

# Convexification of Motion Planning Through Liftings and Hypercomplex Numbers

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## Abstract

The goal of the proposed research is the development of a rigorous mathematical and algorithmic framework to explore the role of parameterization in convexifying complex motion planning problems for controlled, autonomous, dynamical systems. Specifically, we will examine how novel parameterizations of control, configuration space, and logical constraints, can be effectively utilized to develop convex optimization-based solution methods and real-time executable algorithms for complex motion planning problems. The motion planning problems of interest have non-convexities due to control, state, or coupled state-control constraints, and non-linear dynamics. Our project started in 2020 and this talk will introduce some preliminary mathematical problem formulations and theoretical results. These results include: i) Recent results on lossless convexification of nonconvex control constraints; ii) Dual-quaternion based formulation of 6-DoF coupled translational and rotational (attitude) motion planning; iii) Description of conditionally triggered constraints (CTCs). We will also introduce some real-world applications relevant to this project, which will also serve as our prototype examples through the project, such as 6 DoF motion planning for aerial drones and reusable rockets.

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# Interval Reachability Analysis

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## Abstract

This talk will present computationally efficient reachability methods that bound trajectories of dynamical systems with multi-dimensional intervals. Bounding trajectories is important for evaluating the robustness of systems in the face of parametric uncertainty and for verification or control synthesis problems with respect to safety (ensuring that an unsafe set is never reached) or reachability properties (ensuring that a target set is reached). The methods make use of interval analysis, nonlinear systems tools, and a sampling-based, data-driven approach. The talk will also introduce the open-source software, Toolbox for Interval Reachability Analysis (TIRA), that implements some of these methods, as well as a parallelized implementation that takes advantage of high performance computing platforms. Several physically motivated examples will illustrate the scalability of the methods to high dimensional systems.

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# Progress on a perimeter surveillance problem

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## Abstract

In 2008, Kingston, Beard, and Holt [2] described an algorithm for coordinating  $n$  drones surveilling an interval on the real number line, all moving at the same uniform speed, and they considered the problem of bounding the time to synchronization. They divided the algorithm’s behavior into two phases before synchronization, which we will call *phase 1* and *phase 2*. Normalizing units so that a single drone can traverse the interval in one unit of time, they claimed an upper bound of 3 units of time for phase 1 and an upper bound of 2 units of time for phase 2.

In 2019, Davis, Humphrey, and Kingston [1] pointed out that the previous work did not justify the claimed characterizations of the worst-case behavior, and, in fact, they provided a counterexample to the bound for phase 1. As a result, there is currently no rigorously established bound on the time to synchronization that is independent of the number of drones.

We provide partial progress towards this result, including:

- sharp bounds for phase 2, showing that the originally claimed worst-case behavior was indeed correct
- improved lower bounds
- partial results towards bounding the length of phase 1

This is joint work with Floris van Doorn, Department of Mathematics, University of Pittsburgh.

## References

- [1] Jennifer A. Davis, Laura R. Humphrey, and Derek B. Kingston. When human intuition fails: Using formal methods to find an error in the “proof” of a multi-agent protocol. In Isil Dillig and Serdar Tasiran, editors, *Computer Aided Verification (CAV) 2019*, pages 366–375. Springer, 2019.
- [2] Derek B. Kingston, Randal W. Beard, and Ryan S. Holt. Decentralized perimeter surveillance using a team of UAVs. *IEEE Trans. Robotics*, 24(6):1394–1404, 2008.

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# Direct Adaptive Control of Nonlinear Systems with Uncertain Unstable Zero Dynamics\*

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## **Abstract**

Unstable zero dynamics present a serious challenge to nonlinear control in general and adaptive control in particular. This project focuses on the extension of retrospective cost adaptive control (RCAC) to nonlinear systems with unstable zero dynamics. For linear systems, RCAC is known to cancel unmodeled or poorly modeled nonminimum-phase (NMP) zeros. In the case of known NMP zeros, we will describe progress on almost global convergence to a controller that provides internal stability. Extensions to nonlinear systems will be discussed, as well as preliminary results on integrated identification and adaptive control, where the adaptive controller provides sufficient persistency to estimate the crucial modeling information required by RCAC.

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# Graphon Mean Field Games and the GMFG Equations: Equilibrium Theory for Large Populations of Non-Cooperative Agents on Large Scale Networks

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## Abstract

Very large scale (finite) networks (VLSNs) linking large populations of dynamical agents occur in many settings at various scales; three examples are provided by electrical power grids, social media networks and epidemic transmission networks. VLSNs typically present problems of intractable complexity, however the emergence of the theory of graphons for the analysis of the limits of very large scale networks has enabled the development of Graphon Mean Field Game (GMFG) theory. GMFG theory generalizes the standard MFG theory of large populations of non-cooperative agents on completely connected uniform networks to populations distributed over VLSNs. In particular, in work with Minyi Huang, GMFG theory has yielded (i) the existence and uniqueness of Nash equilibria for infinite populations distributed over infinite networks, and (ii) epsilon-Nash equilibria for finite populations of dynamical systems distributed over VLSNs when subject to GMFG strategies. The theory and its methodology are currently being developed in several directions including applications to various classes of systems and networks.

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# Observability of Ensemble Systems with Unknown Population Density

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## Abstract

The talk addresses observability of continuum ensembles of control systems, and takes the ensemble of Bloch equations as a prototype. We assume that both initial states and the population density of the individual systems are unknown. Meanwhile, there is only one finite-dimensional measurement output available at our disposal. The output integrates a certain observation function common to all individual systems. The problem we address is whether one can use the output to estimate the initial states and, moreover, identify the population density. We show that the answer is affirmative if the observation function is a harmonic homogeneous polynomial of positive degree. We demonstrate the use of Lie algebra representation for tackling the problem.

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# Mixed Monotonicity for Reachability and Safety in Dynamical Systems

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## Abstract

A dynamical system is mixed monotone if its vector field or update-map is decomposable into an increasing (cooperative) component and a decreasing (competitive) component. In this talk, we present some recent results on using mixed monotonicity for efficient reachable set computation in both continuous-time and discrete-time systems. Mixed monotonicity is defined with respect to a decomposition function, and this decomposition function allows for constructing an embedding system that lifts the dynamics to another dynamical system with twice as many states but where the dynamics are monotone with respect to a particular southeast order. This enables applying the powerful theory of monotone systems to the embedding system in order to conclude properties of the original system. In particular, a single trajectory of the embedding system provides hyperrectangular over-approximations of reachable sets for the original dynamics. In this way, mixed monotonicity enables efficient reachable set approximation for applications such as optimization-based control and abstraction-based formal methods in control systems. In this talk, we present some recent foundational theory for mixed monotone systems and also demonstrate how mixed monotonicity can be combined with control barrier functions to enable an efficient online filtering scheme for ensuring safety at run-time for a controlled dynamical system.

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# A Switched Systems Approach for Navigation and Control with Intermittent Feedback

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## Abstract

Current visual servo control and image-based state estimation algorithms assume that image feedback is continuously available from a single source. To generalize the persistent staring assumption of adaptive image-based estimators, the ability to arbitrarily switch between different uncertain and nonlinear image dynamics is required. Motivated by this need, switched systems methods were explored as a means to ensure convergence of image-based observers to determine the relative trajectory between a camera and intermittently viewed image features. Such switched systems methods provided insights to develop controllers/observers for a more general class of intermittent feedback problems. Efforts also focused on new advancements in switched systems methods that yield less conservative theorems for differential inclusions.

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# Explicitly Solvable Control Problems with Non-Gaussian Noise

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## Abstract

A linear-quadratic stochastic control problem is formulated and explicitly solved where the noise process is a Rosenblatt process. Rosenblatt processes are continuous, non-Gaussian processes with a long range dependence so they can be appropriate for modeling a variety of physical phenomena. Empirical evidence demonstrates that a Gaussian process is often not appropriate to model the noise in many physical control systems. An explicit expression is obtained for the optimal linear feedback control and the associated optimal quadratic cost.

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# V&V and Optimization using Nonlinear Dynamical Systems Theory in EP Devices

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## Abstract

Space is fundamentally irreproducible, and thus there is a challenge in testing electric propulsion (EP) devices in ground chambers, and extrapolating their behavior. In addition, these devices are stochastic, chaotic systems, which often develop emergent structures in phase space, even in nominally steady state operation, frequently exhibit limit cycle oscillations. Results from recent work in low-dimensional time-lagged embeddings of data from both numerical and experimental sources have proved to be extremely effective in representing nonlinear dynamics. However, the theoretical underpinnings for these efforts are not complete and significantly more robust techniques for global optimization of noisy response surfaces is necessary to further exploit nonlinear system dynamics for data assimilation. This work leverages ideas from Takens' theorem to develop techniques to study the behavior of these devices.

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# Control Systems and Differential Inclusions on Metric Spaces

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## Abstract

In some applied models (of flocking or of the crowd control) it is more natural to deal with elements of a metric space (as for instance a family of compacts in a vector space endowed with the Hausdorff metric, or probability measures over compact subsets endowed with the Wasserstein distance) rather than with vectors in a normed vector space. Because of the lack of the linear structure, the notion of derivative of a function has to be replaced by mutations. A number of results known for ODEs are valid in metric spaces and control systems can be extended to this framework. In this talk I will discuss optimal control of the so called morphological control system whose trajectories are time dependent tubes of subsets of an Euclidean space and show how the theory of Hamilton-Jacobi-Bellman inequalities can be extended to this framework.

In the second part of the talk a novel definition of solutions to differential inclusions involving a multifunction (of dynamics) in the Wasserstein (metric) space of probability measures will be introduced. These solutions are absolutely continuous curves of probability measures whose driving velocity fields are measurable selections of multifunction. In this general setting, founding results of the theory of differential inclusions like Filippov estimates, the Relaxation theorem, and the compactness of the solution set are valid. This leads to new existence result for fully non-linear mean-field optimal control problems with closed-loop (or open-loop) controls. Finally the first order necessary optimality in the form of Pontryagin principle will be presented.

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# Large-scale pursuit-evasion differential games

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## Abstract

Differential game theory provides the right framework to analyze pursuit-evasion problems and, as a corollary, combat games. Pursuit-evasion scenarios involving multiple agents are important but challenging problems in aerospace, control, and robotics. Most pursuit-evasion differential games consider a small number of players due to the curse of dimensionality. We consider an  $N$ -pursuer vs.  $M$ -evader team conflict. This work extends classical differential game theory to simultaneously address weapon assignments and multi-player pursuit-evasion scenarios. Saddle-point strategies that provide guaranteed performance for each team regardless of the actual strategies implemented by the opponent are devised. The players' optimal strategies require the co-design of cooperative optimal assignments and optimal guidance laws. A representative measure of performance is proposed and the Value function of the game is obtained. It is shown that the Value function is continuous, continuously differentiable, and that it satisfies the Hamilton-Jacobi-Isaacs equation – the curse of dimensionality is overcome and the optimal strategies are obtained. The cases of  $N = M$  and  $N > M$  are considered. In the latter case, cooperative guidance strategies are also developed in order for the pursuers to exploit their numerical advantage. This work provides a foundation to formally analyze hybrid and high-dimensional conflicts between teams of  $N$  pursuers and  $M$  evaders by means of differential game theory.

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# Condensed Matter Physics and Hybrid Thermodynamic Control Protocols for Networked Systems with Intermittent Information

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## Abstract

Network systems involve distributed decision making for coordination of networks of dynamic agents and address a broad area of applications in science and engineering. In this work, we develop a thermodynamically-based framework for addressing consensus problems using a hybrid control protocol architecture. The proposed hybrid controller architecture involves the exchange of intermittent state information between agents guaranteeing that the closed-loop dynamical network is semistable to an equipartitioned equilibrium representing a state of information consensus consistent with basic thermodynamic principles. More specifically, the hybrid control protocol architecture involves a dynamic communication topology wherein communication events are triggered via state-dependent resettings inspired by discontinuous thermodynamic phase transitions and critical phenomena from condensed matter physics.

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# Distributed Asynchronous Constrained Optimization in Blocks

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## Abstract

Distributed optimization algorithms have seen widespread development and use in multi-agent systems, and a large segment of the literature is devoted to averaging-based algorithms that interleave optimization and consensus steps. However, such algorithms do not readily apply to problems in multi-agent autonomy, where each agent can only meaningfully update decision variables corresponding to its own states. To remedy this need, this work develops a distributed constrained optimization framework in which decision variables are partitioned into “blocks,” each of which is updated by only a single agent. We use a first-order primal-dual approach, which accommodates parallelization and provides some inherent robustness to asynchrony. Asynchrony in our framework is permitted in both computations and communications, and all delays can be arbitrarily long (but must be finite). We derive convergence rates to a Lagrangian saddle point that depend on the number of operations agents have executed, without specifying when they must occur. These rates are used to show that, contrary to the unconstrained case, it is not always beneficial for agents to execute as many operations as possible in constrained problems. This talk will also show preliminary extensions of block asynchronous optimization to non-convex problems.

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# Variational principles for fluid dynamics on rough paths

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## Abstract

In recent works, beginning with Holm [PRSA-2015], several stochastic geophysical fluid dynamics (SGFD) models have been derived from variational principles. Such models are obtained using the tools of stochastic calculus to construct semi-martingale driven variational principles. In this report, we introduce a new framework for parametrization schemes in GFD. Using the theory of controlled rough paths, we derive a class of rough GFD models as critical points of rough action functionals. These rough GFD models characterize Lagrangian trajectories in fluid dynamics as geometric rough paths (GRP) on the manifold of diffeomorphic maps. The GRP framework preserves the geometric structure of fluid dynamics obtained by using Lie group reduction to pass from Lagrangian to Eulerian variational principles, thereby yielding a rough formulation of the Kelvin circulation theorem. The rough formulation enhances its stochastic counterpart. For example, the rough-path variational approach includes non-Markovian perturbations of the Lagrangian fluid trajectories. In particular, memory effects can be introduced in this formulation through a judicious choice of the rough path (e.g. a realization of a fractional Brownian motion). Preprint at <https://arxiv.org/abs/2004.07829>.

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# Cooperation and Autonomy in Spatially Constrained Environments

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## Abstract

This presentation will give an overview of a cooperative control framework for multiple autonomous systems operating under time-critical constraints in a spatially constrained environment. Different type of coordination constraints are explored, and consequently new coordination laws are derived to accommodate these constraints in respective missions of UAVs. New directions will be presented for exploring contraction theory in combination with L1 adaptive control theory for enabling more versatile coordination strategies in confined spaces with tight constraints. Extensions to swarms as one potential opportunity will be proposed for future.

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# Moment-Based Ensemble Control using Aggregated Measurements

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## Abstract

Recent years have witnessed a wave of research activities in systems science and engineering toward the control of population systems. The driving force behind this shift was geared by numerous emerging and ever-changing technologies from neuroscience, biology, and quantum physics to robotics, where many control-enabled applications involve manipulating a large ensemble of structurally identical dynamic units or agents. Owing to the large scale and complexity of ensemble systems, placing sensors or installing the infrastructure to acquire measurements of individual agents in the population is impractical at best and often impossible. Although measurements from individual agents may be unavailable, the ensemble system can be coarsely monitored at a population level, e.g., through aggregated measurements such as partial snapshots or low resolution images. In this talk, we will introduce a moment-based framework that utilizes the idea of statistical moments induced by aggregated measurements to synthesize a novel approach to controlling an ensemble system through its associated moment system. In particular, we present the connection between the ensemble control problem and the classical moment problem, by which the control-theoretic analysis, e.g., controllability, can be carried over through the study of the moment system. Moreover, we will illustrate how the proposed moment-based method enables a systematic design strategy to close the loop and design feedback laws for ensemble control systems.

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# Al'brekht's Method in Infinite Dimensions

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## Abstract

In 1961 E. G. Albrekht presented a method for the optimal stabilization of smooth, nonlinear, finite dimensional, continuous time control systems. This method has been extended to similar systems in discrete time and to some stochastic systems in continuous and discrete time. In this paper we extend Albrekht's method to the optimal stabilization of some smooth, nonlinear, infinite dimensional, continuous time control systems whose nonlinearities are described by Fredholm integral operators.

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# Prescribed-Time Control and Estimation of PDEs

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## Abstract

In missile guidance the interception of a moving target occurs in finite time. A stronger stabilization concept is pursued in this project where finite-time stabilization occurs in a time interval that is independent of the initial condition. We call it  $\bar{\text{O}}$ prescribed-time stabilization.  $\bar{\text{O}}$  Such prescribed-time stabilization is a simultaneous answer to the mathematical question of null-controllability and to the engineering question of fixed-time stabilization. In this annual review, we review our most recent results on prescribed-time (PT) stabilization of PDEs, which include (1) PT stabilization of parabolic PDE-ODE cascades, (2) PT stabilization of the Stefan system for applications in rapid 3D printing, (3) PT trajectory tracking for coupled systems of reaction-diffusion PDEs, (4) adaptive output-feedback PT stabilization of uncertain strict-feedback nonlinear ODEs, (5) PT stabilization of stochastic nonlinear systems, and (6) fixed-time extremum seeking. In all these results, not only is the state guaranteed to converge to zero but also all the derivatives of the state, ensuring a  $\bar{\text{O}}$ rapid yet gentle $\bar{\text{O}}$  attainment of the control objective. Among many applications for algorithms with such qualities would be satellite proximity operations, including docking.

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# Bayesian Attitude Estimation with Matrix Fisher–Gaussian Distribution

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## Abstract

Since the first implementation to the space precision attitude reference system in 1969, the multiplicative extended Kalman filter (MEKF) has been the workhorse of attitude estimation. Despite numerous successful applications, the performance of MEKF is inherently limited by the fact that the topological structure of the special orthogonal group is not well respected in its design. For instance, it may suffer from singularities in attitude representations or covariance matrices, and it cannot represent a completely unknown initial attitude, thereby limiting its performance especially for large uncertainties. To address these, we present a Bayesian attitude estimation scheme defined globally on the special orthogonal group. This is achieved by formulating a new probability density function, referred to as the *matrix Fisher–Gaussian distribution*, on the product of the special orthogonal group and the Euclidean space, which can represent both of uncertainties in attitude and the associated angular–linear correlations in a global fashion. Utilizing this, a Bayesian attitude estimator and an unscented filter are constructed to estimate the attitude and a gyro bias concurrently. Through an extensive benchmark study, it is illustrated that the proposed approach exhibits similar properties as MEKF under nominal cases, but it outperforms MEKF noticeably for challenging scenarios with large initial errors and uncertainties. Further, we show that the complete attitude can be estimated by single direction measurements, as opposed to the common belief.

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# Scalable Reinforcement Learning of Localized Policies for Multi

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## Abstract

We study reinforcement learning (RL) in a setting with a network of agents whose states and actions interact in a local manner where the objective is to find localized policies such that the (discounted) global reward is maximized. A fundamental challenge in this setting is that the state-action space size scales exponentially in the number of agents, rendering the problem intractable for large networks. In this paper, we propose a Scalable Actor-Critic (SAC) framework that exploits the network structure and finds a localized policy that is a  $O(\rho\kappa)$ -approximation of a stationary point of the objective for some  $\rho \in (0, 1)$ , with complexity that scales with the local state-action space size of the largest  $\kappa$ -hop neighborhood of the network.

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# Multi-scale analysis of deployment and coverage control problems

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## Abstract

As the size of networked systems and swarms dramatically increases, new challenges arise from algorithm performance evaluation and design, and agent-level algorithm implementation. This calls for a consistent, multi-scale approach that can bridge the gap between small and large agent-set cases. With this in mind, this talk presents a new class of proximal descent schemes for the transport of probability measures in the L2-Wasserstein space, which we project onto the underlying Euclidean space to obtain the corresponding transport scheme for the agents. This algorithms can be directly related to previous coverage control algorithms based on Voronoi partitions, and We provide two new insights regarding this: (i) our results establish that in the  $N$  infinity limit, previously considered gradient descent-based transport schemes achieve convergence to global minimizers of typical aggregate objective functions —even if the convergence is only local for any finite  $N$ , and (ii) the most basic distortion performance metric of coverage control does not result in the desired performance in the  $N$  infinity limit.

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# Robust stability of Population Games: new tools, methods and concepts

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## Abstract

Our work focuses on a system-theoretic methodology to analyze the non-cooperative strategic interactions among the members of a large population of bounded rationality agents. We adopt a population-game framework that is pertinent for large-scale decentralized defense systems in which a large number of agents have the authority to select and repeatedly revise their own strategies, such as in autonomous transport swarms and resource allocation. A mechanism that is inherent to the problem at hand or is designed and implemented by a coordinator ascribes a payoff to each possible strategy. We seek to characterize the long-term emerging behavior that arises when the agents prioritize switching to strategies whose payoff is either higher than the current one or exceeds the population average. I will describe some of our recent results that provide, for the first time, methods to systematically obtain such a characterization when the agents consider whether to revise their strategies at different rates, such as when there are intermittent failures in the communication network used to convey information to the agents. I will also illustrate our results numerically and suggest how new tools, which were originally intended for use in computer science problems, can be used to obtain certificates of performance.

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# Conversion of Second-Order HJ PDE Problems into First-Order HJ PDE Problems

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## **Abstract**

A class of nonlinear, stochastic staticization control problems (including minimization problems with smooth, convex, coercive payoffs) driven by diffusion dynamics with constant diffusion coefficient is considered. The nonlinearities are addressed through stat duality, and the second-order Hamilton-Jacobi partial differential equations (HJ PDEs) are converted to first-order HJ PDEs in the dual variables. This has substantial numerical implications. Examples will be included.

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# Estimation and verification of hybrid systems with data-rate guarantees

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## Abstract

We study state estimation and verification problems for dynamical and hybrid systems, from the perspective of computational efficiency and data efficiency. We consider different levels of assumptions about our knowledge of the underlying system model and establish how this knowledge impact the efficiencies in analysis and estimation. First, we show that knowledge of symmetries can enable scalable, and possibly unbounded, verification of multi-agent systems. We develop a new notion of symmetry abstractions for hybrid systems and establish its soundness using a forward simulation relation (FSR). We illustrate an application of this abstraction in making bounded safety verification faster and enabling unbounded verification for linear as well as nonlinear models. Second, we study the state estimation problem for nonlinear dynamical systems where we have additional knowledge about its inputs in terms of maximum variations. We introduce a new notion of topological entropy, to lower bound the bit rate needed for state estimation. We motivate this new notion by showing that the existing notions of estimation entropy diverge for this class of systems. As the bound on the input decreases, we recover a previously known bound on estimation entropy. We present algorithms for state estimation and for computing this new entropy bound.

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# Distributed Control of Multi-Channel Linear Systems Across Networks with Transmission Delays

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## Abstract

It has recently been shown that any jointly controllable, jointly observable multi-channel linear system with a strongly connected neighbor {communication} graph can be exponentially stabilized with arbitrarily fast convergence using a time-invariant distributed linear control. This result was established using a distributed observer-based control architecture analogous to the familiar observer-based control architecture for a centralized controllable, observable, linear system. A key consequence of this finding is that the well-known fixed spectrum {set of fixed modes} of a multi-channel linear system is no longer an obstacle to the system's regulation provided distributed rather than decentralized control is used. In this talk it will be explained how to avoid this obstacle using distributed control without restricting the control architecture to be of the observer-based type. Often overlooked in the study of distributed control are the effects of communication network transmission delays. It will be explained why in the face of such delays, exponential stabilization at a prescribed convergence rate can still be achieved with distributed control, at least for discrete-time, multi-channel linear systems.

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# Tractable Algorithms to Design Nonlinear Systems using Lifting Operators

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## Abstract

This presentation considers optimal control design of a class of affine nonlinear systems whose right-hand sides are analytic functions. Building upon ideas from Carleman linearization that is a nonlinear procedure to transform (lift) a finite-dimensional nonlinear system into an infinite-dimensional linear system with no loss, we show how to lift a Hamilton-Jacobi-Bellman (HJB) equation into an infinite-dimensional quadratic form that resembles the familiar (algebraic) Riccati equation. Then, an efficient method is proposed to calculate solution of the resulting infinite-dimensional equation using one algebraic Riccati equation (of the same dimension as the original nonlinear system) and a series of linear matrix equations in an iterative manner. In every iteration, sub-block matrices of the actual solution to the HJB equation are obtained (with no loss). For a finite number of iterations, one can obtain arbitrarily near-optimal solution using finite truncations techniques. It is shown that the resulting solutions are symmetric. Using these approximate solutions to the HJB equation, we construct approximate optimal control laws. Our case study results assert that our method enjoys high accuracy in compared to the actual optimal feedback control laws and the accuracy increases as higher-order truncations are used.

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# Thermal Management for Aircraft

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## Abstract

The Air Force Scientific Advisory Board published a technical report detailing the substantial thermal management challenges in fielding key Air Force capabilities. Modern aircraft, that use fuel as a heat sink, are beginning to experience thermal issues that can cause the temperature of the fuel at some point in the system to exceed a limit. Exceeding a temperature limit can cause fuel coking, damage to subsystems, or even complete system failure. In some cases, thermal endurance, defined as the time that an aircraft can operate before the fuel temperature at some point in the system exceeds a limit, is sufficiently small to result in incomplete missions. Other more severe cases can arise which could result in complete vehicle loss. Therefore, extending the thermal endurance of aircraft has become a critical need. Many factors contribute to the increased thermal loads, for example, onboard sensors, mission systems, electromechanical actuators, electronic engine accessories, electronic attack systems, and directed energy systems all produce heat loads, while the use of composites for the aircraft structure reduce the ability to transfer thermal energy to the atmosphere, as compared to the high conductivity metals used in older airframes. Fuel onboard the aircraft stores thermal energy and the engine, through burning of the fuel, transfers some of the thermal energy to the atmosphere. This work utilized physics based models developed from first principles that describe the salient features of aircraft thermal behavior. These models, which take into account the transport delays that exist in a fuel system due to moving a fluid through a set of pipes, formed the basis for defining new architectures and control laws that increased the disposal rate of waste thermal energy from an aircraft, thus extending thermal endurance. The current work both simplifies and extends the control scheme to explicitly address operation when flight endurance is limited by thermal constraints. The control design is divided, using time scale separation, into an operation mode controller, tank level controller, and temperature controller.

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# Intelligent Control in Adversarial and Stochastic Environments Multi-Player Pursuit-Evasion Differential Games

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## Abstract

Multi-player games of pursuit-evasion type are addressed, and in particular swarm pursuit-evasion differential games in the Euclidean plane where an evader is engaged by a team of multiple pursuers. All the players have simple motion a la Isaacs/are holonomic. The pursuers might be endowed with finite capture sets, so attention is not confined to point capture. Furthermore, the pursuers, or some of the pursuers, might not be faster than the evader and might have the same speed as the evader. The state space region where under optimal pursuer play capture is guaranteed is delineated, thus solving the Game of Kind. The synthesis of the protagonists' optimal state feedback strategies is addressed, thus providing the solution of the Game of Degree.

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# Security-Aware Control of Systems with Varying Levels of Autonomy

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## Abstract

Increasing set of functionalities, network interoperability, and system design complexity have introduced easily exploitable security vulnerabilities in controllers for cyber-physical systems (CPS). Furthermore, the tight interaction between the information technology and physical world makes these systems vulnerable to attacks beyond the standard cyber-attacks, whereas relying exclusively on conventional security techniques may be unfeasible due to resource-constraints and long system lifetime. Consequently, there is a need to change the way we reason about security for CPS controllers, and start designing platform-aware attack-resilient control components and architectures capable of providing strong safety and performance guarantees even under attack. We present research challenges and our recent efforts in this domain, starting from cyber-physical security techniques that (a) capture effects of attacks on control performance, (b) introduce attack resilience into control & autonomy components and facilitate attack detection, and (c) enable mapping of the desired Quality-of-Control (QoC) under attack guarantees into real-time performance requirements on the underlying platforms in a way that supports design-time tradeoffs between the QoC under attack and security-related overhead. Finally, for systems with varying levels of autonomy and human interaction, we show how to exploit the human power of inductive reasoning and ability to provide context, in order to improve the overall security guarantees.

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# Synthesis and Analysis of Multi-Agent Systems under Spatial and Temporal Specifications

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## Abstract

In the third year of the project we continued the development of theoretical and algorithmic results on the control synthesis and analysis under spatiotemporal (state and time) constraints. In this talk, we will first summarize our results on the control synthesis for nonlinear control-affine systems under spatiotemporal specifications; our approach utilizes Quadratic Programs (QPs) that encode safety and time specifications via the novel class of Fixed-Time Control Lyapunov Functions and Control Barrier Functions (FxT-CLF-CBF). We analyze the feasibility of the corresponding QPs and provide relaxations that guarantee safety at all times and fixed-time convergence under certain conditions. We will also discuss the development of fixed-time gradient flows and their application to a class of convex optimization problems. Lastly, we will present our ongoing work on extending the FxT-CLF-CBF synthesis to multi-agent problems under spatiotemporal constraints.

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# New Results on Risk Sensitive Control for Quantum Linear Systems

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## Abstract

In considering the risk sensitive control problem for quantum linear systems, we first develop a finite-horizon expansion for the system variables using the eigenbasis of their two-point commutator kernel with noncommuting position-momentum pairs as coefficients. This quantum Karhunen-Loeve expansion is used in order to obtain a Girsanov type representation for quadratic-exponential functions (QEFs) of the system variables. Such QEFs are used as risk-sensitive performance criteria for linear quantum stochastic systems driven by multichannel bosonic fields. Using the quantum Karhunen-Loeve expansion, we can then compute the asymptotic infinite-horizon growth rate for the cost corresponding to a stable linear quantum system driven by vacuum input fields. The resulting frequency-domain formulas express the QEF growth rate in terms of two spectral functions associated with the real and imaginary parts of the quantum covariance kernel of the system variables. This result then enables us to consider a numerical solution to a risk-sensitive optimal control problem for a feedback connection of a linear quantum plant with a measurement-based linear classical controller. The control objective is to stabilize the closed-loop system and minimize the infinite-horizon asymptotic growth rate of a quadratic-exponential functional which penalizes the plant variables and the controller output. Variational techniques are used to establish first-order necessary conditions of optimality for the state-space matrices of the controller.

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# Data Driven Systems and Control Framework for Multiway Dynamical Systems

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## Abstract

In biological and engineering systems, structure, function and dynamics are highly coupled. Such multiway interactions can be naturally and compactly captured via tensor-based representations. Exploiting recent advances in tensor algebraic methods, we develop novel theoretical and computational approaches which exploit higher-order correlations and hidden patterns/redundancies for data-driven model learning, analysis, and control of such tensor-based representations. In one line of work, we extend classical linear time-invariant (LTI) system notions including stability, reachability and observability to multilinear time-invariant (MLTI) systems, in which the state, inputs and outputs are preserved as tensors, and express their criteria in terms of more standard concepts of tensor ranks/decompositions. Furthermore, we develop higher-order generalization of balanced proper orthogonal decomposition (BPOD) and its variants for model reduction and identification of MLTI systems. Using tensor train decomposition (TTD), we show that compared to traditional matrix-based BPOD, the higher-order BPOD is computationally more efficient and provides more compact representation of tensor time series data.

In another line of work, we develop the notion of entropy for uniform hypergraphs which can capture higher-order interactions between entities compared to classical graphs. We employ the probability distribution of the generalized singular values, calculated from the higher-order singular value decomposition (HOSVD) of the Laplacian tensors, to fit into the Shannon entropy formula. We show that this tensor entropy is an extension of von Neumann entropy for graphs. In addition, we establish results on the lower and upper bounds of the entropy and demonstrate that it can be used as a measure of regularity and robustness for uniform hypergraphs. Moreover, we employ uniform hypergraphs for studying controllability of high-dimensional networked systems. We propose another tensor-based multilinear system representation to characterize the multidimensional state dynamics of uniform hypergraphs, and derive a Kalman-rank-like condition to identify the minimum number of driver vertices in order to achieve full control of the whole hypergraph.

We demonstrate these new tensor-based theoretical/computational developments on a variety of biological and engineering simulated and experimental datasets including dynamic measurements of human cancer cells using live cell microscopy, human fibroblast reprogramming in form of gene expression and genome structure and mouse brain neuron endomicroscopy.

# 2D3D Vision-Based Control

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## Abstract

While significant work in autonomy and robotics have been made, most recently with advances in machine learning, there still exists no universal framework capable of handling complex real-world scenarios due to “unknown unknown.” From a vision-based modeling perspective, this is often due to lack of image integrity (e.g., features) and rapid onset of highly nonlinear model dynamics. To combat this, we will review our geometric interactive control 2D3D feedback approach based on single or multiple 2D image observations. In doing so, we show that a particular form of curvature (a measure of flatness in geometry) is positively correlated to convergence rates of stability. We will relate how this result fits into a broader thematic result whereby another form of curvature, namely Ricci curvature, can be subsequently utilized to exploit functional robustness and further motivating the open problem of characterizing image integrity. Time permitting, we will generalize such concepts to form a natural predictive signal for stochastic systems. The second half of this talk will focus on recent developments that abandoned “vision” altogether by focusing on radar-based reconstruction. In particular, remote sensing radar techniques provide highly detailed imaging yet do not embed explicit shape information for which classic vision algorithms are employed for reconstruction. As such, we present a feasibility study for geometric shape construction directly from raw data for which inversion has seldom been attempted in radar due to high frequency signals (e.g., tightly packed narrow local minima). From a data fusion perspective, we will detail how this can potentially enable to fuse information at the sensor location in a unified 2D3D modeling framework to provide increased information integrity. We close with future mathematical pursuits as it relates to geometry and control to the problems at hand.

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# Systematic Tools to Satisfy Temporal Logic Specifications in Hybrid Dynamical Systems

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## Abstract

Air Force combat systems require autonomous systems that are agile, reliable, and resilient. The unavoidable combination of the physics, the networks, and the algorithms in such systems leads to hybrid system models, which makes the systematic design of algorithms very challenging. Moreover, the unavoidable presence of obstacles, adversarial elements, and uncertainty in the environment further adds to the complexity and requires design tools that neither discretize the state or the dynamics while guaranteeing given high-level specifications, robustly. While discretization may aid computations in some cases, discretization of the hybrid system models emerging in Air Force missions may not permit to assure that the desired specifications are actually satisfied. In this talk, I will introduce and highlight properties of reachability maps and minimal-time functions used in the certification of temporal logic specifications in hybrid dynamical systems. We introduce reachability maps for such systems. A reachability map is given by a map from the initial set to the possible values that solutions can reach. We reveal properties guaranteeing that reachability maps are bounded, outer semicontinuous, and continuous. We also introduce minimal-time functions with respect to a set. A minimal-time function evaluated at a point is the smallest time that a solution starting from the given point reaches the set. We introduce infinitesimal necessary and sufficient conditions for the minimal-time function to be locally Lipschitz. We show that Lipschitz continuity of the minimal-time function with respect to the boundary of the set where the solutions are defined plays a crucial role on the Lipschitz continuity of the reachable set. Using these results, we show that time-varying barrier functions that depend on the proposed reachability maps and minimal-time functions inherit their regularity properties.

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# Pixel array method using the Catlab framework

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## Abstract

Applied category theory is a growing field, with applications to quantum physics, natural language processing, computer science, and dynamical systems. Its generality and ability to abstract common patterns across all these fields suggests the possibility of implementing general-purpose category-theoretic software that could provide a unified syntax and reusable modules for a large variety of applications, from tensor networks to epidemiological models. A relatively recent project originating in the Statistics Department at Stanford University, called Catlab.jl, has been growing in popularity because of its speed and ease of use, and is being used in several DoD projects. I will discuss recent work in collaboration with the lead developer, including applications to the pixel array method for finding steady states of nonlinear dynamical systems.

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# Perception-based Motion Planning: Preliminary Results

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## Abstract

This project is motivated by human decision-makers' capability of "simplicity-oriented" path planning (e.g., simple and long paths are sometimes preferred to short and complex paths), which often contributes to the reliability and the robustness of human decisions. Due to the lack of a standardized mathematical metric for "simplicity," it is challenging for robots to reproduce human experts' capability of discovering "simple" solutions. Consequently, the task of finding the simplest path is often left for human operators, which significantly restricts the capability of robots in the current practice. To overcome this challenge, we develop the framework of Information-Geometric Path Planning (IGPP), which has a built-in capability of penalizing both path length and path complexity. This can be achieved by introducing a novel information-geometric distance function in the configuration space. The IGPP concept is closely related to the notion of rational inattention introduced by C. Sims (2003). The proposed research project involves the development of efficient algorithms to find the shortest path with respect to the new distance function. In this presentation, we summarize preliminary results we have obtained so far in collaboration with Lockheed Martin Corporation.

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# Analyzing the effect of persistent, abrupt asset changes on the performance of novel online optimization algorithms

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## Abstract

We consider online optimization problems where the assets available to a planner, or algorithm, can change abruptly and persistently. Such situations include formation optimization in UAV ad hoc networks, as well as similar multi-agent systems that relay or gather data, wherein each agent can operate only for a limited time due to limited capacity for power or data. We limit the asset switching frequency through an average dwell-time switching constraint. We characterize an asymptotically stable set for the system when the average dwell-time parameter is zero. Then we indicate how, as this dwell-time parameter grows, the “steady-state” behavior of the optimizing system belongs to a larger and larger neighborhood of the ideal, asymptotically stable set. For these results, we appeal to results pertaining to switched differential inclusions. Since one of our recent, high-performance optimization algorithms corresponds to a hybrid system (presented at last year’s AFOSR meeting) rather than a differential inclusion, we approximate that hybrid algorithm by a continuous-time system. As an independent result, we find that this modified algorithm performs exceedingly well, at times better than the hybrid version. The overall theory and algorithm will be illustrated through an example.

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# Null controllability properties for multi-D fractional heat equations

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## Abstract

In the context of PDEs involving the fractional Laplace operator the number of observability/controllability results is limited. They are essentially as follows:

- **1-D problems:** In fact, 1-D problems can be handled by using Fourier series techniques as well as Ingham type inequalities.
- **Multi-D problems:** The interior null controllability of multi-D fractional Schrödinger equations has been established. This the only model that we can deal with in multi-D.

In this program review, we analyze the controllability properties of the non-local multi-dimensional heat equation  $y_t + (-\Delta)^s y = f\chi_\omega$  in  $\Omega \times (0, T)$  involving the fractional Laplace operator  $(-\Delta)^s$  ( $0 < s < 1$ ). We prove that finite energy solutions are null-controllable in any time  $T > 0$ ,  $\omega \subset \Omega$  a neighborhood of the boundary  $\partial\Omega$ , and  $\frac{1}{2} < s < 1$ . The proof of our result employs several classical techniques in mathematical control theory adapted to the non-local setting. We start by considering the fractional wave equation  $y_{tt} + (-\Delta)^s y = f\chi_\omega$  in  $\Omega \times (0, T)$  and establish a partial null controllability result for  $\frac{1}{2} < s < 1$  of solutions involving low frequency eigenmodes, namely, solutions belonging to the finite-dimensional space generated by the first  $J \in \mathbb{N}$  eigenfunctions of the fractional Laplacian. Moreover, we show that controllability is lost for finite energy solutions, that is, when  $J \rightarrow +\infty$ . These results follow from the Pohozaev identity for the fractional Laplacian which we use to obtain the partial observability from a neighborhood of the boundary  $\partial\Omega$  in a time horizon  $T$  that grows as the number of involved frequencies increases. In a second moment, we apply transmutation techniques to transfer the partial observability of the fractional wave equation into the parabolic setting, thus obtaining a frequency-dependent null controllability result for the fractional heat equation, again for  $\frac{1}{2} < s < 1$ . Finally, we employ a novel idea to deal with the high-frequency components of the fractional heat equation and recover a full null controllability result for finite energy solutions for all  $\frac{1}{2} < s < 1$ . This is the first multi-D controllability result available for the fractional heat and wave equations.

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# Controlled systems involving McKean-Vlasov equations and applications

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## Abstract

Why do we need to treat controlled systems involving McKean-Vlasov equations? We provide several examples to answer the question. One of them is a controlled switching diffusion with mean-field terms, in which we review our progress on solving a fundamental issue. The second example is a class of non-traditional linear quadratic Gaussian (LQG) problems, in which the terminal cost is quadratic not with respect to the terminal state but with respect to the measure associated with the terminal state. This problem will be used as a benchmark example for solving high-dimensional HJB problems in the future. We illustrate our recent work on developing numerical methods for filtering methods. In addition, we briefly summarize our recent work on distributed optimization.

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