

Control & Estimation in the Presence of Adversarial Action and Uncertainty

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Abstract

We consider swarm pursuit-evasion differential games in the Euclidean plane where an evader is engaged by multiple pursuers and point capture is required. All the players have simple motion à la Isaacs and the pursuers are faster than the evader. It is shown that in group/swarm pursuit, when the players are in general position, capture is effected by one, two, or by three critical pursuers, and this irrespective of the size $N (> 3)$ of the pursuit pack. Thus, group pursuit devolves into pure pursuit by one of the pursuers, into a pincer movement pursuit by two pursuers, or cornering by three pursuers, who isochronously capture the evader, a *ménage à trois*. The solution of the Game of Kind is obtained and critical pursuers are identified. Concerning the Game of Degree, the players' state feedback optimal strategies are synthesized and the Value of the game is derived.

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Multi-agent control for safety and defense

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Abstract

In this presentation, two topics involving multi-agent control for safety and self-protection of UAV teams will be discussed. First, we will discuss scalable safe navigation when dealing with uncertainty in other vehicles' trajectories. The scalable solution is found by decomposing the value function of a one-on-one stochastic optimal control problem into its respective hazard and expected time of arrival components. We are able to do this because, in general, the optimal cost-to-go satisfies the same PDE as the expected time to reach the target. This fact enables us to solve for the value function and expected time to go separately, and then find the expected hazard by subtracting the latter from the prior. These components are then used to develop an approximate solution to the multi-vehicle problem by coupling these one-on-one solutions together. The second topic will cover the complete closed form solution to the multi-body active defense differential game. In this game, a defender strives to protect a target from an attacker. The objective function is the terminal separation between the attacker and the target. We provide closed-loop state feedback strategies for all agents and obtain the value function of the game. From this we are able to define the target's escape set and show that the value function is continuous and continuously differentiable and satisfies the HJI over the escape set.

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Explicit Solvability for Stochastic Control and Differential Games for Systems with Gauss-Volterra and Rosenblatt Noise

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Abstract

Some stochastic control and game problems are formulated and explicitly solved for bilinear equations in finite and infinite dimensional spaces with Gauss-Volterra processes that include fractional Brownian motions with the Hurst parameter greater than one-half and some other known Gaussian processes. The controls are restricted to be linear feedback policies and the time horizons can be either finite or infinite. The associated Riccati equations differ from those for the related linear-quadratic control problems with Brownian motions. Some scalar linear-quadratic control problems with infinite time horizons and Rosenblatt process noise are formulated and explicitly solved. The associated quadratic polynomial that determines the optimal feedback control differs from the well known corresponding algebraic Riccati equation for the control problems with Brownian motions. Rosenblatt processes are non-Gaussian and non-Markovian processes with continuous sample paths that have been verified as appropriate models from some data for specific physical systems and the optimal controls seem to represent the initial results for continuous, non-Gaussian processes. Some partially observed differential games for linear equations with Brownian motions and exponential quadratic (risk sensitive) payoffs are explicitly solved.

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Stochastic HJB Equations and Regular Singular Points

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Abstract

We show that some HJB equations arising from both finite and infinite horizon stochastic optimal control problems have a regular singular point at the origin. This makes them amenable to solution by power series techniques. This extends the work of Al'brecht who showed that the HJB equations of an infinite horizon deterministic optimal control problem can have a regular singular point at the origin, Al'brecht solved the HJB equations by power series, degree by degree.

In particular, we show that the infinite horizon stochastic optimal control problem with linear dynamics, quadratic cost and bilinear noise leads to a new type of algebraic Riccati equation which we call the Stochastic Algebraic Riccati Equation (SARE). If SARE can be solved then one has a complete solution to this infinite horizon stochastic optimal control problem.

We also show that a finite horizon stochastic optimal control problem with linear dynamics, quadratic cost and bilinear noise leads to a Stochastic Differential Riccati Equation (SDRE) that is well known. If these problems are the linear-quadratic-bilinear part of a nonlinear finite horizon stochastic optimal control problem then we show how the higher degree terms of the solutions can be computed degree by degree. To our knowledge this computation is new.

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Prescribed-Time Observers for ODEs and PDEs

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Abstract

It is generally well known that, while linear feedback laws ensure asymptotic/exponential stabilization over finite time intervals, sliding mode controllers—and observers—are capable of achieving finite-time convergence, however, the convergence time is larger when the initial condition is larger. To make the convergence time invariant of the initial condition, certain “homogeneous” time-invariant controllers and observers are capable of ensuring that the convergence time not exceed a desired value but are complex and conservative.

As an alternative, and inspired by the “proportional navigation” laws in missile guidance, we propose time-varying observers, both for ODEs and for PDEs, which achieve perfect estimation of the unmeasured state in exactly the prescribed time. This is the observer dual of the concept of “null controllability” but achieved with feedback and robustness, rather than in an open-loop sense. Our prescribed-time controllers and observers can be combined into prescribed-time output feedback laws, i.e., we prove that they satisfy a separation principle.

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Limited Communication and Communication Complexity in Gradient Methods

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Abstract

Distributed optimization increasingly plays a central role in economical and sustainable operation of cyber-physical systems. Nevertheless, the complete potential of the technology has not yet been fully exploited in practice due to communication limitations posed by the real-world infrastructures. Our work investigates fundamental properties of dual gradient methods in distributed resource allocation optimization, where the dual gradient information is quantized and communicated using a limited number of bits. Sufficient and necessary conditions are provided on such a quantization set to guarantee that the methods minimize any convex objective function with Lipschitz continuous gradient and a nonempty and bounded set of optimizers. The minimal cardinality of the quantization set is identified along with specific examples of minimal quantization. Convergence rate results are established that connect the fineness of the quantization and the number of iterations needed to reach a predefined solution accuracy. Lastly, we also investigate communication complexity of the problem, the minimal number of communicated bits needed to solve some classes of resource allocation problems regardless of the used algorithms.

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Entropy Maximization for Efficient Robotic Surveillance

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Abstract

Motivated by robotic surveillance applications, we study a novel entropy optimization problem, i.e, the maximization of the return time entropy of a Markov chain, subject to a graph topology with travel times and subject to a stationary distribution constraint. The return time entropy is the weighted average, over all graph nodes, of the entropy of the first return times of the Markov chain; this objective function is a function series that does not admit in general a closed form. We propose theoretical and computational contributions. First, we obtain a discrete-time delayed linear system for the return time probability distribution and establish its convergence properties. We show that the objective function is continuous over a compact set and therefore admits a global maximum. We then establish upper and lower bounds between the return time entropy and the well-known entropy rate of the Markov chain. To compute the optimal Markov chain numerically, we establish the asymptotic equality between entropy, conditional entropy and truncated entropy, and propose an iteration to compute the gradient of the truncated entropy. Finally, we apply these results to the robotic surveillance problem. Our numerical results show that, for a model of rational intruder over prototypical graph topologies and test cases, the maximum return time entropy chain outperforms several pre-existing Markov chains.

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Cooperative Data-Driven Robust Optimization

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Abstract

We study a class of multiagent stochastic optimization problems where the objective is to minimize the expected value of a function which depends on a random variable. The probability distribution of the random variable is unknown to the agents, so each one gathers samples of it. The agents aim to cooperatively find a solution to the optimization problem with guaranteed out-of-sample performance that improves with the size of the dataset. To tackle this, we adopt a data-driven distributionally robust optimization formulation. We make the problem amenable to distributed algorithm design by reformulating it as a distributed optimization problem, and identify a convex-concave augmented Lagrangian function whose saddle points are in correspondence with the optimizers, provided a min-max interchangeability criteria is met. Our algorithm design then consists of the saddle-point dynamics associated to the augmented Lagrangian. We formally establish that the trajectories of the dynamics converge asymptotically to a saddle point, and hence an optimizer of the problem. Finally, we discuss classes of functions that meet the min-max interchangeability criteria.

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Action-Based Methods in Dynamics and Stability Issues in Networked Systems

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Abstract

Part 1: The Staticization Operator and Solution of Conservative Dynamical Systems

A very promising approach to the solution of problems in conservative dynamical systems is derived from the principle of stationary action, which asserts that a system follows the trajectory that is a stationary point of its action functional. By appending various terminal costs to the action functional, one can use this approach to find fundamental solutions of a wide variety of classes of two-point boundary value problems (as well as initial value problems) for such systems. Fundamental solutions allow for the rapid generation of specific solutions corresponding to given initial and/or terminal data. The key to this new area is the staticization operator, which is a set-valued, nonlinear operator that maps a function to its range values at stationary points. An absolutely essential tool in the development of fundamental solutions is that of inversion of the order of staticization operations. Conditions under which such a reordering is possible will be indicated, and these will be discussed in relation to classes of relevant applications.

Part 2: Stability: Kolmogorov Systems and Networked Systems

We report our findings in two class of problems. Focusing on Kolmogorov systems, the first one is concerned with permanence and extinction of certain populations, which is a long-standing problem in ecology, epidemiology, and other applications. We show how stochastic differential equations connected with differential geometry and geometric control theory can be used to treat the problem to arrive at sufficient and nearly necessary conditions to completely characterize the systems. The second class of problems deals with the stability of stochastic delay differential systems with applications to networked systems and consentability of linear systems with time delays and multiplicative noises. It is shown that multi-agent consentability depends on certain characterizing system parameters, including linear system dynamics, communication graph, channel uncertainties, and time delay of the deterministic term. Second-order integrator multi-agent systems are also examined.

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Estimating the State of a Linear System Across a Network

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Abstract

The problem of estimating the state of a linear system whose measured outputs are distributed across a network has been under study in one form or another for a number of years. Despite this, only recently have provably correct state estimators emerged which solve this problem under reasonably non-restrictive assumptions. The aim of this talk is to describe a newly developed, simply structured, very easy to analyze distributed observer intended to estimate the state x of an m -channel, n -dimensional jointly observable, linear system of the form $\dot{x} = Ax$, $y_i = C_i x$, $i \in \{1, 2, \dots, m\}$. The system's state x is simultaneously estimated by m agents assuming agent i senses y_i and receives state of each of its neighbor's estimators. Neighbor relations are characterized by a directed graph \mathbb{N} whose vertices correspond to agents and whose arcs depict neighbor relations. In sharp contrast to previously proposed observers, the observer to be described in this talk does not depend on any properties of \mathbb{N} other than the requirement that it be a connected graph.

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A compositional approach to network control

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Abstract

In this talk, we discuss a compositional/structural setup to reason about control of networked systems. We argue that adopting such a point of view for networked systems becomes critical as the size of the underlying network increase, necessitating a modular approach for system analysis and control synthesis. We then provide concrete realizations of this setup for diffusion dynamics on graphs, using constructs such as network symmetries, graph products, cographs, and series-parallel networks. The talk also delves into how this approach can be adopted for the purpose of approximation and ordering of networked systems.

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Optimal Control of Action-Dependent Resources: Remote Estimation and Task Scheduling

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Abstract

I will report on recent results to design and analyze the performance of systems in which the operation of a constitutive component is contingent on resource levels that depend on previous actions. Examples include 1) remote estimation systems in which the wireless transmission of data from the sensors to a base station is powered by batteries recharged by energy harvesting, and 2) task scheduling for a server whose effectiveness depends on workload or fatigue levels (such as a human operator). In the first part of the talk, I will describe an overarching formulation that encompasses these problems, and in the rest of the presentation I will discuss some of our results and briefly outline the techniques used. In particular, I will show that, even in very surprising cases, thresholds solely on the action-dependent resource may be optimal. Structural results such as these can be leveraged to reduce the complexity of the design process significantly.

This work is being developed in collaboration with Prof. Richard La (UMD) and Mr. Michael Lin (UMD) who is a Ph.D. student under our supervision.

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A new notion of boundary control for Nonlocal PDEs

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Abstract

We first show that the classical notion of controllability (exact or approximate controllability) from the boundary does not make sense for diffusion equations associated with a nonlocal operator such as the fractional Laplacian. This is also the case for the classical notion of boundary optimal control for PDEs associated with the fractional Laplace operator. In another word, for such problems, the control function cannot be localized on a subset of the boundary of the domain on which the PDE is solved. We next introduce a new notion of boundary control that makes sense for the above mentioned nonlocal problems. More precisely, we show that for nonlocal PDEs involving operators like the fractional Laplacian the right notion is to seek if such a system can be controlled from the exterior of the domain on which the PDE is solved. We discuss what is so far known regarding the controllability (exact and approximate) from the exterior of nonlocal heat and wave type equations. Finally we give some recent results on nonlocal optimal control problems.

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Robustness of Quantum Risk-Sensitive Control

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Abstract

Joint work with Igor Vladimirov and Ian Petersen.

This talk presents new results that resolve a long-standing open question concerning the robustness properties for optimal risk-sensitive controllers for quantum systems. The risk-sensitive optimal control problem for quantum systems was formulated and solved in 2004 and 2005 using a time-ordered exponential criterion. However, the question of robustness was unresolved. Recently, the authors provided fundamental robustness inequalities for a direct non time-ordered exponential criterion. In this talk we show how the time-ordered criterion can be transformed to an equivalent direct exponential, thereby resolving robustness properties. Robustness is important due to the presence of uncertainty, noise and disturbances. Quantum systems differ from classical systems because of their non-commutative structure, and our results give important new insights into the fundamental nature of quantum control systems.

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Synthesis of Data-Driven Modeling, Global Sensitivity Analysis, and Uncertainty Quantification for Robust Control Design of Adaptive Material Systems

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Abstract

In this presentation, we will discuss the role of active subspaces, data-driven model development, and uncertainty quantification to improve robust control designs for AFOSR applications utilizing advanced transductive materials. To motivate this analysis, we consider piezoelectric materials employed for improved flow control and drive mechanisms for micro-air vehicles such as Robobee. We will summarize the development of high-fidelity nonlinear physics-based models and the use of dynamic mode decompositions to construct surrogate models that are appropriate for design and control. We will also discuss the use of parameter subset selection and active subspace techniques to determine subsets and subspaces of parameters that are identifiable in the sense that they are uniquely determined by data. Finally, we will discuss ties between uncertainty quantification, identification, and robust control design that may be utilized in other model-based control problems.

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A Switched Systems Approach for Navigation and Control with Intermittent Feedback

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Abstract

Current visual servo control and image-based state estimation algorithms assume that image feedback is continuously available from a single source. To generalize the persistent staring assumption of adaptive image-based estimators, the ability to arbitrarily switch between different uncertain and nonlinear image dynamics is required. Motivated by this need, we generalize the LaSalle-Yoshizawa Theorem to switched nonsmooth systems. Filippov and Krasovskii regularizations of a switched system are shown to be contained within the convex hull of Filippov and Krasovskii regularizations of the subsystems, respectively. A common candidate Lyapunov function that has a negative semidefinite generalized time derivative along the trajectories of the subsystems is shown to be sufficient to establish LaSalle-Yoshizawa-like results for the switched system. The extension facilitates the analysis of the asymptotic characteristics of a switched system based on the asymptotic characteristics of its subsystems where a non-strict common Lyapunov function can be constructed for the subsystems.

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Stochastic hybrid inclusions applied to global almost sure optimization on manifolds

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Abstract

We present a class of stochastic dynamical systems designed to solve non-convex optimization problems on smooth manifolds. In order to guarantee convergence with probability one to the set of global minimizers of the cost function, the proposed dynamics combine continuous-time flows, characterized by a differential equation, and discrete-time jumps, characterized by a stochastic difference inclusion. The flows are constrained to the manifold through hysteresis-based switching among coordinate charts. By using the framework of stochastic hybrid inclusions, global asymptotic stability in probability of the set of minimizers can be asserted. A simple extension to address learning problems in games defined on manifolds can also be formulated. The framework is illustrated through a numerical example in the setting of a weighted non-cooperative potential game defined on the torus.

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Robust Estimation and Synchronization in Complex Networks

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Abstract

The rapid growth of digital networks has enabled systems at different locations to perform joint tasks, such as coordination, synchronization, and estimation. At the same time, it has brought about analysis and design challenges, mainly due to the presence of events, heterogeneity, distributed information, as well as unavoidable uncertainty in the models and data. In this talk, I will present new tools for the design of decentralized algorithms for estimation and synchronization that effectively handle the combination of continuous and discrete behavior, asynchronous events, Zeno behavior, and the presence of perturbations and delays in complex networks. These new tools are applicable to single as well as to interconnected hybrid dynamical systems, and have been developed as part of this project for the study of a wide range of dynamical properties, including Lyapunov stability, attractivity, pointwise asymptotic stability, and robustness.

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Optimal network resource allocation for monitoring continuous and hybrid systems

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Abstract

This project applies concepts from topological entropy and techniques from nonlinear continuous and hybrid systems to the problems of state estimation and model detection over finite-data-rate communication links. First, we consider a scenario where an output function with finitely many values, such as a one modeling a quantizing sensor, provides measurements of the state of a continuous-time process. We introduce a fundamental quantity called estimation entropy that corresponds to the minimal bit rate needed for estimating dynamical systems. Building up on this concept, we have developed upper and lower bounds on this quantity in terms of some contraction metrics in the general nonlinear case, or matrix eigenvalues in the linear case. Next, we address the problem of estimating the state of the continuous dynamical systems with desired eventual accuracy using coarse output measurements. When sufficient data rate is available, a concrete algorithm for state estimation of continuous systems is constructed. We study the efficiency gaps between the algorithms and the lower bounds.

In parallel threads, we have developed notions of entropy, upper bounds, and preliminary results on corresponding algorithms for solving the state estimation problem in the more challenging setting of nonlinear switched systems and systems with inputs. Even if the uncertainty about the state can be made to decrease over time using sensor measurements, the uncertainty about the input or the switching signal may not decrease. For example, the input can change arbitrarily with few constraints and the continuous effect of the uncertain input prevents the estimation error from going to zero. Thus, we use a weaker notion of estimation, that only requires the error to be bounded by a constant $\epsilon > 0$, instead of exponentially decaying down to zero. For switched systems, the interdependence of the uncertainties in the state and the mode requires the estimation algorithm to simultaneously solve the estimation and mode detection problems: unless a mode is detected, it may be impossible to estimate the state, and vice versa. We present preliminary experimental results on applying the estimation algorithms to linear and nonlinear switched systems, and discuss the implications of the choice of key algorithm parameters.

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Time-Varying Semidefinite Programs

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Abstract

We study time-varying semidefinite programs (TV-SDPs), which are semidefinite programs whose data (and solutions) are functions of time. Our focus is on the setting where the data varies polynomially with time. We show that under a strict feasibility assumption, restricting the solutions to also be polynomial functions of time does not change the optimal value of the TV-SDP. Moreover, by using a Positivstellensatz on univariate polynomial matrices, we show that the best polynomial solution of a given degree to a TV-SDP can be found by solving a semidefinite program of tractable size. We also provide a sequence of dual problems which can be cast as SDPs and that give upper bounds on the optimal value of a TV-SDP (in maximization form). We prove that under a boundedness assumption, this sequence of upper bounds converges to the optimal value of the TV-SDP. We demonstrate the efficacy of our algorithms on applications in optimization and control.

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Theory-Based Engineering of Biomolecular Circuits in Living Cells

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Abstract

The objective of this research is to establish a *data-driven theoretical framework* based on mathematics to enable the robust design of interacting biomolecular circuits in living cells that perform complex decision-making. Microbiology as a platform has substantial advantages with respect to human-made hardware, including size, power, and high sensitivity/selectivity. While the latest advances in synthetic biology have rendered the creation of simple functional circuits in microbes possible, our ability of composing circuits that behave as expected is still missing. This hinders the possibility of designing robust complex decision-making circuits, such as those that recognize and classify chemical signatures and those that program degradation of pre-specified materials upon contact. Overcoming this bottleneck goes beyond the engineering of new parts or new assembly methods. By contrast, it requires a deep understanding of the dynamical interactions among synthetic modules and the cell machinery, a particularly hard task since dynamics are nonlinear, stochastic, and involve multiple scales of resolution both in time and space.

In this project, we propose an interdisciplinary approach merging mathematics, control and dynamical systems theory, electrical circuit theory, and synthetic biology, in order to tackle this problem. We propose to establish a design-oriented theoretical framework that explicitly accounts for interactions among circuits, between the circuits and the cell machinery, and provides engineering solutions to mitigate the undesirable effects of these interactions (compositionality effort). Within this framework, we will develop mathematical tools to quantify the propagation of stochasticity through the nonlinear dynamics of biological networks (stochasticity effort) and to incorporate spatial heterogeneity effects (spatial heterogeneity effort). These research efforts will be focused on solving concrete engineering problems on a prototype experimental system that integrates different sensors to classify a number of chemical signatures.

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Data Driven Systems and Control Framework for Multiway Dynamical Systems

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Abstract

In complex biological and engineering systems, structure, function, and dynamics tend to be highly coupled. Recent advances in biology allow us to observe human genome organization and dynamics at unprecedented resolution. These data can be naturally and compactly captured via tensor based state space representations. However, such representations are not amenable to the standard system and controls framework which require the state to be in the form of vector. In order to address this limitation we explore a class of multiway dynamical system in which the states, inputs and outputs are tensors. As a first step we investigate multi linear time invariant (MLTI) systems, in which the state, input and output matrices are replaced by multilinear operators represented as tensors. We extend classical LTI system notions including controllability and observability for the MLTI system. We also develop methods for MLTI system identification and present efficient numerical methods which leverage advances in computational tensor algebra.

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Decoding and Control of Spatiotemporal Structures in Dynamic Ensembles

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Abstract

The ability to decode and control spatiotemporal structures, e.g., synchronization patterns or stable clusters, forms a critical step toward understanding large and diverse complex networks. In this talk, we will present the geometry inherited from the notion of synchronization in an oscillatory network and show how this geometric interpretation can be used to reveal dynamic features in the network. We will next introduce an optimization-free iterative method for computing optimal inputs that establish desired synchronization patterns in a network of nonlinear oscillators, where such pattern formation problems are formulated as optimal ensemble tracking problems. Practical control designs, including entrainment waveforms for pattern formation in a nonlinear oscillatory network and optimal spiking controls for a neuronal population will be illustrated.

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Using Formal Methods to Find an Error in the “Proof” of a Multi-Agent Protocol

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Abstract

Designing protocols for multi-agent interaction that achieve the desired behavior is a challenging and error-prone process. Proofs of protocol correctness rely on human intuition and require significant effort to develop. Even then, proofs can have mistakes that may go unnoticed after peer review, modeling and simulation, and testing of the resulting system. The potential for errors can be reduced through the use of *formal methods*, i.e. automated or semi-automated mathematically rigorous tools and techniques for system specification, design, and verification. In this work, we apply a type of formal method called *model checking* to a previously published decentralized protocol for coordinating a surveillance task across multiple unmanned aerial vehicles. The original publication provides a compelling proof of correctness, along with extensive simulation results to support it. However, our analysis found an error in one of the lemmas used in the proof. Here, we provide an overview of the protocol, the original “proof” of correctness, the model checking approach we use to analyze the protocol, the counterexample returned by the model checker, and the insight this counterexample provides into why the original lemma was incorrect. We also discuss how the formal modeling process revealed that certain aspects of the protocol were underspecified, and what future efforts would be needed to modify it and fully verify it with formal methods.

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Formal Methods in Analysis and Control Theory

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with Steve Awodey, Stefan Mitsch, and André Platzer*

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Abstract

Formal Methods in Analysis is a joint project that combines domain specific-logics for verification of hybrid systems, interactive theorem proving, and methods and ideas from categorical logic. I will provide a brief overview of the project's components. Then I will focus on interactive theorem proving and explore the role it can help play in the design and verification of complex systems.

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[†]Avigad and Awodey: Philosophy and Mathematical Sciences, Platzer and Mitsch: Computer Science

Synthesis and Analysis of Hybrid Systems under Spatial and Temporal Specifications

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Abstract

In this research program we investigate the design of low-level controllers that accomplish given spatiotemporal high-level mission specifications with certain, provable guarantees. Our ultimate goal is to develop a Multiple Lyapunov-like Barriers framework, where the barriers and underlying switching controllers meet both state and time constraints.

We will discuss (i) our recent results on finite-time stability, which enable the development of controllers that can achieve tasks with time specifications, and their application to safe motion planning using finite-time controllers in the presence of uncertainty, and (ii) our preliminary results on the synthesis of constrained hybrid systems for safe trajectory generation using barriers capturing multi-task specifications.

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A Conservation-Based Distributed Control Architecture for Asymptotic and Finite Time Semistability and Consensus in Random Networks

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Abstract

Due to advances in embedded computational resources over the last several years, a considerable research effort has been devoted to the control of networks and control over networks. Network systems involve distributed decision-making for coordination of networks of dynamic agents and address a broad area of applications including cooperative control of unmanned air vehicles, microsatellite clusters, mobile robotics, battle space management, and congestion control in communication networks. In this presentation, we will present a conservation-based framework for addressing almost sure consensus problems for nonlinear stochastic multiagent dynamical systems with fixed communication topologies. Specifically, we present distributed nonlinear controller architectures for multiagent coordination over random networks with state-dependent stochastic communication uncertainty. The proposed controller architectures involve the exchange of generalized charge or energy state information between agents guaranteeing that the closed-loop dynamical network is stochastically semistable to an equipartitioned equilibrium representing a state of almost sure consensus consistent with basic thermodynamic principles. Furthermore, extensions for the design of semistable protocols over random networks for achieving coordination tasks in finite time are also presented.

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Thermal Management for Aircraft

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Abstract

The Air Force Scientific Advisory Board published a technical report detailing the substantial thermal management challenges in fielding key Air Force capabilities. One of the findings was that thermal considerations can potentially limit the operating performance, and even the practical feasibility, of many key capabilities that the Air Force will need to accomplish its future mission. This work developed fundamental physical models of aircraft thermal systems that were used to estimate and extend aircraft thermal endurance, developed new architectures for thermal management, designed and implemented control laws for optimal thermal system performance, and developed a new electromechanical actuator and control allocation method that significantly reduced the heat loads generated by aircraft flight control systems. The work developed physics based models from first principles that describe the salient features of aircraft thermal behavior. These models formed the basis for defining new architectures and control laws that increased the disposal rate of waste thermal energy from an aircraft, thus extending thermal range and endurance. The analytical, physics based models used in this work provided insight into the thermal behavior of the vehicle, exposed opportunities for design trades between classical aircraft performance parameters and onboard thermal system components, and provided the opportunity to yield real-time implementable results.

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Distributed control for spatial self-organization of large multi-agent swarms

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Abstract

We present distributed control laws for the spatial self-organization of multi-agent, large swarms in 1D and 2D spatial domains. The objective is to achieve a desired density distribution over a simply-connected spatial domain. Since individual agents in a swarm are not themselves of interest and we are concerned only with the macroscopic density objective, we view the network of agents in the swarm as a discrete approximation of a continuous medium and design control laws to shape the density distribution of the continuous medium. The key feature of this work is that the agents in the swarm do not have access to position information. Each individual agent is capable of measuring the current local density of agents and can communicate with its spatial neighbors. The network of agents implement a Laplacian-based distributed algorithm, which we call pseudo-localization, to localize themselves in a new coordinate frame, and integrate this with a distributed control law to converge to the desired spatial density distribution. We start by discussing self-organization in one-dimension, which is then followed by the two-dimensional case.

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Cooperation and Autonomy in Communication-Limited Environments

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Abstract

During the last year of this grant we completed stability and performance proofs for distributed control laws for cooperative missions that have tight time-critical constraints. We have also designed a distributed trajectory generation optimization scheme that leverages the onboard computation capabilities of all vehicles to generate a set of feasible trajectories for all agents involved in the mission. In-depth analysis of the cooperative coordination control laws for a wider spectrum of time-critical specifications has led to a prioritization of the coordination and temporal constraints. Finally, we have used a novel combination of mathematical tools from Lyapunov, algebraic graph theory, and switched systems to derive transient performance guarantees for a set of coordination and temporal constraints.

Exploiting characteristics in approximating feedback solutions to optimal control problems

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Abstract

Optimal feedback control policies for nonlinear continuous time dynamical systems are difficult to synthesize in practice, due to a curse-of-dimensionality that afflicts the numerical solution of the attendant Hamilton-Jacobi-Bellman partial differential equation (HJB PDE). By identifying the value function of an optimal control problem with the minimax solution of its HJB PDE, the optimal control problem solution can be represented via a *generalized Hopf formula*. This generalized Hopf formula expresses the value function as the solution of a time and state dependent optimization problem, whose cost is defined with respect to the generalized characteristics associated with the HJB PDE. Using this representation, the value function may be evaluated at isolated points in the state space, independently of its evaluation elsewhere, and parallel computation is possible. Moreover, as the optimizing variable involved corresponds to the initial costate for the characteristic system, open-loop controls immediately follow via the minimum principle. As these open-loop controls are naturally parameterized by the initial state of the dynamics, a state feedback characterization of the optimal control is also possible, albeit expressed in terms of the aforementioned optimization problem. Application of this optimization based representation is illustrated via specific examples for which the state dimension involved renders grid based methods impractical. This includes the synthesis of control Lyapunov functions via an equivalent Zubov (HJB) PDE, for nonlinear dynamical systems of state dimension 5 and 6, and the robust stabilization of an unstable nonlinear plant using the state feedback characterization proposed.

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Optimal sensor and actuator selection in large-scale dynamical systems

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Abstract

We consider the problem of the optimal selection of a subset of available sensors or actuators in large-scale dynamical systems. By replacing a combinatorial penalty on the number of sensors or actuators with a convex sparsity-promoting term, we cast this problem as a semidefinite program (SDP). The solution of the resulting convex optimization problem is used to select sensors (actuators) in order to gracefully degrade performance relative to the optimal Kalman filter (Linear Quadratic Regulator) that uses all available sensing (actuating) capabilities. Since general-purpose SDP solvers cannot handle typical problem sizes that are of interest in many emerging applications, we develop an algorithmic framework using proximal methods. Our customized algorithms exploit problem structure and allow handling sensor and actuator selection for substantially larger problem sizes than what is amenable to current general-purpose solvers. Computational experiments are provided to demonstrate the merits and the effectiveness of our approach.

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Rapid and Accurate Uncertainty Propagation for Nonlinear Systems using Nonlinear Invariants

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Abstract

We present our recent progress on computing tight enclosures of the solutions of nonlinear ODEs subject to bounded uncertainties. For nonlinear systems with large uncertainties, existing approaches either provide extremely conservative bounds (e.g., simple interval methods) or require cumbersome computations that are not amenable to real-time implementation (e.g., methods using complex set representations). To address this, we use the key idea of adding redundant states and ODEs to the original system, thereby creating a lifted system that obeys algebraic solution invariants by design. Fast and accurate bounds can then be computed by combining efficient interval bounding methods with an algorithm that uses algebraic solution invariants to continuously refine the bounds as they are propagated forward in time. Previously, we presented new theory and algorithms for manufacturing and exploiting affine solution invariants and showed that this can provide very sharp bounds at low cost in many applications. This talk will present new theory and algorithms for manufacturing and using nonlinear invariants, which allows our methods to address important new classes of models. However, a key remaining drawback is that manufacturing effective invariants requires considerable user insight. To address this, we also present a new "mean-value" bounding method that uses first-order sensitivity information to automatically construct approximate nonlinear invariants, resulting in tight bounds for some highly complex systems.

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Design and Control of Attack-Tolerant Networks

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Abstract

Longest Induced Path: Graph diameter, which is defined as the longest shortest path in a graph, is often used to quantify graph communication properties. In particular, the graph diameter provides an intuitive measure of the worst-case pairwise distance. However, in many practical settings, where vertices can either fail or be overloaded or can be destroyed by an adversary and thus cannot be used in any communication or transportation path, it is natural to consider a more general measure of worst-case distance. One such measure is the longest induced path. The longest induced path problem is defined as finding the subgraph of the largest cardinality such that this subgraph is a simple path. In contrast to the polynomially computable graph diameter, this problem is NP-hard. In this research, we focus on exact solution approaches for the problem based on linear integer programming (IP) techniques. We propose three conceptually different linear IP models and study their basic properties.

Fortification against Cascades: Network cascades represent a number of real-life applications: social influence, electrical grid failures, viral spread, and so on. The commonality between these phenomena is that they begin from a set of seed nodes and spread to other regions of the network. We consider a variant of a critical node detection problem dubbed the robust critical node fortification problem, wherein the decision-maker wishes to fortify nodes (within a budget) to limit the spread of cascading behavior under uncertain conditions. In particular, the arc weights - how much influence one node has on another in the cascade process - are uncertain. This problem is shown to be NP-hard, even in the deterministic case. We formulate a mixed-integer program (MIP) to solve the deterministic problem, and improve its continuous relaxation via nonlinear constraints and convexification.

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Surviving the Data Deluge: A Combined Dynamical Systems / Machine Learning Approach

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Abstract

Arguably, one of the hardest challenges faced now by the systems community stems from the exponential explosion in the availability of data, fueled by recent advances in sensing and actuation capabilities. Simply stated, classical techniques are ill equipped to handle very large volumes of (heterogeneous) data, due to poor scaling properties and to impose the structural constraints required to implement ubiquitous sensing and control.

The goal of this research is to explore how this “curse of dimensionality” can be potentially overcome by exploiting the twin “blessings” of self-similarity (high degree of spatio-temporal correlation in the data) and inherent underlying sparsity. While these ideas have already been recently used in machine learning (for instance in the context of dimensionality reduction and variable selection), they have hitherto not been fully exploited in systems theory. By appealing to a deep connection to semi-algebraic optimization, rank minimization and matrix completion, in this talk we will show that, in the context of systems theory, the limiting factor is given by the “memory” of the system rather than the size of the data itself, and discuss the implications of this fact. These concepts will be illustrated examining examples of “easy” and “hard” problems, including identification of low order models in the presence of outliers, identification and model (in)validation of switched systems and data driven control. The talk will conclude by exploring the connection between these ideas and the problem of learning in dynamical systems. In particular, we will show how exploiting sparsity leads to tractable, scalable solutions to the problems of anomaly detection and time series classification.

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A higher-order temporal logic for dynamical systems

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Abstract

Dynamical systems can be designed to perform particular functions, often as components within a larger dynamical system. The larger system generally consists of several heterogeneous components working in concert; its behavior is notoriously difficult to predict—or even consistently describe—in terms of its components’ behaviors. Existing logics, meant to describe and prove properties of such hybrid dynamical systems, are generally ad hoc mixtures of differential equations and first order logic.

Here we present a new higher-order temporal logic and sheaf-theoretic semantics for modeling any sort of behavior, including that of hybrid dynamical systems. The logic is standard higher-order logic, except it is equipped with a single atomic sort called **Time** and a few axioms. From the semantics of this logic we can extract a novel *intervallic monitor* that assesses behavioral properties across time intervals.

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