

# **Hierarchical theoretical methods for understanding and predicting anisotropic thermal transport and energy release in rocket propellant formulations**

**M. Ortiz**

California Institute of Technology

University of Missouri PI: Thomas D. Sewell

Subcontract: EC-SRP-12-0053

Program Review, January 21, 2015



Michael Ortiz  
AFOSR 01/15

# Overview

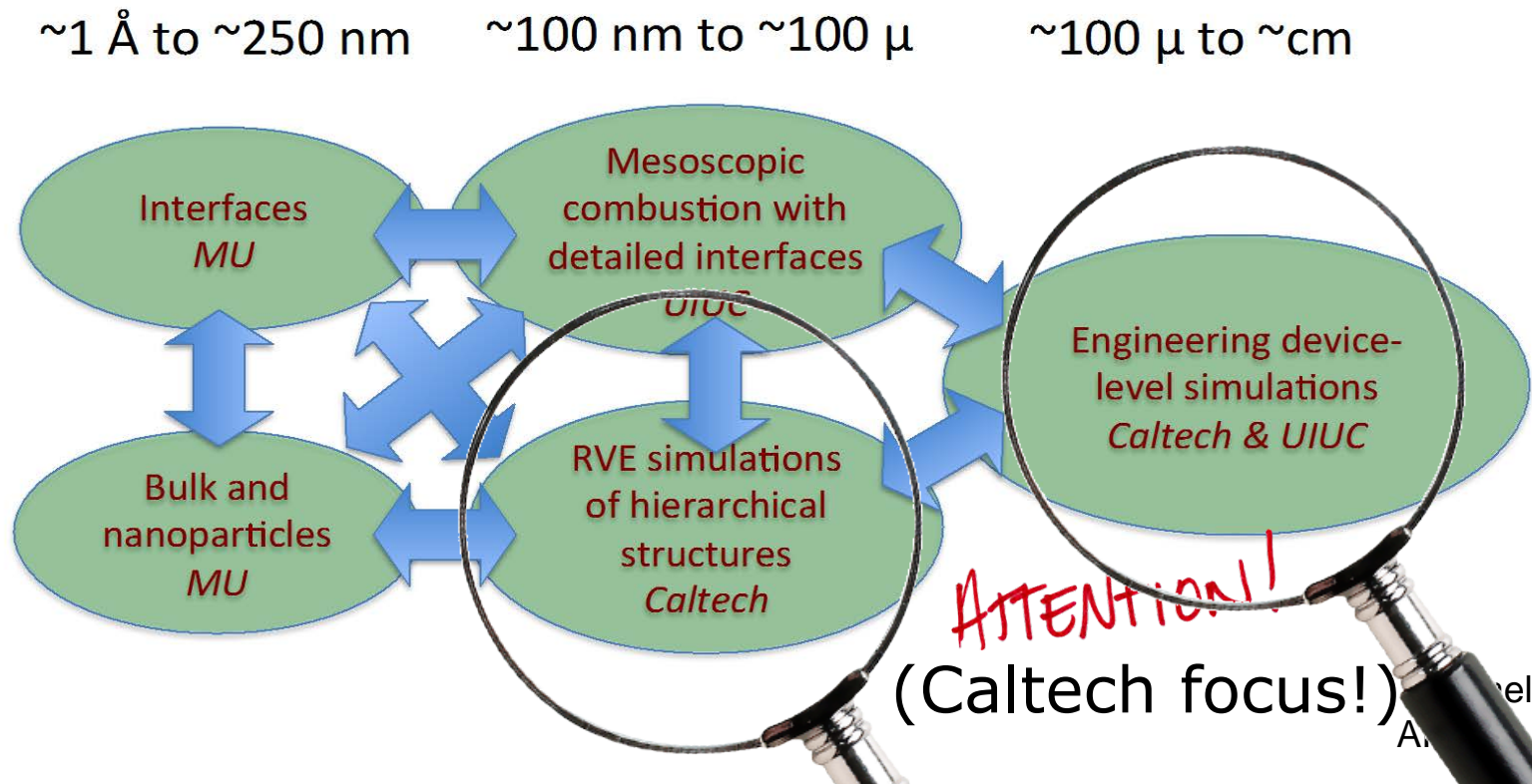
- Research designed to yield understanding and predictive capability regarding ***anisotropic thermal transport*** and ***energy release*** in ***advanced rocket propellants***:
  - *Practical capability for a priori propellant design*
  - *Polymer nanocomposite formulations augmented by non-traditional additives or passivation agents*
- Study and exploit ***anisotropy*** at three levels:
  - ***Intrinsic*** anisotropy at the molecular up to the continuum microscale for pure constituents
  - ***Manufactured*** nano- and microscale anisotropy that is induced by manufacture specifications of the composition
  - ***Mesoscale*** anisotropy persistence during the physico-chemical structural decomposition, mixing, and reaction



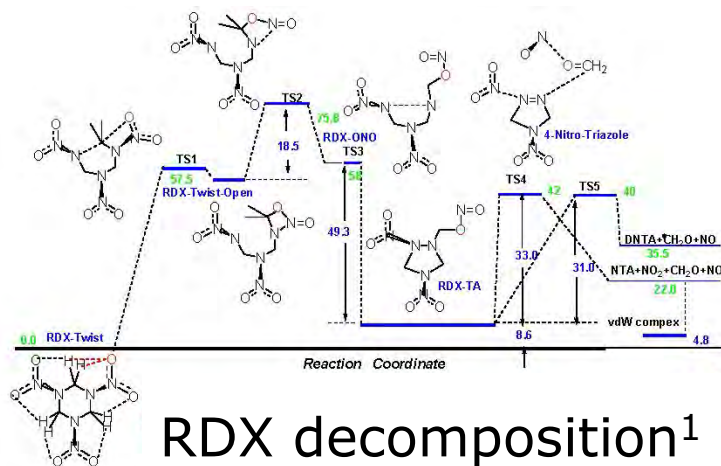
**ATTENTION!**  
(Caltech focus!)

# Approach

- Fundamental information from atomic-scale simulations
- Detailed mesoscopic models of interfaces and reaction fronts
- Microstructure-resolved RVE-simulations to generate homogenized models
- Engineering-scale models for propellant combustion



# The fundamental challenge



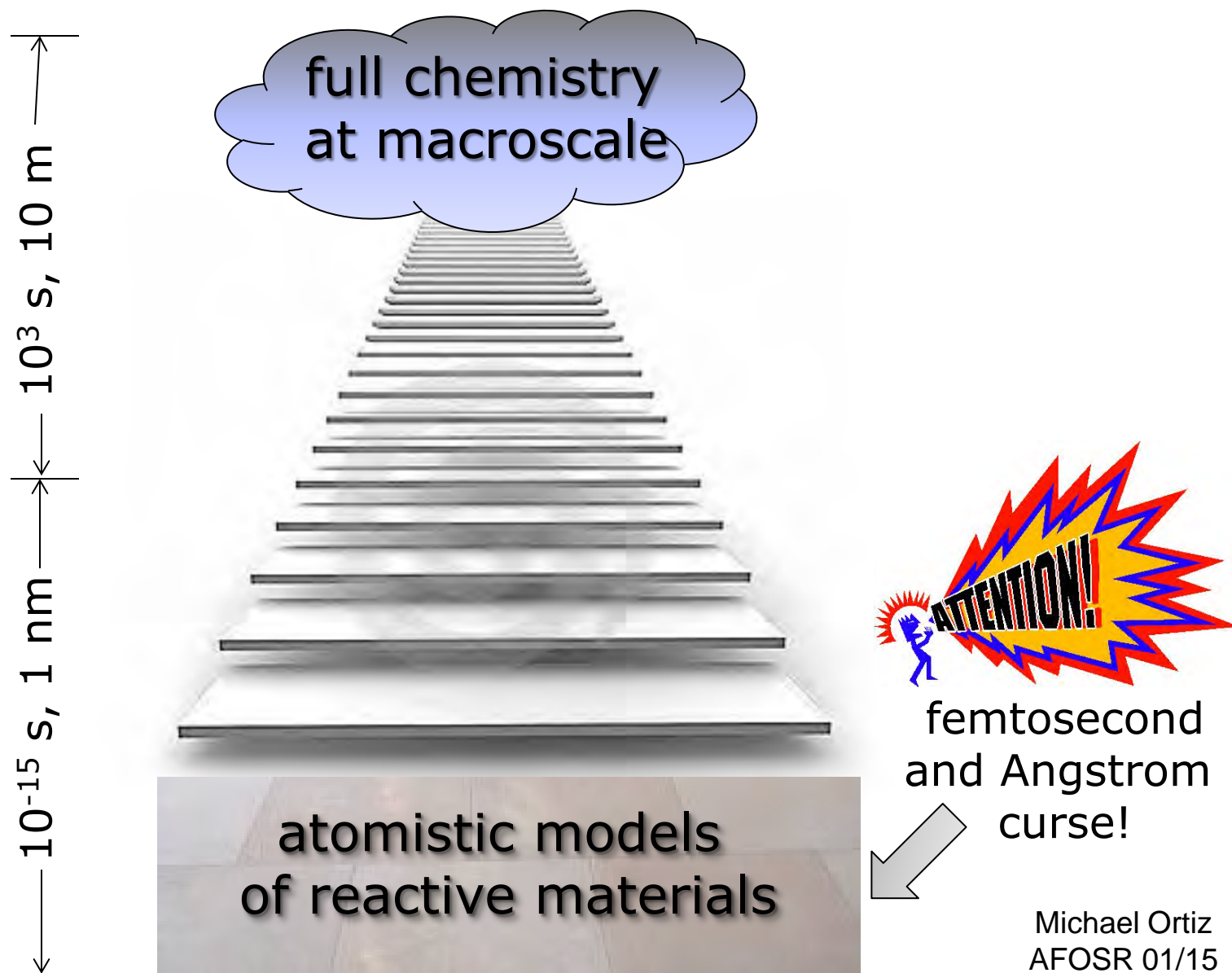
Space Shuttle Atlantis<sup>2</sup>

- Energetic materials undergo complex chemistry coupled to temperature and deformation
- Reactions take place at atomic scale, involve bond breaking and creation of new bonds
- Reaction paths are extremely complex, defy reduce modeling
- Full chemistry, reaction-front speeds on the order of seconds
- Outside scope of straight MD...

<sup>1</sup>R. Asatryan, G. da Silva, J.W. Bozzelli (2008), 20th Intern. Symp. on Gas-Kinetics, Manchester, UK.

<sup>2</sup><http://www1.nasa.gov/images/>

# The fundamental challenge



# Atomistic-to-continuum methods

- The essential difficulty: Multiple scales,
  - *Atomic level rate-limiting processes: Thermal vibrations, bond breaking, collisions, atomic hops...*
  - *Macroscopic processes of interest: Burn rates, kinetics, effect of microstructure, anisotropy, specific impulse...*
- Time-scale gap: From thermal vibrations (femtosecond) to device operation (seconds)
- Spatial-scale gap: From atomic lattice scale (Angstroms) to device dimensions (m)
- Application to solid propellants:
  - *Wish atomistic realism within reaction zone...*
  - *But reaction kinetics is too slow for MD*
  - **Question**: *How to effect space-time coarse-graining (atoms to device) without introducing spurious physics and without essential loss of information?*





# Spacetime atomistic-to-continuum

- *Objectives*: Thermodynamics without all the thermal vibrations; mass transport without all the hops; atomistics without all the atoms...
- Our approach<sup>1,2</sup> (max-ent+kinetics+QC):
  - *Treat atomic-level fluctuations statistically (away from equilibrium) through maximum-entropy principle*
  - *Approximate grand-canonical free entropy using variational meanfield theory*
  - *Append Onsager-like empirical atomic-level kinetic laws (heat and mass transport)*
  - *Treat (smooth) mesodynamics by implicit integration (large time steps  $\gg$  MD!)*
  - *Quasicontinuum spatial coarse-graining*

<sup>1</sup>Y. Kulkarni, J. Knap & MO, *J. Mech. Phys. Solids*, **56** (2008) 1417.

<sup>2</sup>G. Venturini, K. Wang, I. Romero, M.P. Ariza & MO,  
*J. Mech. Phys. Solids*, **73** (2014) 242-268.



# Max-Ent Non-Equilibrium SM

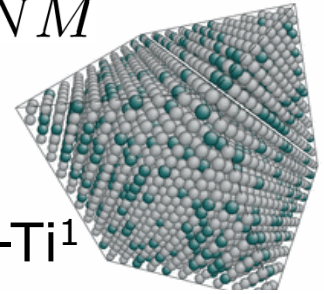
- Grand-canonical ensemble,  $N$  atoms,  $M$  species:

- State:  $(\{\mathbf{q}\}, \{\mathbf{p}\}, \{\mathbf{n}\}) \in \mathbb{R}^{3N} \times \mathbb{R}^{3N} \times \mathcal{O}_{NM}$

- Atomic positions:  $\{\mathbf{q}\} = \{\mathbf{q}_1, \dots, \mathbf{q}_N\}$

- Atomic momenta:  $\{\mathbf{p}\} = \{\mathbf{p}_1, \dots, \mathbf{p}_N\}$

- Occupancy:  $n_{ik} = \begin{cases} 1, & \text{site } i \text{ occupied by species } k, \\ 0, & \text{otherwise.} \end{cases}$



- Ensemble average of observable:  $\langle A \rangle =$

$$\sum_{\{\mathbf{n}\} \in \mathcal{O}_{NM}} \int A(\{\mathbf{q}\}, \{\mathbf{p}\}, \{\mathbf{n}\}) \underset{\substack{\uparrow \\ \text{grand-canonical pdf}}}{\rho(\{\mathbf{q}\}, \{\mathbf{p}\}, \{\mathbf{n}\})} d\mathbf{q} d\mathbf{p}$$





# Max-Ent Non-Equilibrium SM

- Assume  $H = \sum_{i=1}^N h_i$ , (e. g., EAM, TB...)
- Principle of max-ent<sup>1</sup>:  $\mathcal{S}[p] = -k_B \langle \log \rho \rangle \rightarrow \max!$   
 subject to:  $\left. \begin{array}{l} \langle \mathbf{q}_i \rangle = \bar{\mathbf{q}}_i, \quad \langle \mathbf{p}_i \rangle = \bar{\mathbf{p}}_i, \\ \langle h_i \rangle = e_i, \quad \langle n_{ik} \rangle = x_{ik} \end{array} \right\} \text{local constraints!}$
- Lagrangian:      reciprocal temperatures      chemical potentials  

$$\mathcal{L}[p, \{\beta\}, \{\gamma\}] = \mathcal{S}[p] - k_B \{\beta\}^T \{\langle h \rangle\} - k_B \{\gamma\}^T \{\langle \mathbf{n} \rangle\}$$
- Gran-canonical pdf:  $\rho = \frac{1}{\Xi} e^{-\{\beta\}^T \{h\} - \{\gamma\}^T \{\mathbf{n}\}},$

on affine subspace  $\left\{ \langle \{\mathbf{q}\} \rangle = \{\bar{\mathbf{q}}\}, \langle \{\mathbf{p}\} \rangle = \{\bar{\mathbf{p}}\} \right\}$



<sup>1</sup>E.T. Jaynes, *Physical Review Series II*, **106**(4) (1957) 620–630; **108**(2) (1957) 171–190.

# Non-Equilibrium Statistical Mechanics

- From max-ent principle, free entropy:

$$\Phi(\{\beta\}) = k_B \log \int e^{-\{\beta\}^T \{h(q,p)\}} dq dp$$

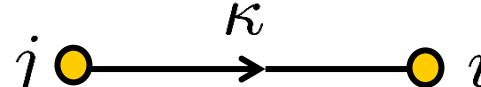
↑reciprocal atomic temperatures
 ↑local atomic Hamiltonians

- Mesososcopic dynamics:

$$\frac{d\bar{q}_i}{dt} = \frac{1}{k_B} \frac{\partial^2 \Phi}{\partial \beta_i \partial \bar{p}_i}, \quad \frac{d\bar{p}_i}{dt} = -\frac{1}{k_B} \frac{\partial^2 \Phi}{\partial \beta_i \partial \bar{q}_i}$$

- Temperature field evolution, discrete heat equation:

$$\underbrace{\frac{d}{dt} \frac{1}{k_B} \frac{\partial \Phi}{\partial \beta_i}(\{\beta\})}_{\text{internal energy of atom } i} = \sum_{j \neq i} \underbrace{\partial \psi(\beta_i - \beta_j)}_{\text{heat flux into atom } i}$$





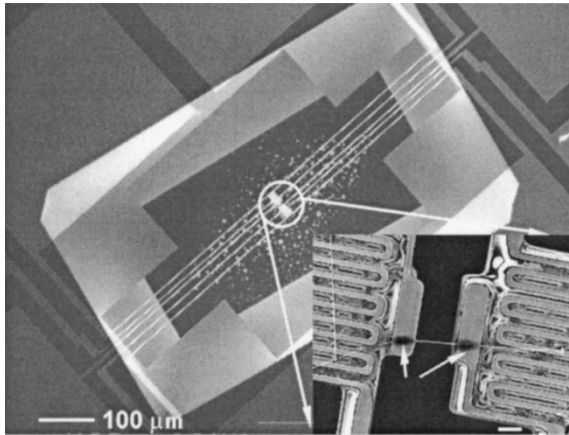
# Non-Equilibrium Statistical Mechanics

	Molecular dynamics	N.E. Stat. Mech.
Configuration space	Phase space $(q,p)$	<ul style="list-style-type: none"> <li>•Temperature field</li> <li>•Atomic-fraction field</li> </ul>
Governing equations	$\Sigma F=ma$	<ul style="list-style-type: none"> <li>•Diffusive transport</li> <li>•Mesodynamics</li> </ul>
Spatial resolution	Atomic lattice	<ul style="list-style-type: none"> <li>•Temperature grads.</li> <li>•Concentration grads.</li> </ul>
Temporal resolution	<ul style="list-style-type: none"> <li>•Thermal vibrations</li> <li>•Transition states</li> </ul>	<ul style="list-style-type: none"> <li>•Mesoscopic dynamics</li> <li>•Diffusional transients</li> </ul>
Time-scale bridging	Non-equilibrium statistical mechanics	
Spatial-scale bridging	Quasicontinuum method	

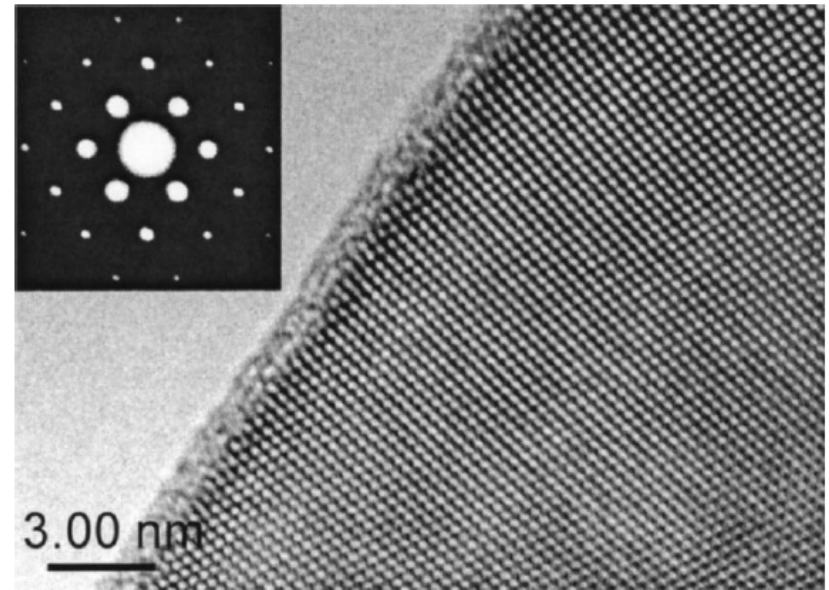
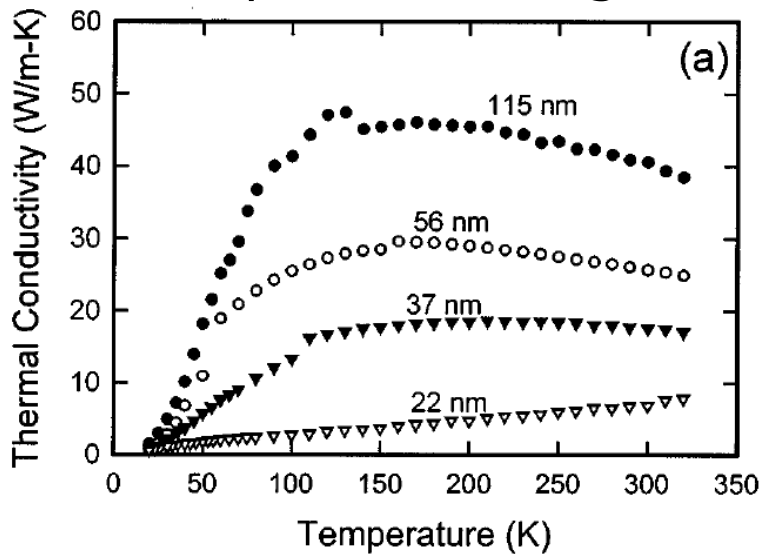
- Paradigm shift: From Newtonian dynamics to diffusional transport (heat and mass)
- Time step limited by diffusional time scale!



# Kinetic validation – Si nanowires



Experimental rig<sup>1</sup>



Amorphous layer in SiNW!



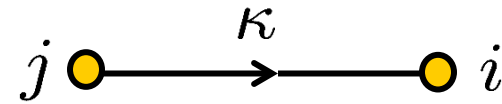
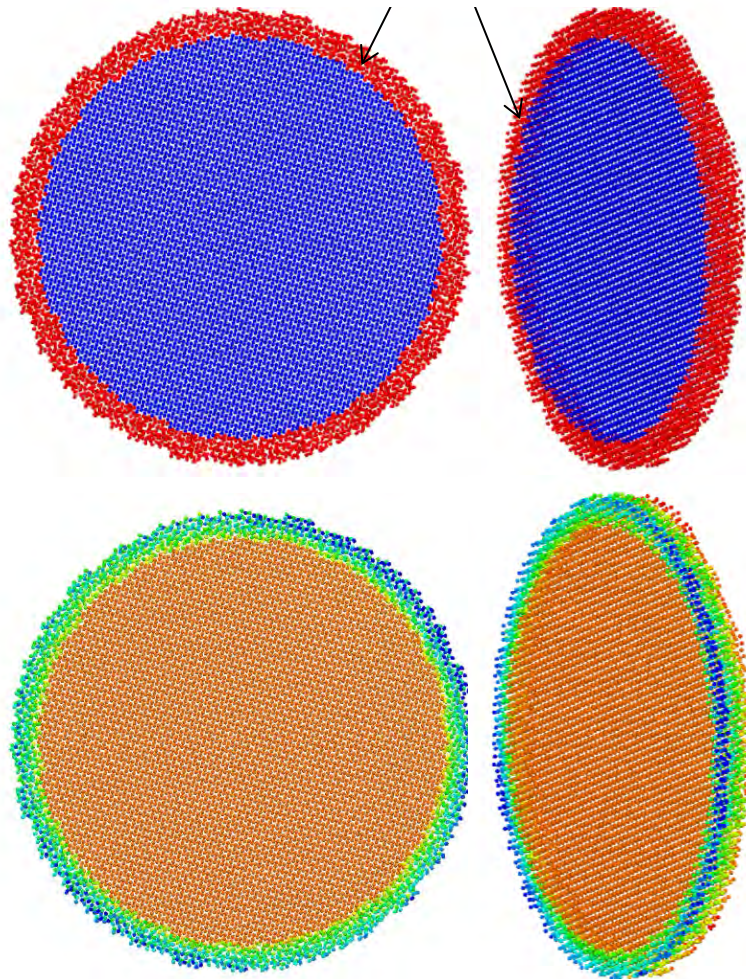
<sup>1</sup>D. Li, Y. Yu, P. Kim, L. Shi, P. Yang and A. Majumdar, *Appl. Phys. Lett.*, **83** (2003) 2934.

Michael Ortiz  
AFOSR 01/15

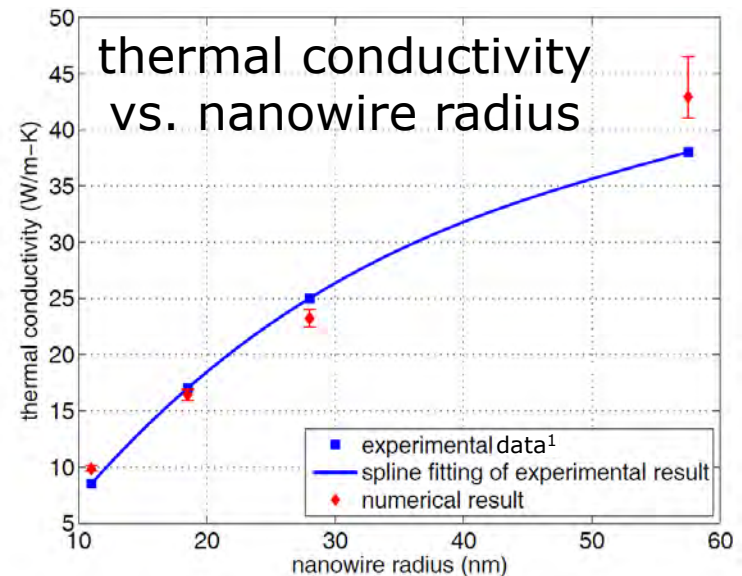


# Kinetic validation – Si nanowires

amorphous layer



- Rigid model, linear kinetics
- $\kappa_{\text{xal}} = 0.09 \text{ nW/K}$ ,  $\kappa_{\text{amo}} = 16 \text{ nW/K}$
- Prescribed temperature gradient
- Output: Average axial heat flux

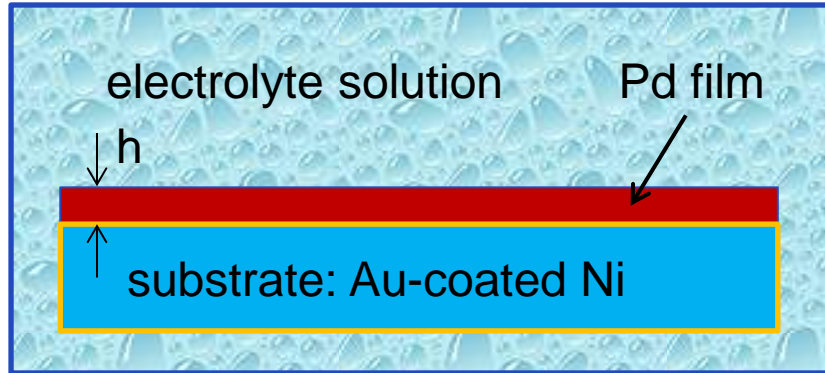


Temperature  
distribution

<sup>1</sup>D. Li, Y. Yu, P. Kim, L. Shi, P. Yang and A. Majumdar, *Appl. Phys. Lett.*, **83** (2003) 2934.

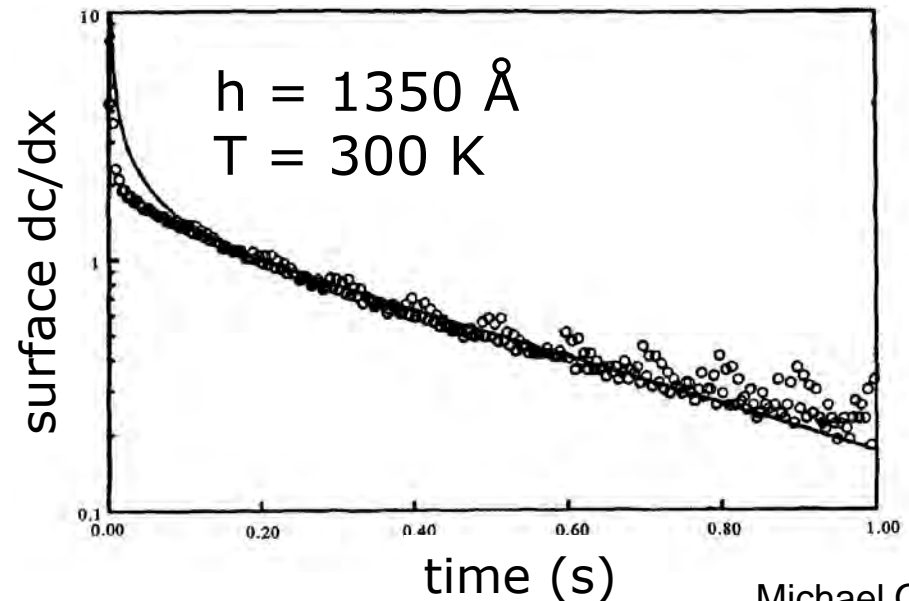
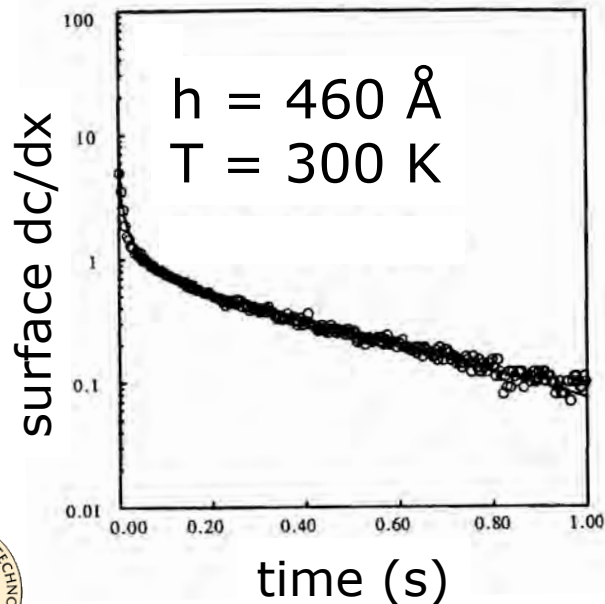
Michael Ortiz  
AFOSR 01/15

# Kinetic validation – H absorption in Pd



- H absorption into (111) Pd foil<sup>1</sup>
- Foil thickness = 460, 1350 Å
- Temperature = 300K
- Measurement: Surface concentration gradient vs. time

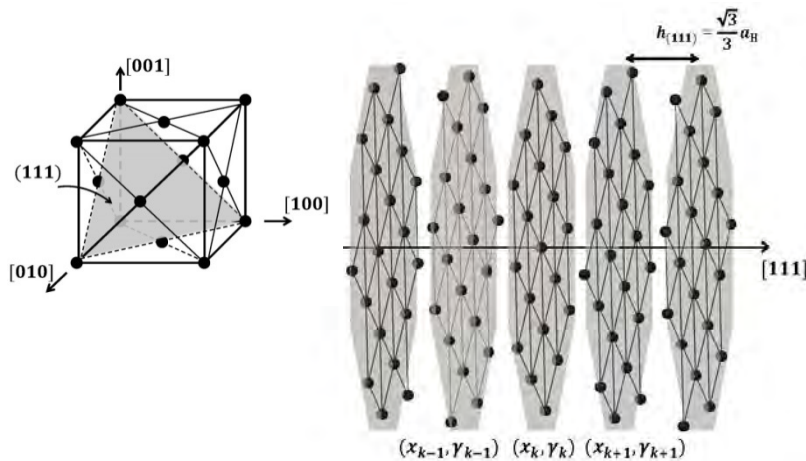
Experimental configuration<sup>1</sup>



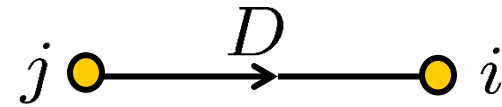
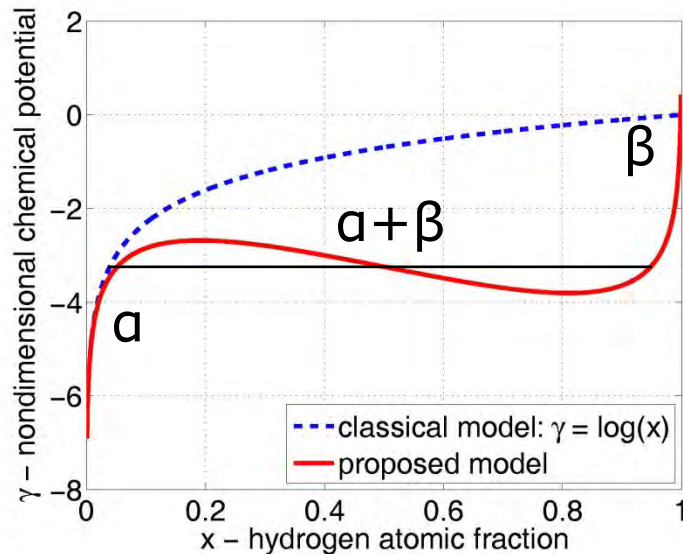
<sup>1</sup>Y. Li and Y.-T. Cheng., *Int. J. Hydrogen Energy*, **21** (1996) 281-291.



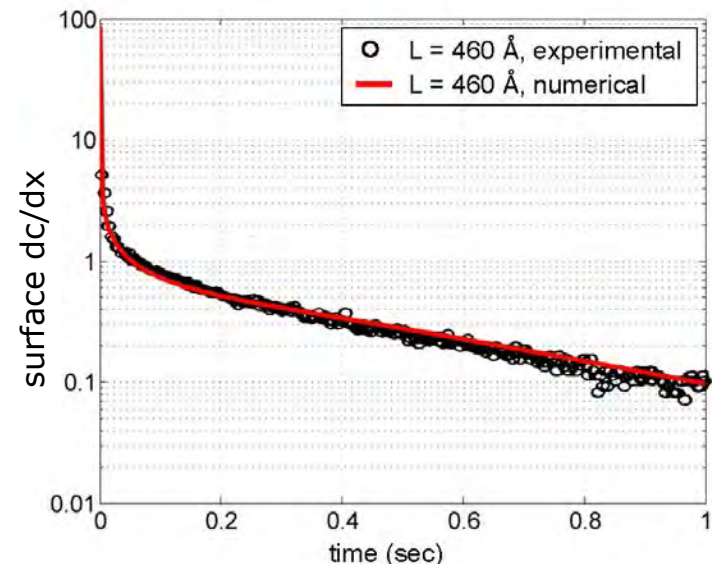
# Kinetic validation – H absorption in Pd



Pd foil crystallography



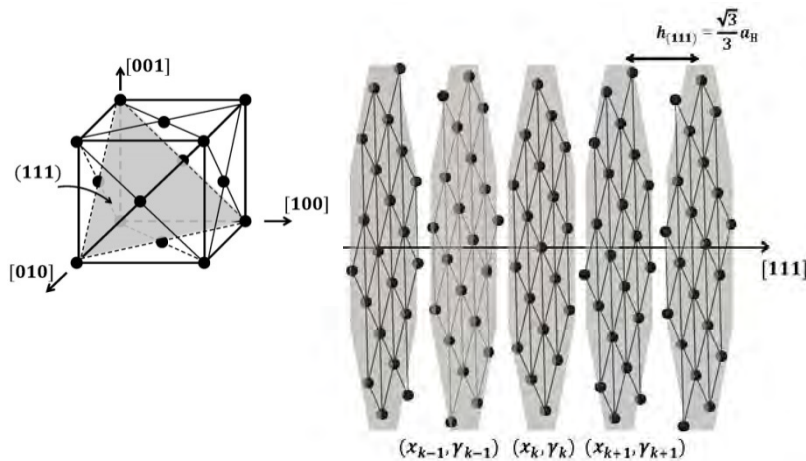
- Linear kinetics,  $D = 2.1 \times 10^5 \text{ \AA}^2/\text{s}$
- Ising-type meanfield model
- Johnson EAM potential<sup>1</sup>
- Prescribed surface concentration
- Output: Surface  $dc/dx$  vs. time



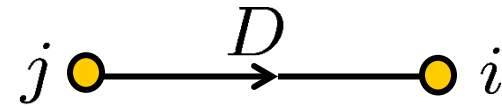
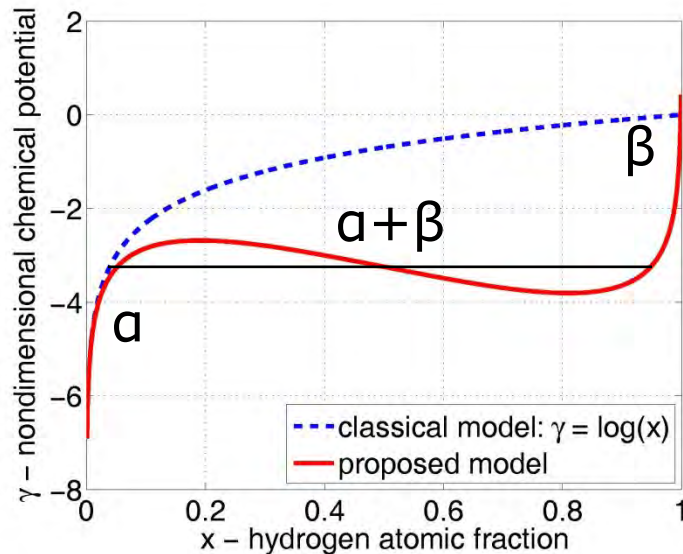
<sup>1</sup>R. Johnson, *Phys. Rev. B*, **39**(17):12554, 1989

<sup>2</sup>Y. Li and Y.-T. Cheng, *Int. J. Hydrogen Energy*, **21** (1996) 281-291.

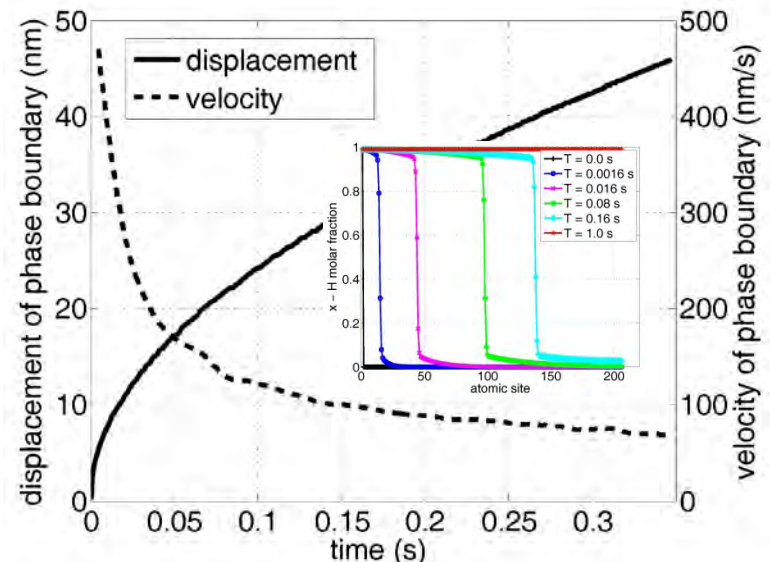
# Kinetic validation – H absorption in Pd



Pd foil crystallography



- Linear kinetics,  $D = 2.1 \times 10^5 \text{ \AA}^2/\text{s}$
- Ising-type meanfield model
- Johnson EAM potential<sup>1</sup>
- Prescribed surface concentration
- Output: Surface  $dc/dx$  vs. time



<sup>1</sup>R. Johnson, *Phys. Rev. B*, **39**(17):12554, 1989

<sup>2</sup>Y. Li and Y.-T. Cheng, *Int. J. Hydrogen Energy*, **21** (1996) 281-291.

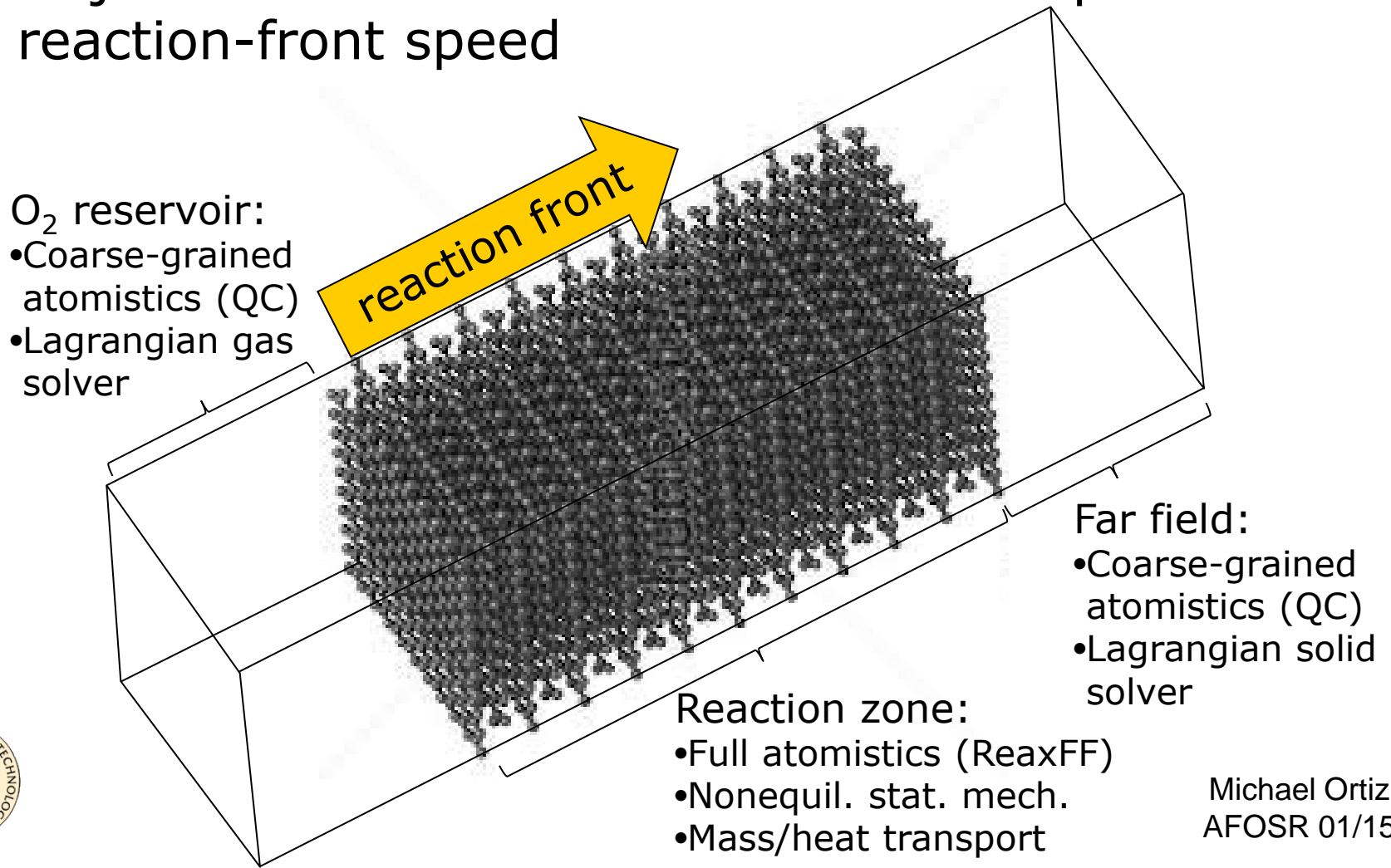
# Non-Equilibrium Statistical Mechanics

- N.E. Stat. Mech. (max-ent + kinetics) provides atomistic realism without the femtosecond curse
- Time step is limited by diffusive scale
- N.E. Stat. Mech. is built on arbitrary interatomic potentials + kinetic models
- ***Challenge***: Applications involving chemical reactions (modeled through reactive force fields, e.g., ReaxFF)
- Expect chemical reaction rates to be controlled by heat transport/anisotropy



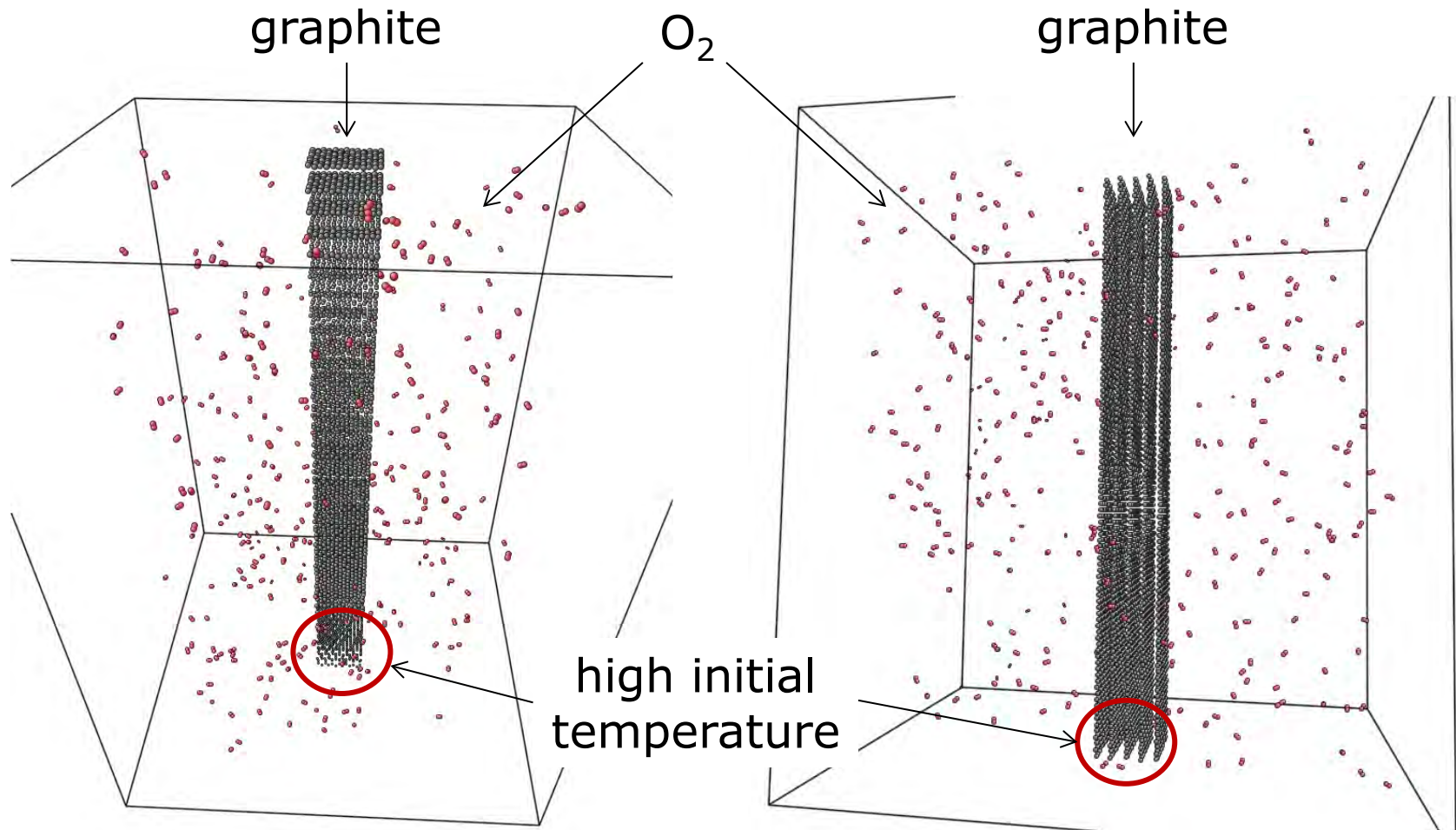
# N.E. Stat. Mech. + Quasicontinuum

- Test case: Combustion of graphite (anisotropy!)
- Objective: Calculation of orientation-dependent reaction-front speed





# Graphite + O<sub>2</sub> – Calculations in progress



- Interatomic potentials: ReaxFF/LAMMPS
- Calibration vs. thermal conductivities

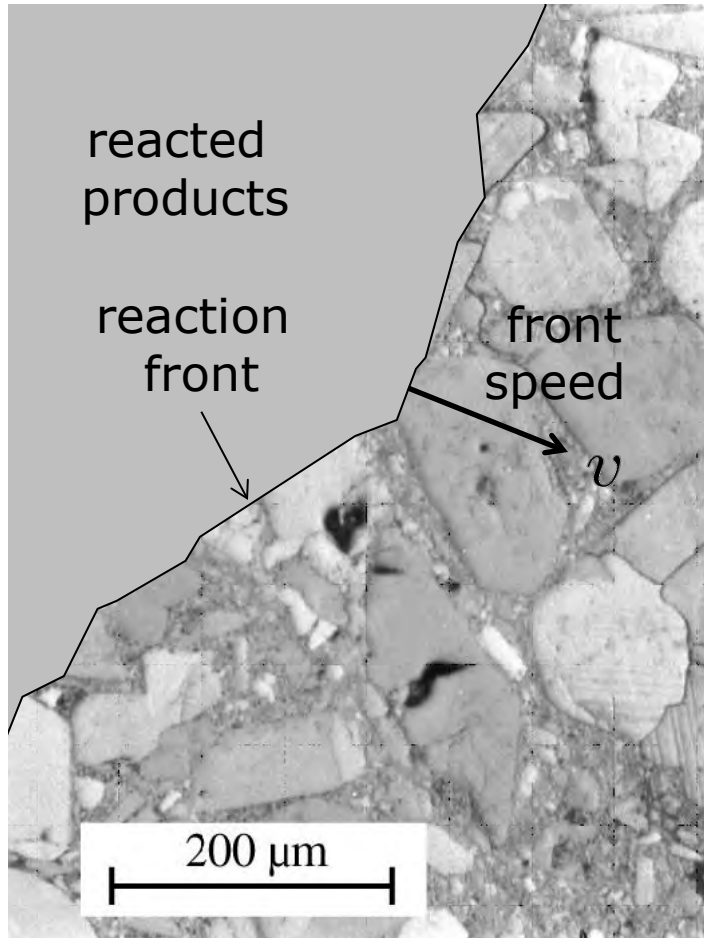
# Outlook: Optimal propellant microstructures

- Build on fundamental information from atomic-scale calculations:
  - *Orientation-dependent reaction-front speed*
  - *Anisotropic thermal conductivity tensor*
- Parameterize microstructures of interest:
  - *Composites: Polymer binder + oxidizer granules + additives (e.g.,  $\text{NH}_4\text{ClO}_4$  Composite Propellant, APCP)*
  - *Parameters: volume fractions, texture, morphology...*
- Simulate passage of reaction fronts through representative volume elements (front tracking)
- Determine effective/macroscopic burning rates
- Optimize microstructure for ***maximum specific impulse***





# Front-tracking (Hamilton/Jacobi) in heterogeneous/anisotropic media



- Fermat's principle (rays):

$$T = \min_{\text{paths}} \int_0^L \frac{ds}{v(x(s), x'(s))}$$

propagation speed  $\uparrow$   
propagation direction  $\uparrow$

- Lagrangian:  $L(x, x') = \frac{1}{v(x, x')}$

- Ray equations:

$$\dot{x}(s) = \nabla_{\xi} H(x(s), \xi(s))$$

$$\dot{\xi}(s) = -\nabla_x H(x(s), \xi(s))$$

- Front (eikonal) equation:

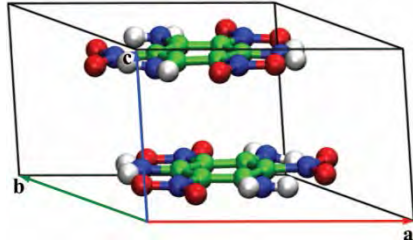
$$H(x, \nabla u(x)) = E$$



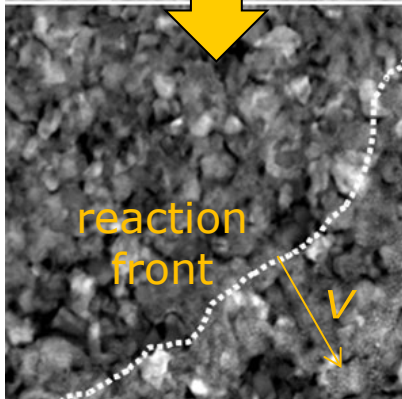
Tan, iMechanica, 2008

Michael Ortiz  
AFOSR 01/15

# Development of methods to identify optimal propellant microstructures



Anisotropic thermal conductivity of TATB  
(Kroonblawd & Sewell, *J. Chem. Phys.*, 2013)

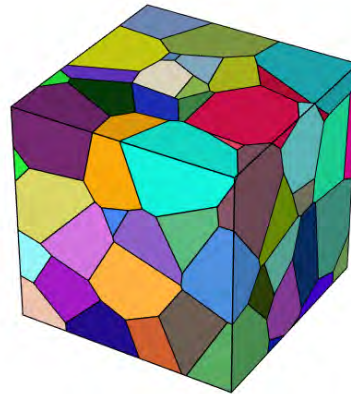


Anisotropic reaction-front propagation speed

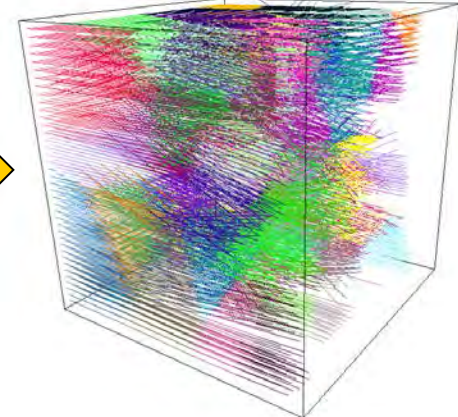
- Microstructure/specific impulse relation?
- Optimal microstructures?

Optimizer  
(e.g., genetic algorithm)

Objective function  
(e.g., mean reaction front speed)



Parameterized microstructure  
(e.g., Voronoi tessellation)

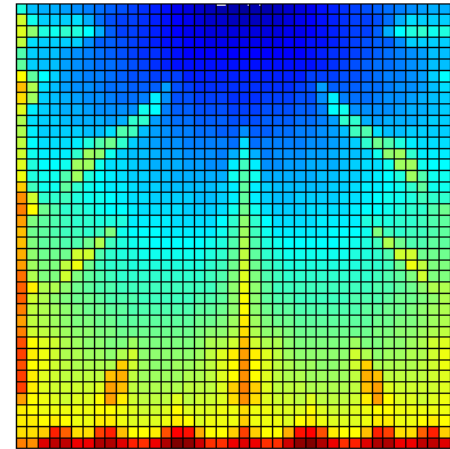
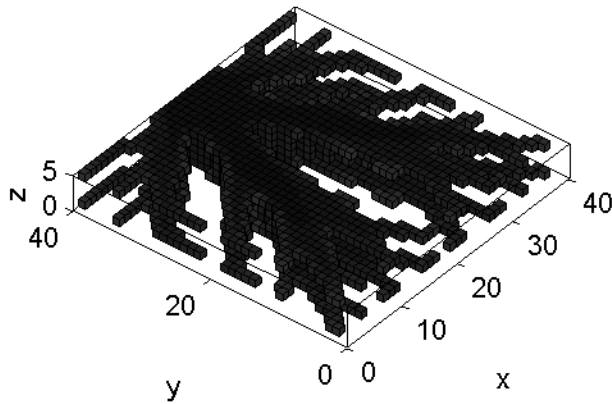


Reaction front tracking calculation  
(ray tracing)

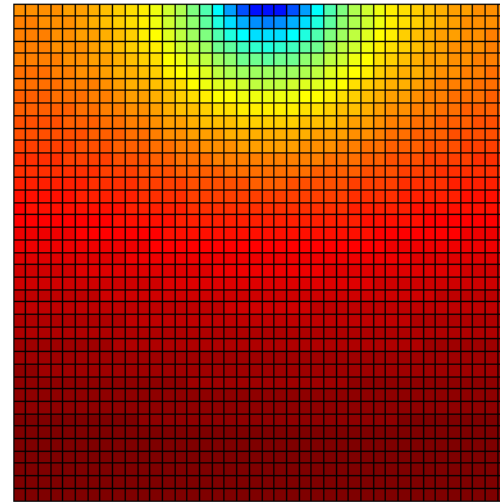
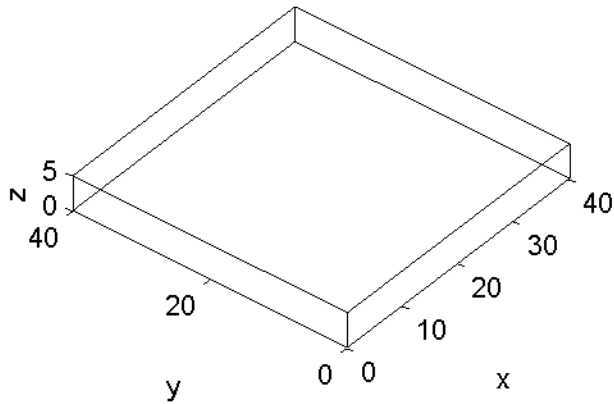
Michael Ortiz  
AFOSR 01/15



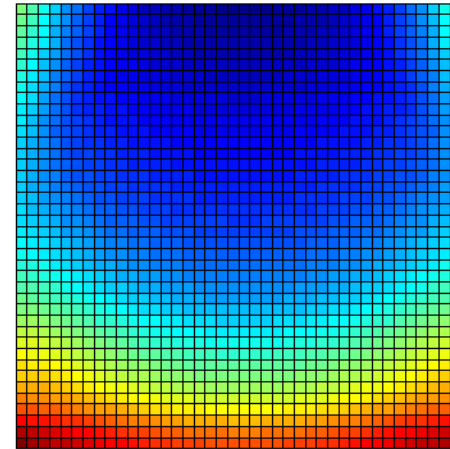
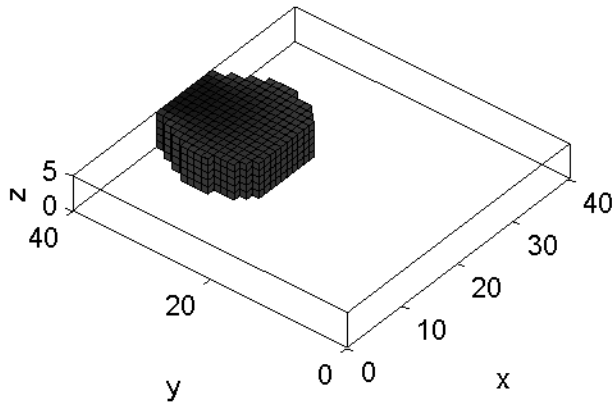
# Test case: Heat Conduction



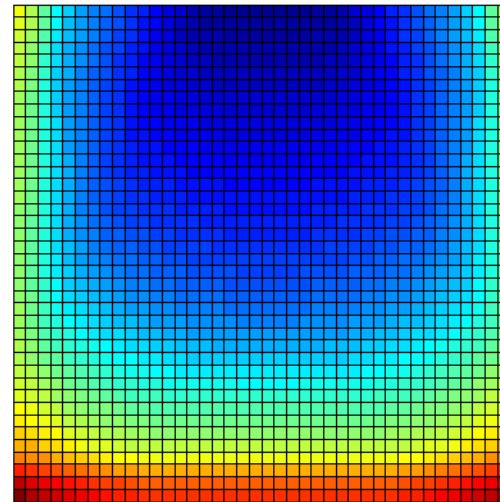
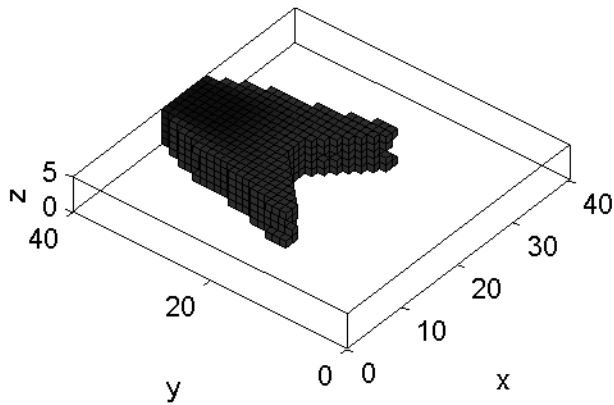
# Test case: Heat Conduction



# Test case: Heat Conduction

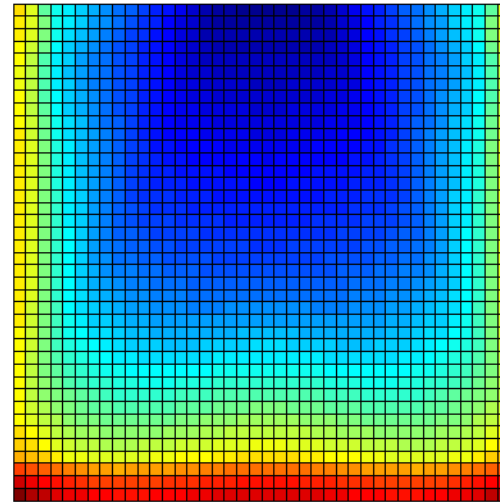
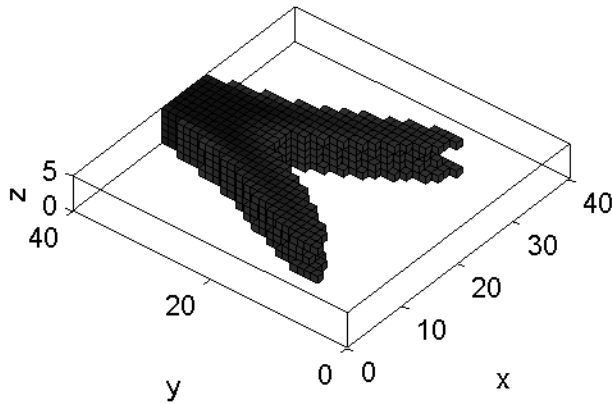


# Test case: Heat Conduction

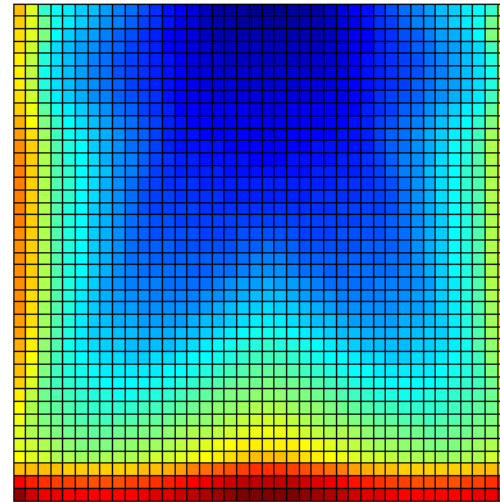
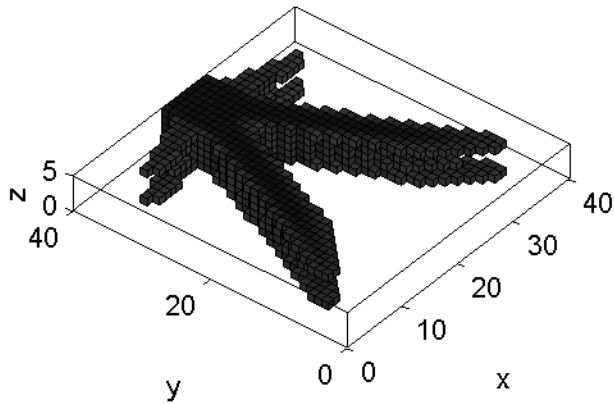




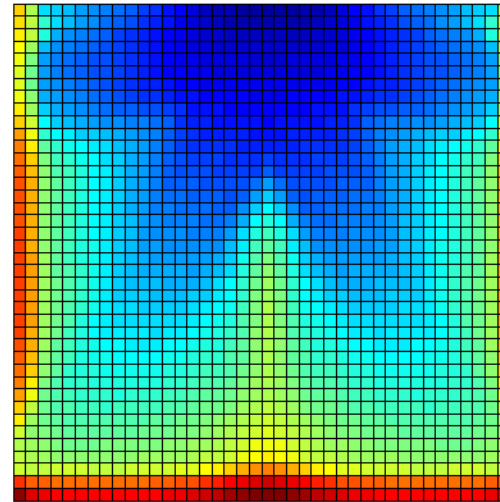
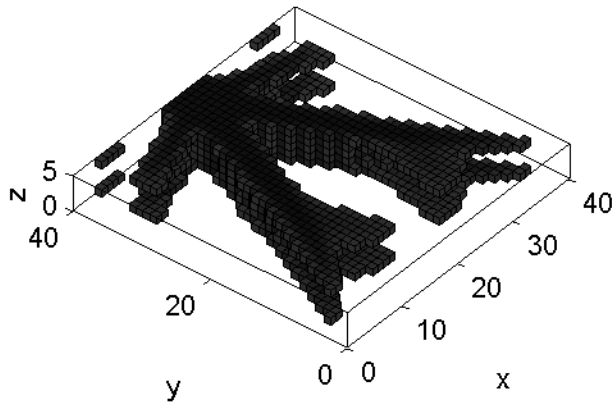
# Test case: Heat Conduction



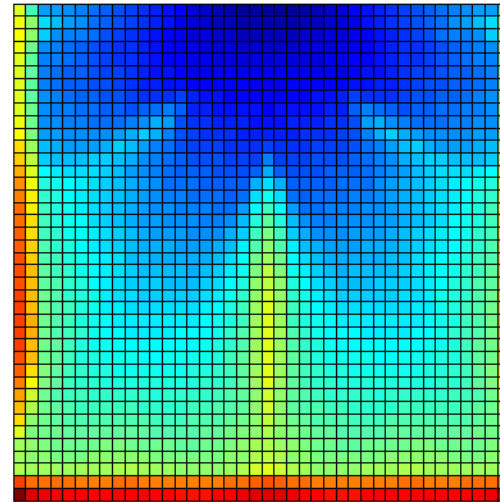
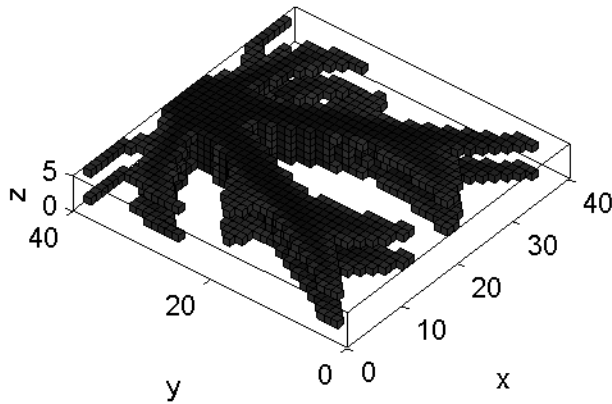
# Test case: Heat Conduction



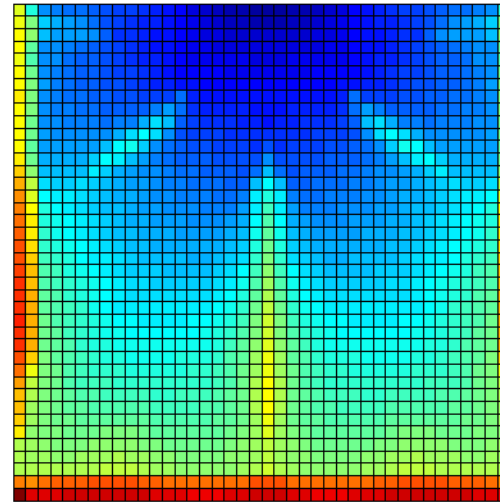
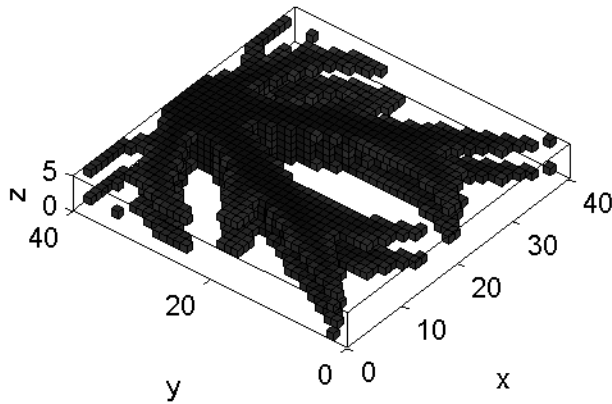
# Test case: Heat Conduction



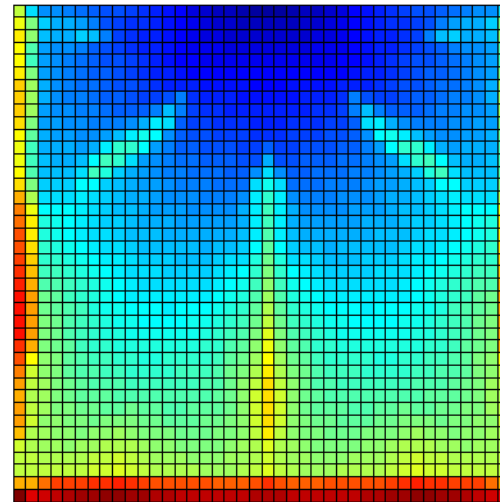
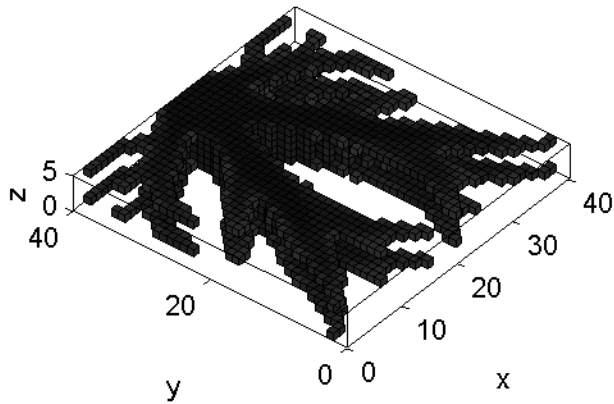
# Test case: Heat Conduction



# Test case: Heat Conduction

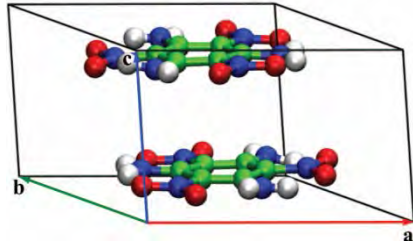


# Test case: Heat Conduction

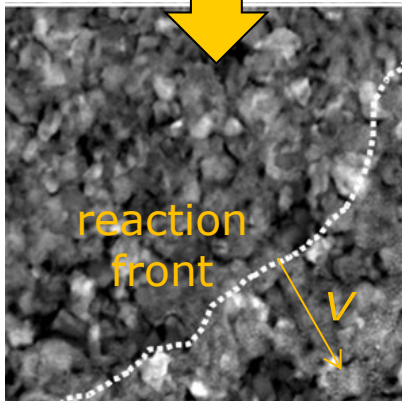




# Development of methods to identify optimal propellant microstructures



Anisotropic thermal conductivity of TATB  
(Kroonblawd & Sewell, *J. Chem. Phys.*, 2013)

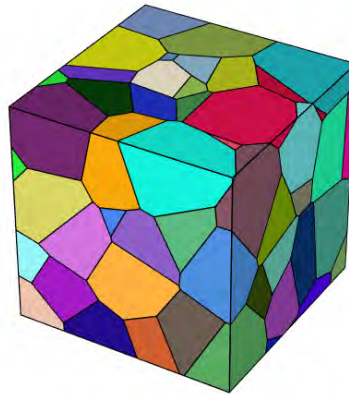


Anisotropic reaction-front propagation speed

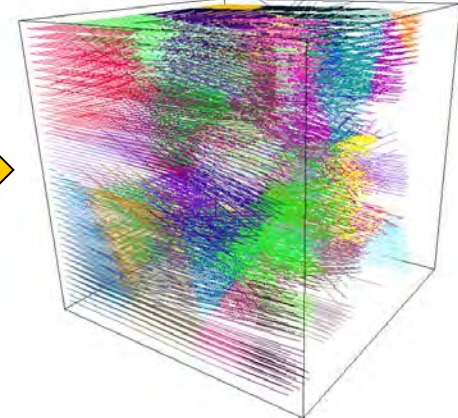
- Microstructure/specific impulse relation?
- Optimal microstructures?

Optimizer  
(e.g., genetic algorithm)

Objective function  
(e.g., mean reaction front speed)



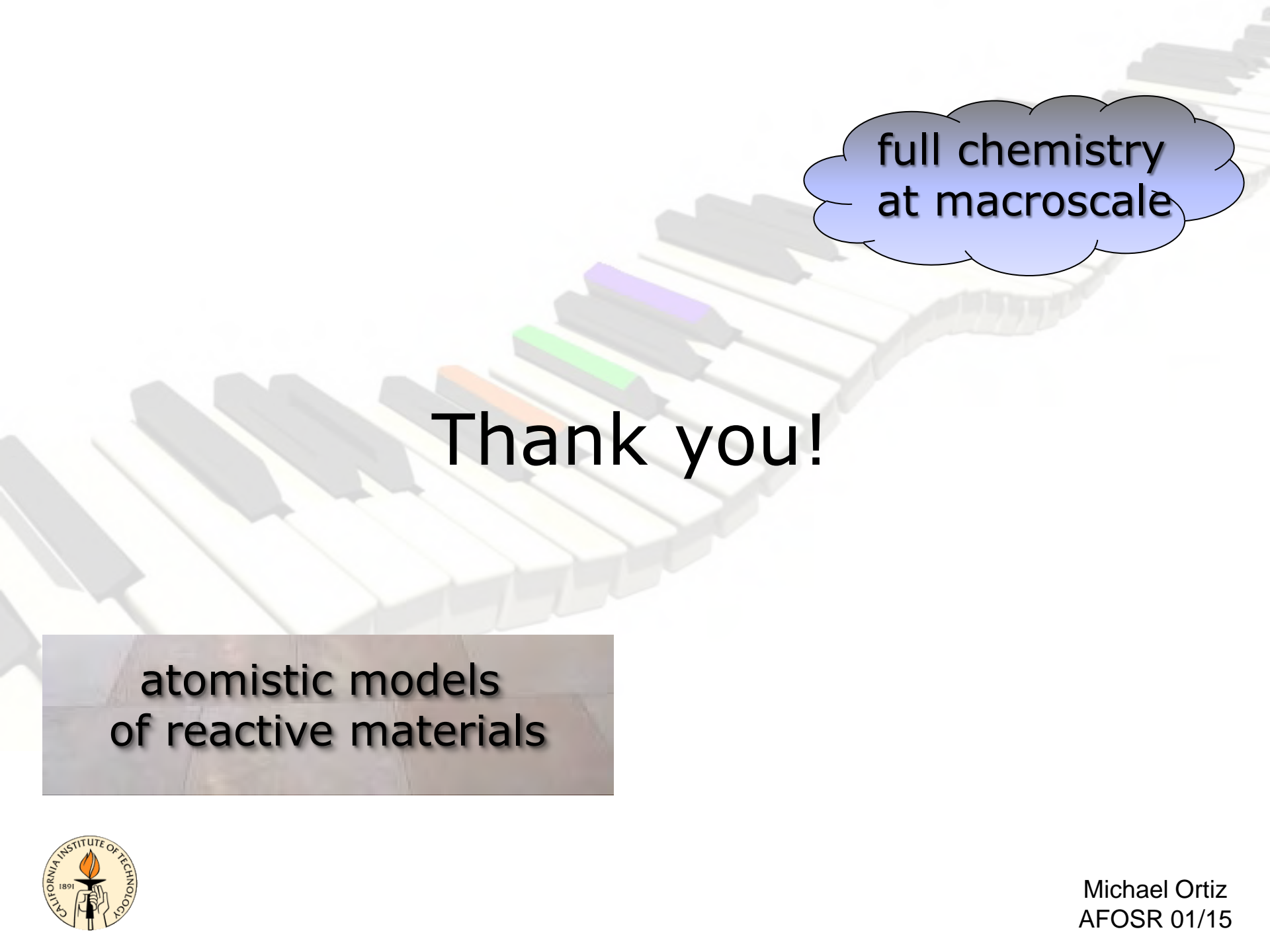
Parameterized microstructure  
(e.g., Voronoi tessellation)



Reaction front tracking calculation  
(ray tracing)

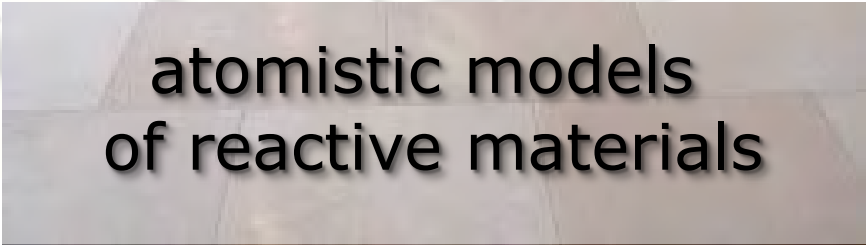
Michael Ortiz  
AFOSR 01/15





full chemistry  
at macroscale

Thank you!



atomistic models  
of reactive materials

