

# ***ENHANCING EARLY-TIME DIFFUSION THROUGH BEAM COLLIMATION IN PULSE PROPAGATION IN SPARSE AND DISCRETE RANDOM MEDIA***

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*sparse media:* mean-free path several orders of magnitude larger  
than average separation between scatterers

*examples:* atmospheric clouds, fog, haze, dust, aerosols

## ■ objective

- ▶ investigate the problem of *imaging* and *communication* through *sparse* and *discrete random media* (such as atmospheric clouds, fog, dust, aerosols, ...) with optical / infrared *pulsed* signals in the framework of radiative transport equation (*RTE*)

## ■ previous work

- ▶ we presented theoretical analysis indicating that for a short infrared/optical pulse propagating in a sparse medium composed of scatterers large compared to the wavelength, there exists, in the time-resolved intensity, an early-time diffusion component which,
  - immediately follows the coherent signal
  - is attenuated proportionally to the non-diffractive cross-section on an average medium constituent (i.e. significantly weaker than the coherent component whose attenuation is governed by the total cross-section)
  - can be extracted by high-pass filtering (no time-gating required)

## ■ current work

- ▶ we investigate modal decomposition of *RTE* and identify dominant propagation modes responsible for early- and late-time diffusion
- ▶ we extend our analysis to sources radiating with arbitrary angular distributions
- ▶ we show that proper angular modulation of the source angular distribution may lead to strengthening of the early time diffusion effect

## ■ *parameters and characteristic length scales*

### ► *medium – atmospheric cloud*

*scatterers*

*water droplets with gamma radius distribution*

*average scatterer radius*

$$a \approx 5 \mu\text{m}$$

*number density*

$$n_0 = 10^9 \text{ m}^{-3}$$

*attenuation length*

$$\ell \approx 6 \text{ m}$$

*(attenuation length determined as  $\ell = 1/(n_0 \sigma_t)$ , where  $\sigma_t$  is the total scattering cross section computed from Mie solution)*

### ► *transmitted pulse*

*carrier frequency*

$$\nu_0 \approx 47.934 \text{ THz}$$

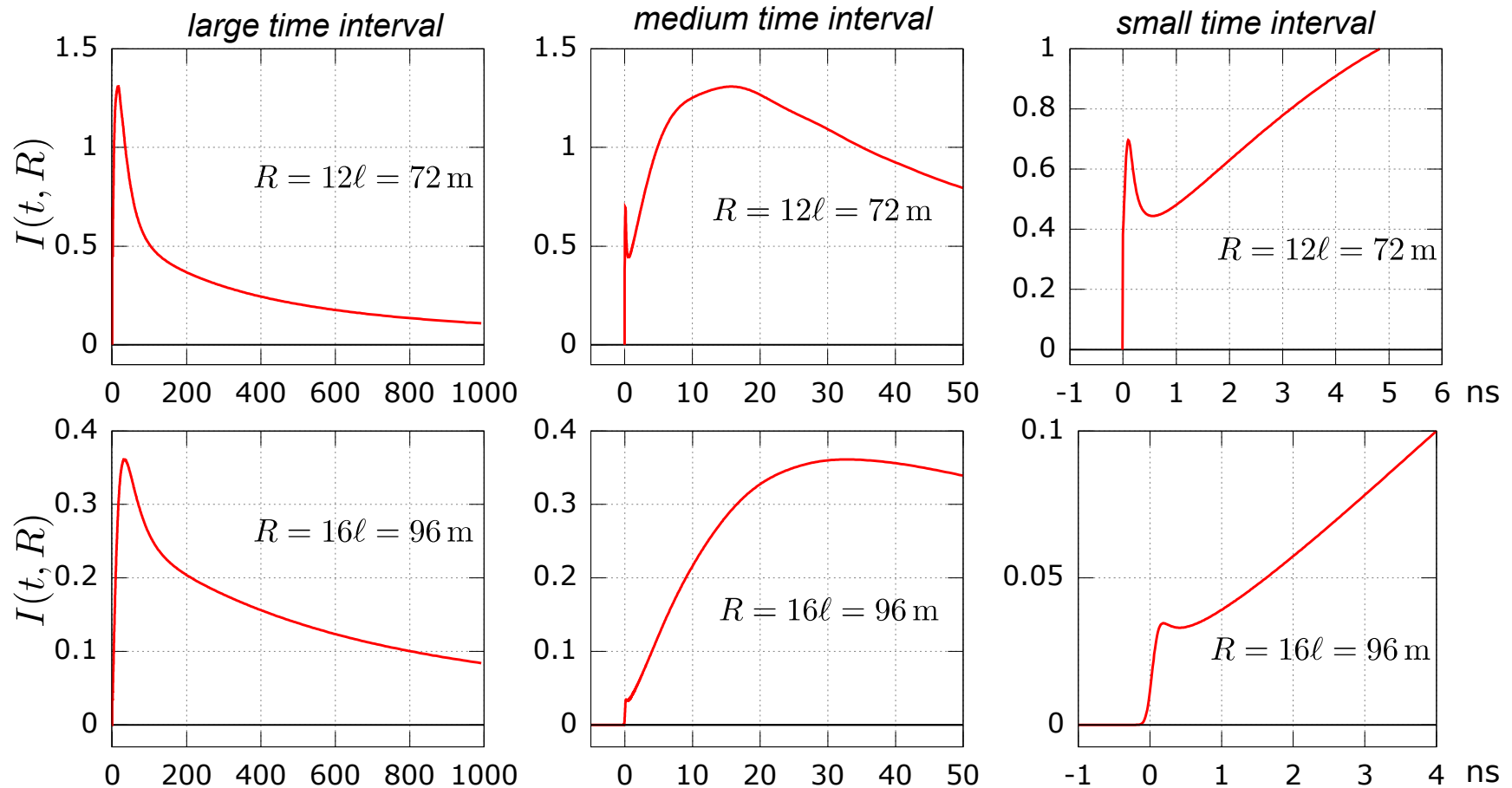
*wavelength*

$$\lambda_0 = 633 \text{ nm}$$

*pulse length*

$$T_p = 60 \text{ ps}, \quad cT_p = 2 \text{ cm}$$

■ *example: time-resolved intensity for two penetration depths, 72 m and 94 m, in an atmospheric cloud (for an omnidirectional source) \**

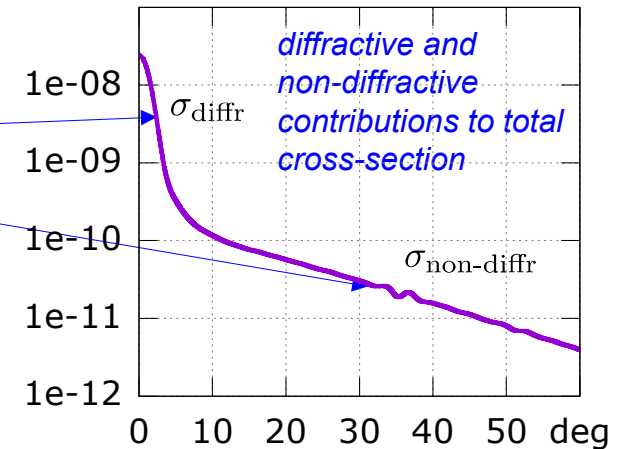


- ▶ *time-resolved intensity exhibits a **sharply rising narrow peak** at an early time*
- ▶ *as the **propagation distance increases**, the the early-time diffusion signal starts to overlap with the late-time diffusion but its **sharply rising edge remains***

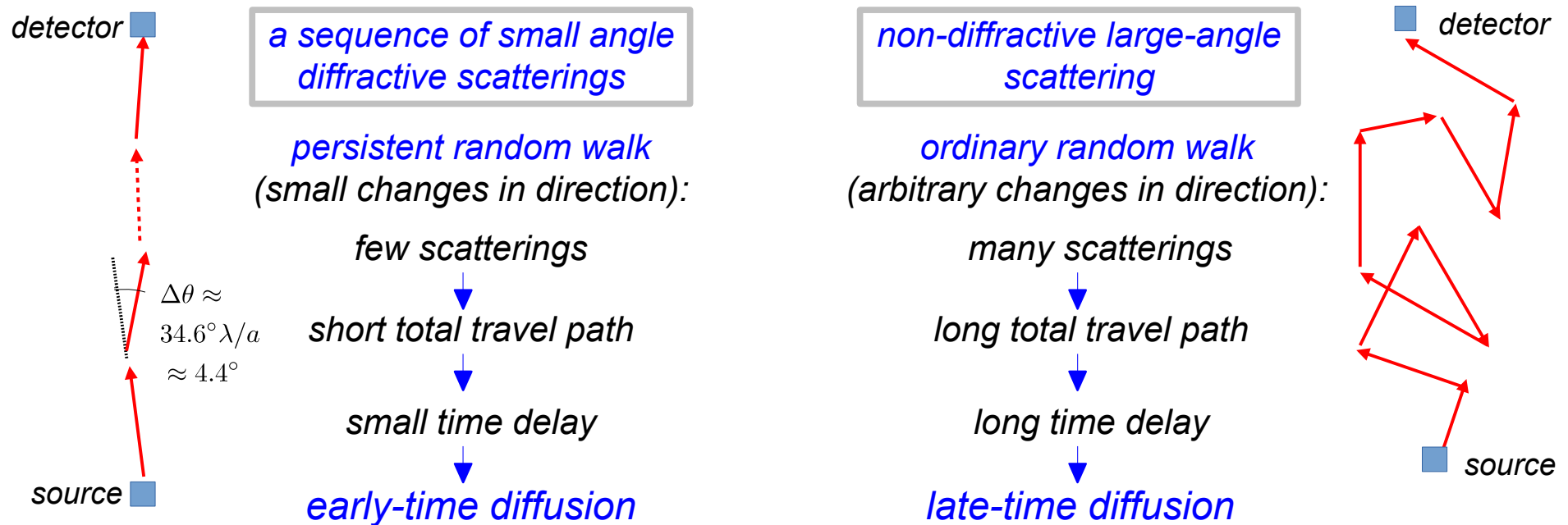
\* *E. Bleszynski, M. Bleszynski, and T. Jaroszewicz, "Early-time diffusion in pulse propagation through dilute random media", Optics Letters, v. 39, pp. 5862-5865, 2014*

## ■ physical origin of the early-time diffusion phenomenon

- ▶ differential cross section on a single water droplet exhibits a **sharp forward peak** and a **flat large-angle tail**



- ▶ this “two slope” behavior leads to two scattering mechanisms in the medium:



## ■ characteristic features of the early-time diffusion signal

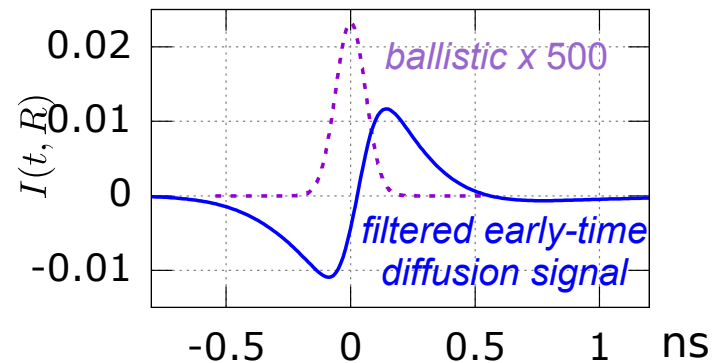
*intensity* of a propagating pulse in addition to the ballistic contribution and a wide, late-time diffusive tail exhibits a narrow *early-time diffusive component* characterized by:

- ▶ *significantly reduced attenuation* in comparison to the ballistic component

attenuation factors: 
$$\begin{array}{ll} \text{early-time diffusion} & \sim e^{-n_0 \sigma_{\text{non-diffr}} R} \\ \text{ballistic} & \sim e^{-n_0 \sigma_{\text{tot}} R} \end{array} \quad \sigma_{\text{non-diffr}} \approx 0.65 \sigma_{\text{tot}}$$

- ▶ *sharp rise* hence rich high frequency content → possibility of extraction from the received signal by filtering - *no time gating required*

*propagation distance in atmospheric cloud ~100 m*



- ▶ *small time width* - hence *no loss of resolution*

## ■ Radiative Transport Equation (RTE)

$$\Gamma(t; \mathbf{R}, \hat{\mathbf{s}}, \hat{\mathbf{s}}')$$

- ▶ rigorous description of radiance propagation in random, discrete scatterer medium can be obtained through Bethe-Salpeter (B-S) equation (practically intractable)
- ▶ assumptions of sparse, stationary, statistically homogeneous medium allow to reduce Bethe-Salpeter equation to RTE describing *propagation of radiance* (specific intensity) as a function time, space-point, and energy flux direction

*integro-differential form of RTE:*

$$\left( \mu_t + \frac{1}{v_0} \partial_t + \hat{\mathbf{s}} \cdot \nabla_{\mathbf{R}} \right) \Gamma(t, \mathbf{R}; \hat{\mathbf{s}}, \hat{\mathbf{s}}') - \int d^2 \hat{\mathbf{s}}'' \tilde{\Sigma}(\hat{\mathbf{s}} \cdot \hat{\mathbf{s}}'') \Gamma(t, \mathbf{R}; \hat{\mathbf{s}}'', \hat{\mathbf{s}}') = \delta(t) \delta^3(\mathbf{R}) \delta^2(\hat{\mathbf{s}} - \hat{\mathbf{s}}')$$

↑  
coherent propagation  
with attenuation
↑  
incoherent scattering

*the quantities involved:*

$\Gamma(t; \mathbf{R}, \hat{\mathbf{s}}, \hat{\mathbf{s}}')$  - probability density of an infinitely short light pulse emitted at  $t=0$  in the direction  $\hat{\mathbf{s}}'$  from a source at the origin  $\mathbf{R} = 0$  to arrive in the direction  $\hat{\mathbf{s}}$  at the time  $t$  at the observation point  $\mathbf{R}$

$G(t, \mathbf{R}; \hat{\mathbf{s}})$  - “coherent Green function” describing propagation of the field and its conjugate in the effective medium

$\mu_t = n_0 \sigma_t$  - attenuation coef

$n_0$  - number density

$\sigma_t$  - total cross-section

$\tilde{\Sigma}(x) = n_0 \sigma(x)$  - scattering kernel

$\sigma(x)$  - assemble-averaged differential cross-section on a medium constituent

subsequently, *specific intensity* at any space-time point is the convolution of the source with the Green function

## ■ solving for the RTE Green function

### ► expand $\Gamma(t; \mathbf{R}, \hat{\mathbf{s}}, \hat{\mathbf{s}}')$

- in plane waves in the variables  $t$  and  $\mathbf{R}$ :  $e^{-i\Omega t} e^{i\mathbf{P}\cdot\mathbf{R}}$

- in (rotated) spherical harmonics in the variables  $\hat{\mathbf{s}}$  and  $\hat{\mathbf{s}}'$ :  $Y_{l,m}(\hat{\mathbf{s}}; \hat{\mathbf{P}}) Y_{l',m'}^*(\hat{\mathbf{s}}'; \hat{\mathbf{P}})$

$$\begin{aligned}\Gamma(t, \mathbf{R}; \hat{\mathbf{s}}, \hat{\mathbf{s}}') &= \int \frac{d\Omega}{2\pi} \int \frac{d^3P}{(2\pi)^3} e^{-i\Omega t} e^{i\mathbf{P}\cdot\mathbf{R}} \tilde{\Gamma}(\Omega, \mathbf{P}; \hat{\mathbf{s}}, \hat{\mathbf{s}}') \\ &= \int \frac{d\Omega}{2\pi} \int \frac{d^3P}{(2\pi)^3} \sum_{l,l'=0}^{\infty} \sum_{m,m'=-l,-l'}^{l,l'} \Gamma_{l,l'}^{m,m'}(\Omega, P) e^{-i\Omega t} e^{i\mathbf{P}\cdot\mathbf{R}} Y_{l,m}(\hat{\mathbf{s}}; \hat{\mathbf{P}}) Y_{l',m'}^*(\hat{\mathbf{s}}'; \hat{\mathbf{P}})\end{aligned}$$

### ► substitute the expanded Green function into RTE, truncate the expansion in $l$ and $l'$

→ obtain a set of matrix equation (one for each  $m$  for the matrices  $\Gamma^m(\Omega, P)$  consisting of coefficients  $\Gamma_{l,l'}^m(\Omega, P)$ )

$$\left[ -i \frac{\Omega}{v_0} \mathbf{I} + M^m(P) \right] \Gamma^m(\Omega, P) = \mathbf{I}$$

### ► diagonalize $M^m(P)$

$$M^m(P) w_j^m(P) = i \frac{\Omega_j^m(P)}{v_0} w_j^m(P)$$

frequency eigenvalue
eigenvector

### ► Green function – resolvent of $M^m(P)$

$$\Gamma^m(\Omega, P) = \left[ -i (\Omega/v_0) \mathbf{I} + M^m(P) \right]^{-1} \quad \Gamma_{l,l'}^m(\Omega, P) = i v_0 \sum_j \frac{w_{j,l}^m(P) w_{j,l'}^m(P)}{\Omega - \Omega_j^m(P)}.$$



## ■ solving for the RTE Green function

### ► supporting notes

$$\int d^2 \hat{s} Y_{l,m}^*(\hat{s}; \hat{\mathbf{P}}) i \mathbf{P} \cdot \hat{s} Y_{l',m}(\hat{s}; \hat{\mathbf{P}}) = i P (\delta_{l,l'+1} b_l^m + \delta_{l',l+1} b_{l'}^m)$$

$$\int d^2 \hat{s} \int d^2 \hat{s}' Y_{l,m}^*(\hat{s}; \hat{\mathbf{P}}) \Sigma(\hat{s} \cdot \hat{s}') Y_{l',m}(\hat{s}'; \hat{\mathbf{P}}) = \delta_{l,l'} \Sigma_l$$

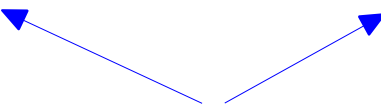
$$b_l^m = \sqrt{\frac{l^2 - m^2}{4l^2 - 1}} \quad \Sigma_l = 2\pi \int_{-1}^1 dx \Sigma(x) P_l(x)$$

$$\left[ -i \frac{\Omega}{v_0} \delta_{l,l'} + (\mu_t - \Sigma_l) \delta_{l,l'} + i P (\delta_{l,l'+1} b_l^m + \delta_{l',l+1} b_{l'}^m) \right] \Gamma^m(\Omega, P) = \delta_{l,l'}$$

## ■ solving for the RTE Green function

► substitute  $\Gamma_{l,l'}^m(\Omega, P)$  into the expression for the Green function

$$\begin{aligned}
 \Gamma(t, \mathbf{R}; \hat{\mathbf{s}}, \hat{\mathbf{s}}') &= \int \frac{d\Omega}{2\pi} \int \frac{d^3P}{(2\pi)^3} e^{-i\Omega t} e^{i\mathbf{P} \cdot \mathbf{R}} \tilde{\Gamma}(\Omega, \mathbf{P}; \hat{\mathbf{s}}, \hat{\mathbf{s}}') \\
 &= \int \frac{d\Omega}{2\pi} \int \frac{d^3P}{(2\pi)^3} e^{-i\Omega t} e^{i\mathbf{P} \cdot \mathbf{R}} \sum_{m=-\infty}^{\infty} \sum_{l, l'=|m|}^{\infty} \left[ i v_0 \sum_j \frac{w_{j,l}^m(P) w_{j,l'}^m(P)}{\Omega - \Omega_j^m(P)} \right] Y_{l,m}(\hat{\mathbf{s}}; \hat{\mathbf{P}}) Y_{l',m}^*(\hat{\mathbf{s}}'; \hat{\mathbf{P}}) \\
 &= v_0 H(t) \sum_m \sum_j \int \frac{d^3P}{(2\pi)^3} \underbrace{e^{-i\Omega_j^m(P)t} e^{i\mathbf{P} \cdot \mathbf{R}} \sum_l w_{j,l}^m(P) Y_{l,m}(\hat{\mathbf{s}}; \hat{\mathbf{P}})}_{\Psi_{j,\mathbf{P}}^m(t, \mathbf{R}, \hat{\mathbf{s}})} \underbrace{\sum_{l'} w_{j,l'}^m(P) Y_{l',m}^*(\hat{\mathbf{s}}'; \hat{\mathbf{P}})}_{\Psi_{j,\mathbf{P}}^{mC}(0, \mathbf{0}, \hat{\mathbf{s}}')}
 \end{aligned}$$


  
 eigenfunctions

## ■ solving for the RTE Green function (cont)

- substitute  $\Gamma_{l,l'}^m(\Omega, P)$  into the expression for the Green function

$$\Gamma(t-t', \mathbf{R}-\mathbf{R}'; \hat{\mathbf{s}}, \hat{\mathbf{s}}') = v_0 \text{H}(t-t') \sum_m \sum_j \int \frac{d^3 P}{(2\pi)^3} e^{-i \Omega_j^m(P) (t-t')} \Psi_{j,\mathbf{P}}^m(\mathbf{R}, \hat{\mathbf{s}}) \Psi_{j,\mathbf{P}}^{mC}(\mathbf{R}', \hat{\mathbf{s}}'),$$

where

$$\Psi_{j,\mathbf{P}}^m(t, \mathbf{R}, \hat{\mathbf{s}}) = e^{-i \Omega_j^m(P) t} e^{i \mathbf{P} \cdot \mathbf{R}} \sum_l w_{j,l}^m(P) Y_{l,m}(\hat{\mathbf{s}}; \hat{\mathbf{P}})$$

each  $\Psi_{j,\mathbf{P}}^m$  represents a *radiance configuration mode* propagating as a plane with the (real) wave number  $\mathbf{P}$  and the (complex) frequency  $\Omega_j^m(P)$ ; its dependence on the flux direction  $\hat{\mathbf{s}}$  is given by a combination of spherical harmonics and remains *independent of time and spatial coordinates*

- interpretation of  $\text{Im } \Omega_j^m(P)$  and  $\text{Re } \Omega_j^m(P)$

$$\mu_j^m(P) \equiv - \frac{\text{Im } \Omega_j^m(P)}{v_0} \geq 0$$

↑  
*P*-dependent attenuation coefficient  
for the *j*-th mode

$$-v_0 \leq v_j^m(P) \equiv \frac{\text{Re } \Omega_j^m(P)}{P} \leq v_0$$

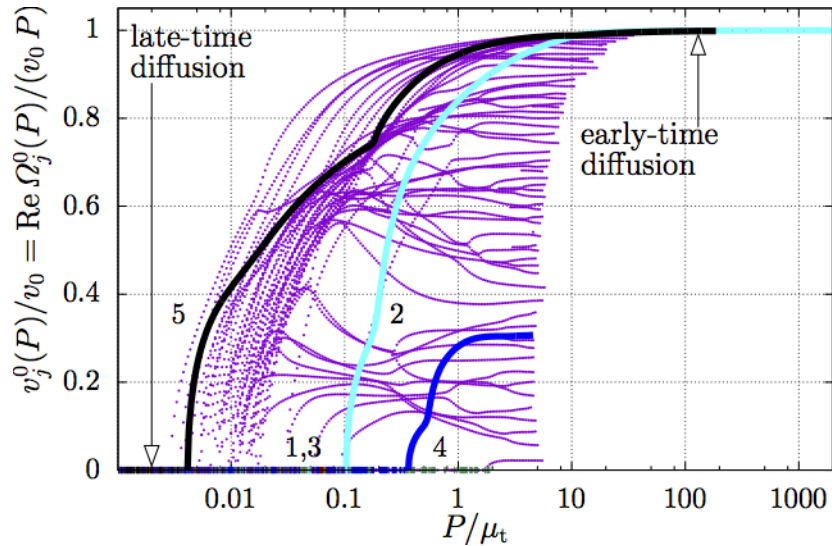
↑  
radiance propagation velocity  
of the *j*-th mode

$\Omega_j^m(P)$  form “trajectories” of eigenvalues moving on the complex  $\Omega$  plane

## ■ some relevant features of frequency eigenvalues and eigenvectors of RTE

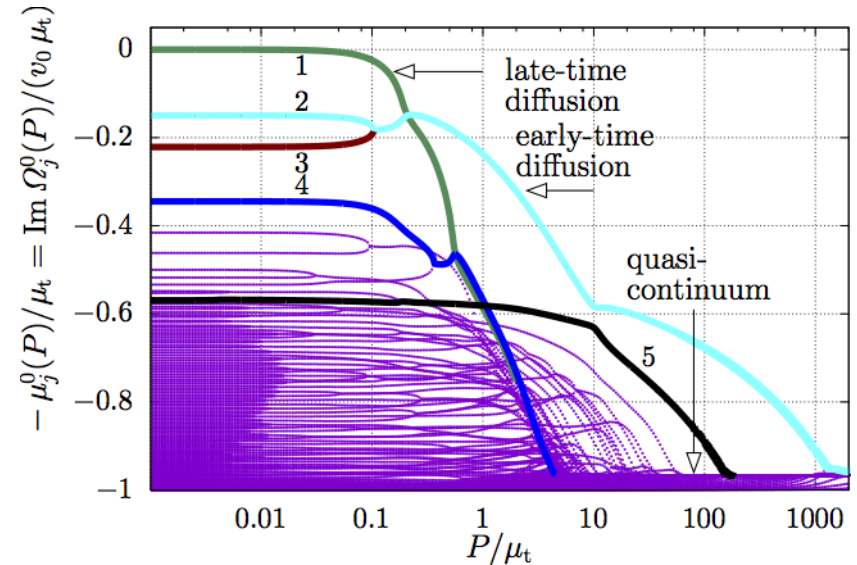
### ► real and imaginary parts of frequency eigenvalues as a function of $P$

$\frac{\text{Re } \Omega_j^0(P)}{P} = \text{radiance propagation velocity of } j\text{-th mode}$



- velocities of only discrete modes plotted
- velocity trajectories terminated for values at which attenuation reaches that of the coherent wave

$\frac{\text{Im } \Omega_j^0(P)}{v_0} = j\text{-th mode attenuation coefficient}$

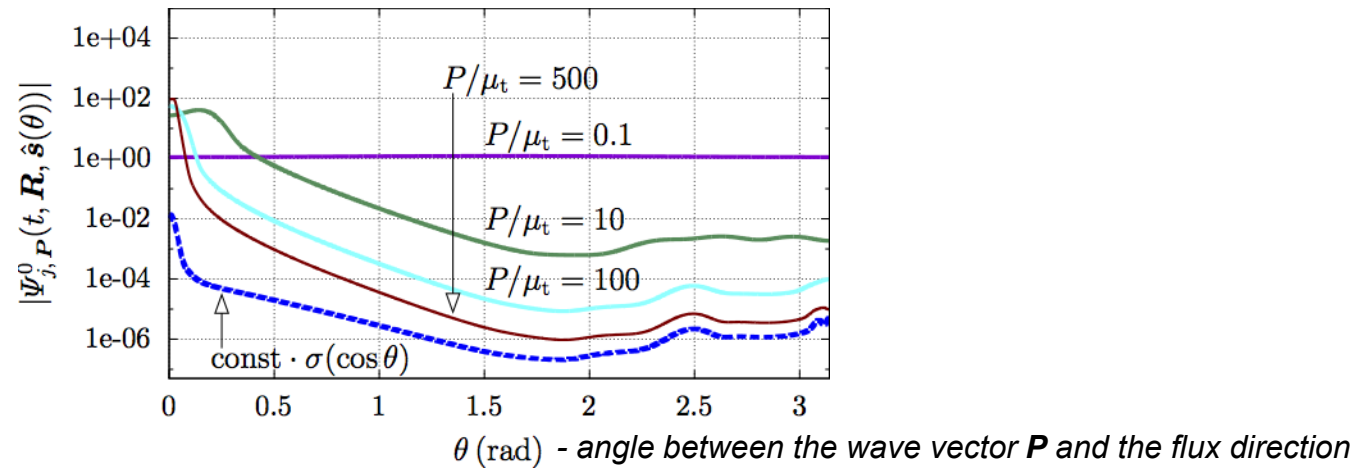


- attenuation coefficients for all the modes (including those belonging to "quasi-continuum")

- at distances small compared to the mean-free path ( $R \lesssim 10 \ell_t$ ) all modes are important
- at large distances the Green function is dominated by a few dominant (least attenuated) modes (marked with thick lines and numbered 1 – 4)
- for  $P/\mu_t < 0.1$  the dominant trajectories are characterized by zero-velocity and their associated eigenmodes have flat, omnidirectional distribution => they represent the ordinary, late-time diffusion
- for  $P/\mu_t \approx 1$  trajectory "2" nearly reaches coherent speed velocity and its associated eigenmode attains narrow angular distribution => clear indication of early-time diffusion

## ■ some relevant features of frequency eigenvalues and eigenvectors of RTE

- ▶ angular distribution of leading (least attenuated) RTE modes for different values of  $P$



for small  $P$  the dominant eigenmodes have nearly flat angular distribution

with the increasing  $P$  the leading modes become strongly peaked for small angles  $\theta$

for large  $P$ , width of the eigenmodes angular distribution approaches the width of the forward peak in the scattering cross-section on a medium constituent

how can we use this information to enhance contribution of the modes associated with early-time diffusion?

## ■ enhancing early-time diffusion by adjusting the source

- ▶ *intensity* (radiance integrated over received flux directions  $\hat{s}$ ) is a convolution of the *Green function* with the *radiance source*

$$I(t, \mathbf{R}) = \int d^2 \hat{s} \int dt' d^3 R' d^2 \hat{s}' \Gamma(t - t', \mathbf{R} - \mathbf{R}'; \hat{s}, \hat{s}') S(t', \mathbf{R}'; \hat{s}')$$

- ▶ *substitute expression of the Green function in terms of its eigenmodes*

$$I(t, \mathbf{R}) = v_0 \sum_j \int \frac{d^3 P}{(2\pi)^3} \int d^2 \hat{s} \Psi_{j,P}^0(t, \mathbf{R}, \hat{s}) \int d^2 \hat{s}' \int_{-\infty}^t dt' d^3 R' \Psi_{j,P}^{0C}(t', \mathbf{R}', \hat{s}'(\theta, \phi)) S(t'; \mathbf{R}'; \hat{s}')$$

- ▶ *assume point-like source, located in the origin, and radiating in the z-direction with axially symmetric flux distribution*

$$S(t', \mathbf{R}'; \hat{s}'(\theta, \phi)) = \delta^3(\mathbf{R}') S(t'; \hat{z} \cdot \hat{s}')$$

*normalization factor* (unit total energy)

$$S(t'; \hat{z} \cdot \hat{s}') = S_0 e^{-t'^2/2 T_p^2} e^{-\theta^2/2 \Theta^2}$$

- ▶ *evaluate intensity for several angular widths of the source*

$$\Theta_\sigma \approx 0.02 \text{ rad} \quad \Theta_\sigma \approx 0.05 \text{ rad} \quad \Theta_\sigma = \pi \text{ rad} \quad (\text{omnidirectional source})$$

$$\Theta_\sigma \approx 0.021 \text{ rad} \quad (\text{angular width of the scattering cross-section})$$

## ■ *example: model of a source*

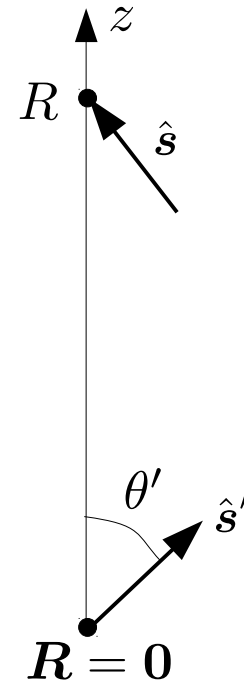
- ▶ *a spatially small source emitting axially symmetric radiation in the z-axis direction with the  $\hat{s}'$  angular distribution*

$$\sim e^{-\theta'^2/2\Theta^2}$$

*and with the width in  $\Theta$  of the order of the scattering cross-section*

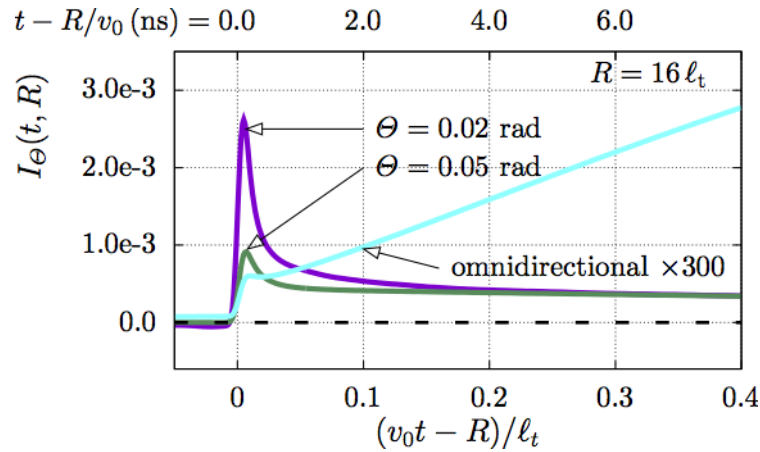
$$\Theta_\sigma \approx 0.02 \text{ rad}$$

- ▶ *observation point on the z axis at a large distance  $R$ , radiance integrated over flux arrival directions  $\hat{s}$*

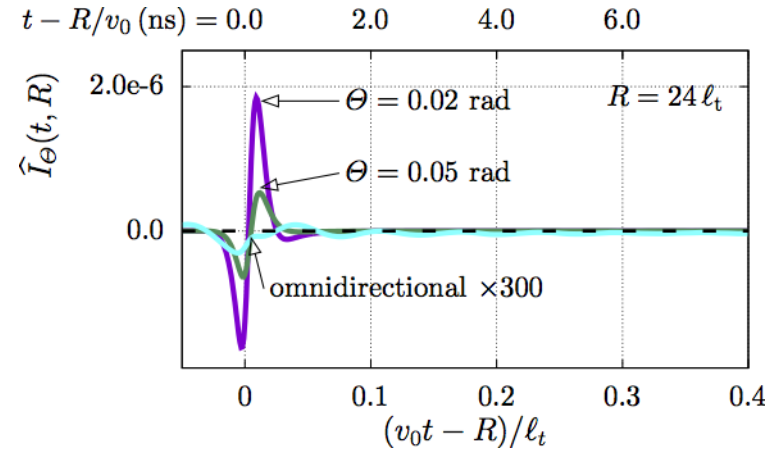
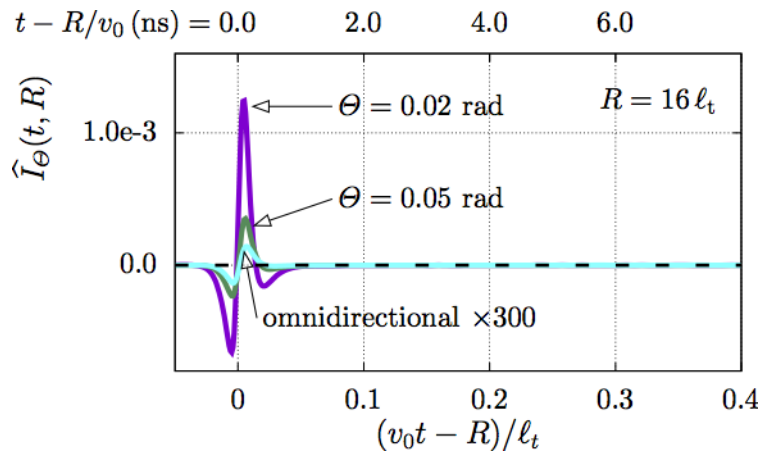
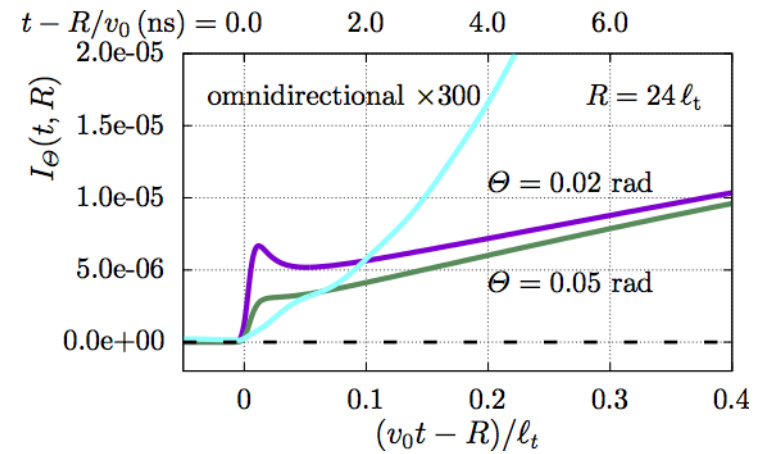


■ *example cont: time-resolved intensities for sources with different angular flux distributions*

*penetration depth ~100m*



*penetration depth ~150m*



*we observe enhancement of the early time diffusion component for sources with angular flux directions distribution comparable to that of the angular distribution of the cross-section on individual medium scatterers*



## ■ *summary*

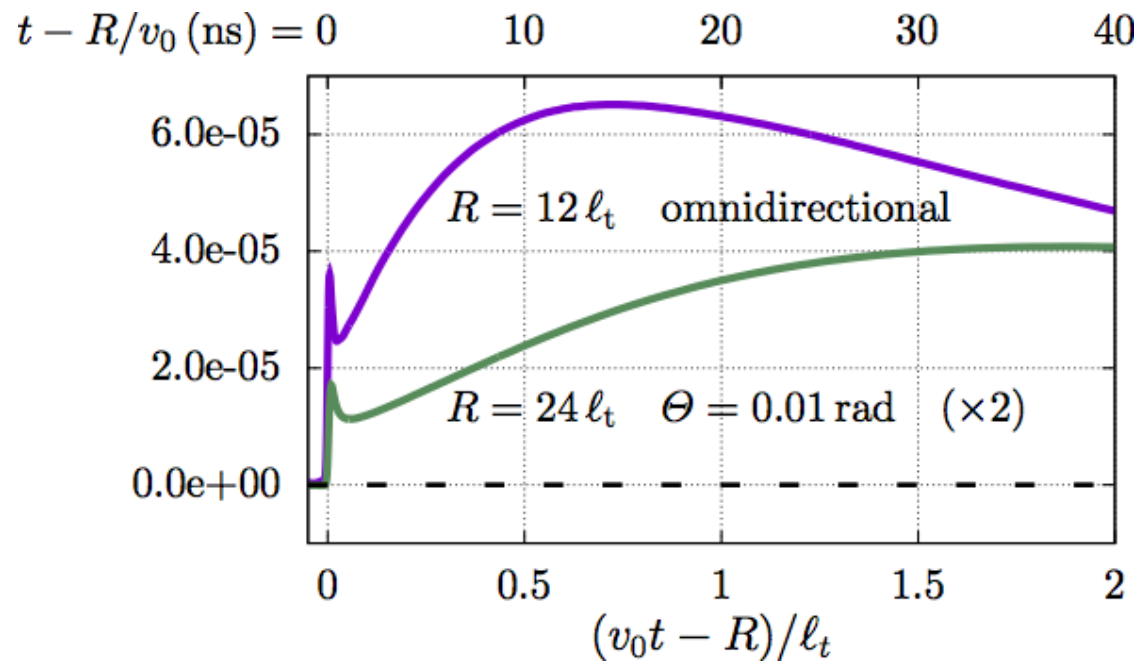
*we extended our work on early time diffusion phenomenon*

*we investigated the possibility to increase relative importance  
of the early-time diffusive component  
through matching the source to dominant propagation modes of the RTE operator*

- ▶ *we identified RTE modes responsible for early and late time diffusion*
- ▶ *we investigated angular flux distribution of the RTE modes*
- ▶ *we achieved enhancement of the early time diffusion component by matching the source angular flux distribution to that of dominant RTE propagation modes responsible for early time diffusion*

## ■ *improvement and future plans*

- ▶ *improvement due to implementation of a source with angular dependence*



- ▶ *investigate the impact of a directional detector*
- ▶ *investigate propagation through a **layer** of clouds*

*THANK YOU !*

## ■ enhancing early-time diffusion by adjusting the source

- radiance (for an coherent or partially coherent source) is a convolution of the Green function (expressed in **eigenmodes of RTE**) with the **radiance source**

$$I(t, \mathbf{R}; \hat{\mathbf{s}}) = \frac{1}{v_0} \sum_m \sum_k \int \frac{d^3 P}{(2\pi)^3} e^{-i \Omega_k^0(P) t} \Psi_{k, \mathbf{P}}^m(\mathbf{R}, \hat{\mathbf{s}}) \underbrace{\int d^3 R' \int d^2 \hat{\mathbf{s}}' \Psi_{k, \mathbf{P}}^{mC}(\mathbf{R}', \hat{\mathbf{s}}') S(\mathbf{R}'; \hat{\mathbf{s}}')}_{S_k^m(\mathbf{P})}$$

- in order to enhance the early-time-diffusion, we try to **maximize** the projections of the source on **early-time** diffusion modes and **minimize** projections on **late-time** diffusion modes
  - under the constraint of a fixed total energy flux of the source, while ensuring positive semi-definiteness of radiance (equivalent to  $S(t, \mathbf{R}, \hat{\mathbf{s}}) \geq 0$ )
- **comment:** late-time diffusion eigenmodes occur for small  $P$

$$\begin{aligned} \text{for those modes} \quad & \Psi_{k, \mathbf{P}}^m(\mathbf{R}, \hat{\mathbf{s}}) \longrightarrow 1 \quad \text{for } P \rightarrow 0 \\ \text{therefore,} \quad & S_k^m(\mathbf{P}) \longrightarrow \int d^3 R' \int d^2 \hat{\mathbf{s}}' S(\mathbf{R}'; \hat{\mathbf{s}}') = \text{total energy flux} = 1 \end{aligned}$$

cannot be small, which makes suppression of late-time diffusion difficult

- however, **early-time** diffusion eigenfunctions have narrow  $\hat{\mathbf{s}}$  distributions and we can **enhance** them by choosing a source distribution also narrow in  $\hat{\mathbf{s}}$