

Computational Considerations for Compact Multi-frame Blind Deconvolution

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Motivation

Big Data Imaging Problems

- Multi-Frame Blind Deconvolution (MFBD) where multi = very large
- MFBD combined with 3D (shape) or 4D (shape and color) reconstructions.

Requirements

- Powerful computers
- More efficient algorithms
 - More efficient use of current algorithms
 - Smarter ways to process massive data sets
- Collaborative/synergistic teams
 - e.g., physics, math, computer science, engineering

Outline

- 1 Introduction
- 2 Past: Compact Multi-Frame Blind Deconvolution
- 3 Present: Global Variable Consensus
- 4 Future: Higher Dimensional Image Reconstruction

Convolution

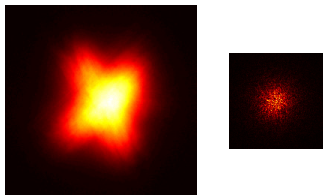
Consider the convolution image formation model:



Deconvolution

Deconvolution: Given

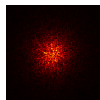
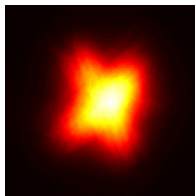
- Blurred image, and
- Point spread function (convolution kernel).



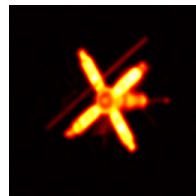
Deconvolution

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\Rightarrow compute \Rightarrow



- Compute estimate of true image.

Blind Deconvolution

Blind Deconvolution: Given

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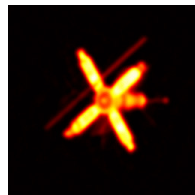
Blind Deconvolution

Blind Deconvolution: Given

- Blurred image.



\Rightarrow compute \Rightarrow

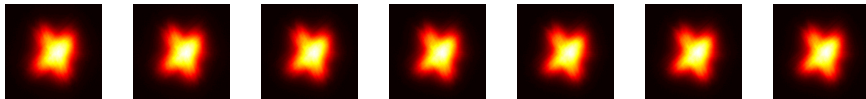


- Compute estimate of true image, and
- Compute estimate of PSF.

Multi-Frame Blind Deconvolution

Multi-Frame Blind Deconvolution (MFBD):

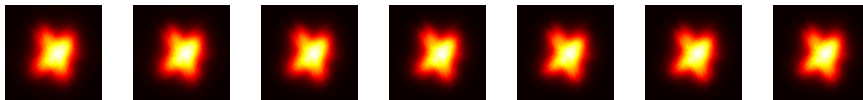
- Given multiple frames of blurred images:



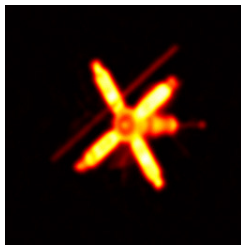
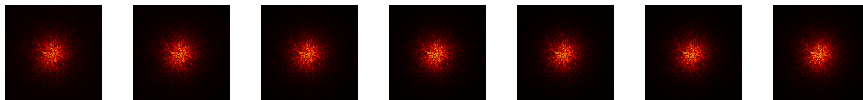
Multi-Frame Blind Deconvolution

Multi-Frame Blind Deconvolution (MFBD):

- Given multiple frames of blurred images:



- Reconstruct PSFs and object:



Single Frame Blind Deconvolution (SFBD) Model

Parameterize point spread function

- Using convolution model: $\mathbf{b} = \mathbf{psf}(\mathbf{y}) * \mathbf{x} + \boldsymbol{\eta}$
- Or, using matrix notation: $\mathbf{b} = \mathbf{A}(\mathbf{y})\mathbf{x} + \boldsymbol{\eta}$

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Example parameterizations:

- PSF pixels $\mathbf{psf} \left(\begin{array}{c} \text{[PSF Image]} \end{array} \right)$



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- Wavefront phase with Zernikes:

$$\mathbf{psf} \left(\begin{array}{c} \text{[Wavefront image]} \end{array} \right) = |\mathcal{F}^{-1}(Pe^{i(y_1z_1 + \dots y_mz_m)})|^2$$

General Mathematical Model

General mathematical model for image formation:

$$\mathbf{b} = \mathbf{A}(\mathbf{y}) \mathbf{x} + \boldsymbol{\eta}$$

where

- \mathbf{b} = vector representing observed image
- \mathbf{x} = vector representing true image
- $\mathbf{A}(\mathbf{y})$ = matrix defining blurring operation

For example,

- Convolution with imposed boundary conditions
- Spatially variant blurs
- \mathbf{y} = vector of parameters defining blurring operation

Goal: Given \mathbf{b} , jointly compute approximations of \mathbf{y} and \mathbf{x} .

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To solve, could consider least squares best fit objective:

$$\left\| \begin{bmatrix} \mathbf{b}_1 - \mathbf{A}(\mathbf{y}_1)\mathbf{x} \\ \vdots \\ \mathbf{b}_m - \mathbf{A}(\mathbf{y}_m)\mathbf{x} \end{bmatrix} \right\|_2^2 = \left\| \begin{bmatrix} \mathbf{b}_1 \\ \vdots \\ \mathbf{b}_m \end{bmatrix} - \begin{bmatrix} \mathbf{A}(\mathbf{y}_1) \\ \vdots \\ \mathbf{A}(\mathbf{y}_m) \end{bmatrix} \mathbf{x} \right\|_2^2 = \|\mathbf{b} - \mathbf{A}(\mathbf{y})\mathbf{x}\|_2^2$$

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Also need regularization, but we omit that complication for now.

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Processing a large number of frames is computationally intensive.

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Compact MFBD (CMFBD)

D. Hope, S. Jefferies, Optics Letters, 36 (2011), pp. 867–869.

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- Identify a small set of *control frames* that contain most independent information.

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Compact MFBD (CMFBD)

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- Identify a small set of *control frames* that contain most independent information.
- Reduce full set of data to small set of control frames, without losing any important information.

CMFBD: Basic Idea of Computational Problem

Use spectral ratios to transform the MFBD least squares objective:
(we assume one control frame for illustration)

$$\left\| \begin{bmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \\ \vdots \\ \mathbf{b}_m \end{bmatrix} - \begin{bmatrix} \mathbf{A}(\mathbf{y}_1) \\ \mathbf{A}(\mathbf{y}_2) \\ \vdots \\ \mathbf{A}(\mathbf{y}_m) \end{bmatrix} \mathbf{x} \right\|_2^2$$

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$$\text{(using orthogonal rotations)} = \left\| \begin{bmatrix} \tilde{\mathbf{b}}_1 \\ \tilde{\mathbf{b}}_2 \\ \vdots \\ \tilde{\mathbf{b}}_m \end{bmatrix} - \begin{bmatrix} \mathbf{W} \\ \mathbf{0} \\ \vdots \\ \mathbf{0} \end{bmatrix} \mathbf{A}(\mathbf{y}_1) \mathbf{x} \right\|_2^2$$

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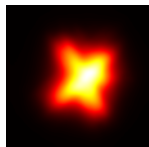
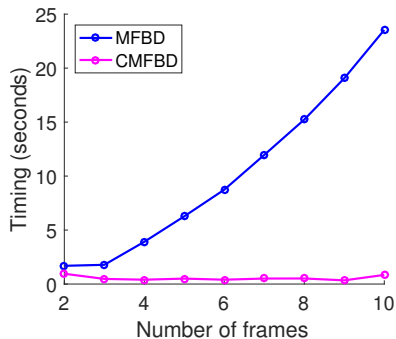
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$$\begin{aligned}
 \left\| \begin{bmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \\ \vdots \\ \mathbf{b}_m \end{bmatrix} - \begin{bmatrix} \mathbf{A}(\mathbf{y}_1) \\ \mathbf{A}(\mathbf{y}_2) \\ \vdots \\ \mathbf{A}(\mathbf{y}_m) \end{bmatrix} \mathbf{x} \right\|_2^2 &= \left\| \begin{bmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \\ \vdots \\ \mathbf{b}_m \end{bmatrix} - \begin{bmatrix} \mathbf{C}_1 \\ \mathbf{C}_2 \\ \vdots \\ \mathbf{C}_m \end{bmatrix} \mathbf{A}(\mathbf{y}_1) \mathbf{x} \right\|_2^2 \\
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 &= \|\tilde{\mathbf{b}}_1 - \mathbf{W} \mathbf{A}(\mathbf{y}_1) \mathbf{x}\|_2^2
 \end{aligned}$$

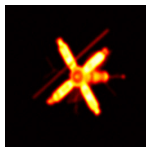
Note that by using orthogonal rotations:

- information in $\mathbf{b}_1, \mathbf{b}_2, \dots, \mathbf{b}_m$ is folded into $\tilde{\mathbf{b}}_1$
- information in $\mathbf{C}_1, \mathbf{C}_2, \dots, \mathbf{C}_m$ is folded into \mathbf{W}

Numerical Illustration of Time Savings



Mean data frame



MFBD



CMFBD

Present: Global Variable Consensus

The MFBD problem can be written as:

$$\min_{\mathbf{y}_i, \mathbf{x}} \sum_{i=1}^m \|\mathbf{b}_i - \mathbf{A}(\mathbf{y}_i)\mathbf{x}\|_2^2 + g(\mathbf{x})$$

where here we include an object regularization term, $g(\mathbf{x})$.

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Remarks:

- Regularization $g(\mathbf{x})$ can be used to enforce nonnegativity, sparsity, etc.

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Remarks:

- Regularization $g(\mathbf{x})$ can be used to enforce nonnegativity, sparsity, etc.
- The unknown \mathbf{x} couples the objective terms $i = 1, \dots, m$
- We can get a partial decoupling by reformulating as:

$$\min_{\mathbf{y}_i, \mathbf{x}_i} \sum_{i=1}^m \|\mathbf{b}_i - \mathbf{A}(\mathbf{y}_i)\mathbf{x}_i\|_2^2 + g(\mathbf{z}) \quad \text{subject to } \mathbf{x}_i = \mathbf{z}, \quad i = 1, \dots, m$$

Global Variable Consensus

Using an augmented Lagrangian approach, and the Alternating Direction Method of Multipliers (ADMM)¹, the optimization decouples:

for $k = 1, 2, \dots$

$$\left[\mathbf{y}_i^{(k+1)}, \mathbf{x}_i^{(k+1)} \right] = \underset{\mathbf{y}_i, \mathbf{x}_i}{\operatorname{argmin}} \left\| \mathbf{b}_i - \mathbf{A}(\mathbf{y}_i) \mathbf{x}_i \right\|_2^2 + \frac{\beta}{2} \left\| \mathbf{x}_i - \mathbf{z}^{(k)} + \mathbf{u}_i^{(k)} \right\|_2^2$$

$$\bar{\mathbf{x}}^{(k+1)} = \frac{1}{m} \sum_{i=1}^m \mathbf{x}_i^{(k+1)}$$

$$\bar{\mathbf{u}}^{(k)} = \frac{1}{m} \sum_{i=1}^m \mathbf{u}_i^{(k)}$$

$$\mathbf{z}^{(k+1)} = \underset{\mathbf{z}}{\operatorname{argmin}} \left\{ g(\mathbf{z}) + \frac{m\beta}{2} \left\| \mathbf{z} - \bar{\mathbf{x}}^{(k+1)} - \bar{\mathbf{u}}^{(k)} \right\|_2^2 \right\}$$

$$\mathbf{u}_i^{(k+1)} = \mathbf{u}_i^{(k)} + \mathbf{x}_i^{(k+1)} - \mathbf{z}^{(k+1)}$$

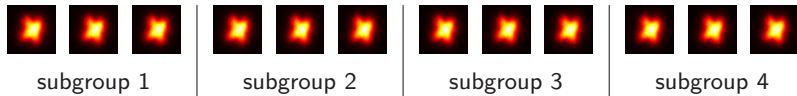
end

¹Good ADMM references: Wahlberg, Boyd, Annergren, Wang, *Proc. 16th IFAC Symposium on System Identification*, 2012, and Boyd, et. al., *Foundations and Trends in Machine Learning*, 2010.

Global Variable Consensus

Advantages:

- Decoupling allows for easy parallel processing of groups of frames

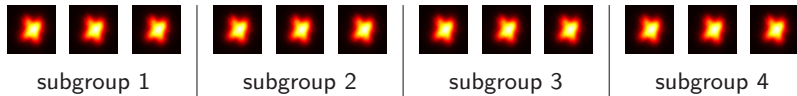


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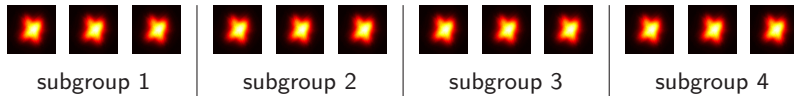


- Can either use standard MFBD on subgroups of frames, or
 - Use CMFBD on subgroups of frames
- Regularization term is also decoupled, allowing users to plug in many options, and it simplifies the computation.

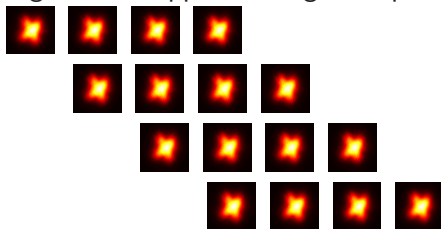
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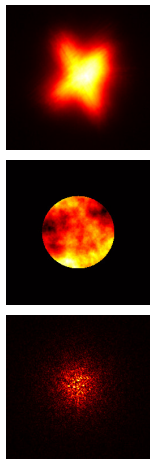
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- Can either use standard MFBD on subgroups of frames, or
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- Sliding window approach might be possible:

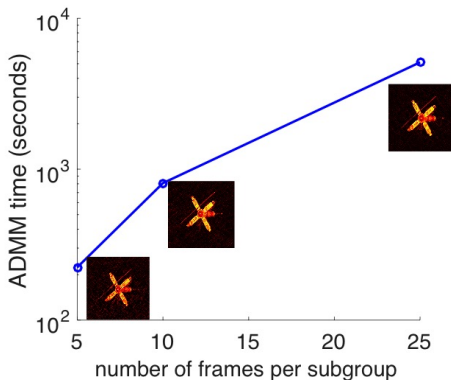


Global Variable Consensus: Numerical Illustration²



50 total frames, split in three different ways

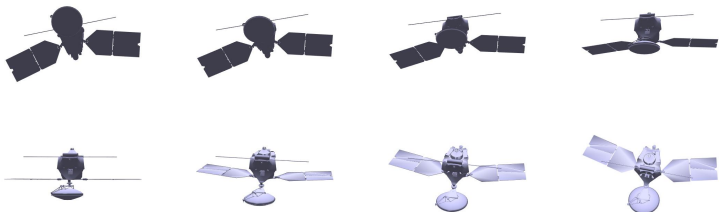
10 subgroups (5 frames each)	5 subgroups (10 frames each)	2 subgroups (25 frames each)
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²J. D. Schmidt, Numerical Simulation of Optical Wave Propagation, SPIE Press Monograph Vol. PM199, 2010

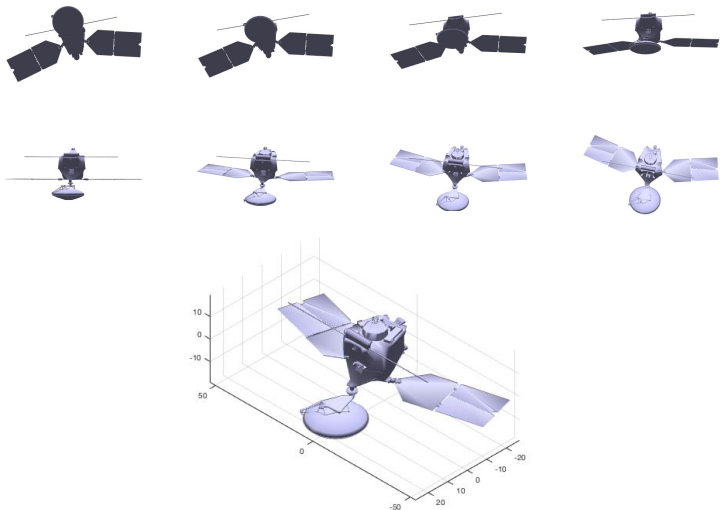
Future: Higher Dimensional Image Reconstruction

Three dimensional reconstruction from two dimensional measurements:



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Three dimensional reconstruction from two dimensional measurements:



Higher Dimensional Image Reconstruction

Some computational challenges:

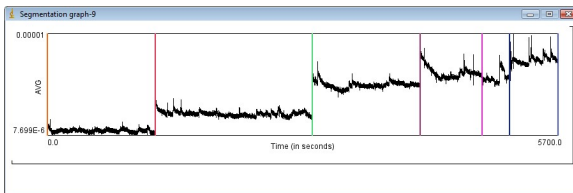
- Requires processing many, many frames of data.
- Mathematical model is similar to MFBD, but
 - Number of unknowns for object significantly increases.
 - Additional unknowns associated with parameters defining object orientation.
- Some related work has been done for molecular structure determination, e.g. in Cryo-EM and x-ray crystallography³

³J. Chung, P. Sternberg and C. Yang, *High Performance 3-D Image Reconstruction for Molecular Structure Determination*, International Journal of High Performance Computing Applications, 24 (2010), pp. 117–135.

Higher Dimensional Image Reconstruction

What further information can be used?

- Possibly assume blocks of data have constant orientation parameters
Idea like this was used in PET brain image reconstruction⁴



- Can use consensus ADMM type approach on blocks of data.
- Use other information (e.g., a frozen flow assumption), or technologies (e.g., laser guide stars).

⁴P. Wendykier, J. Nagy, *Parallel Colt: High Performance Java Library for Scientific Computing and Image Processing*, ACM Transactions on Mathematical Software, 37 (2010), pp. 31:1–31:22

Summary

- Big data, multi-frame image processing requires not only powerful computers, but also new approaches to process massive data sets.

This is especially true for 3D/4D image reconstructions.

- Goal should be to extract as much information as possible from collected data, but to also do it quickly.
- Important to have synergistic collaborations with various expertise.