

# AFRL/RQ: Power and Thermal Core Technology Competency

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## Abstract

The Air Force Research Laboratory's Aerospace Systems Directorate has a mission to pioneer transformative aerospace technologies for our warfighters' decisive advantage. The control, power, and thermal management area constitutes one of the core technology competencies (CTC) of the directorate. To advance the mission, this CTC is currently focused on 1) integrated propulsion, power, thermal management and hypersonic vehicle guidance and control, 2) autonomy, and 3) modeling, simulation, and analysis. In the power and thermal area, air vehicle energy management, efficient propulsion, high power electric actuation and energy systems, and methods to increase the thermal endurance of aircraft are just some of the technologies being investigated. For autonomy, researchers are considering scalable teaming of autonomous systems, human/autonomous system interaction/collaboration, and test, evaluation, validation, and verification of autonomous systems. The modeling, simulation, and analysis area is utilized to define requirements and mature technologies, support development through ground and flight tests, and evaluate the military utility of potential concepts and capabilities in future warfighter environments. This presentation will focus on the main technologies that are currently being investigated along with any key challenge areas that have been identified.

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# Proof of a UxAS Software Service in SPARK

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## Abstract

We present initial results on using SPARK to prove critical properties of OpenUxAS, a service-oriented software framework developed by AFRL for autonomous command and control of teams of cooperating unmanned vehicles. OpenUxAS was originally designed to enable rapid development of multi-vehicle autonomy algorithms; verification was not an explicit concern. However, the complexity of OpenUxAS makes it difficult to verify through traditional approaches such as testing. We are therefore working to use alternative approaches such as formal methods to verify portions of OpenUxAS. This includes SPARK, a programming language that includes a specification language and a toolset for proving that programs satisfy their specifications. Using SPARK, we reimplemented one of the core services in OpenUxAS and proved that a critical part of its functionality satisfies its specification. This successful application will provide a foundation for further applications of formal methods to OpenUxAS.

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# In Search of Scalable Dynamic Multi-vehicle Defense

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## Abstract

In this presentation, we will discuss the solutions to a few multi-agent pursuit-evasion differential games, as well as extensions to the Markov inequality switching rule for teaming coordination. We have found solutions for the two pursuer one evader game, and have provably robust policies for the three pursuer one evader game. The solutions to these differential games provide close-loop feedback strategies that are robust to unknown, non-optimal opponent strategies. Using geometric insights gained from studying the solutions of these games, we are able to take these strategies as building blocks to solve other more complex games. We will look at three in particular: capture-the-flag, a multi-defender football game and border defense game. Furthermore, we extend a Markov inequality switching rule to account for multiple agents to switch between optimal assignments. The switching rule prevents indefinite updates and guarantees a (probabilistic) completion of the tasks.

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# Thermal Management for Aircraft

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## Abstract

The Air Force Scientific Advisory Board published a technical report detailing the substantial thermal management challenges in fielding key Air Force capabilities. Modern aircraft, that use fuel as a heat sink, are beginning to experience thermal issues that can cause the temperature of the fuel at some point in the system to exceed a limit. Exceeding a temperature limit can cause fuel coking, damage to subsystems, or even complete system failure. In some cases, thermal endurance, defined as the time that an aircraft can operate before the fuel temperature at some point in the system exceeds a limit, is sufficiently small to result in incomplete missions. Other more severe cases can arise which could result in complete vehicle loss. Therefore, extending the thermal endurance of aircraft has become a critical need. Many factors contribute to the increased thermal loads, for example, onboard sensors, mission systems, electromechanical actuators, electronic engine accessories, electronic attack systems, and directed energy systems all produce heat loads, while the use of composites for the aircraft structure reduce the ability to transfer thermal energy to the atmosphere, as compared to the high conductivity metals used in older airframes. Fuel onboard the aircraft stores thermal energy and the engine, through burning of the fuel, transfers some of the thermal energy to the atmosphere. This work developed physics based models from first principles that describe the salient features of aircraft thermal behavior. These models, which take into account the transport delays that exist in a fuel system due to moving a fluid through a set of pipes, formed the basis for defining new architectures and control laws that increased the disposal rate of waste thermal energy from an aircraft, thus extending thermal endurance. The analytical, physics based models used in this work provided insight into the thermal behavior of the vehicle, exposed opportunities for design trades between classical aircraft performance parameters and onboard thermal system components, and provided the opportunity to yield real-time implementable results.

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# Control & Estimation in the Presence of Adversarial Action and Uncertainty

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## Abstract

Multi-player pursuit-evasion differential games are addressed. Swarm pursuit-evasion differential games in the Euclidean plane where an evader is engaged by a team/network of multiple pursuers are considered. All the players have simple motion à la Isaacs/are holonomic, and the pursuers are faster than the evader. The classical requirement of point capture is relaxed – the case where the capture range of the pursuers  $l > 0$ , and also the interesting scenarios where not all pursuers are faster than the evader, will also be addressed. The state space region where capture is guaranteed, notwithstanding the fact that the pursuers are not faster than the evader, is delineated, thus solving the Game of Kind. It is shown that in group/swarm pursuit, when the players are in general position, capture is effected by one, two, or by three critical pursuers, and this irrespective of the size  $N (> 3)$  of the pursuit pack. Thus, group pursuit devolves into pure pursuit by one of the pursuers, into a pincer movement pursuit by two pursuers who isochronously capture the evader, or cornering by three pursuers, a *ménage à trois*. The solution of the Game of Kind is obtained and critical pursuers are identified. Concerning the Game of Degree, the protagonists' optimal state feedback strategies are synthesized and the Value of the game is derived.

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# Stochastic Control for Systems with Rosenblatt Noise

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## Abstract

Historically white Gaussian noise (Brownian motion) was used to describe the noise in models for physical systems. These noise models were mathematically tractable though they did not approximate well the noise in many physical phenomena. Subsequently systems were described by semimartingales, Markov processes and Gaussian processes which increased the applicability for noise models. However the physical data often does not support the use of these models for noise. For continuous processes it is necessary to develop optimal control for non-Gaussian noise models. A basic noise model is the family of Rosenblatt processes. These Rosenblatt processes are not Gaussian or Markov but they have some desirable features such as continuous sample paths and a long range time dependence. Recently a stochastic calculus has been developed for these processes which includes stochastic integrals and a change of variables (Ito) formula. These tools have been used to formulate and solve explicitly some optimal control problems for both finite and infinite time horizons where the cost is quadratic in the state and the allowable controls are linear state feedback. Some of these results as well as possible generalizations are described.

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# Adaptive Horizon Model Predictive Control

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## Abstract

A standard way of finding a feedback law that stabilizes a control system to an operating point is to recast the problem as an infinite horizon optimal control problem. If the optimal cost and the optimal feedback can be found on a large domain around the operating point then a Lyapunov argument can be used to verify the asymptotic stability of the closed loop dynamics. The problem with this approach is that is usually very difficult to find the optimal cost and the optimal feedback on a large domain for nonlinear problems with or even without constraints. Hence the increasing interest in Model Predictive Control (MPC). In standard MPC a finite horizon optimal control problem is solved in real time but just at the current state, the first control action is implemented, the system evolves one time step and the process is repeated. A terminal cost and terminal feedback found by Al'brekht's method and defined in a neighborhood of the operating point can be used to shorten the horizon and thereby make the nonlinear programs easier to solve because they have less decision variables. Adaptive Horizon Model Predictive Control (AHMPC) is a scheme for varying the horizon length of Model Predictive Control as needed. Its goal is to achieve stabilization with horizons as small as possible so that MPC methods can be used on faster and/or more complicated dynamic processes.

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# Dual-Threshold Models for Activation and Influence Propagation in Networks

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## Abstract

We consider new models of activation/influence propagation in networks based on the concept of dual thresholds: a node will *activate* if at least a certain minimum fraction of its neighbors are active and no more than a certain maximum fraction of neighbors are active. These models have possible interpretations, for instance, in the context of communication and social networks. In an online social network, consistently with the hypothesis originally mentioned by Granovetter (1978), a person may *activate* (i.e., adopt and/or repost an opinion) if sufficiently many but not too many of their friends (neighbors in a network) have adopted this opinion. In a communication network, a device would transmit information if it has been obtained and deemed reliable by a certain minimum number of neighbors; however, to save power, it would not transmit that information if most of the neighbors already have it. We study several versions of this problem setup and compare it to the popular *single threshold* (i.e., linear threshold) models previously developed in the literature. We also develop optimization models of seed nodes selection for influence maximization.

An overview of other research efforts under this task will also be presented

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# Network Partitioning and Aggregation for Hierarchical Control

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## Abstract

A common model reduction approach to large-scale network control systems is to aggregate the components of the network into a small number of groups and to obtain a reduced model that captures the interaction between these groups. Applications include power networks where generators swinging in synchrony are aggregated into an equivalent machine, and multicellular ensembles where clusters of cells exhibiting homogeneous behavior are represented with a lumped model. Unlike standard aggregation procedures, which rely on weak coupling assumptions between the groups, here we present a new approach that leverages the graph-theoretic notion of “equitable partitions” and introduce an appropriate relaxation of this notion for broader applicability. With this approach we derive a hierarchical control that regulates the behavior of each group and ensures that components in each group closely follow this behavior up to an error bound. We provide theoretical tools to characterize this error and an optimization procedure to decide which components to assign to each group. We demonstrate the results on a practically motivated network control example.

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# Action-Based Methods in Dynamics and Stability Issues in Networked Systems

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## Abstract

### **Part 1: The Staticization Operator, Conservative Dynamical Systems and the Schrödinger Equation**

We will indicate the progress being made on the application of the staticization operator to two-point boundary value problems in conservative systems and the Schrödinger equation. Most notably, the development of both fundamental solutions for conservative systems of indefinitely long duration and the curse-of-dimensionality-free approach to solution of Schrödinger initial value problems relies on commutativity of the staticization operator. This is a deep issue in nonlinear analysis, and the commutativity holds only for specific classes of systems. Fortuitously, these classes include the systems of interest. An additional piece of the puzzle regards existence and uniqueness for solution of complex-valued stochastic differential equations where the drift is defined by a complex extension of the Coulomb potential. This is a substantial generalization of the existing theory. Application of these developments to the systems of interest will be indicated.

### **Part 2: Integrated Analysis and Design of Control and Communications in Networked Systems**

Modern systems are dominantly interconnected and networked. They are exemplified by autonomous vehicles, smart grids, smart cities, gene networks, social networks, energy and material flow networks, to name just a few. Management of such network systems encounters fundamental issues of information, uncertainty, and complexity, and demands integration of control, communications and computing. In this presentation, we will highlight some key motivations, discuss the historical pursuit of integrated control, communications, and

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computing, and explain how communication uncertainties introduce new issues in control. We will summarize some recent efforts and advances in developing new control frameworks and methods to accommodate communication-induced asynchronous operation, switching network topology, and random delays.

# Advances in Stochastic Surveillance and Network Flow Dynamics

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## Abstract

In the first half of the talk, we review recent progress in the formalization and analysis of Stackelberg games arising in the context of robotic surveillance. Specifically, we analyze and optimize stochastic surveillance strategies against an omniscient intruder. Some characterizations on the dominant strategies for both the intruder and surveillance agent are given, which depend on the topology of the robotic roadmap. We obtain analytic solutions as well as conjectures for the optimal surveillance strategies in simple graph topology. Finally, we report various computational results and numerical studies for different prototypical graph topology and realistic roadmaps.

In the second half of the talk, we show some recent progress in the analysis of network flow systems. Specifically, we propose novel graph-theoretic conditions for stability of positive monotone systems based on concepts of input-to-state stability and network small-gain theory. We characterize and compute two forms of input-to-state stability gains for linear positive systems modeled by Metzler matrices, and these two forms of stability gains give rise to necessary and sufficient stability conditions that are closely related to the cycle structure of the flow network. We extend our results to nonlinear monotone systems and obtain sufficient conditions for global asymptotic stability.

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# Identification of critical nodes in large-scale spatial networks

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## Abstract

In this talk, we will present results on a novel dynamical approach to the problem of identifying critical nodes in large-scale networks, with algebraic connectivity (the second smallest eigenvalue of the graph Laplacian) as the chosen metric. Employing a graph-embedding technique, we reduce the class of considered weight-balanced graphs to spatial networks with uniformly distributed nodes and nearest-neighbors communication topologies. Through a continuum approximation, we consider the Laplace operator on a manifold (with the Neumann boundary condition) as the limiting case of the graph Laplacian. We then reduce the critical node set identification problem to that of finding a ball of fixed radius, whose removal minimizes the second (Neumann) eigenvalue of the Laplace operator on the residual domain. This leads us to consider two functional and nested optimization problems. Resorting to the Min-max theorem, we first treat the problem of determining the second smallest eigenvalue for a fixed domain by minimizing an energy functional. We then obtain a closed-form expression for a projected gradient flow that converges to the set of points satisfying the KKT conditions and provide a novel proof that the only locally asymptotically stable critical point is the second eigenfunction of the Laplace operator. Building on these results, we consider the critical ball identification problem and define novel dynamics to converge asymptotically to these points. Finally, we provide a characterization of the location of critical nodes (for infinitesimally-small balls) as those points which belong to the nodal set of the second eigenfunction of the Laplacian operator. We conclude that the location of such critical nodes is at the nodal set of the second eigenfunction of the Laplace operator, which, as we discuss, has an intuitive geometric interpretation in some cases.

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# Constraints and memory-null controllabilities for fractional PDEs

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## Abstract

We show that for PDEs associated with the fractional Laplace operator, the classical notion of controllability from the boundary must be replaced by an exterior control, that is, the control function must be localized outside the domain where the PDE is satisfied. It turns out that this novel notion of controllability is more related to real life phenomena such as (but not limited to): Acoustic testing; Magnetotellurics; Magnetic drug targeting and Electroencephalography.

In the first part, we analyze the controllability properties (from the exterior of the interior) under positivity constraints on the control or/and the state of the heat equation involving the fractional Laplace operator  $(-\Delta)^s$  ( $0 < s < 1$ ). We prove the existence of a minimal (strictly positive) time  $T_{\min}$  such that the fractional heat dynamics can be controlled from any initial datum to a positive trajectory through the action of a positive control, when  $s > 1/2$ . Moreover, we show that in this minimal time, constrained controllability is achieved by means of a control that belongs to a certain space of Radon measures. We notice that, in order to achieve these controllability results, the minimal time  $T_{\min}$  must be large enough. We also give some numerical simulations that confirm our theoretical results.

In the second part, we study the null controllability properties (from the exterior or the interior) of the wave equation with memory associated with the fractional Laplace operator. The goal is not only to drive the displacement and the velocity to rest at some time-instant but also to require the memory term to vanish at the same time, ensuring that the whole process reaches the equilibrium. This is called memory-null controllability. Assuming that the control is acting on an open subset  $\omega(t)$  which is moving with a constant velocity, we obtain that the system is memory-null controllable in a sufficiently large time.

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# Optimal Sensor Location for Distributed Parameter Systems

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## Abstract

The best location of hardware for estimation is dependent on the type of disturbances, the cost to be minimized, and also the extent of nonlinearities. Earlier results on optimal actuator design for linear distributed parameter systems have been extended to several classes of nonlinear partial differential equations. This extends earlier work on optimal design for linear systems. There are algorithms for concurrent actuator/ controller location and similarly for sensor/estimator design. However, the problem of optimal shape is challenging numerically. Efficient methods to compute the optimal shape and controller (or estimator) are needed. This is complicated numerically by the fact that the cost function is often only weak-star continuous in the design. Some results are described.

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# Munitions Aerodynamics, Guidance, Navigation and Control Core Technical Competency

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## **Abstract**

The Munitions Aerodynamics, Guidance, Navigation and Control Core Technical Competency at the Munitions Directorate of AFRL is uniquely responsible for scientific research that provides a fundamental understanding of precision, autonomy, agility, control and maneuver for air launched weapons. The MAGNC CTC defines component, systems and machine-to-machine technologies to achieve any desired effect, even in highly adversarial, confusing, and cluttered environments.

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# First-order optimization algorithms: tradeoffs between robustness and acceleration

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## Abstract

We study the robustness of accelerated first-order algorithms to stochastic uncertainties in gradient evaluation. Specifically, for unconstrained, smooth, strongly convex optimization problems, we examine the mean-square error in the optimization variable when the iterates are perturbed by additive white noise. This type of uncertainty may arise in situations where an approximation of the gradient is sought through measurements of a real system or in a distributed computation over network. Even though the underlying dynamics of first-order algorithms for this class of problems are nonlinear, we establish upper bounds on the mean-square deviation from the optimal value that are tight up to constant factors. Our analysis quantifies fundamental trade-offs between noise amplification and convergence rates obtained via *any* acceleration scheme similar to Nesterov's or heavy-ball methods. To gain additional analytical insight, for strongly convex quadratic problems we explicitly evaluate the steady-state variance of the optimization variable in terms of the eigenvalues of the Hessian of the objective function. We demonstrate that the entire spectrum of the Hessian, rather than just the extreme eigenvalues, influence robustness of noisy algorithms. We specialize this result to the problem of distributed averaging over undirected networks and examine the role of network size and topology on the robustness of noisy accelerated algorithms.

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# Adaptive Dynamical Learning and Control for Flight Vehicles

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## **Abstract**

Flight vehicles are deployed in uncertain highly complex environments and due to sudden unexpected damage or changes to the airframe, the vehicle can deviate significantly from expected behavior. This research develops an innovative control architecture which detects the system-altering change and intelligently switches from an optimal, model-based control scheme to an adaptive controller for performance and robustness benefits. The adaptive controller employs a data-driven integral concurrent learning scheme to estimate aerodynamic coefficients and mass properties online while ensuring stability during the learning phase. After sufficient parameter convergence of aerodynamic estimates, the controller switches back to the model-based scheme which reformulates the control structure and gains online in order to properly compensate for the faulted system. Dwell-time conditions are developed for the adaptive controller which preserves stability with repeated switching between control schemes.

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# Optimality Conditions via Set-Valued Analysis

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## Abstract

Differential inclusions (involving set-valued maps) play a crucial role to model evolution of systems. Control systems, being a parametrized version of differential inclusions, do benefit from achievements of set-valued analysis.

Some tools of the differential calculus of set-valued maps will be presented and used to derive the first and second order optimality conditions.

In the case of control systems this calculus implies the first and second order "linearizations" along optimal solutions that lead in a straightforward way to variational equations.

As a consequence of this approach, the maximum principle can be obtained for infinite dimensional stochastic control systems with "convex" right-hand sides. Second order "linearizations", in turn, lead to second order optimality conditions for stochastic control problems with the state and control constraints, even in the absence of convexity assumptions.

In the case of deterministic systems results are even stronger, thanks to the possibility of convexification (relaxation) of dynamics.

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# A Switched Systems Approach for Navigation and Control with Intermittent Feedback

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## Abstract

Current visual servo control and image-based state estimation algorithms assume that image feedback is continuously available from a single source. To generalize the persistent staring assumption of adaptive image-based estimators, the ability to arbitrarily switch between different uncertain and nonlinear image dynamics is required. Motivated by this need, switched systems methods were explored as a means to ensure convergence of image-based observers to determine the relative trajectory between a camera and intermittently viewed image features. Such switched systems methods provided insights to develop controllers/observers for a more general class of intermittent feedback problems. Efforts also focused on new advancements in switched systems methods that yield less conservative theorems for differential inclusions.

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# Systematic Tools for Satisfying Temporal Logic Specifications in Hybrid Dynamical Systems

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## Abstract

Air Force combat systems require autonomous systems that are agile, reliable, and resilient. The unavoidable combination of the physics, the networks, and the algorithms in such systems leads to hybrid system models, which makes the systematic design of algorithms very challenging. Moreover, the unavoidable presence of obstacles, adversarial elements, and uncertainty in the environment further adds to the complexity and requires design tools that neither discretize the state or the dynamics while guaranteeing given high-level specifications, robustly. While discretization may aid computations in some cases, discretization of the hybrid system models emerging in Air Force missions may not permit to assure that the desired specifications are actually satisfied. In this talk, I will present tools to certify temporal logic specifications in hybrid dynamical systems that do not require discretization of the dynamics or of the state space. These tools rely on sufficient conditions that are given in terms of infinitesimal inequalities, which resemble those in the context of asymptotic stability and safety. Conveniently, these conditions do not involve the trajectories of the system explicitly, but rather just the objects defining the hybrid system.

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# A fast first-order optimization algorithm based on stability theory for hybrid dynamical systems

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## Abstract

We present a novel algorithm with momentum for the solution of convex optimization problems. The novelty of the approach lies in combining Hamiltonian flows, induced by a Hamiltonian field, with an appropriate flow set that forces the flows to decrease the cost, and a class of resetting mechanisms that reset the momentum to zero whenever it generates solutions that do not decrease the cost function. For radially unbounded functions with Lipschitz gradients we establish uniform global asymptotic stability, and for functions that additionally satisfy the Polyak-Lojasiewicz inequality the algorithm yields exponential convergence. Since the flow dynamics are given by Hamiltonian systems, which preserve energy, symplectic integrators with stable behavior under not necessarily small step sizes can be implemented. These leads to a class of discretized algorithms with performance comparable to existing algorithms that are optimal in the sense of generating the fastest possible rates of convergence.

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# Regularizing for Robustness: Accelerated Optimization Dynamics via Hybrid Feedback

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## Abstract

We study the robustness properties of a class of time-varying accelerated optimization algorithms modeled as ODEs. We show that a family of these algorithms related to the continuous-time limit of the Nesterov’s accelerated gradient method can be rendered unstable under arbitrarily small additive disturbances. Indeed, while solutions of these dynamics may converge to the set of optimizers, in general, the time-varying dynamics do not render the set of optimizers uniformly asymptotically stable, even when the cost function is strongly convex. We show this fact by using Artstein’s idea of limiting equations. To address the lack of uniformity in the convergence we propose a framework where we regularize the ODEs by using resetting mechanisms that are modeled by wellposed hybrid dynamical systems. For these systems, we establish uniform asymptotic stability properties and strictly positive margins of robustness, as well as convergence rates similar to those of the non-hybrid ODEs. We further characterize a family of regular discretization mechanisms that retain the stability and robustness properties of the hybrid dynamics. Our results provide the basis for the design and construction of a family of advanced real-time and feedback control algorithms that incorporate acceleration in closed-loop systems.

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\*J. I. Poveda was a postdoc in the School of Engineering and Applied Sciences at Harvard University and now is an assistant professor in the Department of Electrical, Computer & Energy Engineering at University of Colorado at Boulder

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# Robust Formation Control in the Presence of Intermittent Information

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## Abstract

In this seedling lab task, we tackled a formation control problem in the presence of intermittent communication blackouts. Namely, each agent was considered as having linear second-order dynamics representing the linearized physical laws. The agents are allowed to share information across to their local neighbors only at isolated time instances. Due to the continuous and discrete dynamics inherent into this system, we utilize a hybrid systems framework to model the closed-loop system. Then, we recast this problem as a set stability problem and leverage recent results using Lyapunov functions for hybrid systems to show that, indeed, the desired formation is globally asymptotically stable. Moreover, the set of points governing the formation is also shown to be robust to various perturbations. The next step under this lab task is to consider the case when the formation is being actively attacked and to develop an attack monitor which detects such attacks.

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# Entropy and quantized control of switched systems

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## Abstract

We will discuss a notion of estimation entropy for general continuous-time systems, formulated in terms of the number of functions that approximate all system trajectories up to an exponentially decaying error. This entropy notion characterizes minimal data rates for state estimation and control. Another problem in which it plays a role is that of determining, from quantized state measurements, which of several competing models of a dynamical system is the true model. The focus of the talk will be our ongoing work on computing entropy for switched linear systems whose individual matrices satisfy suitable commutation relations. Implications of these results for quantized control of switched systems will also be discussed.

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# Formal Methods in Analysis and Control Theory

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## Abstract

*Formal Methods in Analysis* is a joint project that combines interactive theorem proving, domain specific-logics for verification of hybrid systems, and methods and ideas from categorical logic.

We have extended the multivariate analysis library of the Lean theorem prover with asymptotic reasoning, the Fréchet derivative, and the Bochner integral. We have also formalized algebraic constructions based on the notion of a *quotient of a polynomial functor*, to facilitate the construction of inductive types, coinductive types, and arbitrary nestings of these. With Alexander Bentkamp, we have begun to develop infrastructure for expressing optimization problems formally, transforming them in a verified way, and checking certificates from external solvers.

For the purposes of advancing the KeYmaera X prover and its integration with other proof assistants, we have continued the VeriPhy prover pipeline which achieves provably correct execution of hybrid system controller sandboxes via a chain of proofs in KeYmaera X, Isabelle/HOL, and HOL4. This pipeline exploits the capabilities in these provers from hybrid systems verification over metatheory proofs to verified compilation resulting in machine code. In order to weld over the seams in the pipeline, we have completed a (partial) integration of Isabelle/HOL and HOL4. We provide a virtual HOL4 instance within Isabelle/HOL that replaces the HOL4 kernel by operations on the Isabelle/HOL kernel and transfers the proved theorems in the virtual HOL4 context back to Isabelle/HOL.

Finally, work in categorical logic has focused on extensions of homotopy type theory with modal operators for cohesion, providing synthetic representations of geometric and differential structure. A new notion of a “modal category with families” has been developed on the basis of a homomorphism of natural models, as a framework for the semantics of modal type theories.

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<sup>†</sup>Avigad and Awodey: Philosophy and Mathematical Sciences, Platzer and Mitsch: Computer Science

# Monadic decision processes and hierarchical planning for autonomous systems

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## Abstract

We discuss a generalization of Markov Decision Processes, which we call  $M$ -DPs, where the notion of probability distribution and reward are replaced by an arbitrary monoidal monad  $M$ . Such a monad can be used to formalize not only probability distributions and rewards in  $\mathbb{R}$ , but also possibility, rewards in arbitrary monoids, task-based planning, exception handling, etc.

$M$ -DPs form the objects of a category  $M$ -DP, where the morphisms are hierarchical action-planners. For example, at a high-level we might take the action “go from the bedroom to the kitchen”. But in order to accomplish this task, it must be translated to a low-level action plan based on the more precise location within the bedroom that we are currently in (e.g. our course of action depends on whether we’re in bed, or in the chair). This is irrelevant to the high-level planner, but relevant to the low-level planner. This process repeats on many scales, e.g. as our bodies plan the exact motions of muscles that accomplish the goal. The category  $M$ -DP collects all of this information. We may briefly discuss the connection to lenses, a new paradigm in functional programming that is currently taking the field by storm.

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# Structural Decomposition for Quantum Two-level Systems

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## Abstract

The dynamics of a two-level system immersed in a dissipative environment can be described by its system Hamiltonian  $H$  and coupling operator  $L$ , both of which can be parametrized in terms of Pauli matrices. In this paper, we propose other parametrization methods of  $H$  and  $L$  so that the resulting system matrices may look simpler than that by means of Pauli matrices. The proposed parametrization methods make direct use of controllability and observability properties of the system matrices, reminiscent of the Kalman decomposition for linear quantum systems.

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# Reduction of Decoherence in Quantum Information Systems Using Direct Adaptive Control of Infinite Dimensional Systems

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## Abstract

Using Quantum Mechanical systems to store and retrieve information and use it in quantum computing is a new aspect of physical science. These quantum systems are inherently infinite dimensional systems and their dynamic behavior is not well known. What must be controlled, are the quantum gates that do all the computational work but should remain reversible because the gates are expected to be unitary operators in the Hilbert space of quantum system states. These gates will suffer some decoherence of this unitarity because they are open systems and subject to interaction/entanglement with other related quantum systems. Consequently, they will not operate as the ideal systems they are expected to be and will produce significant errors. Direct adaptive control does not use detailed information about the gates, but can still be used to reduce decoherence.

Our overall direction is on using our research in adaptive control of infinite dimensional systems to explore how these feedback control ideas in conjunction with quantum gates and quantum error correction can reduce decoherence in quantum information and computing. It has been shown that there are decoherence-free subspaces in the Hilbert space of quantum states. This presentation will focus on our current approach using adaptive control of infinite dimensional systems to guide quantum systems into these decoherence-free subspaces.

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# Advanced Space Resilient Technologies Future Direction

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## **Abstract**

The US Space Enterprise is continuing to pivot towards Space Warfighting, with an emphasis on resilient, disaggregated, proliferated platforms, agile business practices, and emphasis on partnerships. Challenges with operating in a contested domain combined with the need to move to more integrated multi-domain operations for near-peer conflicts present significant challenges for space systems. The Air Force Research Lab has taken a leading role in advancing all of these areas through science and technology, new strategies such as Space Accelerators for engaging nontraditional partners using novel business mechanisms, and broad international cooperation.

This presentation will provide a top level overview of the technology direction that the Air Force Research Laboratory, Space Vehicles Directorate is taking to support this change in direction as well as the impacts of these changes to the Advanced Spacecraft Resilient Technologies CTC, which is a broad, pervasive technology area that includes core satellite bus technologies such as power generation and storage, structures, thermal, and electronics. The goal of the presentation will be to provide a top level understanding of the changing space environment, the new research directions with the CTC, and potential areas of interest specific to the Dynamical Systems and Control Theory Program.

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# Interactive 2D3D Vision-Based Operator Control

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## Abstract

While significant work in autonomy and robotics have been made, most recently with advances in machine learning, there still exists no universal framework capable of handling complex real-world scenarios due to “unknown unknown.” In particular, during black-swan events for which loss-of-life must be mitigated, operators often mistrust and abandoned their autonomous counterparts in favor of (computationally intractable) experience for which errors may still arise yet accountability is upheld. While such operators are experts in their domain, they can be considered non-experts given their obfuscation to the autonomous model construction leading to mistrust especially in situations of duress. This leads to an information gap plaguing even the most sophisticated learning approaches. To combat this, we introduce a geometric interactive control 2D3D feedback approach towards imaging systems in order to properly incorporate a 3D human operator based on single or multiple 2D image observations. From a vision-based autonomy perspective, we will show a 2D3D geometric framework that does not require well-defined features often realized in low contrast and adverse imaging conditions (e.g., space, high-altitude), but is also able to unify image observations to their real-world counterpart through a differential and projective geometric lifting procedure. From this, we develop an interactive feedback control framework that is able to reconcile highly non-convex problems under duress for which perceived global minima is not consider optimal choice due operator mistrust and experience. Ultimately, given the ubiquitous definition of trust under ambiguous situations, we aim to provide the non-expert expert as needed intervention without abandoning their autonomous counterpart. In doing so, we show that a particular form of principle curvature (a measure of flatness in geometry) is positively correlated to convergence rates of stability. This is similar to previous work for which we have shown another form of curvature, namely Ricci curvature, relates to network robustness. Finally, we will give some recent results, highlight potential avenues in PNT and GPS denied environments, and close with future mathematical pursuits as it relates to geometry and control to the problem at hand.

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# Tight Coordination with Time-Varying Temporal Constraints for a Fleet of Heterogeneous Vehicles

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## Abstract

This talk focuses on a cascaded architecture that decouples the low-level control from the higher-level autonomy-enabling algorithms for time-critical coordination scenarios. The framework unencumbers the high-level algorithms from the vehicle dynamics, which facilitates the analysis of the system stability and expands the capabilities that the high-level protocols can impose under transient and steady-state guarantees. The talk provides a brief overview of the control augmentation system, path following algorithm, and speed controllers that make this possible. A novel time-critical coordination algorithm that enforces a time-varying temporal window along an entire mission is introduced. The theory developed to prove that the new protocol behaves as expected combines tools from nonlinear systems, switched systems, algebraic graph theory, and finite state machine theory. Technology transition efforts from NASA LaRC Autonomous Integrated Systems Branch summarize the presentation.

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# Sparse approximation methods for optimal control and stationary action problems

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## Abstract

Optimal control and stationary action problems share a common objective in seeking to render an associated cost or integrated Lagrangian (action) functional stationary in a general calculus of variations sense. By applying the tools of optimal control, or their generalization for stationary action problems, the dynamics of an optimally controlled process or conservative physical system can be encapsulated by the characteristic flow of an attendant Hamilton-Jacobi-Bellman partial differential equation (HJB PDE). Individual open-loop trajectories can subsequently be identified as corresponding solutions of a two-point boundary value problem (TPBVP) constrained by this flow, while a general feedback characterization of all possible such open-loop trajectories can be described via the solution of the aforementioned HJB PDE. While robustness considerations prioritize use of this latter feedback characterization, HJB PDEs are difficult to solve in practice, as numerical methods that approximate their solution routinely suffer from a curse-of-dimensionality that limits their feasibility. In this research, the algebraic properties of dynamic programming and HJB PDEs are explored with the objective of developing new theory and methods for attenuating the curse-of-dimensionality via sparse solution representations and approximations. Theory and examples corresponding in particular to idempotent methods, optimization methods, and relaxation methods, are elucidated. Details include the interplay of idempotent linearity, duality, and semigroup properties in an idempotent method for providing a sparse basis approximation for HJB PDE solutions, the utility of a generalized Hopf formula in yielding sparse approximations via an optimization method, and the role of non-quadratic cost lifting and approximation in a relaxation method.

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# Synthesis and Analysis of Hybrid Systems under Spatial and Temporal Specifications

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## Abstract

In the second year of the project we developed theoretical and algorithmic results on the synthesis and analysis of systems under spatiotemporal (state and time) constraints. In this talk, we will first present our results on finite-time stability of nonlinear switched and hybrid systems; we show that under certain conditions, existence of a finite-time mode and its activation for a given *cumulative* time period guarantees finite-time stability for the switched or the hybrid system, respectively. Next, we will present our approach on control synthesis for nonlinear control-affine systems under spatiotemporal specifications. The specifications (encoded via Signal Temporal Logic) involve reaching a given goal set within a prescribed (user-defined) time while staying in a given safe set at all times. The prescribed-time control synthesis is initially addressed using a control Lyapunov function approach. Then, we generalize our method using a quadratic programming (QP)-based approach, which computes minimum-effort control inputs that satisfy the spatiotemporal specifications and input constraints. We analyze the feasibility of the corresponding QP and provide conditions under which the solution satisfies the spatial, temporal and input constraints. Then, we will highlight our results on novel gradient flow schemes with fixed-time stability guarantees for convex optimization problems. Lastly, we present our plans on the future work for the third year of the project.

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# A Solution to the Distributed Linear Quadratic Optimal Regulator Problem

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## Abstract

An solution is presented to the distributed linear quadratic optimal regulator problem for an  $m > 1$  channel, jointly controllable, jointly observable,  $n$ -dimensional, continuous time, linear system whose sensed outputs are distributed across a time-invariant network with a strongly connected graph. The solution relies on a centrally designed optimally state feedback law, a centrally designed distributed observer, and certainty equivalence to attain a fully distributed, exponentially stable, feedback control system. The observer consists of  $m - 1$  local estimators, each of dimension  $n$ , plus one additional local estimator of dimension  $n + m - 1$ . The distributed observer's spectrum is freely assignable, just as in the case of a centralized observer.

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# Linearization and Optimal Feedback Design via Lifting Operators

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## Abstract

We utilize ideas from Carleman linearization to lift a given finite-dimensional nonlinear system into an infinite-dimensional linear system. Finite-order truncations of the resulting infinite-dimensional linear system is studied and connections between (local) stability properties of the original nonlinear system and its finite-order truncations are established. Then, we consider optimal feedback control design for a class of nonlinear control systems with polynomial right-hand sides.

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# Spectral Bayesian Estimation for Stochastic Hybrid Systems

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## Abstract

We present computational techniques for Bayesian estimation of general stochastic hybrid system. The Fokker-Planck equation that describes the evolution of uncertainty distributions is solved by spectral techniques, where an arbitrary form of probability density of the hybrid state is represented by a mixture of Fourier series. The method is based on splitting the Fokker-Planck equation represented by an integro-partial differential equation into the partial differentiation part for continuous diffusion and the integral part for discrete transition, and integrating the solution of each part. The propagated density function is utilized to estimate the hybrid state for given sensor measurements. The unique feature of the proposed technique is that the probability density describing the complete stochastic properties of the hybrid state is constructed without relying on the common Gaussian assumption. The efficacy of the proposed method is illustrated by a bounding ball model and Dubins vehicles.

Next, we propose a symmetry reduction based approach to treatment of unobservable subspaces in the context of uncertainty quantification using Kalman filtering. This proof of concept demonstrates that the symmetry reduction of the input-output representation of a system with unobservable directions can yield noticeable improvements in the quality of the uncertainty quantification. We aim to combine this approach with the symmetry reduction of hybrid systems and the more sophisticated spectral and real harmonic analysis based representations to address uncertainty quantification in the presence of unobservable directions in hybrid systems evolving on Lie groups.

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# Permutation- and Graph-Based Representations of Lie Brackets: New Approaches to Controllability and Moment-Based Control

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## Abstract

Tools from Lie theory have been shown to be suitable and powerful for analyzing controllability of nonlinear systems, and the most notable development was, unarguably, the Lie algebra rank condition (LARC). In this talk, we will first present an alternative and effective algebraic approach to investigate controllability of right-invariant bilinear systems. The central idea is to map Lie bracket operations of the vector fields governing the system dynamics to permutation multiplications on a symmetric group. In this way, controllability and the controllable submanifold can be characterized by permutation orbits. We will further show that this symmetric group based method is directly applicable to examine controllability of systems defined on undirected graphs, which in turn reveals a graph-based representation of controllability through graph connectivity. We will next introduce a novel moment-based control strategy for manipulating a population system using aggregated feedback, where the task of formation control in the population is mapped to the so-called moment problem.

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# Notions of Heterogeneity in Networked Systems

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## **Abstract**

Three distinct variations on the theme of heterogeneity are examined for networked systems. First, I examine how time-scales effect performance analysis on networks and offer new insights on network synthesis problems with node time-scales as optimization parameters. I will then discuss notions of learning for networks comprised of homogeneous and heterogenous nodes. Lastly, I report on how notions of distributed optimization can be extended to scenarios where the network supports coordination of non-cooperating teams of agents.

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# Data Driven Systems and Control Framework for Multiway Dynamical Systems

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## Abstract

In biological and engineering systems, structure, function and dynamics are highly coupled. Such interactions can be naturally and compactly captured via tensor based state space dynamic representations. By leveraging the tensor unfolding operation, we extend classical linear time invariant (LTI) system notions including stability, reachability and observability to multilinear time invariant (MLTI) systems, in which the state, inputs and outputs are preserved as tensors. While the unfolding-based formulation is a powerful theoretical construct, we also establish results which enable one to express these tensor unfolding based criteria in terms of more standard notions of tensor ranks/decompositions. We propose Higher-Order Balanced Proper Orthogonal Decomposition and Higher-Order Dynamic Mode Decomposition-based model reduction and identification approaches for MLTI systems. Additionally, this work explores multiway dynamical systems as a means to rigorously capture the interplay of function, structure and dynamics inherent in complex biological systems. We obtain data from human fibroblasts at discrete time points during cell division or specialization processes, in the form of gene expression, genome structure, and imaging of processes using live cell microscopy. We use these data to construct a tensor that will guide estimation of the parameters for the dynamics of structure and function. We also discuss how to prove the global existence of periodic behavior in biological and synthetic biological systems such as the repressilator.

# Prescribed–Time Control and Estimation of PDEs

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## Abstract

Motivated by missile guidance, where the interception of a moving target must occur in a finite time, finite–time stabilization of a dynamical system is of interest. A more relevant and stronger notion is when the finite stabilization time can be prescribed by a user independently of the system’s initial condition, which is the focus of this work. The applicability of such prescribed–time stabilization is not limited to missile guidance: it is desired in applications with strict time–performance requirements (e.g., ABS braking). In this presentation, we review some of our results over the past year for unstable reaction–diffusion PDEs and for the linear Schrödinger PDE with boundary actuation. We first show novel boundary controllers and state observers for unstable reaction–diffusion systems which guarantee prescribed–time output–feedback stabilization for which the certainty equivalence principle holds. The trajectories of the resulting closed–loop systems have flat approaches to zero as time approaches the terminal time. This stabilization feature ensures that time–derivatives of the desired quantities also converge to zero in the prescribed time. Importantly, we verify the boundedness of the controllers and observer output measurement injections that we propose. Finally, we present prescribed–time output–feedback stabilization of the Schrödinger equation.

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# Nonlinear-Nonquadratic Optimal Control, Implications of Dissipativity, and Universal Feedback Regulators for Stochastic Systems

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## Abstract

In this work, we present a unified framework to address the problem of optimal nonlinear analysis and feedback control for nonlinear stochastic dynamical systems. Specifically, we provide a simplified and tutorial framework for stochastic optimal control and focus on connections between stochastic Lyapunov theory and stochastic Hamilton-Jacobi-Bellman theory. In particular, we show that asymptotic stability in probability of the closed-loop nonlinear system is guaranteed by means of a Lyapunov function which can clearly be seen to be the solution to the steady-state form of the stochastic Hamilton-Jacobi-Bellman equation, and hence, guaranteeing both stochastic stability and optimality. In addition, we develop optimal feedback controllers for affine nonlinear systems using an inverse optimality framework tailored to the stochastic stabilization problem. These results are then used to provide extensions of the nonlinear feedback controllers obtained in the literature that minimize general polynomial and multilinear performance criteria. Furthermore, using the newly developed notion of stochastic dissipativity we present a return difference inequality to provide connections between stochastic dissipativity and optimality as well as establish guaranteed stability margins for optimal nonlinear stochastic feedback regulators.

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# Scalable Analysis and Control of Dynamic Flow Networks

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## Abstract

We discuss the objectives and early results of a new AFOSR Young Investigator Award to study dynamic flow networks. This project aims to develop fundamental theory and domain-driven techniques for controlling and designing dynamic, physical flow systems that model the flow of physical material among a network of interconnected components. This modeling approach is well-suited for large classes of cyber-physical systems including vehicular transportation networks, air traffic networks, and civil infrastructure. In such systems, the individual node dynamics are often highly nonlinear and hybrid, and these nonlinearities compound due to network interactions. This project focuses on using and extending two powerful tools from nonlinear system analysis, namely, monotone systems theory and contractive systems theory to study physical flow networks. In this talk, we will present recent results for extending contraction theory to hybrid dynamical systems. We demonstrate our approach on a physical flow network in which the flow between adjacent cells exhibits a hysteresis effect.

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# Formal Safety Verification for Nonlinear Systems Using Advanced Reachability Analysis Methods

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## Abstract

We present our recent progress on solving formal safety verification problems formulated as backward reachability problems for nonlinear dynamic systems. Forward reachability analysis refers to the computation of rigorous enclosures of the set of trajectories consistent with a given set of uncertain inputs and/or initial conditions. In contrast, backward reachability analysis aims to characterize the subset of uncertain parameters that generate trajectories that reach a specified target set while avoiding specified obstacles. Existing backward reachability algorithms for general nonlinear systems fall into two broad categories. The first involves the solution of a Hamilton-Jacobi-Bellman PDE and scales exponentially in both the number of states and uncertain parameters. The second, which we develop further in this talk, applies a recursive sequence of forward reachability computations within a branch-and-bound (B&B) algorithm. Although this approach scales worst-case exponentially in the number of uncertain parameters, its performance in practice is largely dictated by the speed and accuracy of the embedded forward reachability calculations. Unfortunately, for nonlinear systems with large uncertainties, existing forward reachability algorithms either provide extremely conservative bounds (e.g., simple interval methods) or require cumbersome computations that are not amenable to fast implementation (e.g., methods using complex set representations). To address this, this talk investigates the performance of the B&B-based backward reachability algorithm when coupled with advanced forward reachability algorithms previously developed with AFOSR support. These methods are based on the key idea of adding redundant states and ODEs to the original system, thereby creating a lifted system that obeys algebraic solution invariants by design. Fast and accurate bounds can then be computed by combining efficient interval bounding methods with an algorithm that uses algebraic solution invariants to continuously refine the bounds as they are propagated forward in time. After presenting some new advances in these methods, we show that embedding them within a B&B-based backward reachability algorithm enables the efficient computation of backward reachable sets for several challenging examples in chemical process safety verification and aircraft collision avoidance. In some cases, the use of advanced forward reachability algorithms reduces the cost of the backward reachability problem by orders of magnitude relative to the current state of the art.

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