



***Integrity ★ Service ★ Excellence***

# **Exploiting Resonances To Find Tunnels Using Synthetic Aperture Radar**

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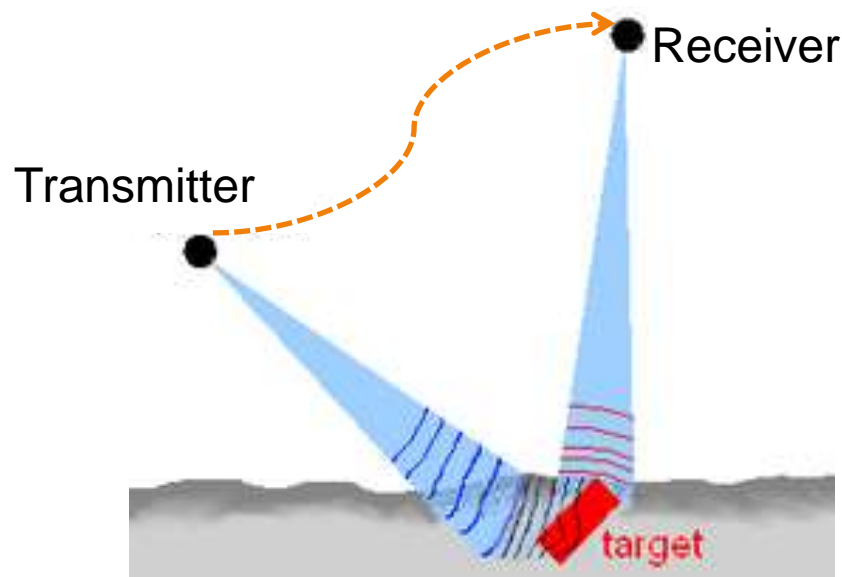
# OUTLINE



- Introduction
- Progress
- Examples
- Future Work
- Conclusion



# INTRODUCTION



**GOAL:** Derive a method to detect and image underground facilities using Synthetic Aperture Radar data.

## • SYNTHETIC APERTURE RADAR

- Collects large amount of information (data sets)
- Difficult to disambiguate targets of interest vs. clutter
- A lot of the signal gets absorbed by target if target is not made from specific materials
- Has ability to penetrate through clouds, trees, building at a cost to data



# Introduction



## Why is this interesting???

Large amounts of radar data is collected for many purposes but there is no possible way to form an image of every radar data point for every application!



If we a priori model what we want from the data using this method, then we may more efficiently find desired:

- **Geometric Structures**
- **Material properties**
- **Local geometric information**



# Introduction

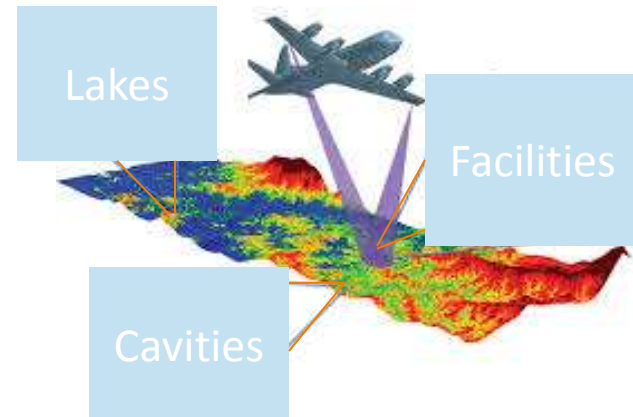


## Importance to Air Force

- 1) Provide war-fighter w/situational awareness.
- 2) Develop detection & imaging methods that may improve existing ISR Radar Technologies.
- 3) Develop algorithms that may be utilized to estimate material properties, find locations of cavities, find potential buried structures, and image data efficiently.

LARGE DATA SETS HAVE DISCRETE SUBSPACES THAT ARE SPECIFIC TO SCENE OBJECT OF INTEREST

EXPLORE STATE-OF-THE-ART “ANALYTICS” OR “DATA MINING” APPROACHES BEFORE MAKING IMAGES SAVES VALUABLE TIME



Imaging this entire scene takes valuable resources & time

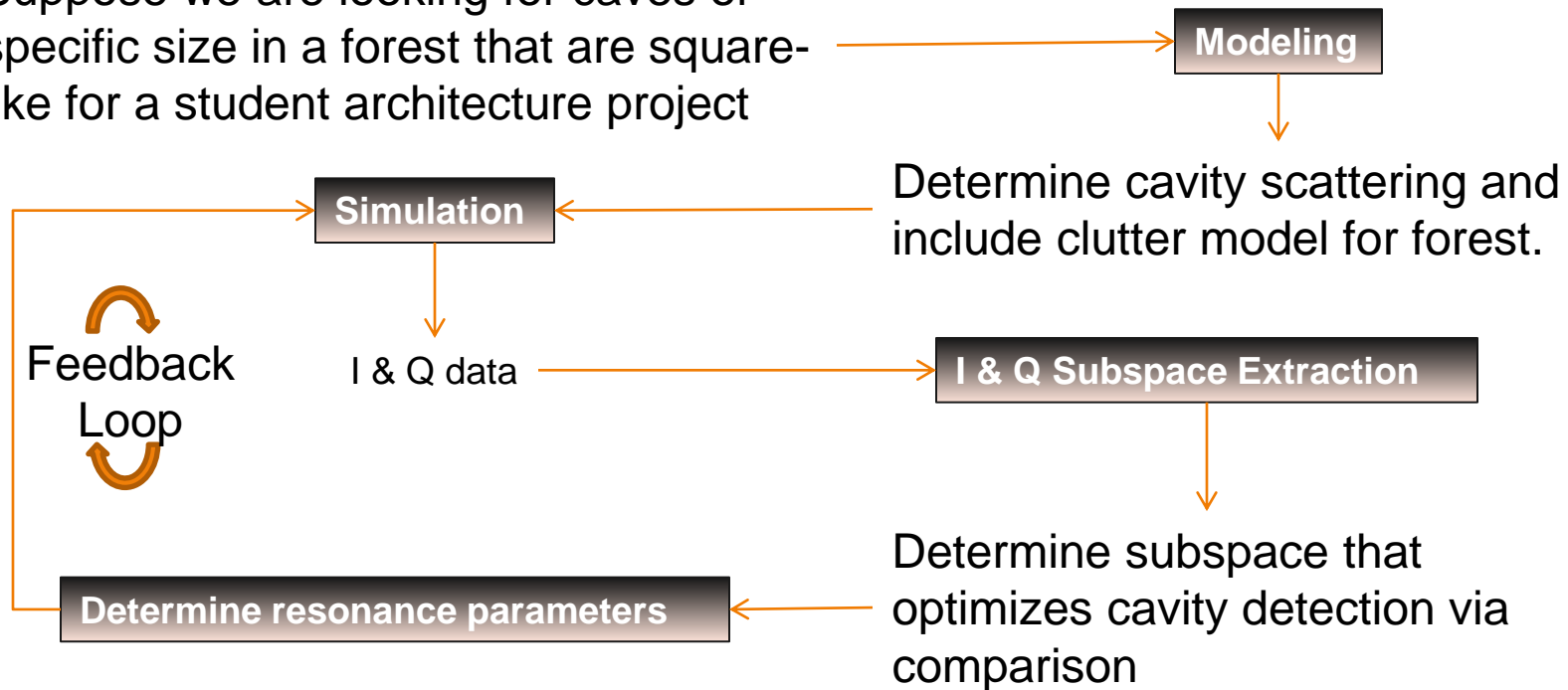


# Progress



## Oversimplified Overview of Tunnel Detection/Imaging Process

Suppose we are looking for caves of specific size in a forest that are square-like for a student architecture project



- Feedback loop ends when optimized subspace converges to linear/quasi-linear Kernel.
- Localized detection/imaging of cavity from far-field radar data begins now.



# Progress



We mathematically model two data sets, one containing a cavity ( $d_i$ ) and one that does not contain a cavity ( $d_j$ ). We perturb both data sets by using some independent identically distributed random function. Let  $\rho_{ij} = \mathbb{E}[(d_i/\sigma(d_i) - \mu_i)(d_j/\sigma(d_j) - \mu_j)] \in \mathbb{R}^{m \times m}$  where  $\mu_i = \mathbb{E}(d_i)$  and  $\sigma = (\mathbb{E}[(d_i - \mathbb{E}(d_i)) \cdot (d_i - \mathbb{E}(d_i))])$ . If we can determine some  $k(\omega)$  related to  $d$  that guarantees that  $|d_i(t, \gamma(t)) - d_j(t, \gamma(t))| \geq \frac{1}{\epsilon}$  for  $\epsilon$  small then we may design a radar that guarantees the detection of a cavity of dimensions similar to that of the model.



# Progress



Example...here we have a rectangular cavity model

$$d(\gamma(s), t, s) = - \int_{\Omega} e^{-i\omega t - i\mathbf{p} \cdot \gamma(s) + \frac{|\mathbf{x} - \gamma(s)|}{2}(t' - \frac{1}{t'})} \sum_n \frac{T_n \sin(n\pi \mathbf{x}) [1 - r_n]}{i\pi |\mathbf{x} - \gamma(s)| t'^2} d\mathbf{x} d\omega dt'$$

Here  $\mathbf{x}$  is a 2-D vector,  $\mathbf{p}$  is the propagation direction,  $T_n$  denotes a transmission coefficient,  $r_n$  denotes a reflection coefficient that is related to a propagation constant  $k_n$  and  $\gamma(s)$  denotes the 2-D flight path.

$$k_n = \sqrt{k^2 - (n\pi)^2}$$

$$r_n = e^{-2iknd}$$

$$k = \frac{\omega}{c}, \mathbf{p} = (\cos\alpha, \sin\alpha)$$

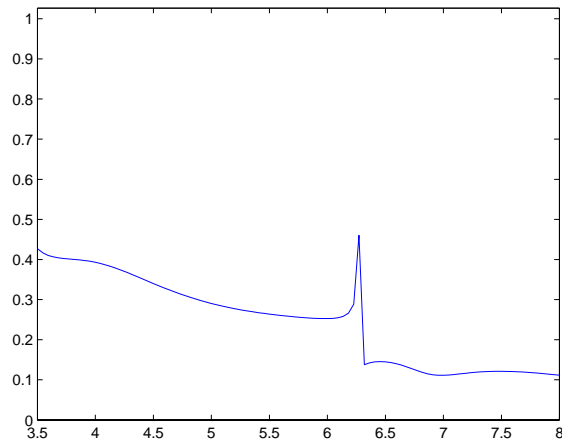




# Progress

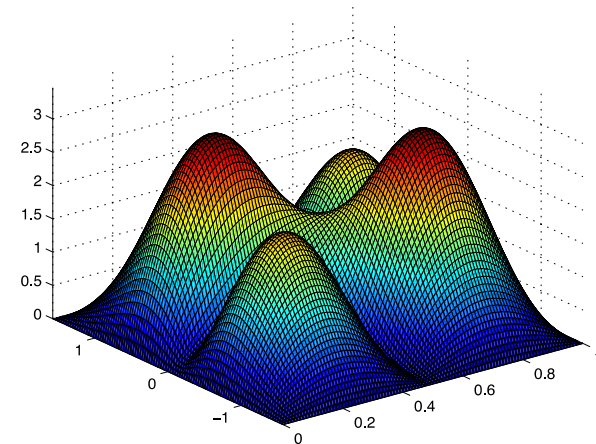


This is a model of the data for the cavity (reflectivity function)



Far Scattered Field response:

- resonance occurs at specific frequency ( $k \sim 6$ )
- peak amplitude changes with aspect angle but always occurs at the same location



Total field response:

- varying  $k$ -values
- Multiple peaks occur at different locations for distinct  $k$ -values
- spreading is occurring at specific locations/may be reason for aspect angle dependence



# Progress



Comparing our rectangular cavity waveguide and a standard half-space with additive (Rician) clutter reflectivity's we find

Important parameters to consider:

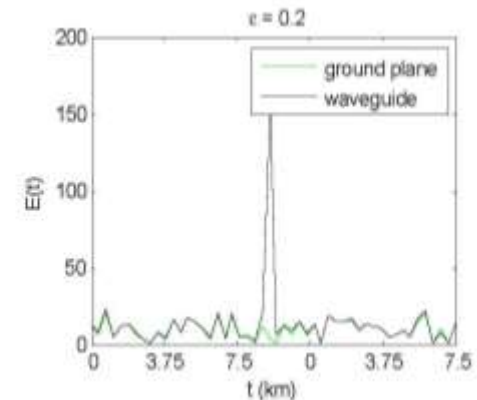
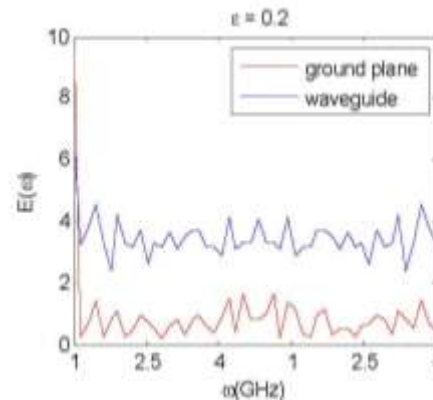
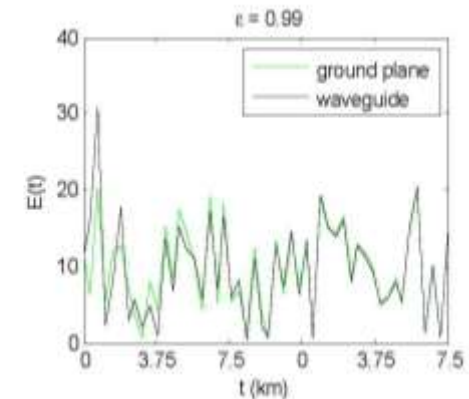
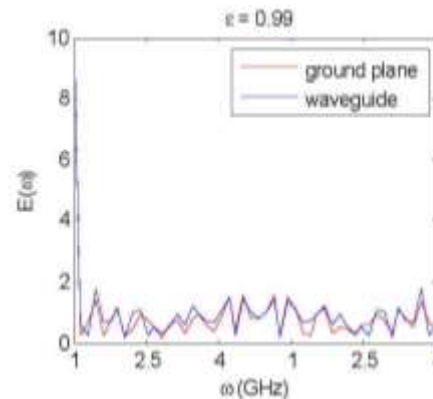
$\gamma = \text{phase shift in material}$

$\mu = \text{phase shift in air}$

In order to achieve desired separation:

$$\frac{\csc(\gamma)}{\cot(\mu) - i} \approx \varepsilon$$

$$|\varepsilon| \leq 0.5$$





# Progress



For the right flight paths, radar waves bouncing around inside geometric cavities like these may be found using modeling and simulation.



- We may apply these techniques to several situations where we have local information such as topographical maps, optical images, or other multispectral data.
- Complex valued frequencies (natural resonances of materials) not explored in this work.



# Progress



**Even when we have narrowed down the data points of interest...**

- **difficult to disentangle effect of material from effect of geometry**
- **want to obtain image depending on 3 variables, but typical SAR data depends on 2 variables**
  - **analogy with circular SAR**
  - **may need wide aperture**
  - **expect resolution tradeoffs**





# Progress



**We need to now figure out how to find and image the targets of interest....**

- **Develop model for radar data:  $d = M V$  ← reflectivity**
  - include flight path, waveforms
  - measure range & Doppler from different antenna positions (“3D” data)
- **Form image by matched filtering  $\text{Image} = M^* d$**   
where  $M^* = \text{Adjoint of } M$
- **Analyze image:  $\text{Image} = M^* M V$** 
  - want  $M^* M$  to be close to identity operator



# Progress



$$d = M V$$



$$\mathcal{E}_B^{sc}(t, \gamma(t)) = -\frac{1}{2\pi} \int e^{-i\omega t} \int \frac{e^{i\omega|\gamma(t)-\mathbf{y}|/c}}{4\pi|\gamma(t)-\mathbf{y}|} V(\omega, \mathbf{y}) \int \frac{e^{i\omega[t'+|\mathbf{y}-\gamma(t')|/c]}}{4\pi|\mathbf{y}-\gamma(t')|} f(t') dt' d\mathbf{y} d\omega$$

measure on  
moving antenna

transform to  
time domain

propagator

frequency- and  
location-dependent  
scattering

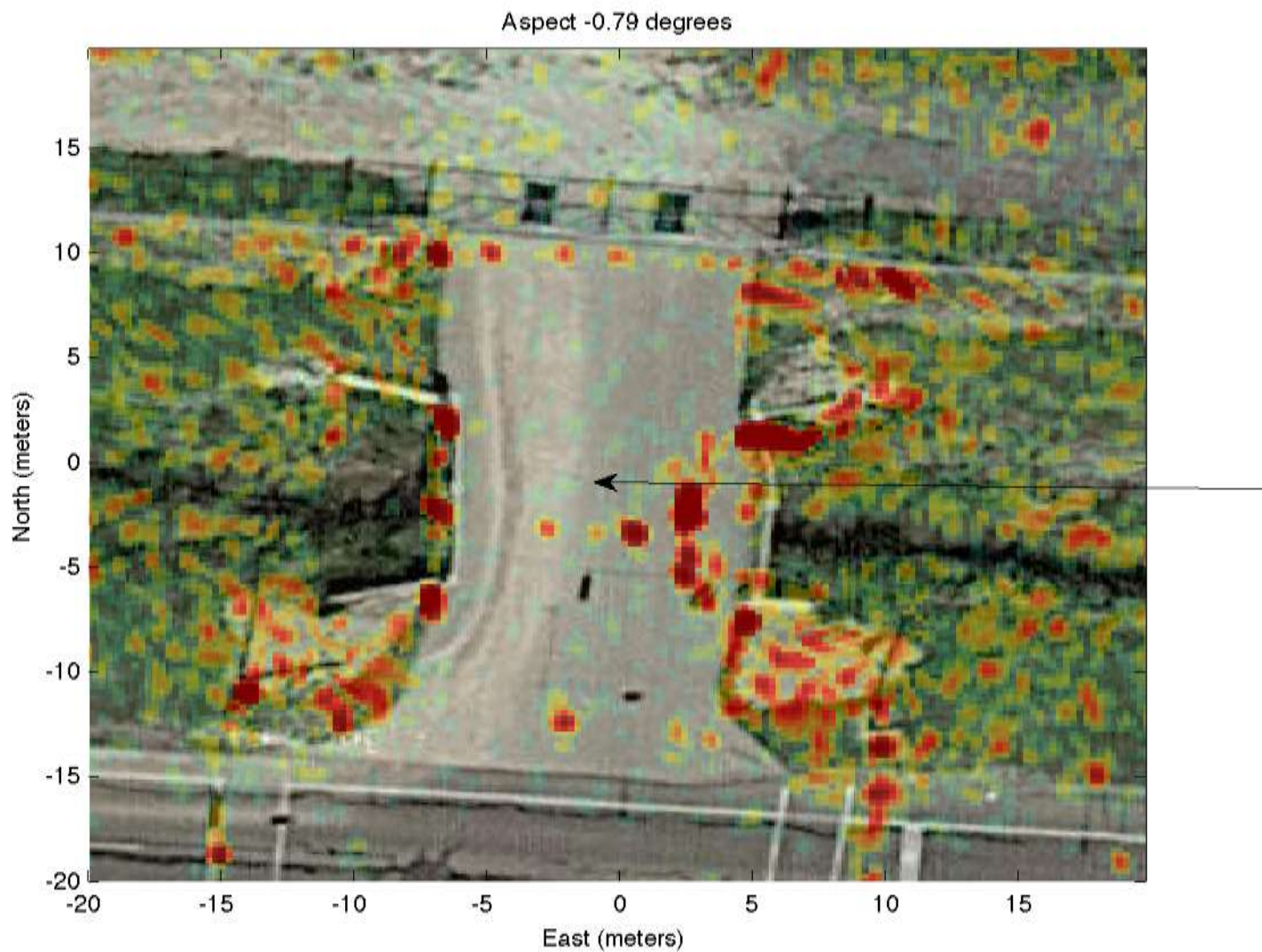
propagator

transmitted  
waveform



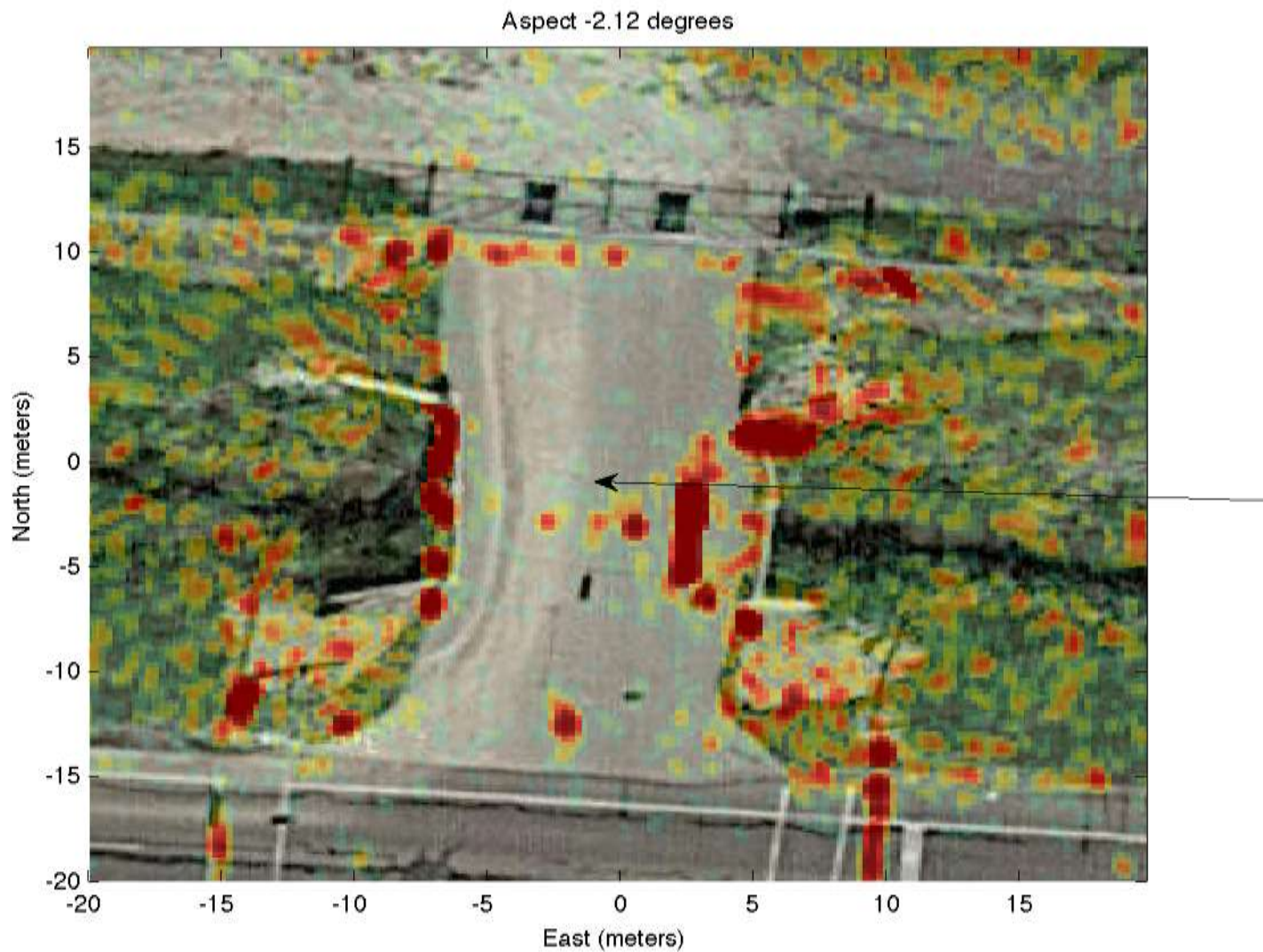


# SAR flying clockwise: CPI 1





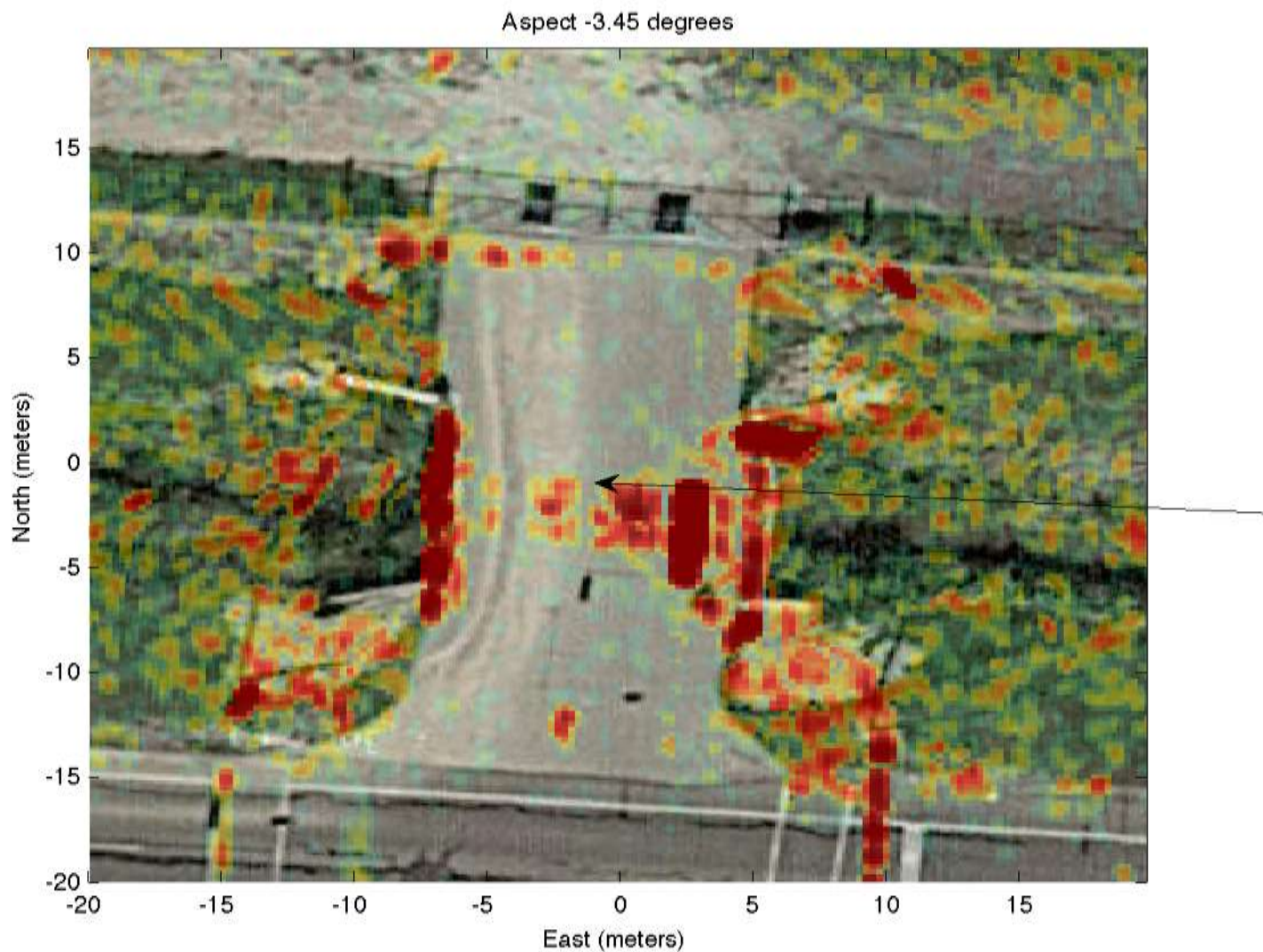
# SAR flying clockwise: CPI 2





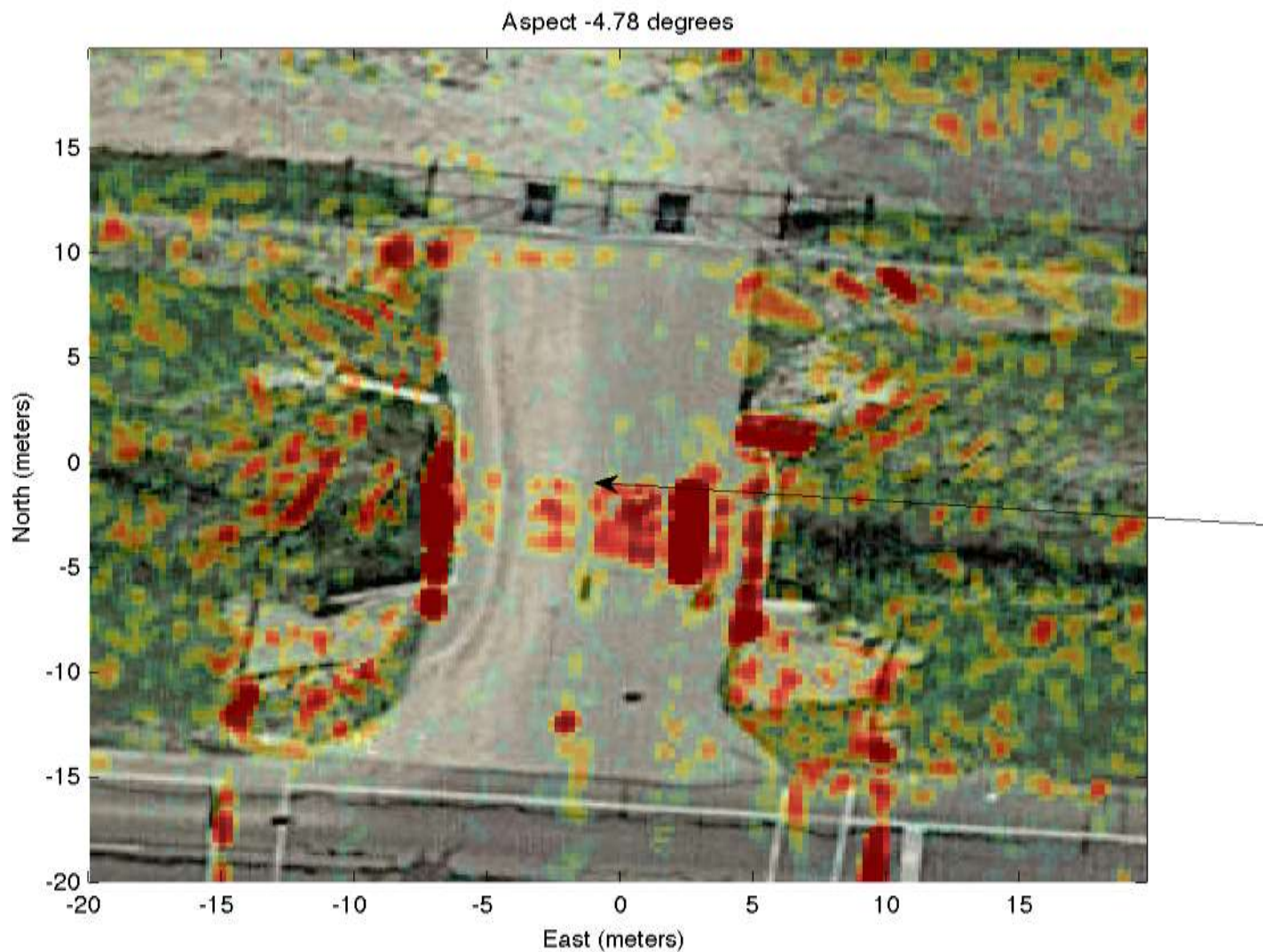


# SAR flying clockwise: CPI 3





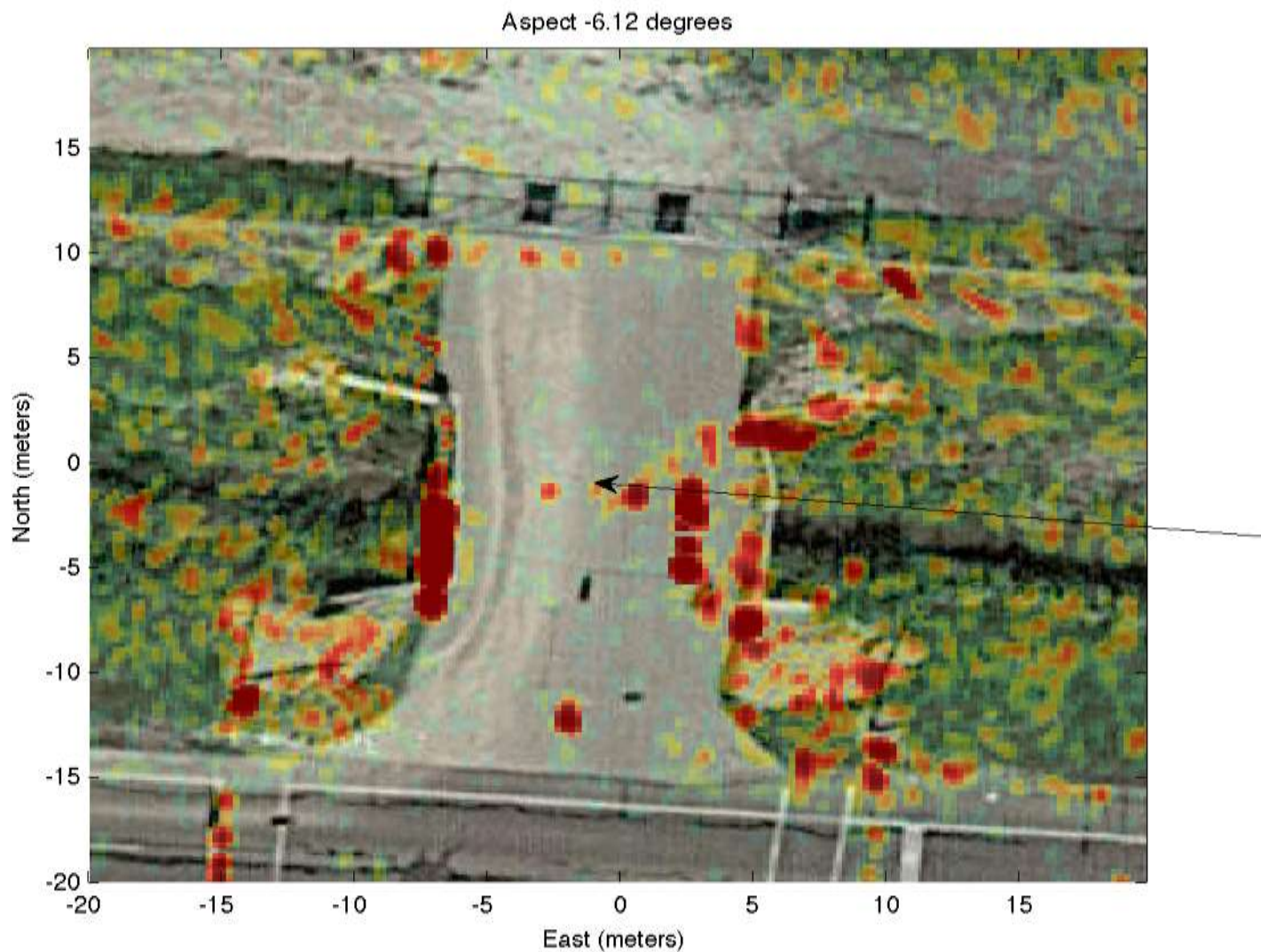
# SAR flying clockwise: CPI 4





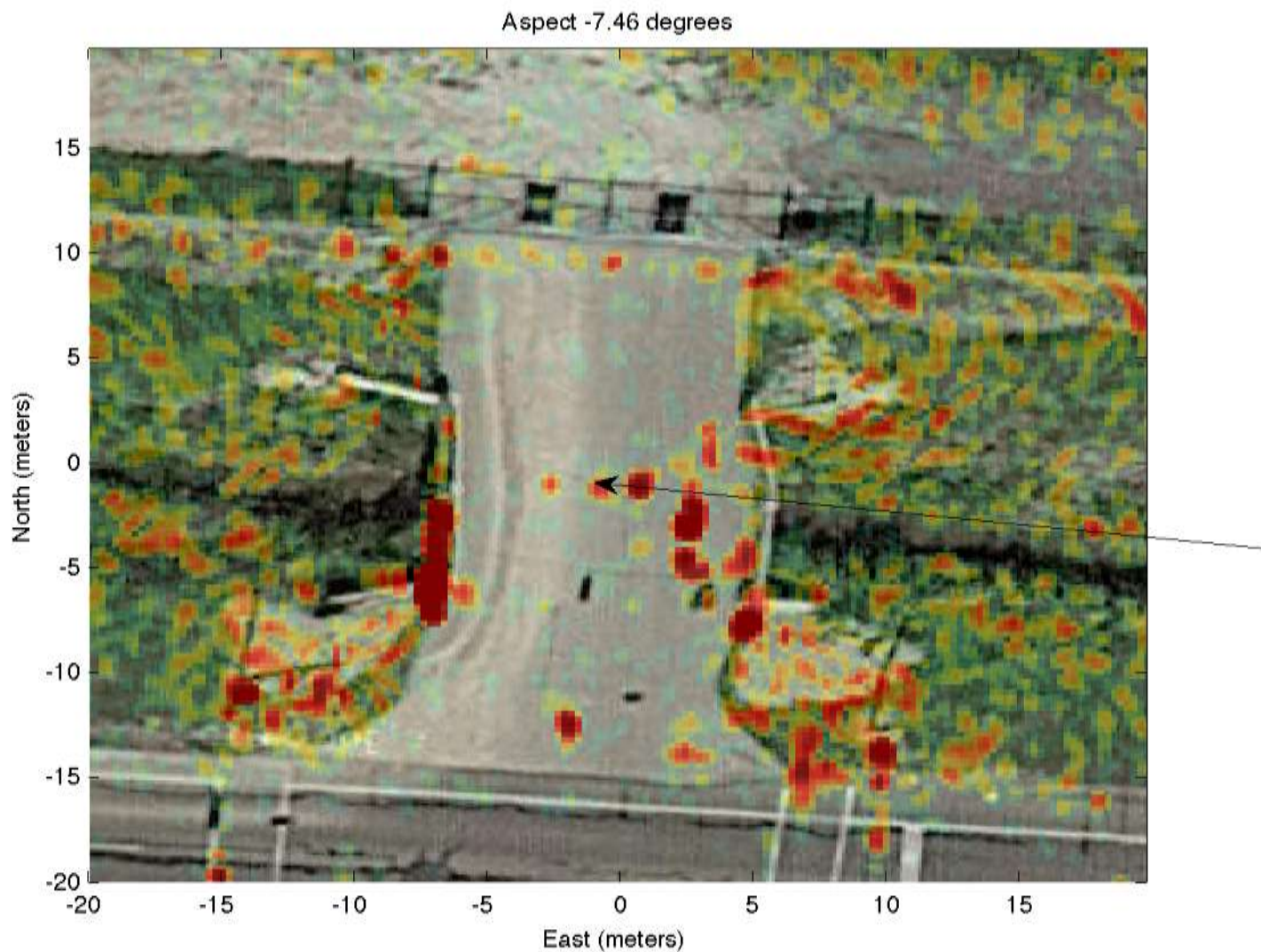


# SAR flying clockwise: CPI 5





# SAR flying clockwise: CPI 6





# CONCLUSION



- **Developed theory to find cavities in large radar data sets**
- **Modeled data for a simple case and showed how theory is useful**
- **Developed more realistic linear forward model that may be inverted to develop a preliminary imaging method for imaging cavities**
- **Showed examples how cavities may be found using SAR**



# Future Work



- **Extend detection theory to include complex variables**
- **Extend imaging method to include**
  - **system that is continuously transmitting**
  - **antenna beam pattern**
  - **simulate one-dimensional test case**



# QUESTIONS



**QUESTIONS ?**