Laser Propagation in Reactive Media
Existence and Computation

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Research Group

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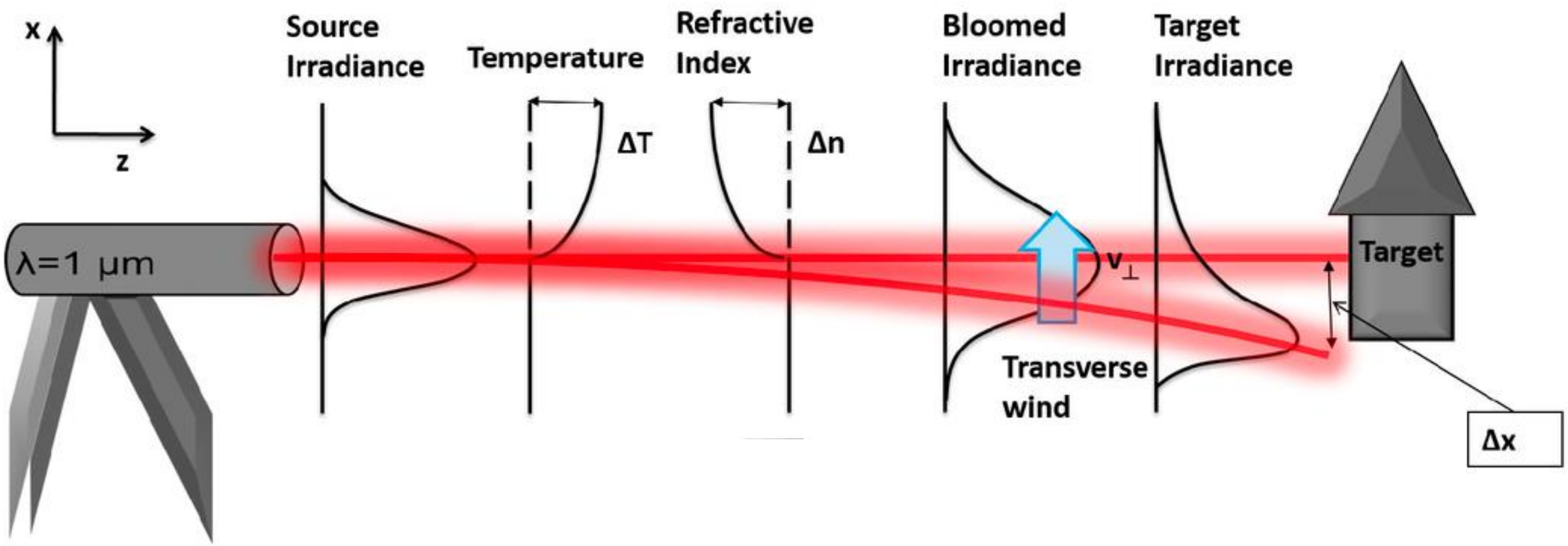
Physical Problem

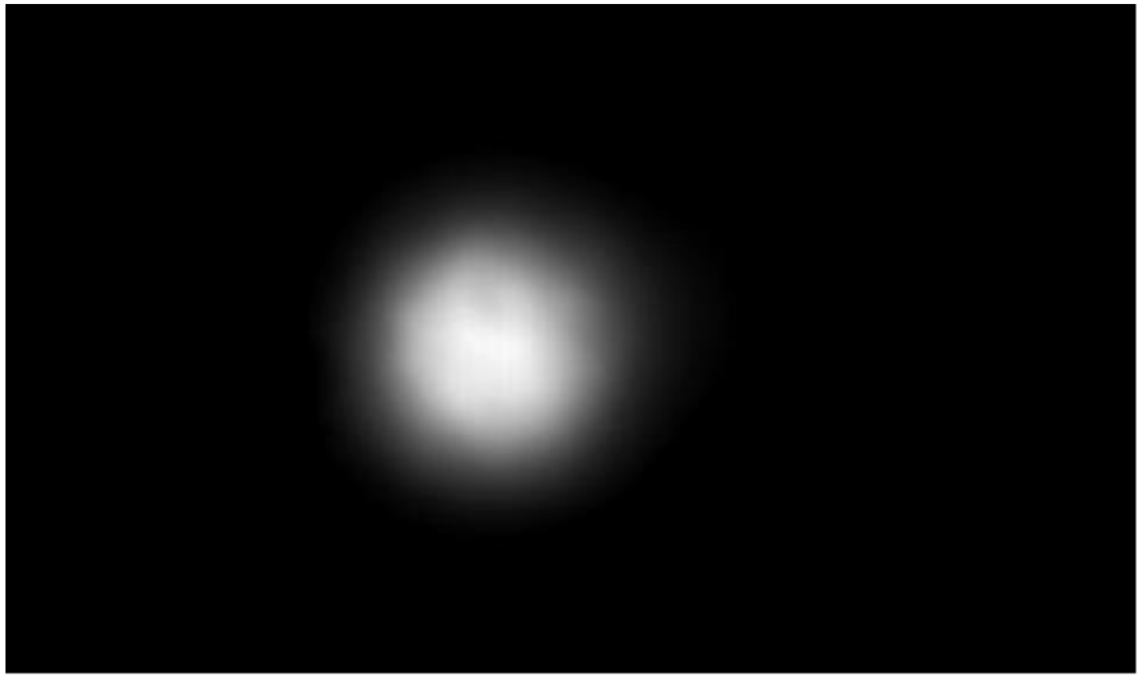


(Spencer, 2020)

Prescribed velocity: Smith 1970's, Gebhardt 1990's, Sprangle, Penano 2000's

Experiments

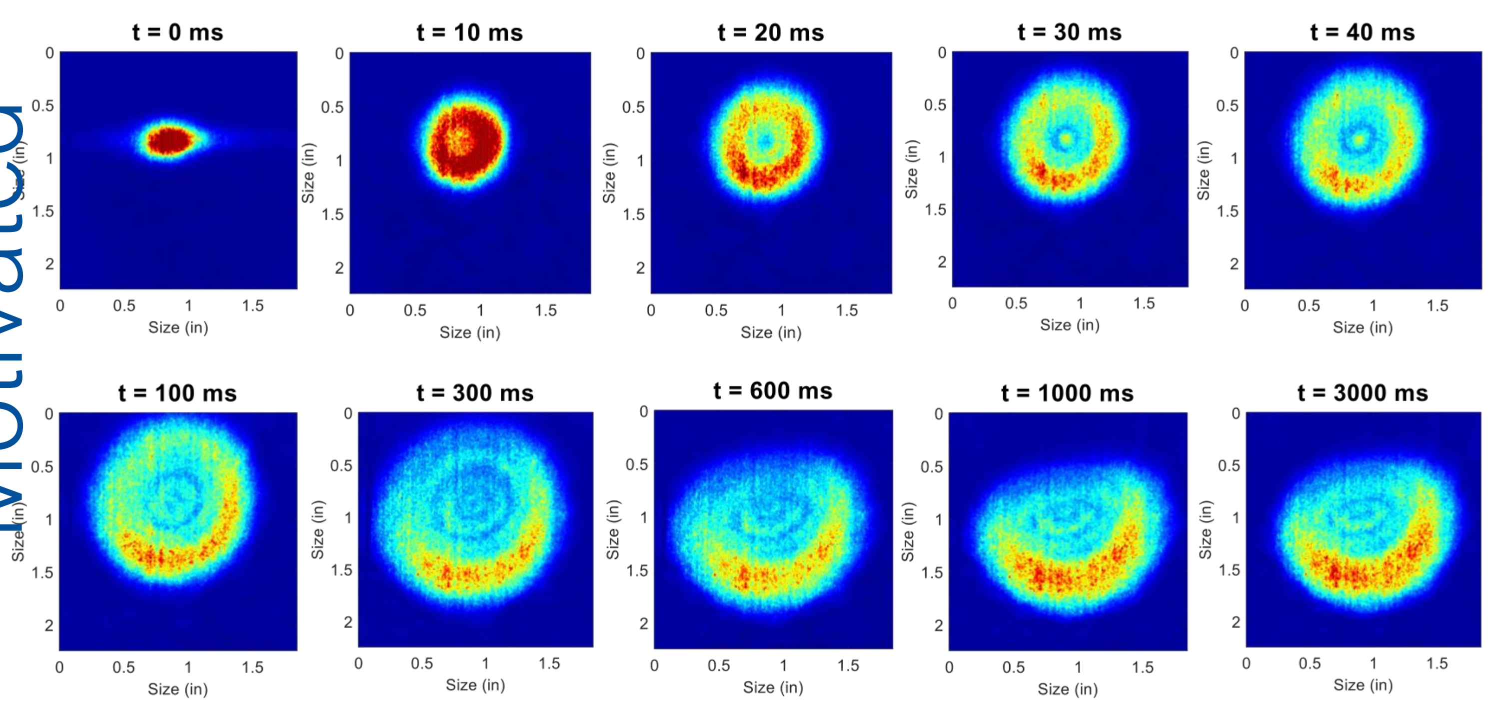
Motivating



Wick and Lloyd, (2010)

- 1cm spot (initially)
- 3m path
- 300W
- $1.07\mu m$
- Smoke

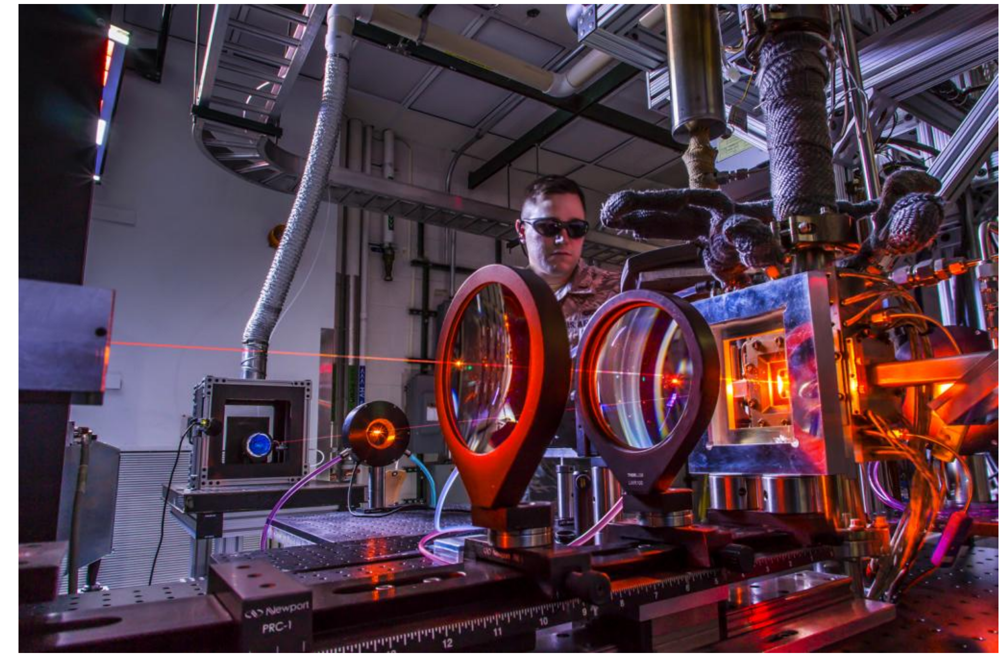
Motivated



Cook and Richardson, (2020)

- 1cm spot (initially)
- 5m path
- 5W
- 1944.867nm
- 50% humidity

Laser Equations



Maxwell + $E = \tilde{E}(x, y, z)\exp(i\omega t)$

↓

Helmholtz + $\tilde{E} = A(\epsilon x, \epsilon y, \epsilon^2 z)\exp(ikz)$
$n = n_0 + \epsilon^2 n_1$

↓

Paraxial

$$iA_Z + \frac{1}{2k}(A_{XX} + A_{YY}) + 2n_0 n_1(T)A = 0$$

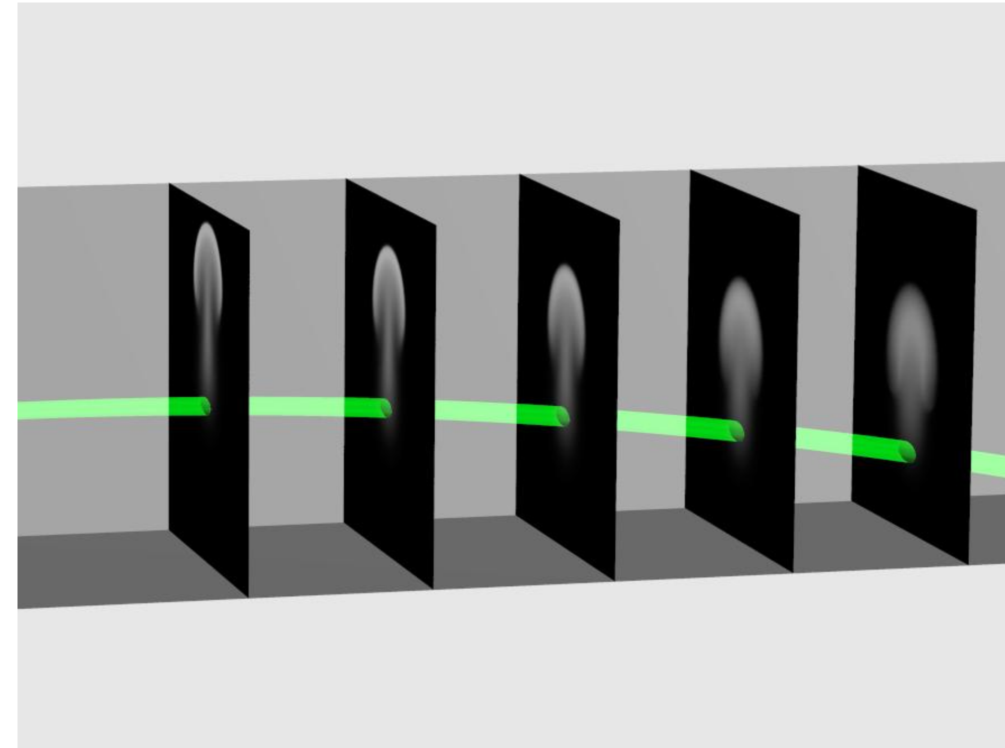
$$A(X, Y, 0) = f(X, Y)$$

Fluid Equations

$$\vec{u}_t + (\vec{u} \cdot \nabla)\vec{u} = -\nabla p + \frac{1}{Re}\Delta \vec{u} + \frac{1}{Fr^2} T e_2$$

$$\nabla \cdot \vec{u} = 0$$

$$T_t + (\vec{u} \cdot \nabla) T = \frac{1}{Pe}\Delta T + St|A|^2$$



$$Re = \frac{UL}{\nu}, Pe = \frac{UL}{\eta}, Fr^2 = \frac{U^2}{gL}$$

$$St = \frac{V_0^2 \beta L}{T_0}$$

Flow created by laser $\longrightarrow$

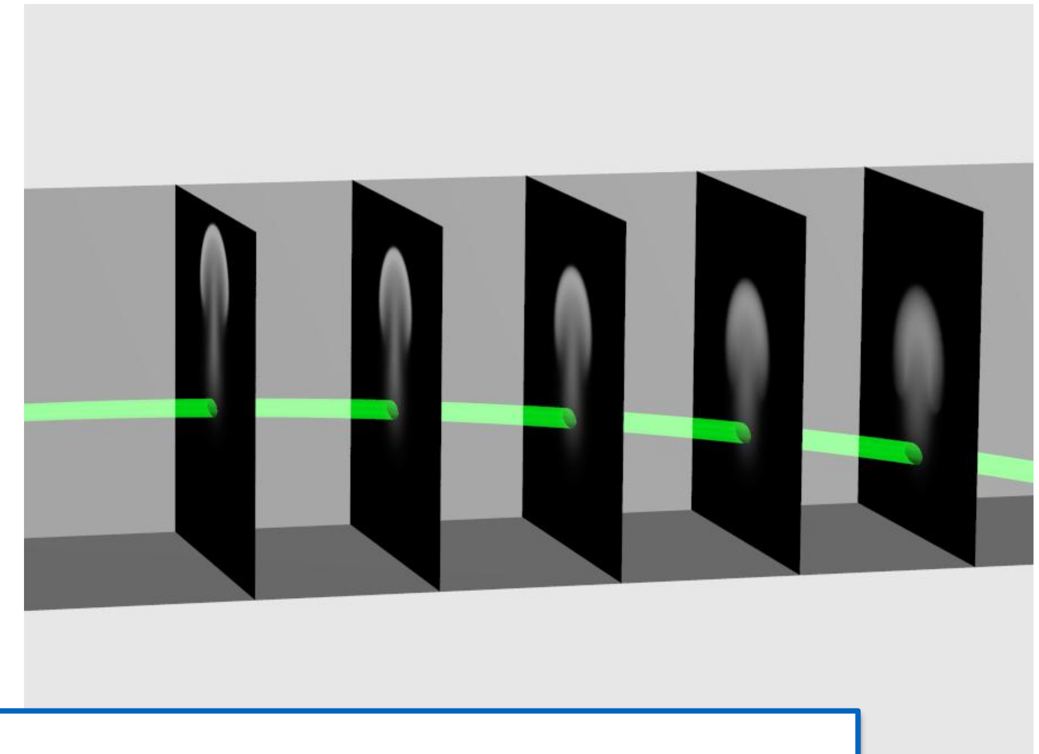
- $L =$ *Beam Diameter*
- $\partial_z u = O(\epsilon^2)$
- *Quiescent initial conditions*

Fluid Equations (Part II)

$$\omega_t + \psi_y \omega_x - \psi_x \omega_y = \frac{1}{Re}\Delta\omega + \frac{1}{Fr^2}T_x$$

$$T_t + \psi_y T_x - \psi_x T_y = \frac{1}{Pe}\Delta T + St|A|^2$$

$$\Delta\psi = \omega$$



Stream Function $\quad \psi_y = u_1, \psi_x = -u_2$

Vorticity $\quad \omega = u_{2,x} - u_{1,y}$

Flow created by laser →

- $\partial_z u = O(\epsilon^2)$
- *Quiescent initial conditions*

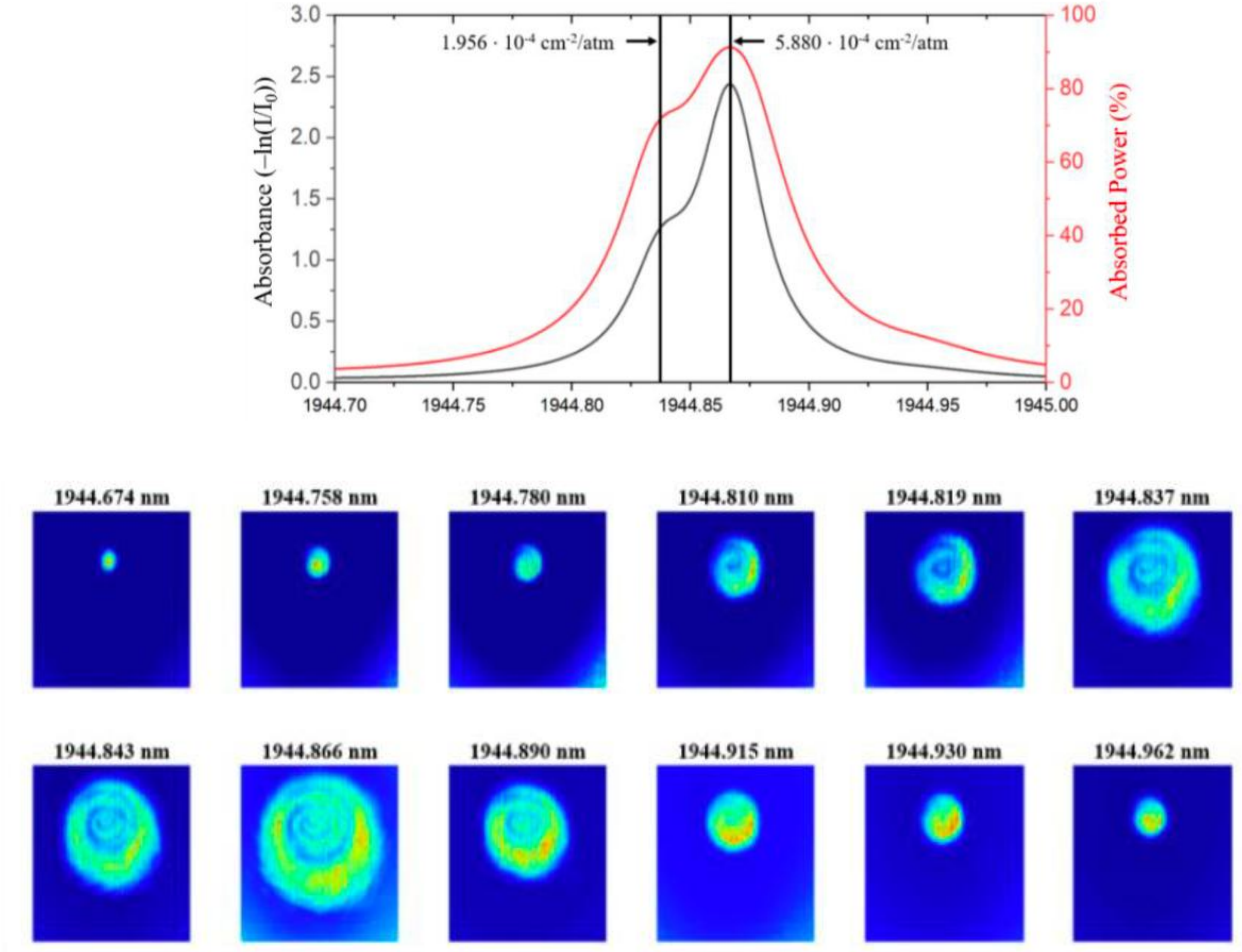
Initial Data vs Forcing

This work

Large Initial Flow

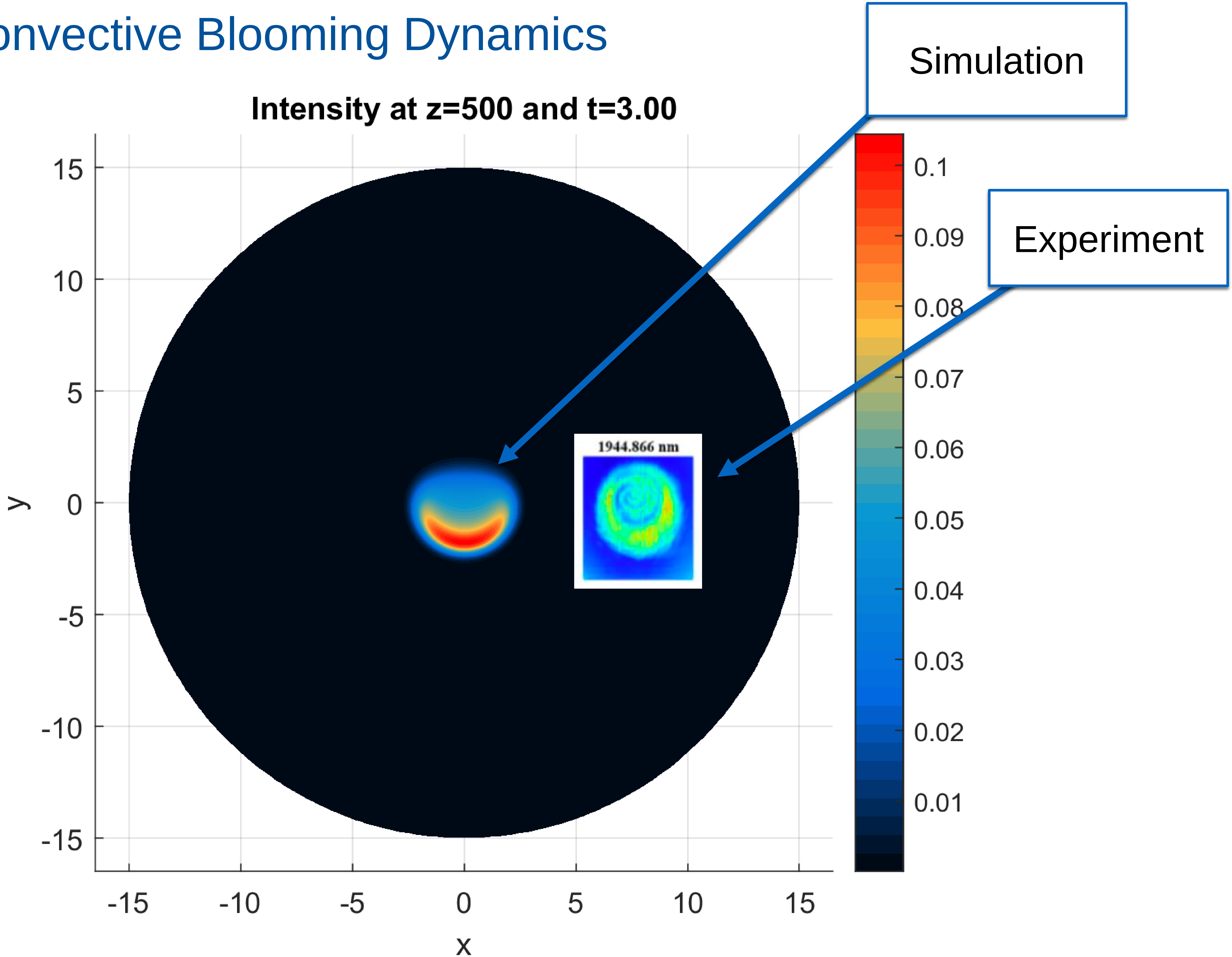
Large Energy Deposition

Experiments at UCF †

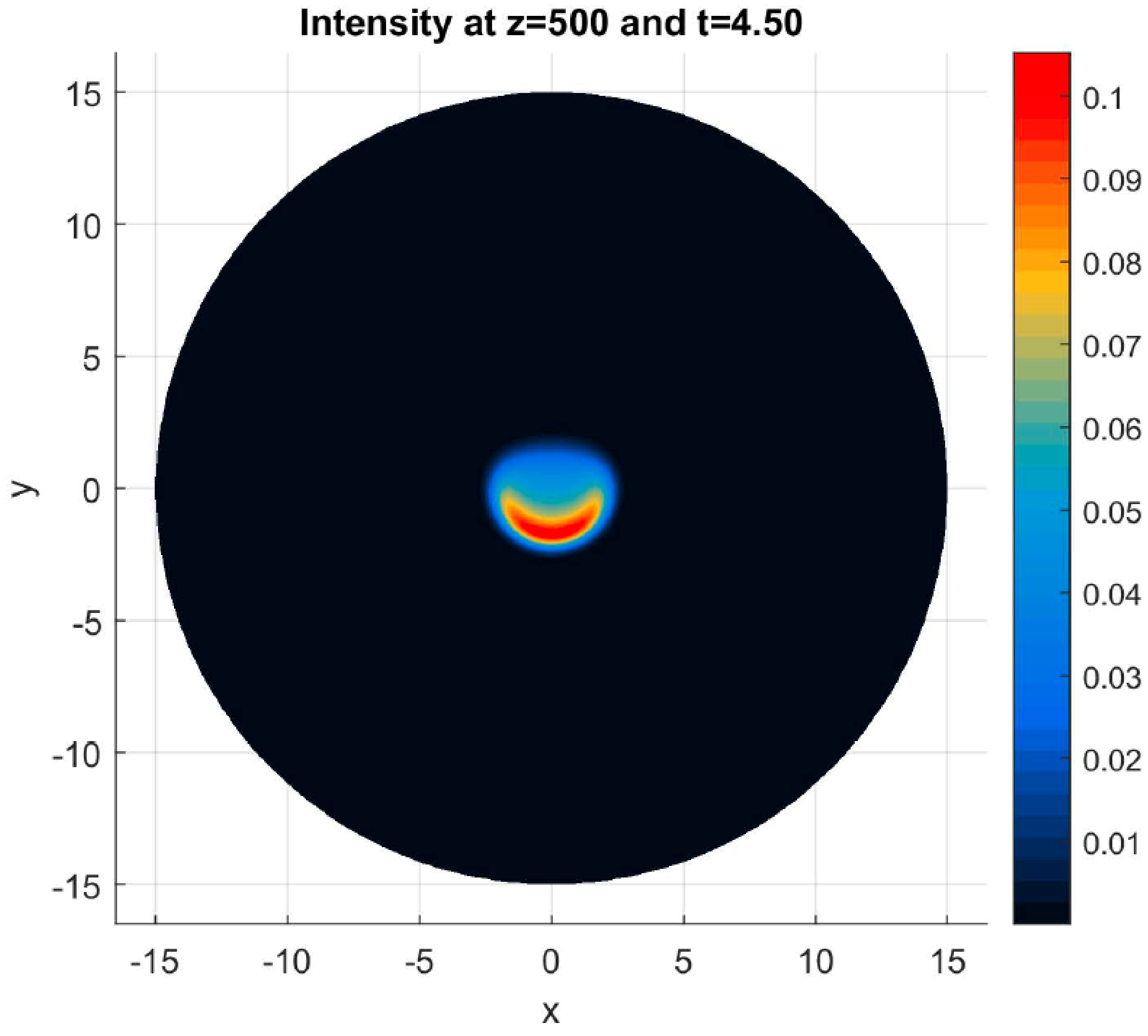


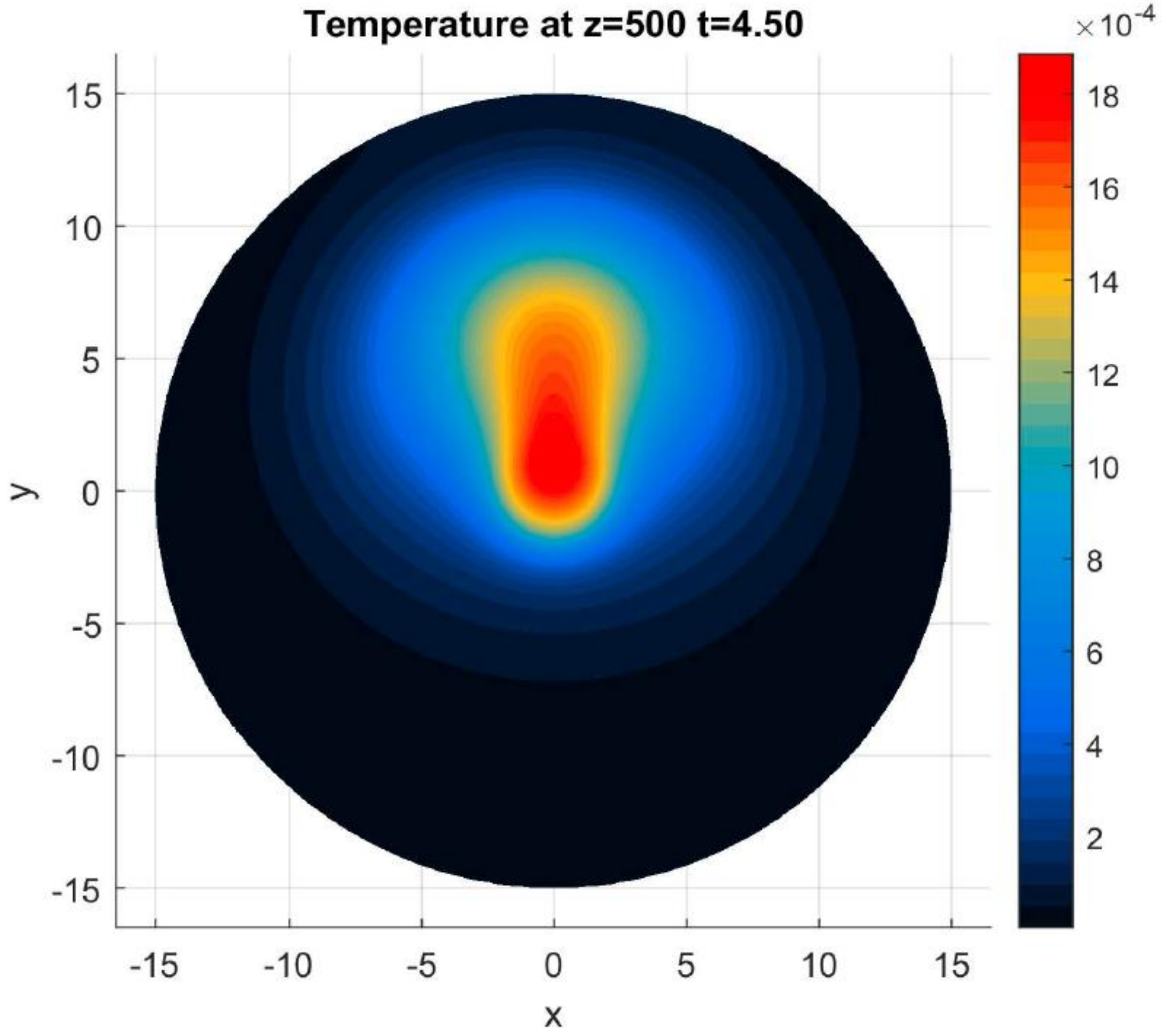
†Cook, et. al., "Narrow line width 80W tunable thulium-doped fiber laser", *Optics and Laser Tech.* (2022)

Convective Blooming Dynamics



Convective Blooming: Time Dynamics





Steady State Convective Blooming

$$X_t = F(X) \Rightarrow F(X) = 0$$

$$\psi_y \omega_x - \psi_x \omega_y - \frac{1}{Re}\Delta\omega - \frac{1}{Fr^2}T_x = 0$$

$$\psi_y T_x - \psi_x T_y - \frac{1}{Pe}\Delta T - St|A|^2 = 0$$

$$\Delta\psi = \omega$$

$$iA_Z + \frac{1}{2k}\Delta_\perp A + 2n_0 n_1(T)A = 0$$

$$\Rightarrow F(\psi, \omega, T, A) = 0$$

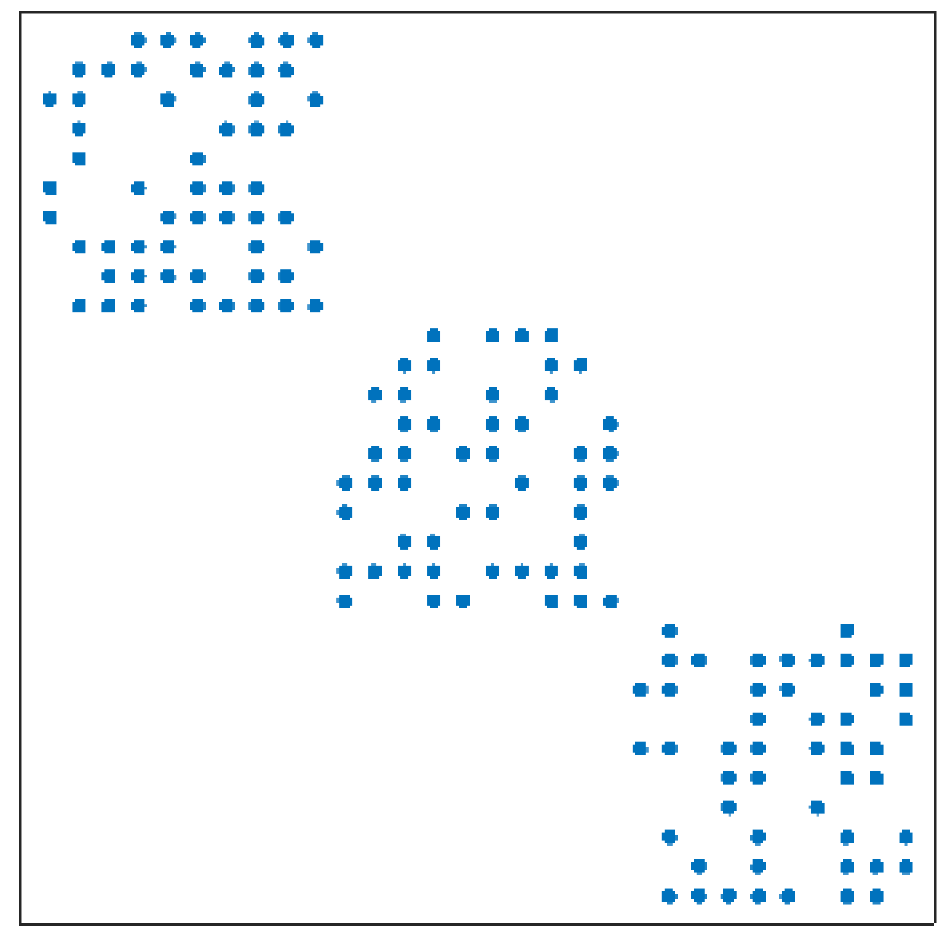
A Fixed Point Iteration for St=ϵ

$$\frac{1}{Pe}\Delta T_{n+1} = \epsilon\left(\psi_{n,y}T_{n,x} - \psi_{n,x}T_{n,y} - |A|^2\right)$$

$$\frac{1}{Re}\Delta \omega_{n+1} = \epsilon\left(\psi_{n,y}\omega_{n,x} - \psi_{n,x}\omega_{n,y}\right) - \frac{1}{Fr^2}T_{n,x}$$

$$\Delta\psi_{n+1} = \omega_n$$

$$A_0 x_n = b_n$$



- A_0 inverted only 1x
- A_0 is block (and sparse)
- Cost: $O(M^2 \log M)$
- Computations for $\epsilon < \epsilon_*$
- Existence by contraction mapping

** Lane & Akers, "Two-Dimensional Steady Boussinesq Convection: Existence, Computation and Scaling", Fluids, (2022)*

Stokes Expansion

$$T = \sum_{n=1}^{\infty} \epsilon^n T_n(x,y)$$

$$\omega = \sum_{n=1}^{\infty} \epsilon^n \omega_n(x,y)$$

$$\psi = \sum_{n=1}^{\infty} \epsilon^n \psi_n(x,y)$$

$$A_0 x_n = \tilde{b}_n$$

$$\left(\tilde{b}_n\right)_1 = Pe \sum_{\ell=1}^{n-1} (\partial_y \psi_\ell \partial_x T_{n-\ell} - \partial_x \psi_\ell \partial_y T_{n-\ell})$$

$$\left(\tilde{b}_n\right)_2 = Re[-Ri\partial_x T_n + + \sum_{\ell=1}^{n-1} (\partial_y \psi_\ell \partial_x \omega_{n-\ell} - \partial_x \psi_\ell \partial_y \omega_{n-\ell})]$$

$$\left(\tilde{b}_n\right)_3 = -\omega_n$$

- Same A_0 as fixed point
- Cost of $\tilde{b}_n$ > Cost of b_n
- Compute corrections once for all $\epsilon < \epsilon^\dagger$

Stokes Expansion

$$T = \sum_{n=1}^{\infty} \epsilon^n T_n(x, y)$$

Parametric Analyticity

$$\| T_n \|_{H^2} \leq C_1 \frac{D^{n-2}}{(n+1)^2}$$

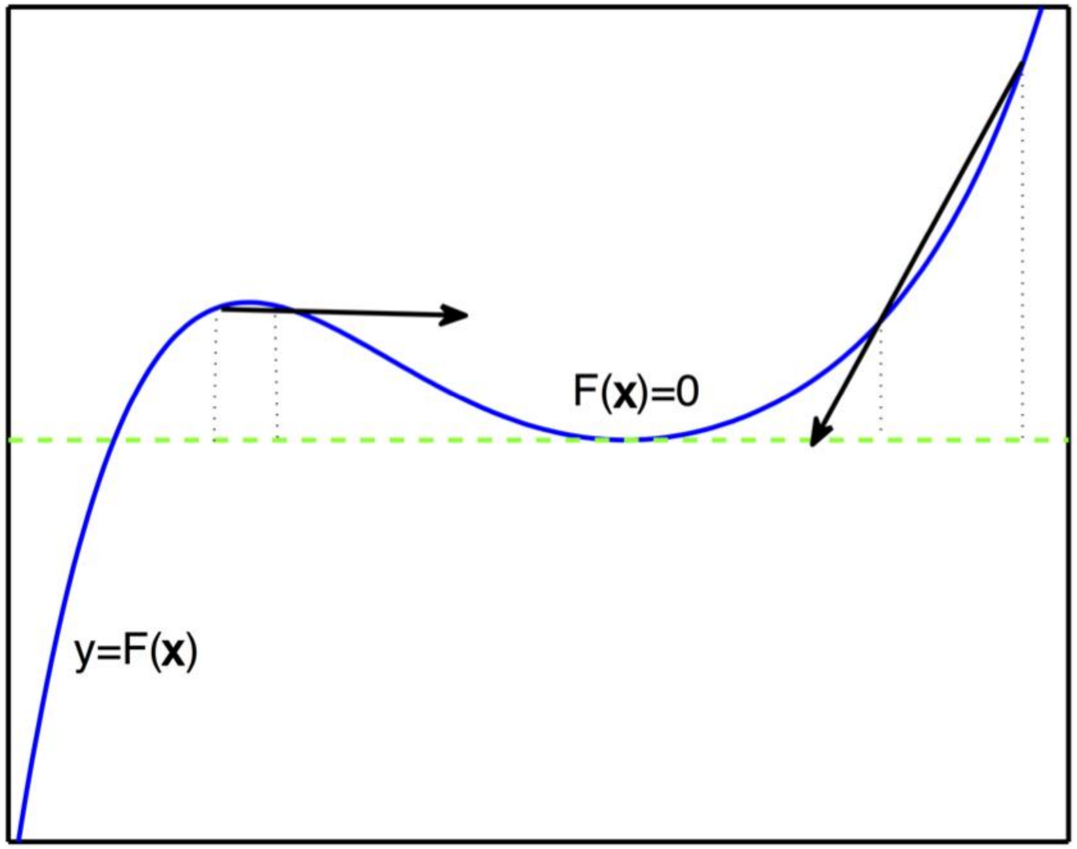
Padè Approximates

$$T(x, y, \epsilon) = \frac{\sum_n \epsilon^n p_n(x, y)}{\sum_n \epsilon^n q_n}$$

- Analytic for $\epsilon < \epsilon^{\dagger} = \frac{1}{D}$
- Extends to first real pole of a Padè approximate
- Trefethen’s SVD algorithm for p_n, q_n

Newton Iteration

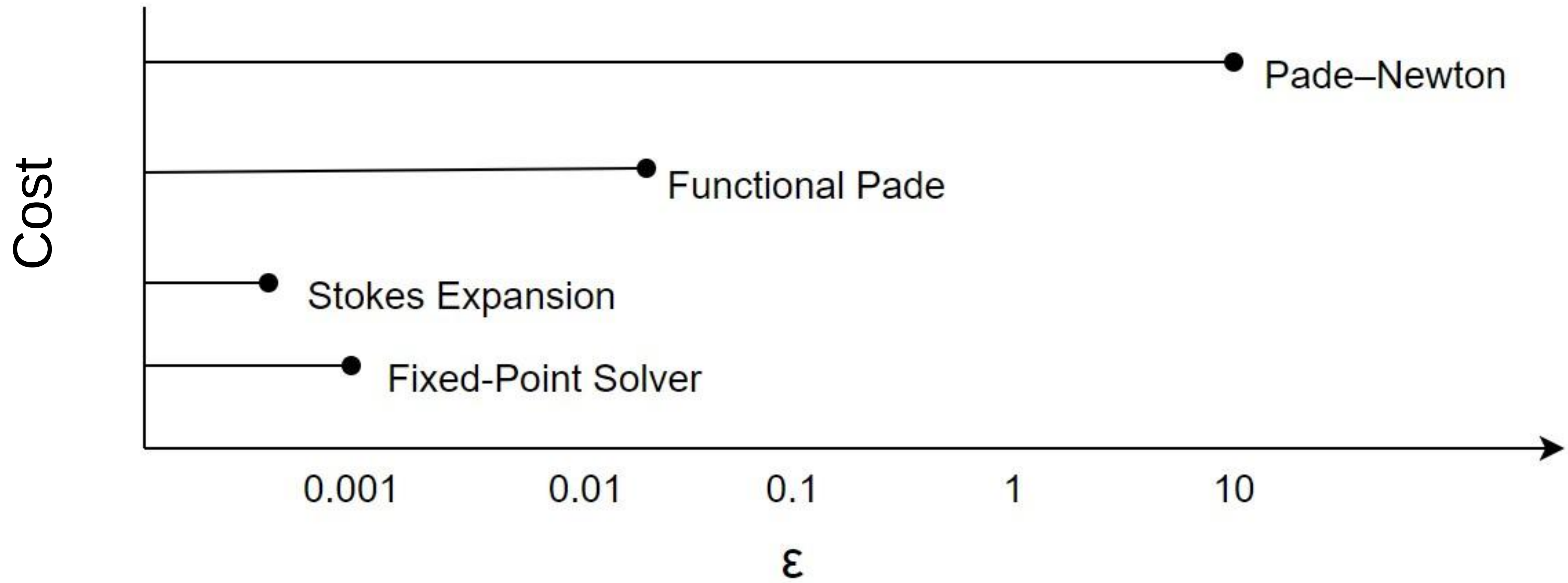
$$X_{n+1} = X_n - J(x_n)^{-1}F(x_n)$$



- Use Padè for initial guess
- Permuted Jacobian has bandwidth $3M$,

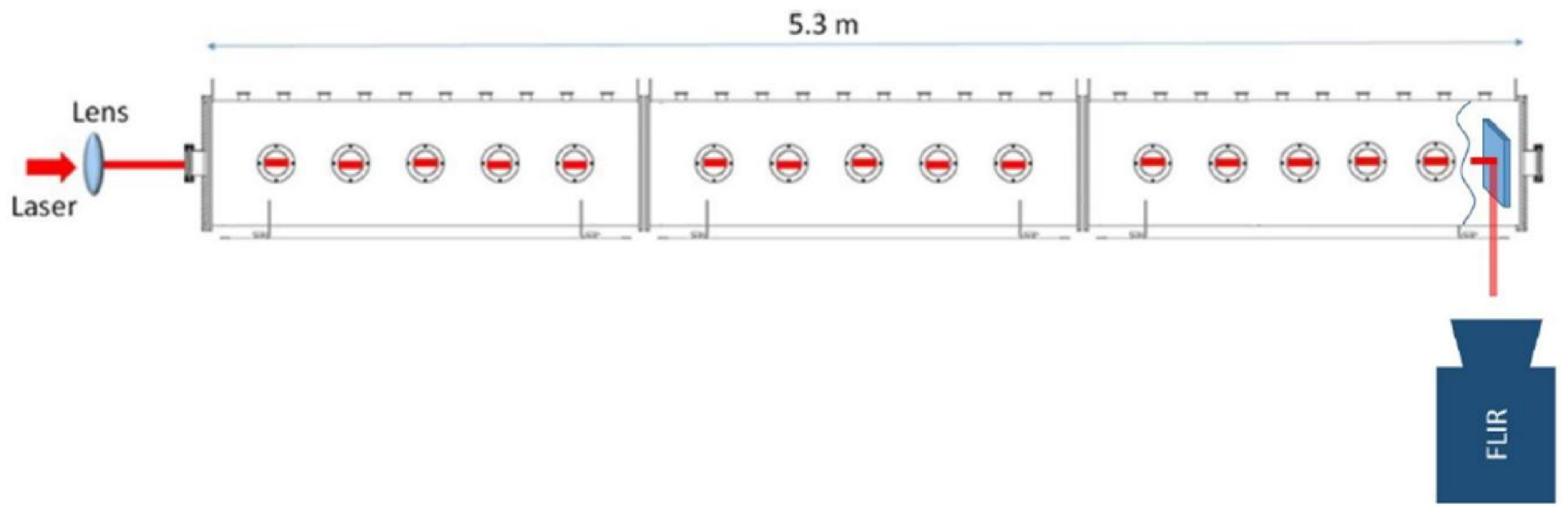
  $(M = N_x = N_Y)$
- Cost: $O(M^4) \ll O(N_x^3 N_y^3) = O(M^6)$

Method Comparison

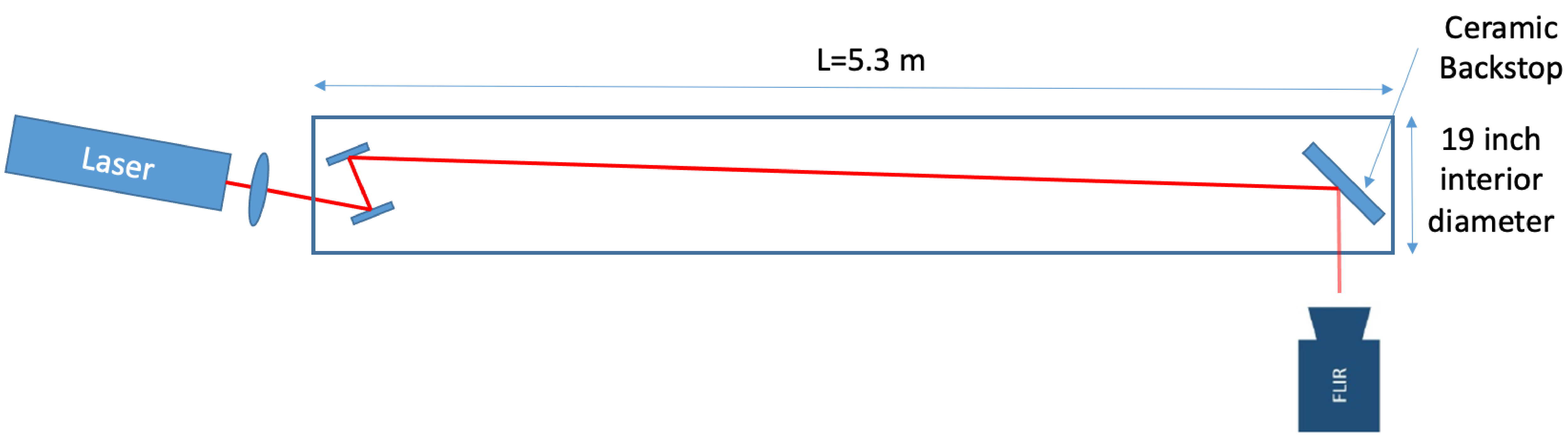


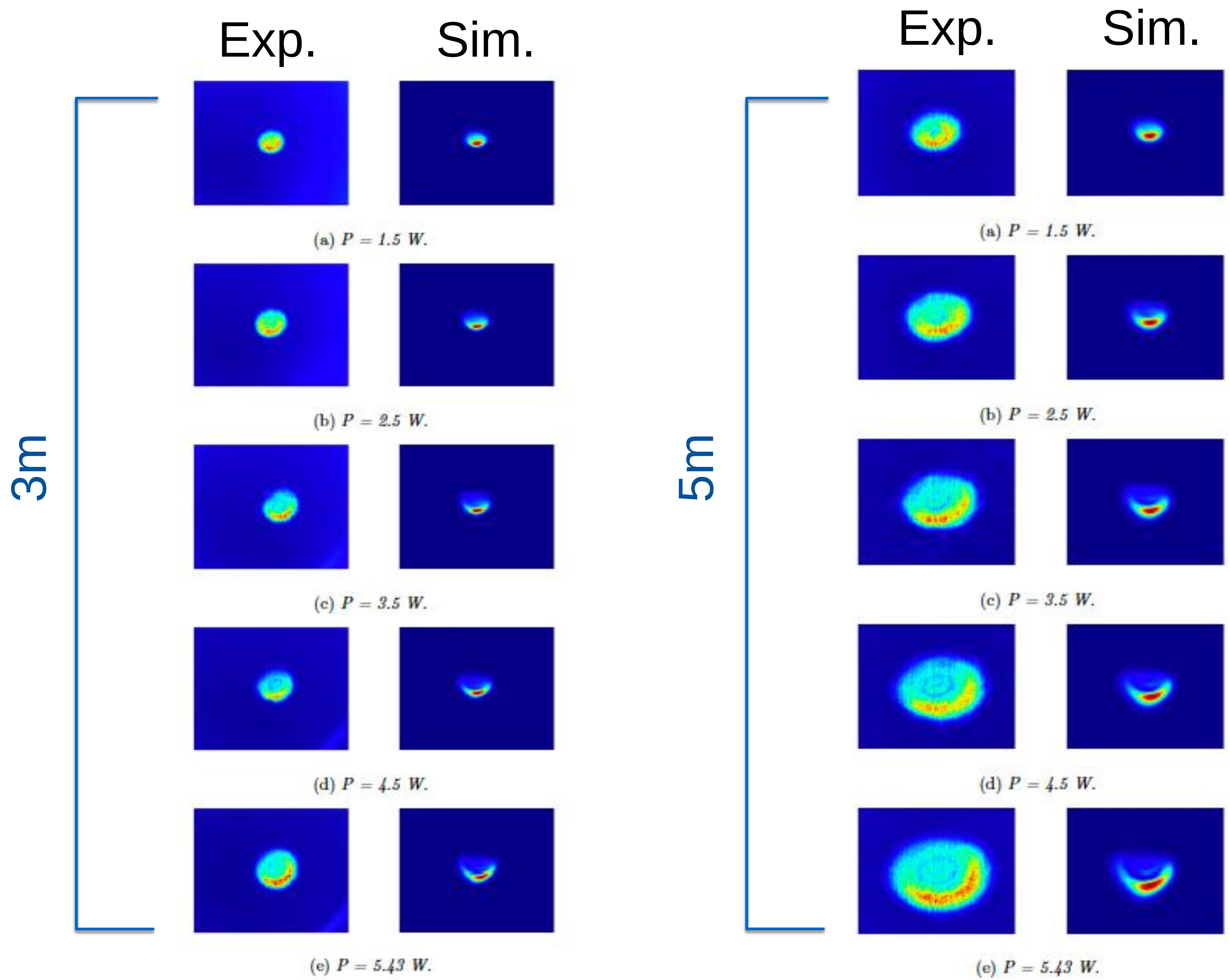
Experimental Setup (UCF/Richardson)

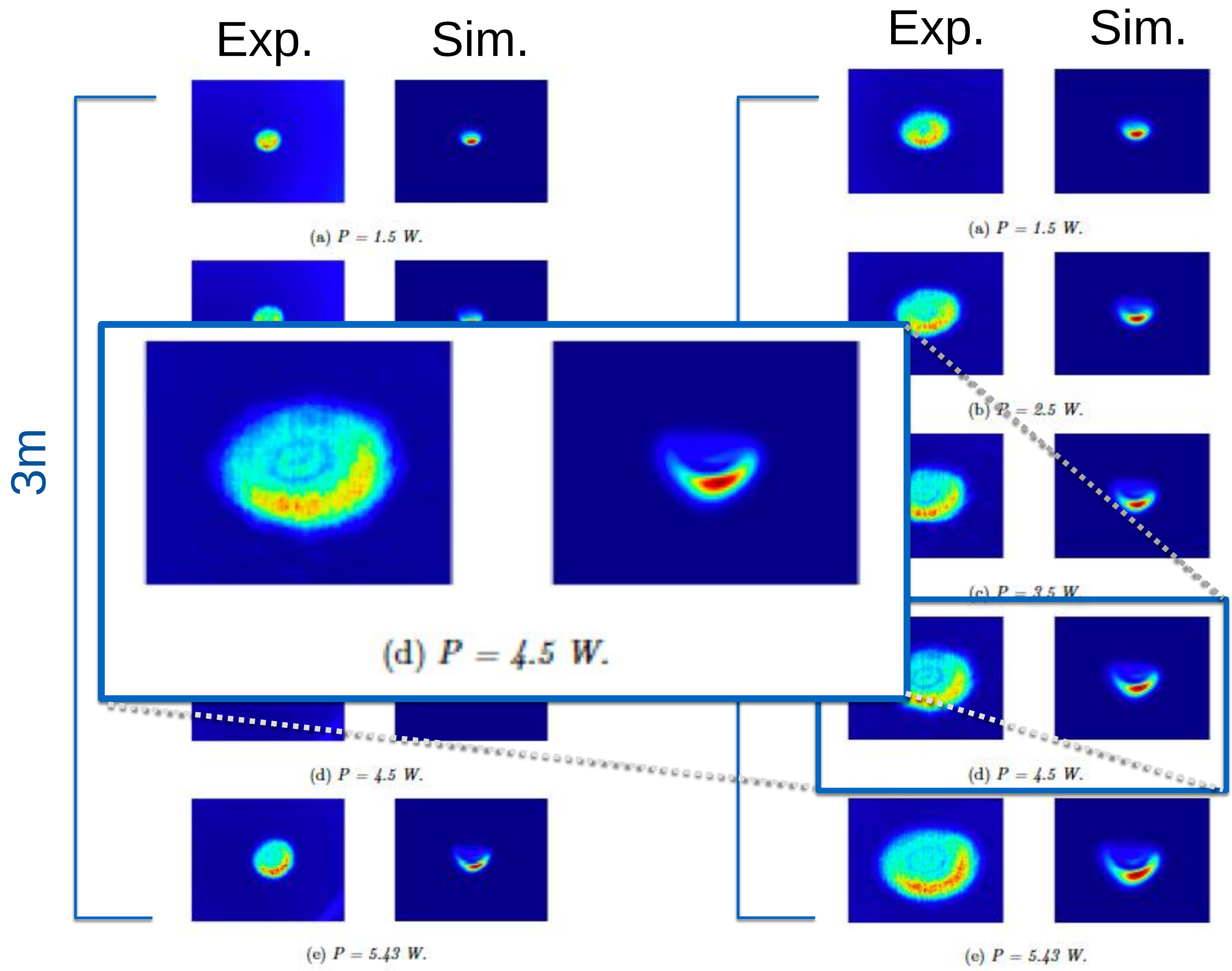
Side View



Top View

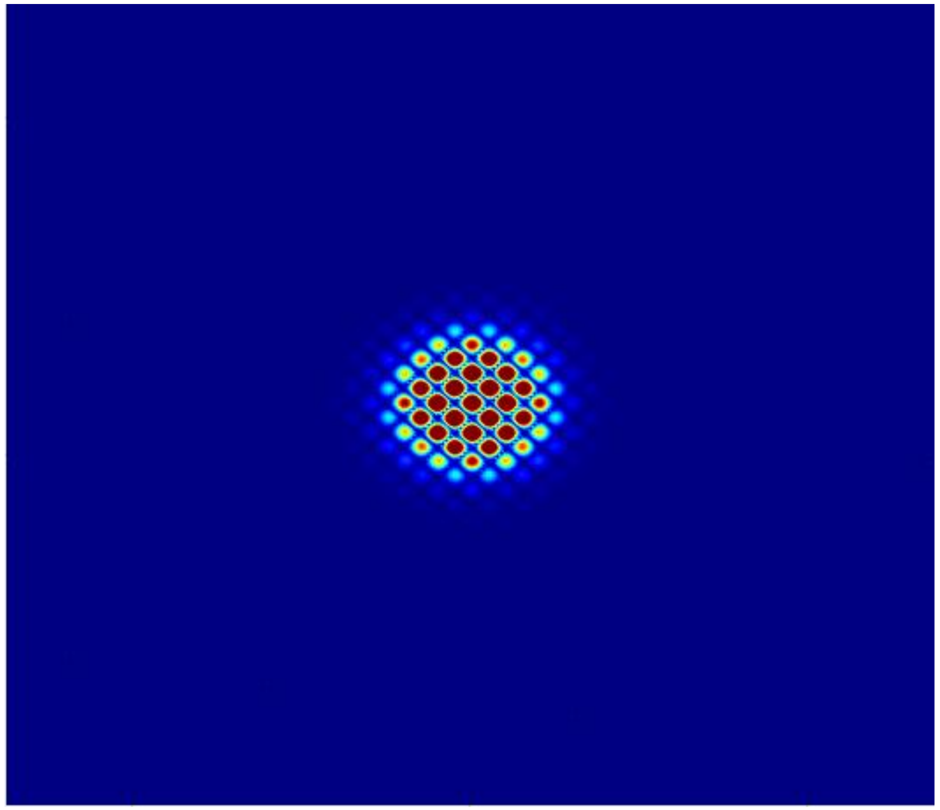


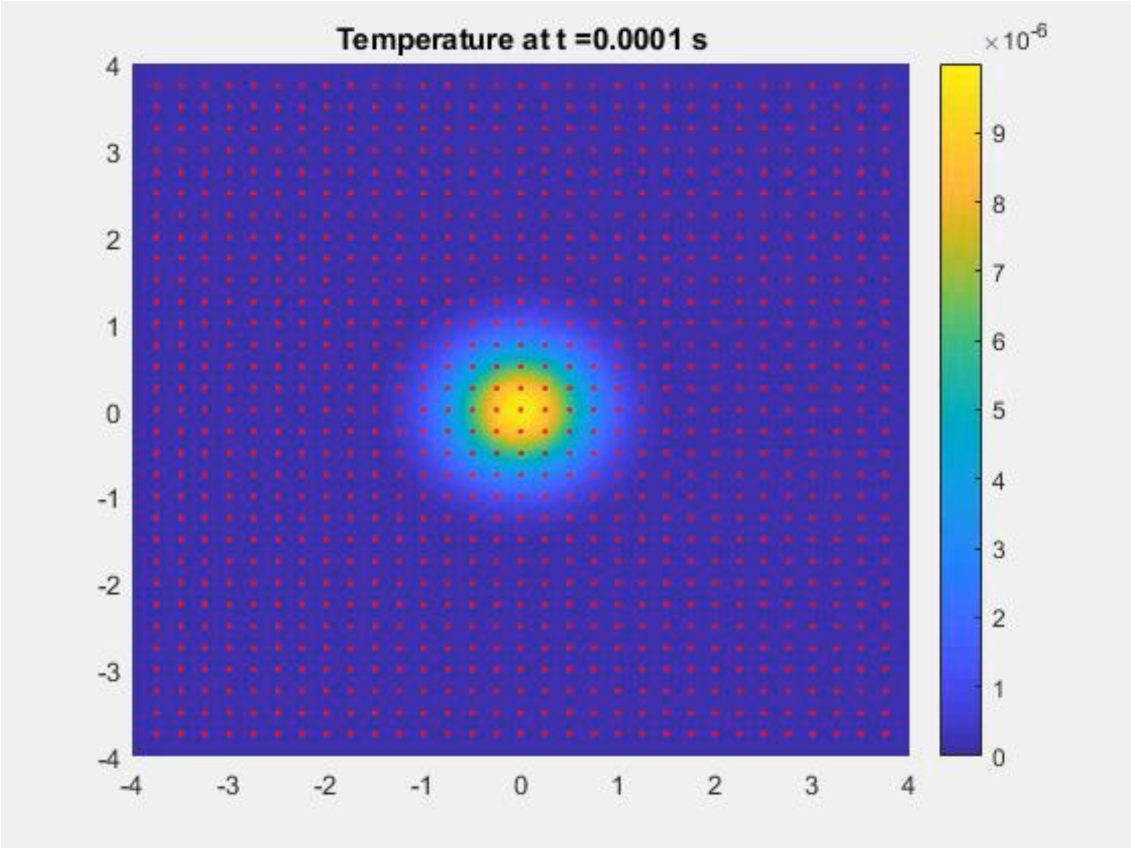

Exp.
Sim.
3m
(a) $P = 1.5$ W.
(b) $P = 2.5$ W.
(c) $P = 3.5$ W.
(d) $P = 4.5$ W.
(e) $P = 5.43$ W.
Exp.
Sim.
5m
(a) $P = 1.5$ W.
(b) $P = 2.5$ W.
(c) $P = 3.5$ W.
(d) $P = 4.5$ W.
(e) $P = 5.43$ W.

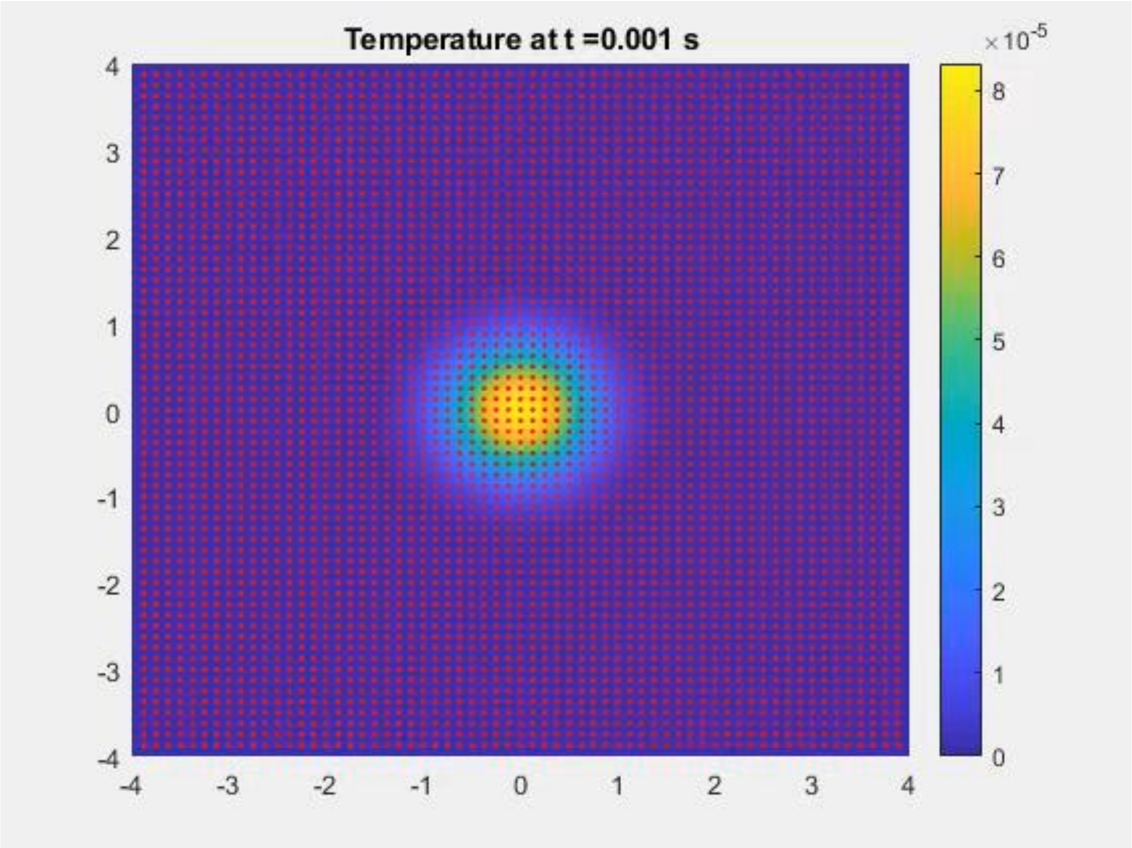

Exp.
Sim.
3m
(a) P = 1.5 W.
(d) P = 4.5 W.
(e) P = 5.43 W.
Exp.
Sim.
(a) P = 1.5 W.
(b) P = 2.5 W.
(c) P = 3.5 W.
(d) P = 4.5 W.
(e) P = 5.43 W.
(d) P = 4.5 W.

Current & Future Work

- Shape dependence
- Aerosol Effects
- Scaling laws
- Beam Combining







Thank You!

References:

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