Nonlinear states and switching in magneto-optical lattices

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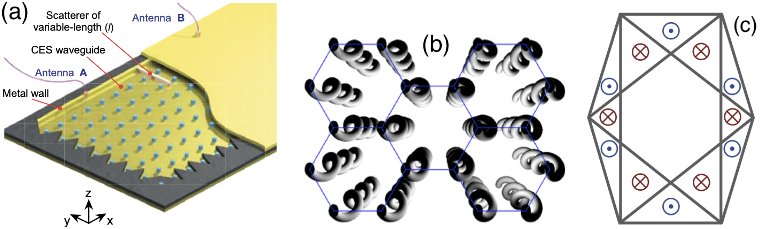
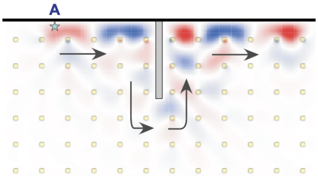
Motivation: Chern Insulators



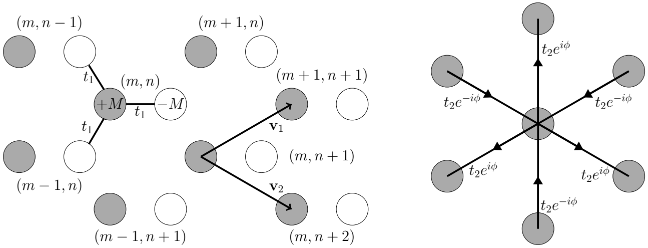


[1]Z. Wang, Y. Chong, J.D. Joannopoulos, M Soljačić, Nature **461** (2009)
[2]M.C. Rechtsman et al., Nature **496** (2013)
[3]G. Jotzu et al., Nature **515** (2014)

The Haldane Model - Quantum Hall Effect

Discrete "toy model" on honeycomb lattice

a-sites (gray discs) and b-sites (white discs)



$M \in \mathbb{R}$, self-interaction, inversion parameter

$t_1 \in \mathbb{R}$, nearest neighbor coefficient

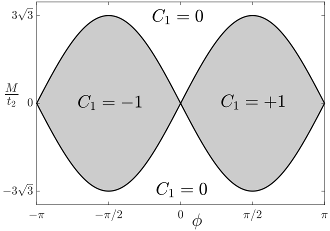
$t_2 \geq 0$, next-nearest neighbor magnitude

$\phi \in [0, 2\pi)$, next-nearest neighbor "flux", time-reversal parameter

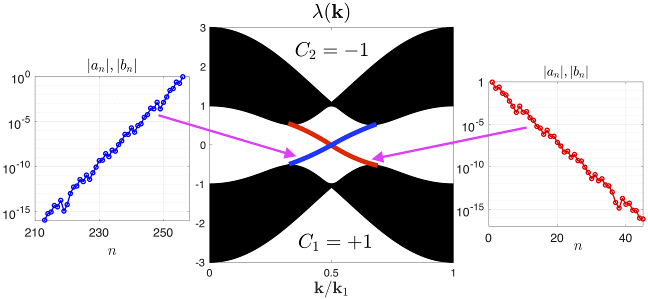
[4] F. D. M. Haldane, Phys. Rev. Lett. **61** (1988)

Chern Insulator Features

topological regions



edge band diagrams



Topological Insulators in Magento-optical Media

Sourceless and current free Maxwell's equations

$$\nabla \times \boldsymbol{H} = \frac{\partial \boldsymbol{D}}{\partial t}, \qquad \nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t},$$
$$\nabla \cdot \boldsymbol{D} = 0, \qquad \nabla \cdot \boldsymbol{B} = 0$$

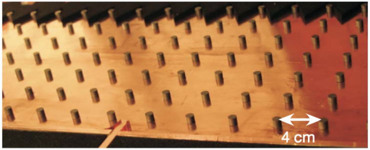
transverse magnetic (TM) polarized, time-harmonic waves

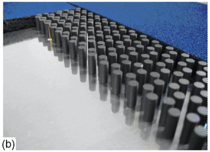
$$\boldsymbol{E} = \begin{pmatrix} 0 \\ 0 \\ E_z(\boldsymbol{r};\omega) \end{pmatrix} e^{i\omega t} + \mathrm{c.c.}, \qquad \boldsymbol{H} = \begin{pmatrix} H_x(\boldsymbol{r};\omega) \\ H_y(\boldsymbol{r};\omega) \\ 0 \end{pmatrix} e^{i\omega t} + \mathrm{c.c.}$$

Isotropic permittivity tensor $\boldsymbol{D}(\boldsymbol{r};\omega) = \varepsilon(\boldsymbol{r})\boldsymbol{E}(\boldsymbol{r};\omega)$

Gyrotropic Media

Apply a strong external magnetic field to array of ferrite (YIG) rods





$$\begin{pmatrix} B_x(\boldsymbol{r};\omega) \\ B_y(\boldsymbol{r};\omega) \\ 0 \end{pmatrix} = \begin{pmatrix} \mu(\boldsymbol{r}) & i\kappa(\boldsymbol{r}) & 0 \\ -i\kappa(\boldsymbol{r}) & \mu(\boldsymbol{r}) & 0 \\ 0 & 0 & \mu_0 \end{pmatrix} \begin{pmatrix} H_x(\boldsymbol{r};\omega) \\ H_y(\boldsymbol{r};\omega) \\ 0 \end{pmatrix}$$

Gyrotropic permeability tensor breaks time-reversal symmetry

[5] Z. Wang, Y. Chong, J. Joannopoulos, and M. Soljačić, Nature **461**, p. 772 (2009)
[6] Y. Poo, R. Wu, Z. Lin, Y. Yang, and C. Chan, Phys. R. Lett. **106**, 093903 (2011)

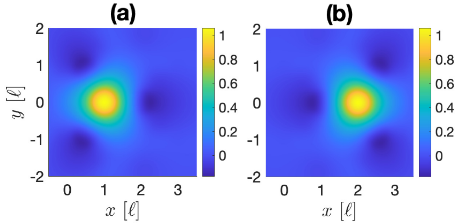
Master Equation and Wannier Expansion

Governing eigenvalue problem

$$-\nabla^2 E_z + \mathcal{M}(\boldsymbol{r}) \cdot \nabla E_z = \omega^2 \varepsilon(\boldsymbol{r}) \eta(\boldsymbol{r}) E_z$$

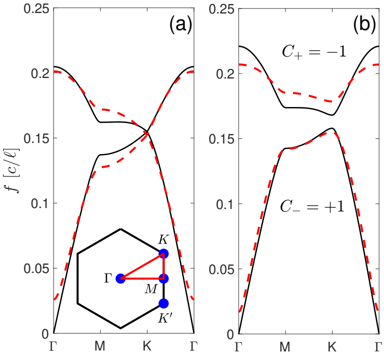
Tight-binding approximation in terms of "borrowed" Wannier basis

$$E_z(\boldsymbol{r}; \boldsymbol{k}, \omega) = \sum_{m,n} a_{mn}(\boldsymbol{k}, \omega) W^a_{mn}(\boldsymbol{r}) + b_{mn}(\boldsymbol{k}, \omega) W^b_{mn}(\boldsymbol{r})$$



Tight-binding Results

Comparison between numerics (solid) and discrete approx. (dashed)



Main result: next-nearest neighbor model is the (gen.) **Haldane model**

Significance: first (derived) tight-binding model for this system

[7] M. Ablowitz and **J. Cole**, Phys. Rev. A **109** (2024)

The Chern Insulator Model

The Haldane model is an effective tight-binding model for:

- Magneto-optical lattices[7]
- Ultracold driven fermionic systems[3]
- Photonic Floquet lattices[8]
- Electron gas[9]

The Haldane model is the <u>universal discrete Chern model</u>

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[7] M. Ablowitz and **J. Cole**, Phys. Rev. A **109** (2024)
[3] G. Jotzu et al., Nature **515** (2014)
[8] M. Ablowitz, S. Nixon, and **J. Cole**, SIAM J. Appl. Math. **151** (2023)
[9] S. Lannebère and M. Silveirinha, Phys. Rev. B **97** (2018)

Effects of Nonlinearity

Polarization effects: $\mathbf{P} = \varepsilon(\mathbf{r})\mathbf{E}(\mathbf{r}, t) + \mathbf{P}^{\mathrm{NL}}$

Third-order susceptibility

$$P_i^{\mathrm{NL}} = \sum_{jkl} \chi^{(3)}_{ijkl} E_j E_k E_l$$

Natural nonlinearity: on-site Kerr-type

$$i\frac{d\psi_n}{dz} + \sum_m H_{mn}\psi_m + \sigma|\psi_n|^2\psi_n = 0$$

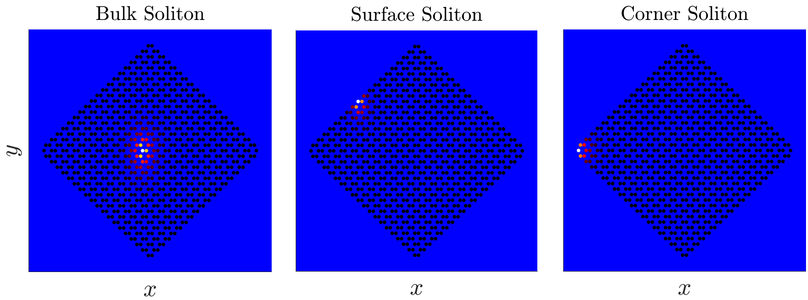
H = linear Hamiltonian

[10] D. Christodoulides and R. Joseph, Opt. Lett. **13** (1988)

[11] M. Ablowitz and **J. Cole**, Physica D, **440** (2022)

Upper/lower Gap Solitons - On-site

Stationary solitons exist in upper (lower) gap for $\sigma > 0$ ($\sigma < 0$)

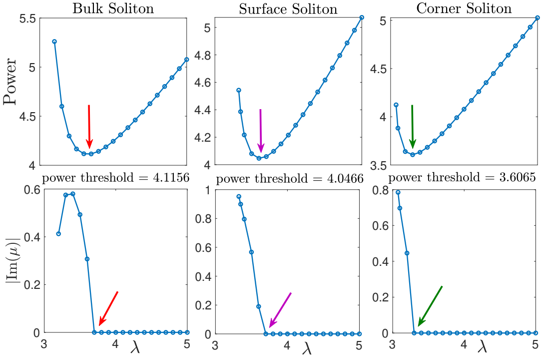


Fished by fixed-point renormalization method

$$\mathbf{v}^{(k+1)} = (H - \lambda I)^{-1}(N[\mathbf{v}^{(k)}]\mathbf{v}^{(k)})$$

Upper Gap Soliton Stability and Topology - On-site

Vakhitov-Kolokolov stability criterion



Results resemble those of simple nonlinear lattices

Trivial topology (zero local Chern number)

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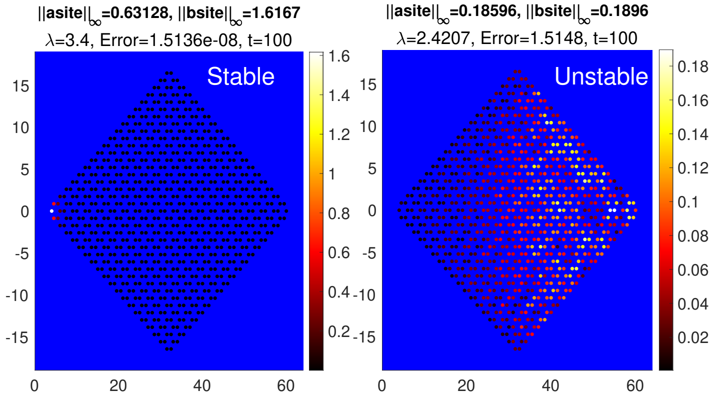
[12] R. Vicencio et al., Phys. Lett. A **364** (2007)

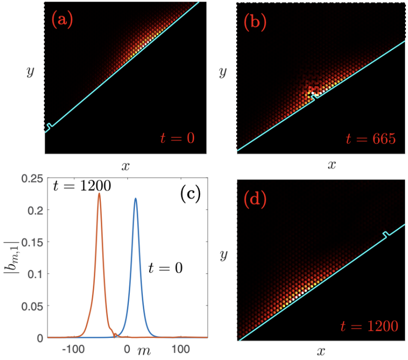
[13] X. Wang et al., Phys. Rev. Lett. **98** (2007)

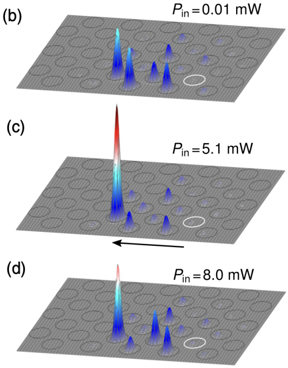
Stability Results



What About Traveling Solutions?

Is it possible to construct nonlinear traveling soliton modes?





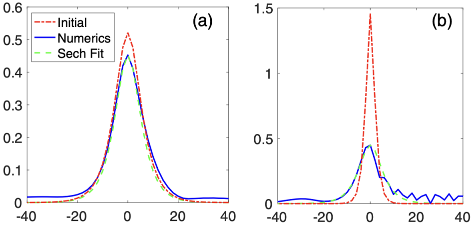
[7] M. Ablowitz and **J. Cole**, Phys. Rev. A **109** (2024)
[14] S. Mukherjee and M. Rechtsman, Phys. Rev. X **11** (2021)

Peierls-Nabarro (PN) Energy Barrier

Weak nonlinearity: generalized NLS model

$$i\frac{\partial C}{\partial Z} + i\alpha'(k_*)\frac{\partial C}{\partial Y} + \frac{\alpha''(k_*)}{2}\frac{\partial^2 C}{\partial Y^2} + \alpha_{\mathrm{nl}}|C|^2 C \approx 0$$

Strong nonlinearity: PN radiation



[15] M. Ablowitz, C. Curtis, and Y.P. Ma, Phys. Rev. A **90** (2014)
[16] L. J. Maczewsky et al., Science **370** (2020)
[17] M. Ablowitz, **J. Cole**, P. Hu. and P. Rosenthal, Phys. Rev. E **103** (2021)
[18] M. Ezawa, Phys. Rev. B **106** (2022)

Classic Discrete Nonlinear Equations

Discrete Nonlinear Schrödinger

$$i\frac{d\psi_n}{dt} + \psi_{n+1} + \psi_{n-1} + |\psi_n|^2\psi_n = 0$$

Peierls-Nabarro energy barrier $\Rightarrow$ (generically) no traveling modes

Ablowitz-Ladik

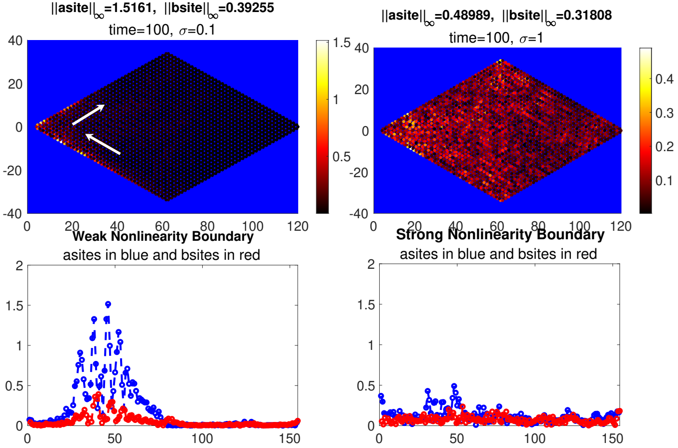
$$i\frac{d\psi_n}{dt} + \psi_{n+1} + \psi_{n-1} + |\psi_n|^2(\psi_{n+1} + \psi_{n-1}) = 0$$

integrable (exactly solvable), admits traveling solutions

[19] Y. Hadad, A. Khanikaev, A. Alù, Phys. Rev. B **93** (2016)

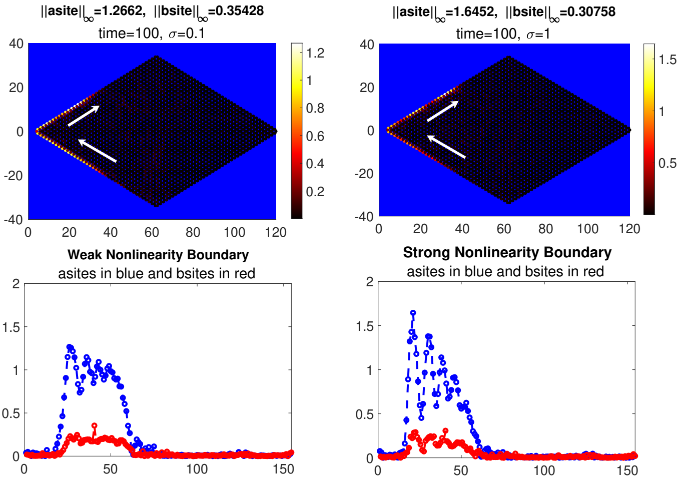
On-site nonlinearity

- propagation for weak nonlinearity
- substantial degradation for strong nonlinearity



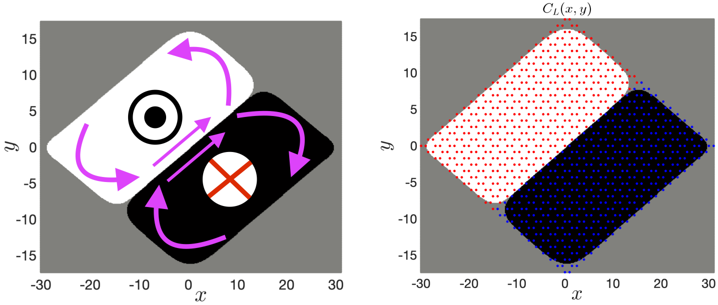
Nearest neighbor nonlinearity

- coherent propagation for weak and strong nonlinearity
- break-up into soliton-like structures



Chern Insulator Interface

Magneto-optical interface between two oppositely-oriented magnetic field



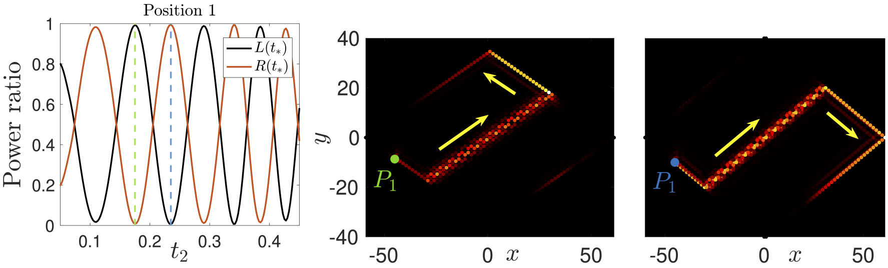
(right) Local Chern number: $C_L = +1$ (white) and $C_L = -1$ (black)

[20] M. Ablowitz, S. Nixon, and J. Cole, Opt. Lett. **49** (2024)

[21] A. Cerjan and T. Loring, Phys. Rev. B **106** (2022)

Switching in Magneto-optical Lattice

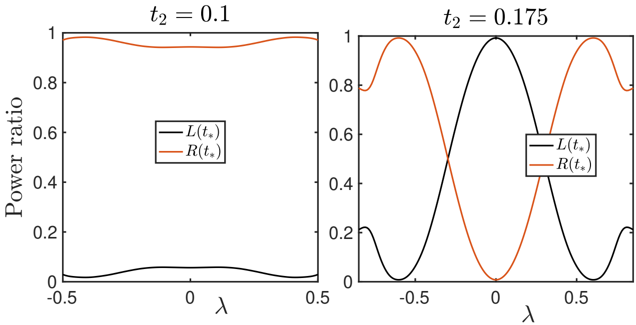
Single antenna source exciting localized interface mode



Result 1: Switching can be controlled by adjusting strength of external magnetic field

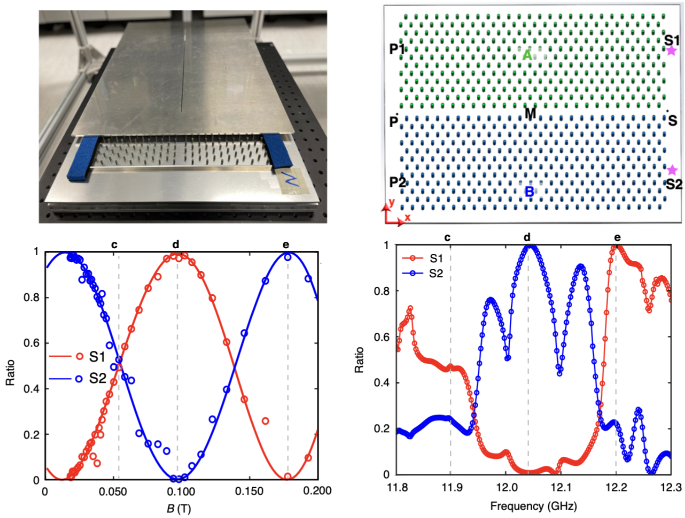
Switching in Magneto-optical Lattice cont.

Single antenna source exciting localized interface mode



Result 2: Switching can be controlled by adjusting the frequency of the antenna source

Experimental Realization



[22]W. Tang et al., Light: Science & Applications **13** (2024)

Conclusions

Nonlinearity

- On-site nonlinearity supports stationary solitons
- Traveling solutions for weak (on-site) nonlinearity
- Ablowitz-Ladik-type nonlinearity admits traveling solitons

Switching

- Interface between two oppositely-oriented chiral regions
- Switching controlled via: magnetic field or frequency