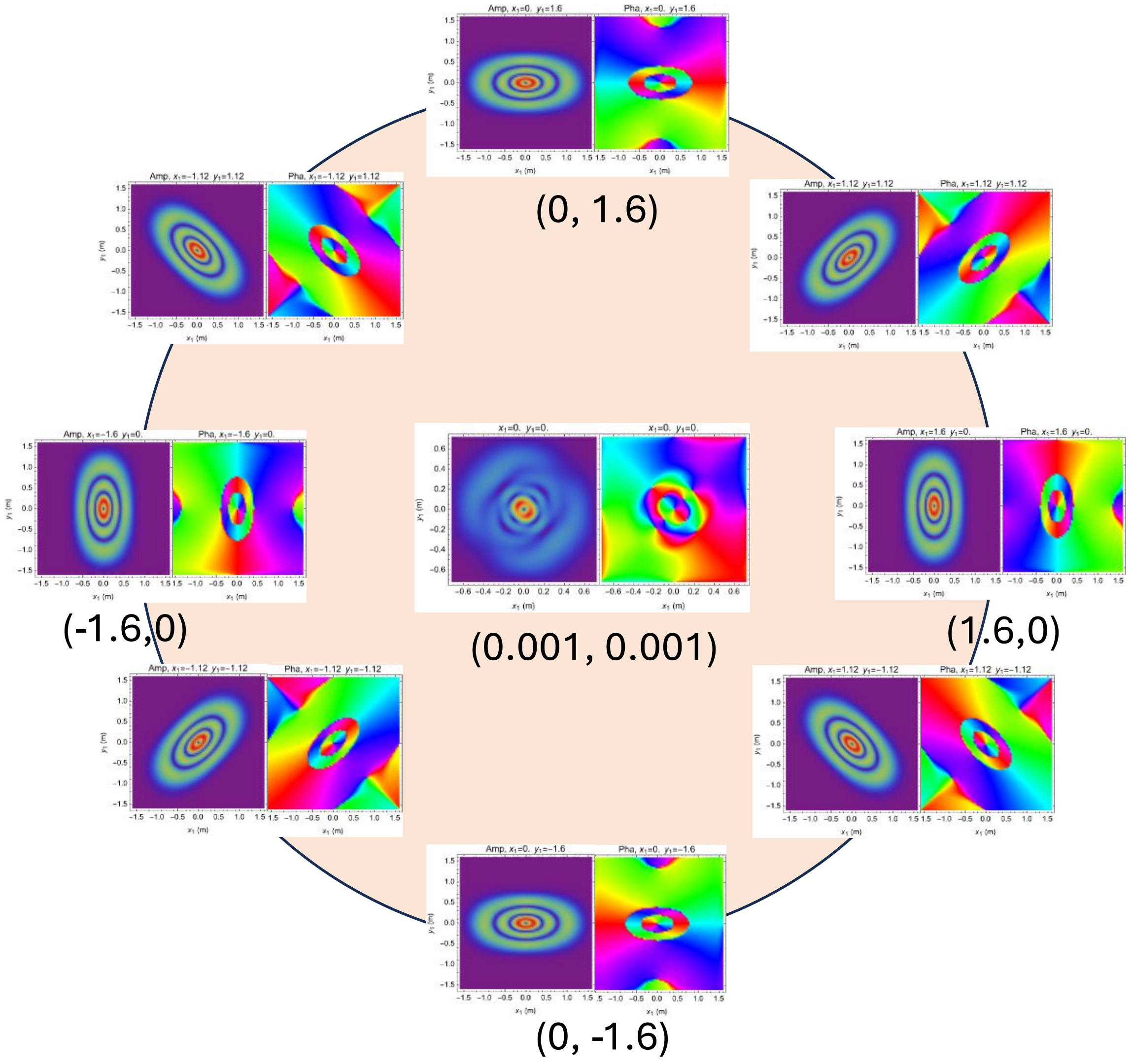




Cross-spectral density in far zone: LG_{21}

- $z = 5000\ \mathrm{m}$
- Spectral density
  - Scaled version of the source plane spectral density.
- CSD under fixed observing points
  - Vortex phase structures are intact.
  - Phase rotates 90° from the source plane, under all observing points.





Deterministic vortex revivals

Can we make a beam for which the deterministic vortex (at least approximately) revives itself periodically? Using Bessel beams, apparently we can!

Let us consider the set of Bessel beams:

$$U_m(\mathbf{r}, z; k_t) = J_m(k_t r) e^{im\phi} e^{ik_z z}$$

where $k_z = \sqrt{k_0^2 - k_t^2}$

First, construct a coherent Bessel beam superposition (quasi-coherent mode) that periodically vanishes at the transverse origin, e.g.:

$$U_{coh}(\mathbf{r}, z) = U_0(\mathbf{r}, z; k_{t1}) - U_0(\mathbf{r}, z; k_{t2}) \qquad k_{t1} \neq k_{t2}$$

Constructing the revival beam

$$U_{coh}(\mathbf{r}, z) = U_0(\mathbf{r}, z; k_{t1}) - U_0(\mathbf{r}, z; k_{t2})$$

This field will periodically have zeros at the transverse origin when

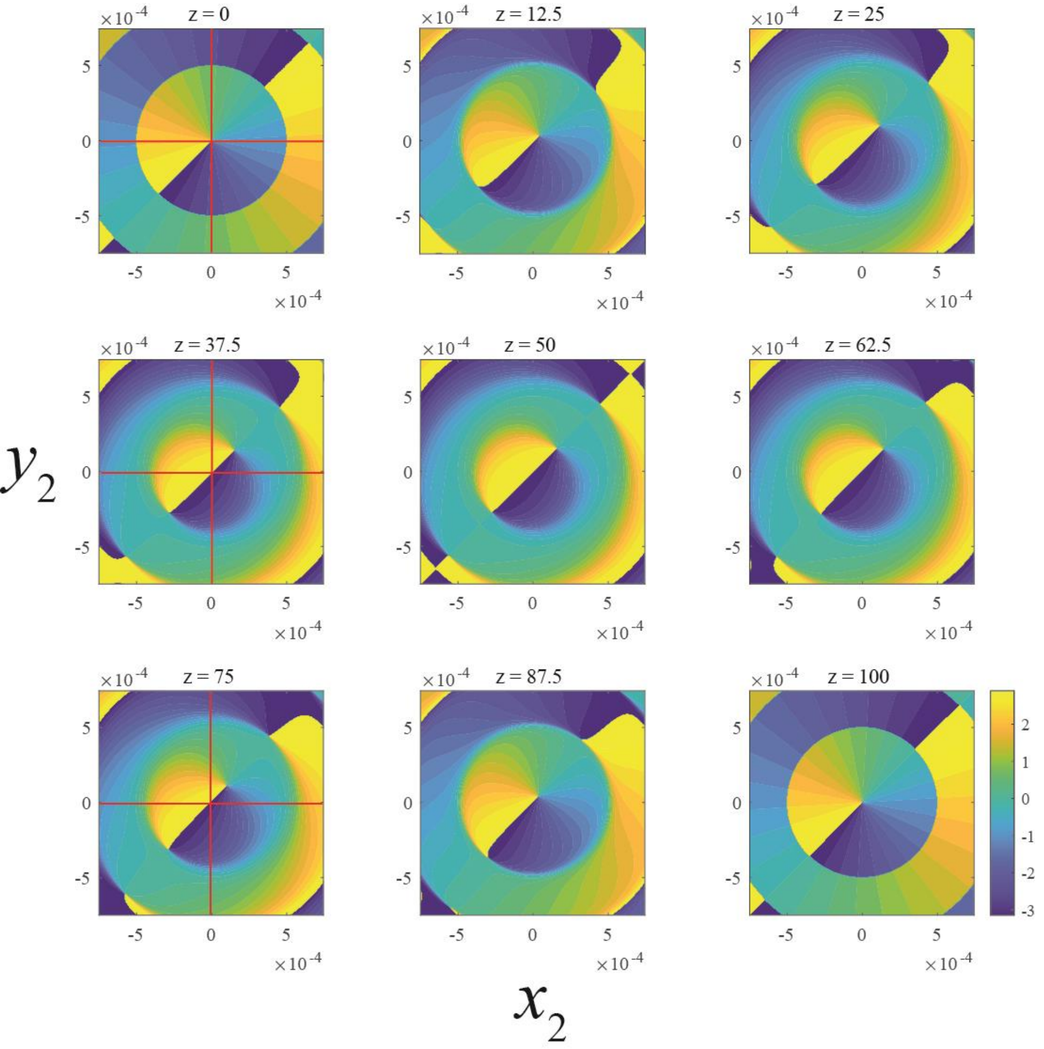
$$(k_{z1} - k_{z2})z = 2\pi n$$ with n an integer

Now we take an incoherent superposition of this beam with a Bessel vortex beam, i.e.

$$W(\mathbf{r}_1, \mathbf{r}_2, z) = U^*_{coh}(\mathbf{r}_1, z)U_{coh}(\mathbf{r}_2, z) + U^*_m(\mathbf{r}_1, z; k_{t3})U_m(\mathbf{r}_2, z; k_{t3})$$

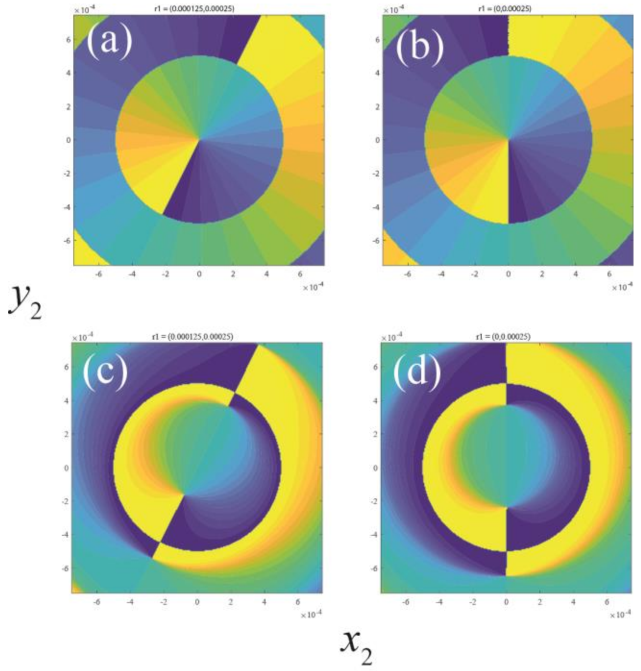
At the special *z*-values, the coherent part of the beam will vanish at the transverse origin and the beam will then manifest a pure intensity zero and *m*th order vortex.

Example of beam revivals



Evolution of the phase of the cross-spectral density with m = 1, w = 0.5 mm , λ = 500 nm , z_r = 100 m, and r_1 = (0.5w, 0.5w). Blue indicates a phase of −π, while yellow indicates +π. Red lines have been added to the first column to show the position of the central beam axis.

Dependence of the phase of the cross-spectral density on the choice of observation point, for (a) z = 0, r1 =(0.25w, 0.5w), (b) z = 0, r1 = (0, 0.5w), (c) z = 50 m,r1 = (0.25w, 0.5w), (b) z = 50 m, r1 = (0, 0.5w).



R. Qi and G. Gbur, “Correlation Vortices in Both the Source and the Far-zone of A Partially Coherent Elliptical Laguerre-Gauss Beams,” to be submitted to Opt. Lett.

Summary

- Partially coherent vortex fields are typically expected to lose their deterministic vortex structure on propagation
- However, we can construct multiple beams where this is not the case!
- Circularly coherent and separable phase beams maintain a deterministic vortex for all propagation distances
- Rankine vortex beams evolve deterministic vortex in far zone
- “Deterministic vortex beams” can be designed to manifest a deterministic vortex at any propagation distance
- “Elliptical basis beams” can be constructed to manifest a deterministic vortex at source and in far zone
- “Deterministic vortex revivals” can be designed using Bessel beams to manifest deterministic vortices periodically on propagation!
- **Plans: put all of them through turbulence to test their robustness!**