Rough Euler-Poincaré equations and their calibration

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Abstract

We recall our framework for variational principles for fluid dynamics on rough paths, in which advection is constrained to be the sum of a smooth and geometric rough-in-time vector field. The corresponding rough Euler-Poincare equations satisfy a Kelvin circulation theorem. These equations conserve circulation for incompressible fluids and, otherwise, nontrivial circulation dynamics is generated by gradients of advected quantities arising from broken relabelling symmetry of inhomogeneous initial conditions. By parameterizing the fine-scale fluid motion with a rough vector field, we establish a flexible framework for parsimonious non-Markovian parameterizations of subgrid-scale dynamics.

We will discuss progress toward data-based calibration of these models, including a new result on the estimation of the noise coefficient of fractional Brownian motion-driven equations using a type of scaled quadratic variation.

This talk is based on joint work with Dan Crisan and Darryl Holm at Imperial College London, UK, and Torstein Nilssen at the University of Agder, Norway.

*Supported by the Air Force Office of Scientific Research under the Award No: FA8655-21-1-7034,
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Defense Differential Games and Leader-Follower Ring Formation

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Abstract

The first problem of this presentation considers the defense of a static but rotating turret from a cooperative attack by two adversaries. The interesting case where one attacker, the penetrator, is able to reach the turret thanks to the cooperative strategy implemented by its teammate, the runner, is analyzed. The runner seeks to engage and distract the turret away from the penetrator. The attackers cooperate in order to maximize the runner-turret-penetrator angle at the time instant where the runner is neutralized by the turret. This strategy optimizes the penetrator performance of reaching the turret and accomplishing the mission. The second problem considers the leader-follower ring formation scenario. Algorithms are developed where the desired position of the follower in the formation is a set of points. The leader is free to move in 3-Dimensional space while the follower seeks to converge to any position on a subspace relative to the leader, here this subspace is a ring radius. By adding the approximated Hessian into the potential field formulation the follower's performance in the ring formation is improved. The gradient descent is damped and allows for the follower to converge to the desired ring without oscillations.

*Partially supported by the Air Force Office of Scientific Research under the Award No. 21RQ-COR084, e-mail: eloy.garcia.2@us.af.mil

Many-on-One Pursuit-Evasion

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Abstract

We first show that optimal control problems are well posed when the terminal manifold is of co-dimension 1. In the context of pursuit-evasion differential games, the proper treatment of “point capture" requires the consideration of a terminal manifold which is a sphere of radius $0 < l << 1$ centered at the origin and point capture means letting $l \rightarrow 0$. This renders the optimal control problem well posed and is in tune with the engineering practice of using a finite tolerance. The mathematically correct stipulation of a terminal manifold of co-dimension 1 brings out critical features of the optimal control problem which are hidden/obscured when “point capture" is considered. Furthermore, the criticality of first identifying the Usable Part (UP) of the terminal manifold is emphasized. From a theoretical point of view, by solving the attendant HJI hyperbolic PDE using the method of characteristics, as opposed to employing the Pontryagin maximum principle, one avoids the need to solve a TPBVP. The agents' optimal controls are globally optimal and are obtained in state feedback form – open loop optimal control makes no sense in a differential game setting. And eschewing point capture allows us to address realistic pursuit-evasion differential games where the pursuers are endowed with effectors whose range l is specified. Thus, attention is given to the operational relevant scenario of multi-player pursuit-evasion differential games where the two pursuers are endowed with circular capture disks with radius l. The off the beaten path scenario where the holonomic players have equal speed is addressed, and also the case is discussed where the evader is faster than the pursuers – in this case, point capture is a no-no and the pursuers must be endowed with capture disks of radius $l > 0$. The state space region where capture is guaranteed is delineated, that is, the solution of the Game of Kind is provided. The players' *optimal* state feedback strategies are synthesized, yielding the solution of the Game of Degree. “Optimal" $\equiv$ a game theoretic saddle point/Nash equilibrium is achieved.

*Partially supported by the Air Force Office of Scientific Research under Project Task: 3004/FY19, e-mail: `meir.Pachter@afit.edu`

Stochastic Control and Differential Games for Systems with non-Gaussian Noise

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Abstract

This presentation focuses on some important problems of stochastic control and stochastic differential games for systems driven by noise processes called Rosenblatt processes that are continuous and non-Gaussian with a useful stochastic calculus and that can be considered as a natural generalization of fractional Brownian motions. Some new problems in control and games for linear systems with Rosenblatt processes and having quadratic cost criteria are formulated and explicitly solved. Some related topics for stochastic analysis for Rosenblatt processes are also described.

*Partially supported by the Air Force Office of Scientific Research under the Award No: FA9550-12-1-0384, e-mail: duncan@ku.edu, bozenna@ku.edu

Safe-by-design planner-tracker synthesis with a hierarchy of system models

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Abstract

Autonomous vehicles and unmanned aerial vehicles must operate subject to complex safety and performance requirements in changing environments. Designing controllers that meet such requirements in real-time may be computationally intractable due to large system dimension and nonlinearities in a high-fidelity dynamical model of the system. This talk will present a safe-by-design trajectory planning and tracking framework for nonlinear dynamical systems using a hierarchy of system models. The planning layer uses a low-fidelity model to plan a feasible trajectory satisfying the planning constraints, and the tracking layer utilizes the high-fidelity model to design a controller that restricts the error states between the low- and high-fidelity models to a bounded set. The simplicity of the low-fidelity model enables the planning to be performed online and the tracking controller and error bound are derived offline. The error bound is accounted for by the planner to ensure safety for the combined planner-tracker system. To provide freedom in the choice of the low-fidelity model, we allow the tracking error to depend on both the states and inputs of the planner. This is achieved by accounting for the jumps in the error variable that are induced by jumps in the input between time steps. The talk will illustrate the results with examples, including a design for vehicle obstacle avoidance. It will also make a connection between the planner-tracker framework and the notion of abstractions.

*Partially supported by the Air Force Office of Scientific Research under the Award No: FA9550-21-1-0288, e-mail: `arcak@berkeley.edu`

Population Games: New Methods And Applications

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Abstract

Our work focuses on a system-theoretic methodology to analyze the non-cooperative strategic interactions among the members of a large population of bounded rationality agents. We adopt a population-game framework that is pertinent for large-scale decentralized defense systems in which a large number of agents have the authority to select and repeatedly revise their strategies, such as in autonomous transport swarms and resource allocation. A mechanism inherent to the problem at hand or designed and implemented by a coordinator ascribes a payoff to each possible strategy. We seek to characterize the long-term emerging behavior that arises when the agents prioritize switching to strategies whose payoff is higher than the current one or exceeds the population average.

In previous meetings, I discussed the work carried out during the first and second years of our project. The work sought to provide, for the first time, methods to systematically characterize the stability of population games when the agents' revision opportunity rates depend on the current strategy, such as when certain strategies cause intermittent failures in the communication network used to convey information to the agents.

In the upcoming PI meeting, I will outline our progress over the last year, building on our first and second years' results.

Given the limited time available to present the work, I will focus on one particular category of results in which we consider the case where each strategy entails multiple steps. As I will explain, alowing multi-step strategies is relevant for several applications. From a technical standpoint, our work is important because the multi-step strategies case violates the so-called "Poisson-clock revision assumption" and, hence, requires new models and methods.

I will also describe a new class problems in which we were able to couple population games and epidemic compatmental models to control the endemic equilibrium of an epidemic in which a population's agents choices affect the transmission rate. From a technical viewpoint, this work required new models and analysis tools that effectively fuse passivity methods with Lyapunov techniques used to establish the stability of epidemic compartmental models.

*Partially supported by the Air Force Office of Scientific Research under the Award No: FA9550-19-1-0315, e-mail: `nmartins@umd.edu`

Inverse Optimal Safe Control

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Abstract

CBF-QP safety filters are pointwise minimizers of the control effort at a given state vector, i.e., 'myopically optimal' at each time instant. But are they optimal over the entire infinite time horizon? (They are not.)

Furthermore,

- What does it even mean for a controlled dynamic system to be "optimally safe" as opposed to, conventionally "optimally stable"?
- When disturbances—deterministic and stochastic—have unknown upper bounds, how should safety be defined to allow a graceful degradation under disturbances?
- Can safety filters be designed to guarantee such weaker safety properties as well as the optimality of safety over the infinite time horizon?

I answer these questions for general systems affine in control and disturbances, illustrate the answers with examples, and, using the existing QP safety filters, as well as more general safety-ensuring feedbacks, show how one generates entire families of safety filters which are optimal over the infinite horizon though they are conservative (favoring safety over 'alertness') relative to the standard QP.

*Partially supported by the Air Force Office of Scientific Research under the Award No: FA9550-22-1-0265, e-mail: krstic@ucsd.edu

Reaching a Consensus with Reduced Information

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Abstract

In its simplest form the well known consensus problem for a networked family of autonomous agents is to devise a set of protocols or update rules, one for each agent, which can enable all of the agents to adjust or tune their "agreement variable" to the same value by utilizing real-time information obtained from their "neighbors" within the network. The aim of this paper is to study the problem of achieving a consensus in the face of limited or reduced information transfer between agents. By this it is meant that instead of each agent receiving an agreement variable or real-valued state vector from each of its neighbors, it receives a linear function of each state instead. The specific problem of interest is formulated and provably correct algorithms are developed for a number of special cases of the problem.

*Partially supported by the Air Force Office of Scientific Research under the Award No: FA9550-20-1-0037, e-mail: as.morse@yale.edu

Model-free Parameter Estimation

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Abstract

Complex rocket devices suffer from diagnostic inaccessibility and computationally intractable models. Since not all the physics is known, the disparate time scales make high fidelity models challenging. This motivates efforts to leverage observed dynamical systems behavior coupled with parameter estimation techniques, to learn usable models for these devices. Our work uses a machine learning model that accepts the current state and some parameters as arguments, to produce the forward state. This surrogate model is trained using time-delayed scalar measurements of the system while in different operating conditions (parameters). The architecture for this surrogate model was chosen to be a simple Multi-Layer Perceptron, with 3 hidden layers and 2000 nodes per layer. This approach is novel in that the time delay relieves the dependence on state variables, yet resulting in reasonable predictions of the forward state. The end result is access to a predictive surrogate model that is correct locally in time, thus allowing for parameters estimation through gradients of the surrogate model.

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Optimal Linear-Time Algorithms in Allocation Problems

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Abstract

Designing distributed algorithms for multi-agent problems is vital for many emerging application domains. Game-theoretic approaches have gained traction as a useful paradigm to design such algorithms. However, much of the emphasis of the game-theoretic approach is on the study of equilibrium that result from distributed algorithms and not on the transient behavior. Therefore, in this talk, we study the transient efficiency guarantees in the context of resource-allocation games, which are used to model a variety of engineering applications. Under a natural class of utility designs, we characterize the optimal efficiency guarantees that can occur in a best-response process in linear-time. Surprisingly, the resulting performance guarantees are relatively close to the optimal asymptotic performance guarantees. Furthermore, we characterize a trade-off that results when optimizing for both asymptotic and transient efficiency through various utility designs.

*Partially supported by the Air Force Office of Scientific Research under the Award No: FA9550-20-1-0054, e-mail: `jrmarden@ucsb.edu`

Achieving global objectives through local control with unknown system dynamics

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Abstract

Achieving objectives such as stability or optimizing performance criteria in multi-agent systems classically requires the designer to possess the knowledge of the agent dynamics and objectives and assumes that the designer can synthesize the controller centrally. In rapidly varying systems, such assumptions may be onerous. While reinforcement learning can relax these assumptions, that field is currently unable to provide desired guarantees for distributed control. We discuss some recent results where combining classical control properties such as dissipativity and the fundamental lemma with data driven algorithms can provide such guarantees.

*Partially supported by the Air Force Office of Scientific Research under the Award No: FA9550-21-1-0231, e-mail: `gupta869@purdue.edu`

Embedded Graphon Mean Field Games

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Abstract

Very large networks linking dynamical agents are now ubiquitous and the need to analyse, design and control them is evident. The emergence of the graphon theory of large networks and their infinite limits (where graphons may be informally viewed as the limits of adjacency matrices) has enabled the formulation of a theory of centralized control of dynamical systems distributed over asymptotically infinite networks [Gao and Caines, CDC 2017, 2018; IEEE TAC 2020; IEEE TCNS 2021]. The study of the decentralized control of such systems was initiated in [Caines and Huang, CDC 2018, 2019; SICON 2021], where Graphon Mean Field Games (GMFG) and the GMFG equations were formulated for the analysis of non-cooperative dynamical games on unbounded networks. In that work existence and uniqueness results were established for the GMFG equations together with an epsilon-Nash theory for GMFG systems, where the latter relates the infinite population equilibria on infinite networks to finite population equilibria on finite networks.

At any instant of time $t \in [0, T]$ and graph location $\alpha \in G$, a GMFG for the set of controlled dynamical systems S_α with individual performance functions $J(x_\alpha, u_\alpha, t)$ interlinked via a graphon $g(\alpha, \beta)$ will (subject to conditions) have a set of Nash equilibrium values $N(x_\alpha, \alpha, t)$.

In current work, critical nodes are defined for a GMFG on G as those where the Nash equilibrium values are maximal or minimal. In order to formalize this notion and apply the calculus, the concepts of vertexons, embedded graphs and their graphons have been introduced; these describe sets of graph vertices and their edges embedded in the unit m cube in R^m, together with their limit objects in the case of sequences of increasingly complex graphs. As a result, the analysis of critical nodes and other features involving differentiation with respect to node location becomes meaningful [Foguen-Tchuendom, Caines, Huang, IEEE CDC 2021]. A further feature of vertexon limits is that they provide a form of node population density.

*Partially supported by the Air Force Office of Scientific Research under Award No: FA9550-19-1-0138, e-mail: peterc@cim.mcgill.ca,peter.caines@mcgill.ca

Mutational Control Systems on Metric Spaces

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Abstract

Control systems on metric spaces do arise, for instance, when investigating dynamic behavior of subsets of a vector space or of curves of probability measures driven by controlled continuity equations. Dynamics of trajectories with values in a metric space can be described using the notion of transition. Mutational equations associated to transitions play the role of ODEs on metric spaces. The aim of this talk is to discuss viability, invariance and Filippov's type theorem in the mutational framework. They are useful to build the Hamilton-Jacobi theory on metric spaces. Namely, optimal control problems on metric spaces involve the notion of the value function that satisfies Hamilton-Jacobi inequalities. Once the final time condition is imposed, under some technical assumptions, solution to such inequalities is uniquely defined. In the last part of the talk I will describe some additional regularity properties of the value function of an optimal control problem with nonlocal dynamics stated on the Wasserstein space of Borel probability measures.

[1] Bonnet B. & Frankowska H. (2022) *Sensitivity analysis of the value function of mean-field optimal control problems and applications*, Journal de Mathématiques Pures et Appliquées, 157, 282-345

[2] Badreddine Z. & Frankowska H. (2022) *Solutions to Hamilton-Jacobi equation on a Wasserstein space*, Calculus of Variations and PDEs, 61:9

[3] Frankowska H. & Lorenz T. (to appear) *Filippov's theorem for mutational inclusions in a metric space*, Annali della Scuola Normale Superiore di Pisa, Classe di Scienze

[4] Badreddine Z. & Frankowska H. (in revision) *Viability and invariance of systems on metric spaces*

[5] Frankowska H. & Lorenz T. (submitted) *Invariance theorem for mutational inclusions in a metric space*

*Partially supported by the Air Force Office of Scientific Research under the Award No: FA9550-18-1-02542, e-mail: helene.frankowska@imj-prg.fr, helene.frankowska@gmail.com

Autonomous guidance and targeting by semi-algebraic methods: From local to global shooting

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Abstract

The goal of this project is to adapt local optimization methods to tackle certain classes of nonconvex optimization problems globally, meaning with guarantees on the optimality of the solutions. The requirements for this approach are that the problem be constrained to a compact domain and that the objective function satisfies some "nice" regularity conditions. Our approach consists of two main steps, first we approximate the objective function with an algebraic construction, a polynomial or rational function, then we compute all of the critical points of this approximant. To provide guarantees that none of the solutions were missed, we seek an effective Stone-Weierstrass theorem which would give us the necessary conditions in order to construct an accurate enough approximant of the original objective function. Using state of the art polynomial system solving methods implemented in the package `Msolve`, we are able to find all critical points of the approximant, which in turn correspond to critical points of the original problem. Our main motivation comes from motion planning problems in optimal control theory, we will show how the shooting method can be adapted to find all locally optimal trajectories on some motion planning with obstacle avoidance examples.

*Partially supported by the Air Force Office of Scientific Research under the Award No: FA8665-20-1-7029 , e-mail: `Georgy.Scholten@lip6.fr`

Control of ill-posed fractional diffusion equations

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Abstract

We consider optimal control problems associated to ill-posed fractional diffusion equation, that is, fractional diffusion equations with missing data. Firstly, we approximate the ill-posed state equation with a family of well-posed one and show that solutions of the latter one converge to solutions of the former one. We also obtain the precise rate of convergence. Secondly, we investigate the minimization problem associated with the approximated state equation. We prove the existence and uniqueness of minimizers that we also characterize with the optimality systems. Finally, we show that minimizers and optimality systems of the approximated control problem converge to the optimal controls and optimality systems of the ill-posed state equation. It turns out that the optimatity system associated to the ill-posed control system is singular. Our method uses the techniques of the so called quasi-reversibility developed by Lattès and Lions. The method we developed can be also applied to general local diffusion equations.

*Partially supported by Air Force Office of Scientific Research under the Award No: FA9550-18-1-0242, e-mail: `mwarma@gmu.edu`

Lyapunov-Like Functions for Almost Global Convergence in Discrete-Time Systems*

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Abstract

This progress report focuses on developments in almost global convergence theory under the project "Direct Adaptive Control of Nonlinear Systems with Uncertain Unstable Zero Dynamics". The analysis methods in this progress report are applicable to adaptive control techniques under investigation in this project. The next progress report will focus on adaptive control.

This progress report presents new results for demonstrating almost global convergence to an invariant set for discrete-time dynamic systems. Notably, these results do not require that the right-hand side of the state-variable difference equation is nonsingular with respect to the measure μ. First, we present density-function-based sufficient conditions for almost global convergence. However, constructing density functions can be difficult. To overcome this difficulty, we leverage the density function results to develop sufficient conditions for almost global convergence using Lyapunov-like functions rather than density functions. Finally, we demonstrate how these new methods can be applied.

*Partially supported by the Air Force Office of Scientific Research under the Award No: FA9550-20-1-0028, e-mail: `jesse.hoagg@uky.edu, dsbaero@umich.edu`

Optimal Control, Inverse Optimality, Dissipativity, and Stability Margins for Nonlinear Discrete-Time Stochastic Regulators

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Abstract

In this work, we present a unified framework to address the problem of optimal nonlinear analysis and feedback control for nonlinear stochastic discrete-time dynamical systems. Specifically, we provide a simplified and tutorial framework for discrete-time stochastic optimal control and focus on connections between stochastic Lyapunov theory and stochastic Bellman theory. In particular, we show that asymptotic stability in probability of the closed-loop nonlinear system is guaranteed by means of a Lyapunov function which can clearly be seen to be the solution to the steady-state form of the stochastic Bellman equation, and hence, guaranteeing both stochastic stability and optimality. In addition, we develop optimal feedback controllers for nonlinear affine systems using an inverse optimality framework tailored to the stochastic stabilization problem. Furthermore, using the newly developed notions of stochastic dissipativity for discrete-time systems we present a return difference inequality to provide connections between stochastic dissipativity and optimality as well as establish guaranteed stability margins for optimal nonlinear discrete-time stochastic feedback regulators.

*Partially supported by the Air Force Office of Scientific Research under the Award No: FA9550-20-1-0038, e-mail: wm.haddad@aerospace.gatech.edu

Learning-based Planning and Control with Persistent Safety for UAS

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Abstract

We introduce a new class of robust-adaptive controllers based on the $\mathcal{L}_1$-adaptive control architecture that is robust to the errors in distributions. This extension of $\mathcal{L}_1$-adaptive control allows to capture broader class of systems described by nonlinear stochastic differential equations, subject to random perturbations. Such systems accurately model random nonlinear processes and dynamics that are learned using data-driven machine learning tools. We pose a new paradigm where we attempt to be robust against the errors between the nominal and true distributions that induce the system trajectories. Retaining the key properties of $\mathcal{L}_1$-adaptive control, we show that the true distribution is bounded uniformly around nominal distributions in the Wasserstein metric. We provide recent work where such uniform bounds enable safe control of uncertain stochastic systems. We then discuss ongoing extensions of these results enabled by considering robustness on the level of distributions and links with the dual-problem of estimation in the area of robust deep learning.

*Partially supported by the Air Force Office of Scientific Research under the Award No: FA9550-21-1-0411, e-mail: nhovakim@illinois.edu

Developments in Suboptimal Model Predictive and Constrained Control

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Abstract

When implemented sub-optimally using time-distributed optimization, the closed-loop properties of model predictive and other optimization-based constrained controllers (such as reference and command governors) become dependent on the optimization algorithm, the number of iterations per time step, and on the warm-starting strategy. In the shrinking horizon MPC setting (when the end time of the maneuver is fixed and the prediction horizon over which the underlying optimal control problem is solved decreases with time) this leads to questions on the extent of inexactness, the number of iterations that need to be performed and the properties of the cost function which ensure that the trajectory enters a specified terminal set or cumulative sub-optimality of the maneuver is acceptable. In the more conventional, receding horizon setting, similar questions relate to the number of iterations that ensures closed-loop stability and to the characterization of the regions of attraction. Several developments aimed at answering these questions will be described. Additionally, we will consider the use of metric regularity, a tool from variational analysis, for constrained control in presence of bounded unmeasured disturbances in implicit systems. Sub-optimal strategies for inexact implementation of optimization-based command governors and those which exploit reference governors for computationally efficient implementation of model predictive controllers will also be described.

*Partially supported by the Air Force Office of Scientific Research under the Award No: FA9550-20-1-0385, e-mail: ilya@umich.edu

Uncertainty Propagation for Stochastic Hybrid System on a Lie Group

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Abstract

We present a computational framework for uncertainty propagation of general stochastic hybrid systems evolving on a Lie group. The Fokker-Planck equation that describes the evolution of uncertainty distributions is solved by spectral techniques, where an arbitrary form of probability density of the hybrid state is represented by non-commutative harmonic analysis. The unique feature of the proposed technique is that the probability density describing the complete stochastic properties of the hybrid state is constructed in a global fashion on a Lie group without relying on the common Gaussian assumption. The efficacy of the proposed method is illustrated by a rigid body pendulum colliding with a planar surface. Further, alternative ways to formulate a probability density function on Lie group are presented.

We also present a Lie group variational integrator for hybrid Hamiltonian systems, where the dynamics is piecewise Hamiltonian, and the switching surface is described as the zero level set of a function. This approach is applied to the collision dynamics of a rigid body evolving in three-dimensional space under the influence of gravity. The proposed method exhibits good total energy preservation properties after multiple collisions, and nontrivial transfer between the rotational and translational kinetic energies. We also demonstrate a simple but robus regularization approach that allows us to consider the collision dynamics of nonsmooth convex rigid bodies, without the need to resort to nonsmooth analysis, and normal and tangent cones.

*Partially supported by the Air Force Office of Scientific Research under the Award No: FA9550-18-1-0228, e-mail: `tylee@gwu.edu`

†Partially supported by the Air Force Office of Scientific Research under the Award No: FA9550-18-1-0288, e-mail: `mleok@ucsd.edu`

Black-box data-driven modeling via occupation kernels

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Abstract

This talk focuses on different applications of the operator-theoretic framework of Liouville operators and occupation kernels. The first application is motion tomography, the goal of which is to recover a description of a vector flow field using information on the trajectory of a sensing unit. A predictor corrector algorithm designed to recover vector flow fields from trajectory data with the use of occupation kernels.

The second application is dynamic mode decomposition of higher order systems. Conventionally, data driven identification and control problems for higher order dynamical systems are solved by augmenting the system state by the derivatives of the output to formulate first order dynamical systems in higher dimensions. In this talk, a method that utilizes the framework of signal-valued reproducing kernel Hilbert spaces is used for a direct analysis of higher order dynamical systems using higher order Liouville operators.

In addition, preliminary results regarding guaranteed-error data-driven identification of nonlinear systems are presented using the framework of Carleman linearization. A majority of existing methods for nonlinear system identification do not provide systematic approaches to meet pre-defined error bounds. The developed method utilizes Carleman linearization-based lifting of the nonlinear system to an infinite dimensional linear system. The linear system is then truncated to a suitable order, computed based on the prescribed error bound, and parameters of the truncated linear system are estimated from data.

*Partially supported by the Air Force Office of Scientific Research under the Award No: FA9550-20-1-0127, e-mail: `rushikesh.kamalapurkar@okstate.edu`

†Partially supported by the Air Force Office of Scientific Research under the Award Nos: FA9550-20-1-0127 and FA9550-21-1-0134, e-mail: `rosenfeldj@usf.edu`

Convergence Guarantees for DMD with Kernel Methods

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Abstract

DMD methods attempt to model an unknown dynamical system using approximations of certain dynamic operators, such as Koopman and Liouville operators. Most existent convergence theories rely on ergodic theorems, which provide at best probability one results. Hence, for any particular selected point in the state-space, there is no direct guarantee of convergence at that point by ergodic methods, and indeed, probability one convergence results may still leave an uncountable number of points where algorithms do not converge.

Recent progress in kernel-based methods enable algorithms that converge everywhere, owing to the dominance of the supremum norm by the Hilbert space norm in a reproducing kernel Hilbert space (RKHS). However, the rigidity of the structure of the functions in and operators over a RKHS can create difficulties in finding dynamics that lead to well behaved operators (i.e. compact).

We will outline a methodology that allows for compact dynamic operators and leads everywhere convergent algorithms.

*Partially supported by the Air Force Office of Scientific Research under the Award No: FA9550-21-1-0134, e-mail: rosenfeldj@usf.edu

Inferring entropy production rate from partially observed Langevin dynamics under coarse-graining

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Abstract

The entropy production rate (EPR) measures time-irreversibility in systems operating far from equilibrium. Using the framework of stochastic thermodynamics, bounds on the total entropy production are derived. I will present a recently developed theoretical toolkit for estimating the dissipation from partial information, based on detecting time-irreversibility from asymmetry in waiting-time distributions (WTD) in observed 2^{nd}-order semi-Markov processes. We estimate the irreversibility in partially observed systems following oscillatory dynamics governed by coupled overdamped Langevin equations. We coarse-grain an observed variable of a nonequilibrium driven system into a few discrete states and estimate a lower bound on the total EPR. By invoking the underlying time-irreversibility, we calculate a lower bound on the total EPR from the Kullback-Leibler divergence (KLD) between WTD. The results of this work will advance new analytical and numerical approaches for quantifying nonequilibrium dynamics in stochastic systems when only partial information is available.

*Partially supported by the Air Force Office of Scientific Research under the Award No: FA9550-20-1-0426, e-mail: `bisker@tauex.tau.ac.il`

Model-Based Machine Learning Methods for Optimal Feedback Control

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Abstract

Designing optimal feedback controllers for high dimensional nonlinear systems is a well-known challenging problem due to the curse of dimensionality in solving Hamilton-Jacobi-Bellman equations. The overarching objective of this project is to develop a *model-based data-driven* approach that combines strengths of computational open-loop optimal control with deep learning techniques to mitigate the curse of dimensionality in optimal feedback control design. In this talk we present results developed during the first year of this project, including: 1) A control theory guided neural network feedback control design with built-in closed-loop stability. We present a suite of new neural network feedback controls for infinite horizon optimal control problems. Those new neural network based controls enjoy a theoretically guaranteed local asymptotic stability of the closed-loop system, while maintaining accurate global approximation of the optimal control. 2) A physics-informed neural network architecture that integrates sparse structure of compositional functions. Such NN architecture has a polynomially growing complexity that is independent of the input dimension of the compositional function. The result provides a potential way to explain why deep neural networks seem to be able to mitigate the curse of dimensionality, a phenomenon that is frequently observed in practice. 3) An actor critic type of method for learning non-smooth viscosity solutions of HJB equations. The method combines an unsupervised actor-critic algorithm with the supervised training by anchoring known accurate data during the unsupervised learning stage to improve the training performance.

*Partially supported by the Air Force Office of Scientific Research under the Award No: FA9550-21-1-0113, e-mail: qgong@ucsc.edu

Chance-Constrained Stochastic Optimal Control and Numerical Methods to HJB PDEs with Dirichlet Boundary Conditions

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Abstract

In this project, we develop a novel path planning algorithm that allows an autonomous mobile agent to navigate itself through an obstacle-filled continuous state space with minimum sensing costs. In our approach, the expected sensing cost required to follow the path is estimated purely from the information-geometric nature of the planned path, without the need for particular sensor models. To this end, we first formulate a shortest path problem on a Gaussian belief space equipped with a quasi-pseudometric representing the minimum information gain required to steer the belief state. An RRT*-based algorithm is then applied to find the shortest path numerically. Finally, the obtained belief path is used as a reference trajectory in the real-time path-following control phase.

This presentation will summarize our recent effort on continuous-time stochastic optimal control (SOC) problems with chance-constraints that frequently appear in the aforementioned motion planning scenarios. Specifically, we discuss numerical approaches to solve such SOC problems using the Hamilton-Jacobi-Bellman (HJB) partial differential equations (PDEs). We first convert the chance-constrained (risk-constrained) SOC problem into a risk-minimizing SOC problem through the Lagrangian relaxation. Leveraging the time-additive Bellman structure of the resulting risk-minimizing SOC problem, we show that the problem is equivalent to solving an HJB PDE with the Dirichlet boundary condition in which the boundary condition represents the probability of failure. We investigate a few numerical approaches including the path integral and finite difference methods to solve the class of risk-minimizing SOC problems whose associated HJB PDE is linearizable via the Cole-Hopf transformation.

*Partially supported by the Air Force Office of Scientific Research under the Award No: FOA-AFRL-AFOSR-2019-0003, e-mail: `ttanaka@utexas.edu`

Data-guided Learning and Control of Higher Order Structures

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Abstract

We discuss a principled data-guided framework for learning and control of higher order structure, function, and dynamics. Underlying our framework are the concepts of hypergraphs, tensor algebra, nonlinear dynamics and bifurcation control. We will build our framework on biological data from cell and genome dynamics, anticipating direct applications in cell reprogramming and wound healing. This biology-inspired data-guided framework, beyond having a high impact on the human genome, is generally applicable to other complex systems.

Multicellular human tissue is a beautiful example of systems of systems exhibiting higher order structure and function. Every cell type in a tissue has unique genome geometry (form) and a gene expression (function) signature. These features along with multiway interactions within the genome and between cells over time give rise to specific cellular functions, or phenotype. Recent technological advancements enable measurement of genomic form and function using biochemical techniques and genome-wide imaging across a tissue. The theoretical and computational foundations developed in this project will enable data-driven dynamic modeling, analysis and intervention/control for a wide range of other applications including supply chain networks, opinion dynamics, synchronization and disease spread.

Partially supported by the Air Force Office of Scientific Research under the Award No: FA9550-22-1-0215, e-mail: `indikar@umich.edu`

Design of robust and accurate biosensing systems

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Abstract

Engineering biology promises revolutionary changes to the way we approach problems in areas ranging from energy and environment to health and medicine. In particular, engineering cells to concurrently sense multiple molecular species and compute a response based on these is going to be critical in a number of applications, including biosignature classification. In this project, we focus on the design of robust and accurate multi-input biosensors that compute the ratio between the levels of different molecular species. Despite tremendous progress in sensor design, capabilities for tracking the ratios of multiple biomarkers in a simple and deployable format have not been realized. Yet, ratiometric biomarker signatures carry key information about stress, fatigue, and cognitive overload in challenging environments. Additionally, although today we can, in principle, build complex genetic circuits comprising multiple genes, such circuits are often fragile and inaccurate.

In this project, we have designed a synthetic genetic circuit to perform ratiometric computation. Specifically, we have used a feedforward network architecture that allows to achieve robustness of the output ratio computation to any changes in the intra-cellular environment that affect availability of cellular resources. We also demonstrate experimentally our ability to quantitatively tune the input/output function of the sensor based on mathematical model predictions. These predictions come from an ordinary differential equation (ODE) model that is simple enough for design. We obtained this convenient model following a model order reduction process, leveraging time scale separation, starting from a highly dimensional partial differential equation (PDE) model that described the spatial dynamics of genetic circuits in bacterial cells. We used the robustness result of contraction theory to perform this reduction and to obtain approximation bounds. As a concrete application of biosensing, and due to the COVID pandemic, we considered rapid detection of airborne viruses, focusing on SARS-CoV2 as a specific case study.

*Supported by the Air Force Office of Scientific Research under the Award No: FA9550-20-1-0044,
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Network motifs and responses of nonlinear systems

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Abstract

This project develops novel theoretical and analytical tools to characterize the responses of nonlinear systems to external inputs, with an emphasis on rigorous mathematical formulations and results. The focus is on how network structure constrains finite-time, transient, behaviors. Of interest are qualitative features that are unique to nonlinear systems, such as non-harmonic responses to periodic inputs, bimodal dose-responses, or invariance to transformations on inputs. These properties play a key role as tools for model discrimination and reverse engineering, as well as in characterizing the robustness of complex systems to disturbances.

The proposed research is motivated by a variety of application areas, largely biological problems at all levels of scale, from the molecular (e.g., extracellular ligands affecting signaling and gene networks), to cell populations (e.g., resistance to chemotherapy due to systemic interactions between the immune system and tumors; drug-induced mutations; sensed external molecules triggering activations of specific neurons in worms), to interactions of individuals (e.g., periodic or single-shot non-pharmaceutical "social distancing" interventions for epidemic control and minimizing transient effects on hospital capacities; adaptive strategies for distancing mandates).

Given the universality of control and dynamical systems concepts, there are shared principles that emerge from such applications, and the theoretical concepts and computational approaches developed can be expected to have applications to other (non-biology) fields where robust performance and transient behaviors are critical.

The presentation will outline the goals of the project, which is in its first year, as well as review recent developments.

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*Partially supported by the Air Force Office of Scientific Research under Award No: FA9550-21-1-0289, e-mail: e.sontag@northeastern.edu, eduardo.sontag@gmail.com

Functional and Distributional Control of Ensemble Systems

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Abstract

Applications concerning with the control of ensemble systems have emerged in pervasive areas of science and technology and have been driving the advancement of theory of ensemble control. In this talk, we will introduce the notions and problems of functional and distributional control of ensemble systems with repect to different types of aggregated measurements. We will present the moment representation of ensemble control systems and show how this leads to dynamically-equivalent moment systems. We will further illustrate the use of moment systems to enable and facilitate systems-theoretic analysis and control design of ensemble systems. Examples of quantum pulse design will be presented to demonstrate the power of the moment-based framework. The generalization of the moment representations beyond ensemble control to information geometric learning, data-driven control, Koopman operator theory, and control on fibered manifolds will also be highlighted.

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Controllability Issues of Linear Ensemble Systems

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Abstract

In this talk, we address a long-standing open problem in ensemble control theory: Whether there exist controllable linear ensemble systems over multi-dimensional parameterization spaces? We provide a negative result: Any real-analytic linear ensemble system is not L^p-controllable, for $2 \leq p \leq \infty$, if its parameterization space contains an open set in $\mathbb{R}^d$ for $d \geq 2$. Because of the duality between controllability and observability for linear ensemble systems, the above negative result also shows that real-analytic linear ensemble systems are not observable if their parameterization spaces are multi-dimensional. Furthermore, the negative result has several implications for approximation theory and operator theory, which will also be presented in this talk.

*This work is supported by the Air Force Office of Scientific Research under the Award No: FA9550-20-1-0076. Email: `xudong.chen@colorado.edu`.

Analysis of Non-convex Optimization Using Control Theory

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Abstract

In this talk, we study two problems regarding non-convex optimization using tools from nonlinear control theory. First, we consider the matrix sensing problem, which is the key to understanding several problems in machine learning as well as polynomial optimization. As the first work in the literature, we study the gap between the information-theoretic complexity and the optimization complexity of this problem. We construct a sub-class of matrix sensing problems that have a low information-theoretic complexity, and then show that they have a high optimization complexity since each of those problems has an exponential number of local minima and finding the global minimum among them using local search is practically impossible. We prove this result by constructing a dynamical system (acting as the gradient algorithm) which has an exponential number of equilibria where the volume of the region of attraction of each equilibrium is bounded away from zero. By defining a distance metric over the dynamical system, we propose a new complexity metric that measures how hard it is to find a global minimum of a matrix sensing problem via local search methods. In the second part of the talk, we focus on finding and tracking the global minimum of non-convex time-varying optimization problems. We analyze a series of dynamical systems, each of which modeling the projected gradient algorithm applied to an instance of the time-varying problem, and study the evolution of the region of attraction of the global solution. This allows us to bound the dynamic regret for the optimization problem, which quantifies the complexity of finding the global minimum and tracking it in the long run.

*Partially supported by the Air Force Office of Scientific Research under the Award No: FA9550-21-1-0250, e-mail: lavaei@berkeley.edu

Contraction Theory for Network Systems

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Abstract

We survey recent progress on the application of the Banach contraction principle to dynamical systems over normed spaces. We introduce and characterize weak pairings as a generalization of inner products; weak pairings lead to integral characterizations of contractivity with respect to arbitrary norms. Special emphasis is given to the ℓ_1 and ℓ_∞ non-Euclidean norms.

Using weak pairings, we establish equivalent characterizations for contraction, including the one-sided Lipschitz condition for the vector field as well as logarithmic norm and Demidovich conditions for the corresponding Jacobian. We elaborate on the robustness properties of strongly contracting dynamical systems.

Finally, we briefly review applications to the study of monotone dynamical systems, recurrent neural networks, implicit learning models, and neural-synaptic networks.

*Partially supported by the Air Force Office of Scientific Research under the Award No: FA9550-22-1-0059, e-mail: `bullo@ucsb.edu`

Stochastic hybrid systems: from robust decision-making networks to distributed command for suppression of persistent adversaries

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Abstract

We formulate a multiple-adversary suppression problem in which the control objective is to repeatedly return to the situation where the states of the adversaries are small. The objective is to be achieved through decentralized communication and coordination among a network of agents that are able to attenuate the size of an adversary's state only by working together to overwhelm the adversary. We show that the goal can be achieved when the number of agents is sufficiently large and there is a sufficiently large (hysteresis) gap between the size of an adversary's state when it is deemed problematic compared to when it is deemed (temporarily) subdued. This gap is related to the time constants of the agents' dynamics and the expected value of the time it takes for the (stochastic, hybrid) decision-making algorithm to single out a problematic adversary. The overall closed-loop system is modeled as a stochastic hybrid dynamical system. Success at achieving the objective is characterized through *recurrence* of an appropriate open set for this system. Recurrence is established via Lyapunov-Foster techniques for stochastic hybrid systems.

*Partially supported by the Air Force Office of Scientific Research under the Award No: FA9550-21-1-0452, e-mail: `teel@ucsb.edu`

Dynamic feedback stabilization and fibre bundles

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Abstract

We study the global feedback stabilization problem from a topological point of view, focusing on the power and limitations of dynamic feedback. A dynamic controller can be thought of as a dynamical system whose state is used to control a system of interest. Let M be the state-space of the system, and F the state-space of the controller. Under standard assumptions, the state-space of the closed-loop system (consisting of the system and the controller) has the topology of a fiber bundle with fiber F and base M. The prevalent assumption in the existing literature is that it is in fact the trivial bundle $M \times F$, but this need not be the case in general. We (1) show that dynamic feedback over twisted bundles can offer better performance that its counterpart over trivial bundles, and (2) provide topological obstructions, based on homotopy theory, to the existence of continuous stabilizing dynamic feedback.

*Partially supported by the Air Force Office of Scientific Research under the Award No: FA9550-20-1-0333, e-mail: `belabbas@illinois.edu`

Topological encoding of graph representations of time series

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Abstract

Observations of dynamical systems can be represented using weighted graphs obtained using ordinal partition networks (OPN). These graphs encode information on the dynamical system's flow across certain regions of the state space, however care must be taken to encode information from the graphs in a way that retains the structure inherent in the attractor. We demonstrate how the resulting graph representation can be combined with tools from Topological Signal Processing (TSP) to provide a noise-resilient approach for studying the shape of the resulting graphs thus allowing dynamic state classification of the underlying system. Specifically, the encoding successfully performs two-state classification (chaos versus regular) using the developed pipeline that demonstrate the capabilities of our approach even for noisy signals with unknown models. We also show a connection between our graph-based approach and the traditional delay-embedding based method, and we highlight some advantages of the former in terms of computational speed and scalability for long data streams.

*Partially supported by the Air Force Office of Scientific Research under Award No: FA9550-22-1-0007, e-mail: `khasawn3@msu.edu, muncheli@msu.edu`

Linear Quadratic Gaussian Synthesis for a Heated/Cooled Rod Using Point Actuation and Point Sensing

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Abstract

We consider a rod that is heated/cooled and sensed at multiple point locations. To stabilize it to a constant temperature we set up a Linear Quadratic Regulator that we explicitly solve by integration by parts and completing the square to find the optimal linear state feedback for the point actuators. But we don't assume that the whole state is measureable so we construct an infinite dimensional Kalman filter to estimate the whole state from a finite number of noisy point measurements. These two components yield a Linear Quadratic Gaussian (LQG) Synthesis for the heat equation under point actuation and point sensing. If time permits we will discuss extensions of this work to wave, beam and nonlinear reaction diffusion equations where the coefficients are spatially varying.

*Partially supported by the Air Force Office of Scientific Research under the Award No: FA9550-20-1-0318, e-mail: ajkrener@ucdavis.edu

Convexification of Motion Planning through Liftings and Hypercomplex Numbers

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Abstract

Our research examines the role of parameterizations in complex guidance and control synthesis problems. In particular, we develop a framework where novel parameterizations of control, the configuration space, and logical constraints, can be effectively utilized for real-time implementation. Using this setup, our goal is develop efficient optimization-based solution methods and autonomously executable algorithms for complex motion planning. After providing a brief overview of our research accomplishments in the previous phase of the project, namely, parametrization of the configuration space via lifting and the corresponding lossless convexification process, we delve into new (first order) algorithmic/numerical approaches to handle challenging state constraints in guidance problems. Lastly, we discuss our recent works on using differential geometric methods for constructing algorithms in distributed estimation and (nonconvex) policy optimization.

*Supported by the Air Force Office of Scientific Research under the Award No: FA9550-20-1-0053,
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Target Tracking Subject to Intermittent Measurements using Attention Deep Neural Networks

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Abstract

We present a novel estimator and predictor framework for target tracking applications that estimates the pose of a mobile target that intermittently leaves the field-of-view (FOV) of a mobile agent's camera. Specifically, the framework uses an attention deep motion model network (DMMN) to estimate the dynamics of the target when the target is in the agent's FOV and uses the DMMN to predict the position, orientation, and velocity of the target when the target is outside the agent's FOV. A Lyapunov-based stability analysis is performed to determine the maximum dwell-time condition on target measurement availability, and experimental results are provided to demonstrate the performance of the proposed framework.

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Finite-Section Approximations of Carleman Linearization and Its Exponential Convergence

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Abstract

The Carleman linearization is one of the mainstream approaches to lift a finite-dimensional nonlinear dynamical system into an infinite-dimensional linear system with the promise of providing accurate finite-dimensional linear approximations of the original nonlinear system over larger regions around the equilibrium for longer time horizons with respect to the conventional first-order linearization approach. Finite-section approximations of the lifted system has been widely used to study dynamical and control properties of the original nonlinear system. In this context, some of the outstanding problems are to determine under what conditions, as the finite-section order (i.e., truncation length) increases, the trajectory of the resulting approximate linear system from the finite-section scheme converges to that of the original nonlinear system and whether the time interval over which the convergence happens can be quantified explicitly. In this talk, I will present explicit error bounds for the finite-section approximation and prove that the convergence is indeed exponential as a function of finite-section order. For a class of nonlinear systems, it is shown that one can achieve exponential convergence over the *entire* time horizon up to infinity. Our results are practically plausible, including approximating nonlinear systems for model predictive control and reachability analysis of nonlinear systems for verification, control, and planning purposes, as our proposed error bound estimates can be used to determine proper truncation lengths for a given sampling period.

*Partially supported by the Air Force Office of Scientific Research under the Award No: AFOSR FA9550-19-1-0004, e-mail: `motee@lehigh.edu`

Is **Poly** the true language of computation?

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Abstract

Complex systems are built from simpler parts that selectively interact. To properly account for the structure and fractal nature of such interactions, one needs an appropriate mathematical language. A category theoretic framework, in particular the operad of wiring diagrams, offers the ability to zoom into a system and investigate how its behavior emerges from the interaction of its parts. However, if one wants the connection pattern between parts to be dynamically selected in real time, according to the situation on the ground, one needs more.

In the pursuit of a suitable language, I found the category **Poly** of polynomial functors to be extraordinarily rich. As I have continued to work with it, the language has shown its value in accounting for not only interacting dynamical systems, but also for foundational aspects of computing: programming languages, dependent types, machine learning, syntax parsing, database migration, cellular automata, and state machines (e.g. Turing machines). The fact that these all emerge from natural operations within a singular language invites the possibility that this language could found computation itself. It is a bold claim, and one that I cannot yet honestly make, but I will explain in this talk why it even makes sense to ask, i.e. why **Poly** should be considered a contender.

*Partially supported by the Air Force Office of Scientific Research under the Award No: FA9550-20-1-0348, e-mail: `david@topos.institute`

Systematic Tools to Satisfy Temporal Logic Specifications in Hybrid Dynamical Systems

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Abstract

Air Force combat systems require autonomous systems that are agile, reliable, and resilient. The unavoidable combination of the physics, the networks, and the algorithms in such systems leads to hybrid system models, which makes the systematic design of algorithms very challenging. Moreover, the unavoidable presence of obstacles, adversarial elements, and uncertainty in the environment further adds to the complexity and requires design tools that neither discretize the state or the dynamics while guaranteeing given high-level specifications, robustly. While discretization may aid computations in some cases, discretization of the hybrid system models emerging in Air Force missions may not permit to assure that the desired specifications are actually satisfied. In this talk, characterizations of signal temporal logic formulas for hybrid dynamical systems are presented. Hybrid dynamical systems are given in terms of differential and difference inclusions, which capture the continuous and discrete dynamics or events, respectively. For such systems, linear temporal logic formulas involving the until operator corresponding to signal temporal logic formulas are proposed. Then, employing characterizations of the linear temporal logic formulas, characterizations of the signal temporal logic formulas in terms of dynamical properties of hybrid systems are presented – in particular, invariance and attractivity properties. As a result, sufficient conditions that guarantee the satisfaction of signal temporal logic formulas are presented by involving the data of hybrid systems and an appropriate choice of barrier functions or Lyapunov-like functions.

*Partially supported by the Air Force Office of Scientific Research under the Award No: FA9550-19-1-0053, e-mail: ricardo@ucsc.edu

Cislunar Catalog Maintenance and Characterization

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Abstract

The nonlinear astrodynamics in the cislunar space beyond the geosynchronous belt (xGEO), encompassing secular, resonant, chaotic, close-encounter, and manifold dynamics, is dramatically different than the weakly perturbed Keplerian approach used for over a half century for the detection and tracking of objects near Earth. The extreme range, difficult observing geometries, and intrinsic sensitivity of cislunar trajectories create particular challenges associated with conducting space domain awareness (SDA) in the cislunar environment. Developing and maintaining a space object catalog (SOC) for xGEO, in particular, will require significant improvements in our understanding of the way objects behave in this complicated restricted four-Âbody problem (R4BP) environment under a realistic ephemeris model. We provide a dynamical cartogÂraphy of the entire cislunar "cone of shame" region (i.e., the lunar exclusion zone) using the intuitive concept of piecewise osculating elements (geocentric or selenocentric, as the physical picture dictates). We employ herein the full mathematical machinery of the Hamiltonian formulation for invariant manifold computations, action-angle variable descriptions, symplectic mappings, and constraints on uncertainty propagation through canonical plane projections. Outcomes of the analysis inform on viewing geometries, required observations campaigns, revisit rates, and surveillance volumes for improved xGEO catalog maintenance.

*Partially supported by the Air Force Office of Scientific Research under the Award No: FA9550-21-1-0191, e-mail: ajrosengren@eng.ucsd.edu, ajrosengren@gmail.com

Hybrid Dynamics for Efficient Learning in Network Games: Momentum, Coordination, and Stability

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Abstract

Multi-time scale techniques have played a significant role in the development of robust distributed control algorithms for network systems. Such techniques intrinsically rely on the uniform asymptotic stability properties of the dynamics that evolve in each of the time scales of the closed-loop system. When these properties are absent, the synthesis of such algorithms is more challenging and requires other regularization mechanisms that preserve robust stability properties without deteriorating the transient performance of the system. We discuss recent progress in the development of these regularization mechanisms in the context of momentum-based pseudo-gradient dynamics in non-cooperative games. Specifically, we introduce a new class of distributed and hybrid Nash set-seeking (NSS) algorithms that synergistically combine dynamic momentum-based flows with coordinated discrete-time resets. The reset mechanisms can be seen as restarting techniques that allow individual agents to choose their own momentum restarting policy to achieve better transient performance. The resulting closed-loop system is modeled and analyzed as a set-valued hybrid dynamical system. By using existing multi-time scale tools for (deterministic) hybrid systems, the proposed algorithms are further extended to settings where agents have access only to partial information of the game. Finally, we discuss potential extensions to multi-agent systems with stochastic communication graphs using new tools recently developed for multi-time scale stochastic hybrid systems with randomness acting in the jumps.

*Partially supported by the Air Force Office of Scientific Research under the Award No: FA9550-22-1-0211, e-mail: `jorge.poveda@colorado.edu`

Entropy in Biology and Other Subjects of Interest

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Abstract

Entropy is a fundamental concept in science and technology. Perhaps the main feature is the ***second law of thermodynamics***, which states that in a thermal system total entropy can never decrease (the change is always non-negative). In this talk, for certain types of systems of interest in biology (regularity gene networks), we give an exact lower bound in terms of the Wasserstein distance from optimal mass transport theory (OMT). Moreover, we consider the second derivative of entropy from which geometry emerges. This relates to some classical work on so-called "entropic gravity." Very importantly, these ideas lead to network notions of curvature, which among several other things, characterize hubs versus bridges (bottle-necks) in networks. This may have consequences in better elucidating some natural processes such as differentiating normal aging from more catastrophic processes such as cancer.

*Partially supported by the Air Force Office of Scientific Research under the Award No: FA9550-20-1-0029, e-mail: `allen.tannenbaum@stonybrook.edu, tryphon@uci.edu`

Runtime Assurance for Safety-Critical Systems from Fast, In-the-Loop Reachability

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Abstract

We show how efficient reachability methods enable runtime assurance (RTA) for safety-critical systems. We focus on interconnected and/or high dimensional systems and we leverage reachability techniques enabled by mixed monotone systems theory. Mixed monotonicity decomposes a dynamical system's vector field into cooperative and competitive elements, resulting in a larger dimensional monotone system for which powerful results from monotone systems theory for, e.g., reachability and invariance are applicable. Notably, these methods offer two key properties: they enable reachable set over-approximations that can be computed very fast for, e.g., inclusion at runtime in feedback controllers, and they scale to high dimensional systems such as neural networks. We demonstrate how both of these appealing features enable RTA mechanisms with provable guarantees for learning-enabled control systems.

*Partially supported by the Air Force Office of Scientific Research under the Award No: FA9550-19-1-0015, e-mail: sam.coogan@gatech.edu