Fluids on geometric rough paths

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Abstract

This presentation is based on earlier work with Dan Crisan, Darryl Holm, and Torstein Nilssen. In that work, we derived variational principles for fluid dynamics on geometric rough paths, which involved working directly with the Lagrangian's reduced form and constraining advected variables to follow the sum of a smooth vector field and geometric rough-in-time vector field. Critical points imply a system of rough PDEs, a perturbed version of the Euler-Poincaré equations introduced by Holm, Ratiu, and Marsden (1998). This novel class of equations opens the path for developing non-Markovian subgrid-scale fluid models. We will begin the presentation by briefly recalling this framework.

We will then focus on the existence and uniqueness problem for linear and non-linear rough transport equations. An extension of Di-Perna Lion's theory for linear equations to rough transport equations will be presented alongside a well-posedness result for a broad class of non-linear equations. As an illustration, we will prove the well-posedness of the 2D rough Euler model with bounded initial data, thus expanding the Yudovich theory to the rough setting. This part is based on work in progress with T. Nillsen and Lucio Galeati.

Lastly, we will discuss parameter estimation for rough differential equations using scaled quadratic variations. We will present an estimator for the parameters of the control vector field and the Hurst exponent in the fractional Brownian motion case, discussing the consistency and rate of convergence as the mesh of the partition for which we observe the solution tends to zero. Numerical experiments will corroborate theoretical analysis for a range of finite-dimensional RDEs and 2D rough Euler.

The presentation will primarily draw from three recent publications:

- Leahy, J.M. and Nilssen, T., 2023. Scaled covariation and parameter estimation for pathwise differential equations driven by fractional Brownian motion. to appear.
- Crisan, D., Holm, D.D., Leahy, J.M. and Nilssen, T., 2022. Variational principles for fluid dynamics on rough paths. Advances in Mathematics, 404, p.108409.
- Crisan, D., Holm, D.D., Leahy, J.M. and Nilssen, T., 2022. Solution properties of the incompressible Euler system with rough path advection. Journal of Functional Analysis, 283(9), p.109632.

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Formalizing an optimization problem for inferring the entropy production from observed statistics

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Abstract

Nonequilibrium processes break time-reversal symmetry and generate entropy. Living systems are driven out-of-equilibrium at the microscopic level of molecular motors that exploit chemical potential gradients to transduce free energy to mechanical work, while dissipating energy. The amount of energy dissipation, or the entropy production rate (EPR), sets thermodynamic constraints on cellular processes. Practically, calculating the total EPR in experimental systems is challenging due to the limited spatiotemporal resolution and the lack of complete information on every degree of freedom. Here, we propose a new inference approach for a tight lower bound on the total EPR given partial information, based on an optimization scheme that uses the observed transitions and waiting times statistics. We introduce hierarchical bounds relying on the first- and second-order transitions, and the moments of the observed waiting time distributions, and apply our approach to two generic systems of a hidden network and a molecular motor, with lumped states. Finally, we show that a lower bound on the total EPR can be obtained even when assuming a simpler network topology of the full system.

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Optimal Control of a Notional Hypersonic Missile

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Abstract

The optimal control of hypersonic air vehicles with a global reach is addressed. From the point of view of differential geometry, the flight paths of long range hypersonic air vehicles which hold altitude and speed are spherical curves. It is therefore tempting to address the synthesis of optimal trajectories of hypersonic air vehicles using the Frenet-Serret formulae: the fundamental theorem of differential geometry of curves states that using the Frenet-Serret equations, every regular curve in three-dimensional space has its shape, up to a translation and rotation, completely determined by its curvature κ and torsion τ time histories. But the geometric concepts of flight path curvature κ and torsion τ are *not* in a direct and one-to -one correspondence with an air vehicle's physical load factor n/"pitch stick" and bank angle ϕ/"roll stick" flight control inputs. We maintain that differential geometry on its own + the theory of optimal control, are not enough to yield the optimal trajectories of air and space vehicles. In the flight scenario under consideration, what in the parlance of differential geometry is the the proverbial "particle" which traces out a curve, is not in fact a geometric point as befits geometry, is not even a " point mass", but is a rigid body - the hypersonic air vehicle itself. The underlying *physical* situation entails dynamics, as opposed to kinematics, where geometry alone carries the day.

• Dynamics & Control is a cut above kinematics/geometry

While differential geometry and the Frenet-Serret equations have a role to play, there is more to it than geometry and optimization only. The flight mechanics equations of a long range hypersonic missile which holds altitude and speed are developed. The novel dynamics equations and concepts from the differential geometry of curves are used to fully characterize the optimal minimum time trajectories. Minimum time optimal trajectories of hypersonic missiles are synthesized: it is shown that generic min time optimal trajectories consist of a concatenation of three arcs: an initial and a terminal circular arc of equal radii $r < R_e$, where R_e is the radius of the Earth, and an intermediate trajectory segment which is an arc of a great circle on Earth. The small circular arcs are where the air vehicle is maneuvering and the aerodynamic load factor n is maximal - the air vehicle pulls the maximum number of g's - while along the intermediate great circle arc the air vehicle is coasting - the latter is a *singular*

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optimal trajectory arc. When the terminal heading is not specified/is free, the optimal trajectory consists of a small circular arc where the hypersonic missile pulls the maximum number of g's, followed by flight along a great circle arc. The solution of the optimal control problem comes down to the solution of a set of 5 quadratic equations in 5 variables. Using the method of Grobner bases from algebraic geometry, the closed form solution of the min time optimal control problem associated with the synthesis of optimal trajectories for hypersonic missiles with a global reach is obtained.

Stochastic Thermodynamics: From Dissipativity to Accumulativity and Energy Storage to Entropy Production

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Abstract

In this work, we develop an energy-based dynamical system model driven by a Markov input process to present a unified framework for stochastic thermodynamics predicated on a stochastic dynamical systems formalism. Specifically, using a stochastic dissipativity, losslessness, and accumulativity theory, we develop a nonlinear stochastic port-Hamiltonian system model $\mathcal{G}$ characterized by energy conservation and entropy nonconservation laws that are consistent with statistical thermodynamic principles. In particular, we show that the difference between the average stored system energy and the average supplied system energy for our stochastic thermodynamic mode $\mathcal{G}$ is a martingale with respect to the system filtration, whereas the difference between average system entropy production and the average system entropy consumption is a submartingale with respect to the system filtration. Furthermore, we show that the average entropy of the final state of an adiabatically isolated stochastic port-Hamiltonian system is greater than or equal to the entropy of its initial state leading to a generalized Clausius *maximum entropy* principle. Finally, we show that the steady-state value of the energy of the adiabatically isolated system is stochastically semistable and uniformly distributed over all the subsystems of $\mathcal{G}$, leading to an *equipartition of energy* principle.

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Stochastic Systems with non-Gaussian Noise

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Abstract

Motivated by practical applications of control models with the evidence of non-Gaussian moise, the focus of this research has been on developing stochastic calculus and modeling applications with Rosenblatt processes and some generalizatiolns as a noise model for stochastic systems. Some problems and their solutions for controlled stochastic systems driven by Rosenblatt processes and some generalizations are described. Rosenblatt processes have continuous sample paths and are non-Gaussian having a stochastic calculus that can be considered as a non-Gaussian generalization of fractional Brownian motions with their long range dependence. Prediction problems for Rosenblatt processes are explicitly solved as well as prediction problems for some generalizations of Rosenblatt processes called Rosenblatt-Volterra processes. Linear-quadratic and mean field games with Rosenblatt noise processes are formulated and some solutions are described. Some absolute continuity problems for Rosenblatt measures for transformations of stochastic systems with Rosenblatt noise are formulated and explicitly solved and their generalization of absolute continuity for Gaussian measures is noted. This absolute continuity result provides a direct method for the solution of some optimal control problems for nonlinear systems driven by Rosenblatt processes and the explicit calculation of mutual information for stochastic signals and these signals with Rosenblatt noise. For many of the problems presented, the Rosenblatt process noise can be replaced by a more general Rosenblatt-Volterra noise process that is analogous to the extension of fractional Gaussian processes to fractional Gauss-Volterra processes.

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Population Games and Evolutionary Dynamics: Towards a More Realistic Theory

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Abstract

Recently, powerful system-theoretic tools and concepts such as passivity have been employed to analyze and design population games. However, models remain to a great extent simplistic, and existing stability results often have limitations that hinder the applicability of population games to real-world problems. In our recent research, we have made substantial progress in overcoming these limitations. In particular, we've made progress in allowing for strategy revision clocks beyond widely-used "Poisson clocks", which are inadequate for most applications, and we have considered the case when an agent's current strategy influences the rate of revision. Additionally, we've started developing a methodology that uses system-theoretic passivity to design a payoff mechanism that can guide the population game towards desirable states, even in the presence of additional dynamics. During this PI meeting, I'll provide an overview of these recent results and highlight some open problems that we plan to tackle over the next three years. Our ultimate goal is to use system-theoretic methods, concepts and tools to generalize the theory of population games so that it can cope with more realistic models, and obtain stability results and design methods subject to fewer assumptions. Moreover, we will study the mean-field limit for more general models to obtain suitable deterministic approximations and concrete probabilistic error bounds when the number of agents is finite. We will also discuss AFOSR relevant applications to illustrate our recent and future work.

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Ensemble PDE Control

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Abstract

Ensemble PDEs are uncountably infinite (continuum) collectives of PDEs. In comparison to PDEs in dimension higher than one, the ensemble variable in an ensemble PDE is like one of the spatial variables in a conventional PDE except that the system model has no derivatives in the ensemble variable but may have integrals.

Ensemble PDEs arise in several applications. In addition to fluids (from chemical engineering to oil drilling) and elastic structures, it is in traffic flows, with their continuously parametrized "classes" of drivers and vehicles, or in epidemiological models with parametrized populations, that ensemble PDEs arise.

Hyperbolic PDEs have only one derivative in space and this makes them a natural class from which to start the study of control of ensemble PDEs. We introduce a result which generalizes previous results on stabilizing feedback designs for an arbitrary finite number of coupled hyperbolic PDEs to the infinite/ensemble PDE case. The significance of acquiring this control capability is that, while being for systems on two-dimensional domains (one space variable and one ensemble variable), such an algorithm is implementable using a single scalar control, whereas the stabilization of conventional 2D PDEs requires, in general, functional actuation on a 1D subset of the boundary.

Extensions to multi-dimensional ensembles, as well as to parabolic PDEs, are subjects of future research.

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Two-phase Differential Games and Cooperative Engagements in the Beyond Visual Range Domain

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Abstract

The first part of this presentation discusses a two-phase reach-avoid differential game. For each phase, the optimal strategies are obtained in closed form and the singular surface of the second phase of the game is characterized. The strategies for the transition phase between the first and second phases are proposed. Based on these solutions and extensions, the solution to the game of kind is obtained for the complete differential game. Then, we address the problem of cooperative engagement in the beyond visual range domain. A faster and more lethal player is tasked to engage two cooperative, slower players which also have lower range weapons. The faster player, which is turn-rate constrained, also has a directional capture zone with maximum range greater than the capture circle radius of the slower players. The reachable regions between the faster player and each slower vehicle are approximated by considering the tradeoffs and effects of each property: speeds, capture zones, and turning rates. Lastly, we consider the engagement zone avoidance problem where an UAV employs passive countermeasures while flying in a contested area. In more detail, the UAV designs its optimal path in order to minimize risks associated with weapon engagement zones (WEZ) by surface-to-air missiles. Four optimal control problems are addressed which consider arrival time and partial or full avoidance of the WEZ.

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Advances in Contraction Theory for Neural Networks and Convex Optimization

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Abstract

We review recent advances in the development of contraction theory for dynamical systems, control, computation, and learning. We will review a subset of recent results on
(1) fundamental theory, including semicontraction theory, non-Euclidean monotone operator theory, and k-contractivity;
(2) example systems, including Euclidean contractivity of neural networks with symmetric weights, semicontractivity of Kuramoto-Sakaguchi oscillator networks, and primal-dual gradient dynamics; and
(3) applications to the method of successive approximations for optimal control, and time-varying convex optimization.

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Collaborative Coalitions in Multi-Agent Systems

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Abstract

The emergence of new communication technologies allows us to expand our understanding of distributed control and consider collaborative decision-making paradigms. With collaborative algorithms, certain local decision-making entities (or agents) are enabled to communicate and collaborate their actions with one another to attain better system behavior. By limiting the amount of communication, these algorithms exist somewhere between centralized and fully distributed approaches. In this talk, we explore this collaborative paradigm and identify potential opportunities associated with collaborative decision-making in distributed resource allocation problems. In particular, we will provide a characterization for how the level and structure of collaboration impacts the efficiency of the emergent collective behavior. Note that having such a characterization is essential for identifying whether or not the communication cost necessary to facilitate the desired collaborations is worth the gains in system welfare.

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Utilizing control-oriented properties in learning

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Abstract

We study three problems in which control-oriented properties are utilized to design learning algorithms that are more useful for application in the real world. In the first problem, we utilize control barrier functions to identify a reinforcement learning algorithm that ensures safety and yet is provably convergent. Since such functions are not easy to design, we also provide some initial results on sample complexity of learning a control Lyapunov function from trajectory data. In the second problem, we present a reinforcement learning algorithm that is provable robust in the sense that while it utilizes data from an approximate model of the system, the controller it designs is provably stabilizing for the real system. In the final problem, we provide and analyze deep learning algorithms for data that display geometric properties since they lie on manifolds.

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Persistence of Graphs: Time-dependence and Directionality

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We present a framework for studying temporal networks using zigzag persistence, a tool from the field of Topological Data Analysis (TDA). The resulting approach is general and applicable to a wide variety of time-varying graphs. For example, these graphs may correspond to a system modeled as a network with edges whose weights are functions of time, or they may represent a time series of a complex dynamical system. Starting with unweighted and undirected temporal graphs we use simplicial complexes to represent snapshots of the networks that can then be analyzed using zigzag persistence. We show two applications of our method to dynamic networks: an analysis of commuting trends on multiple temporal scales, e.g., daily and weekly, in the Great Britain transportation network, and the detection of periodic/chaotic transitions due to intermittency in dynamical systems represented by temporal ordinal partition networks. Our findings show that the resulting zero- and one-dimensional zigzag persistence diagrams can detect changes in the networks' shapes that are missed by traditional connectivity and centrality graph statistics.

We then present a new filtration for incorporating directionality information when measuring the structure of directed graphs. These sorts of graphs arise in many of the same settings above, although much of the available work, particularly in terms of the use of TDA for network analysis, tends to simply throw out the directed information. In this work, we use directed paths as a marker for inclusion of a simplex in the filtration, and show that distinguishes a larger class of directed graphs than previous directed graph TDA methods.

We conclude the talk with some preliminary work on detecting Phenomenological (or P-type) bifurcations in stochastic systems using "homological bifurcation plots." These bifurcations correspond to structural changes in the joint probability density function of the state space variables which can include transitions from a mono-stable state to multi-stable states, or the appearance of stochastic limit cycles.

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Critical Nash Values for Graphon Mean Field Games

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Abstract

Graphon Mean Field Games (GMFGs) constitute generalizations of Mean Field Games to the case where the agents form subpopulations located at the nodes of large graphs. This talk first presents the fundamentals of Mean Field Game theory; it then introduces Graphon Mean Field Games and the central existence results for Nash equilibria in such systems. Previous work analyzed the stationarity of equilibrium Nash values with respect to node location for large populations of non-cooperative agents with linear dynamics on large graphs and their limits embedded in Euclidean space. That analysis is extended here to agent systems in the class of control affine non-linear systems. Illustrative simulations for the LQG case are presented and future extensions to the important category of sparse graphs is indicated.

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Flow-dependent Lyapunov functions and contraction analysis

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Abstract

In this talk, we will discuss an approach that we have developed, in joint work with M. Ali Al-Radhawi and David Angeli, to understand the dynamics of biological interaction networks. This approach is based on "flow-dependent Lyapunov functions" which depend on state variables only through flows among variables, and more specifically through reaction rates. They can be equivalently defined as common Lyapunov functions for finite families of linear systems, which leads to an effective computational package. We will illustrate the the power of the approach in establishing stability and ensuring safety constraints in several biological applications, and explore implications to contraction analysis.

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Global Minimization of Analytic Functions through Polynomial Approximations

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joint work with Mohab Safey El Din and Emmanuel Trélat *
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Abstract

We propose a new method for minimizing analytic functions over compact domains through the use of polynomial approximations. This is in essence an effective application of the Stone-Weierstrass Theorem, as we seek to construct a polynomial approximant of an analytic function f over a compact domain. As we construct increasingly accurate polynomial approximations of f, the critical points of the approximant converge towards the critical points of f. We then use exact methods from computer algebra to compute all critical points of the approximant with guarantees of missing none. Our Main Theorem provides conditions of probabilistic nature on the local minima of the objective function and on the accuracy of the polynomial approximation sufficient to guarantee that all local minima located in the interior of the compact domain are captured by the critical points of the polynomial. We provide an implementation of this method, made up of three steps. We construct a discrete least-squares polynomial approximant of the lowest possible degree, then compute its critical points and initialize local minimization methods on the objective function f at these points, in order to recover the totality of the set of local minima of f on the compact domain.

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Boundary Arc and Boundary Point Stabilization of the Heat Equation on the Unit Disk via LQR

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Abstract

The uncontrolled heat equation on the unit disk under homogeneous Neumann boundary conditions is only neutrally stable as zero is an eigenvalue. To stabilize the heat equation to a uniform temperature which we conveniently take to be zero, we pose and solve a Linear Quadratic Regulator (LQR).

We consider two types of control actuation, boundary arc control and boundary point control. With boundary arc control we assume that there are m arcs on the boundary of the disk where we can control the heat flux uniformly across each arc so the control is m dimensional. Boundary point control is the idealized limit of boundary arc control as the length of each arc shrinks to zero but the gain factor multiplying the control inversely goes to infinity. In effect we are controlling the heat flux at m points on the boundary.

For each type of actuation we derive a Riccati PDE whose solution is the Fredholm kernel of the optimal cost of the LQR. We also compute the Fredholm kernel of the corresponding optimal feedback and the first few closed loop eigenvaues. We show that each optimal feedback moves the neutrally stable eigenvalue into the open left half plane but has little effect on the other eigenvalues as they are already sufficiently stable.

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Direct Adaptive Control of Nonlinear Systems with Uncertain Unstable Zero Dynamics

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Abstract

In this talk we review progress on various topics relating to retrospective cost adaptive control (RCAC) as well as new developments on predictive cost adaptive control (PCAC). For RCAC, we completed the analysis of the retrospective cost decomposition for linear, time-varying (LTV) systems, and this work is under journal review. The main result shows that the retrospective cost is the sum of a performance term and a model-matching term. This work motivated input-to-state stability (ISS) analysis of LTV systems. This work is in the final stages, and a journal version will be submitted soon. In connection with nonminimum-phase zeros, we submitted a journal paper on the relationship between initial undershoot and unstable zero dynamics in nonlinear discrete-time systems. The connection between LTV and nonlinear systems is motivated by model predictive control (MPC) of nonlinear systems using state-dependent coefficients (SDC's). Control of nonlinear systems using MPC with SDC's is achieved by iterating the backward-propagating Riccati equation iteration (quadratic programming can also be used), where, in each iteration, the SDC's are treated as an LTV model. A deadbeat observer is used to facilitate output-feedback control, which avoids the need for full-state measurements. Although the use of SDC's is heuristic, the numerical results show that, within the context of MPC, this formalism works reliably if the horizon is sufficiently large, with better performance than the Jacobian. Finally, PCAC is applied to Lur'e systems with self-excited oscillations. Numerical studies show that, under linear closed-loop system identification and linear controller adaptation, the closed-loop system asymptotically satisfies the discrete-time circle criterion for absolute stability. This is surprising in view of the mismatch between the linear modeling and nonlinear dynamics.

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Learning of Dynamical Systems with More Corrupt Data than Clean Data using Nonlinear Optimization

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Abstract

In the first part of the talk, we study the system identification problem for linear discrete-time systems under adversaries and analyze two lasso-type estimators. We study both asymptotic and non-asymptotic properties of these estimators in two separate scenarios, corresponding to deterministic and stochastic models for the attack times. Since the samples collected from the system are correlated, the existing results on lasso are not applicable. We show that when the system is stable and the attacks are injected periodically, the sample complexity for the exact recovery of the system dynamics is $O(n)$, where n is the dimension of the states. When the adversarial attacks occur at each time instance with probability p, the required sample complexity for the exact recovery scales as $O(\log(n)p/(1-p)^2)$. This result implies the almost sure convergence to the true system dynamics under the asymptotic regime. As a by-product, even when more than half of the data is compromised, our estimators still learn the system correctly. This work provides the first mathematical guarantee in the literature on learning from correlated data for dynamical systems in the case when there is less clean data than corrupt data.

In the second part of the talk, we discuss the design of optimization algorithms for structured problems using tools from control theory. We introduce the notion of matrix sensing over graph to address this problem.

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Learning-Based Planning & Control with Persistent Safety for UAS

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Abstract

We introduce the notion of distributional robustness and motivate it as a generalized notion of robustness that forms the crux of classical control theory. Distributional robustness offers a way to unify the structured classical design methods with the probabilistic testing methodologies in realistic scenarios. By lifting the notion of robustness to the space of distributions over outcomes, we attempt to bring the test and design spaces closer, thus enabling more holistic feedback in the design-test loop. Furthermore, the notion of distributional robustness allows generalizing deterministic nonlinear controllers, moving towards more realistic systems, which opens further directions for safe, quantifiable, and predictable control. We analyze the $\mathcal{L}_1$ adaptive control feedback on Markov processes and show how we can guarantee uniform bounds in the space of Borel measures over the state space. We further show how the feedback is implementable in realistic scenarios and does not require unrealizable conditions for operations. Finally, we discuss the numerous directions that distributionally robust adaptive control opens up.

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Developments in Suboptimal Model Predictive and Constrained Control

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Abstract

When implemented sub-optimally using time-distributed optimization, the closed-loop properties of model predictive controllers become dependent on the optimization algorithm, the number of iterations per time step, and on the warm-starting strategy. In the shrinking horizon MPC setting (when the end time of the maneuver is fixed and the prediction horizon over which the underlying optimal control problem is solved decreases with time) this leads to questions on the extent of inexactness, the number of iterations that need to be performed and the properties of the cost function which ensure that the trajectory enters a specified terminal set or cumulative sub-optimality of the maneuver is acceptable. In this presentation, a strategy for implementing shrinking horizon Model Predictive Control (MPC) using a limited number of optimization iterations at each timestep will be described along with offline and online certification methods for determining a time-varying bound on the number of optimization iterations so that the closed-loop system (with additional physical disturbances) satisfies a specified terminal constraint. For the more conventional receding horizon MPC, two supervisory reference governor schemes will be described: A Feasibility Governor (FG) for preview-based MPC (PMPC) for the control of constrained linear systems with measured disturbances and a Stability Governor (SG) for constrained linear systems under Model Predictive Control (MPC) without terminal constraints. These schemes ensure asymptotic stability of the target equilibrium for a large set of initial states even when a short prediction horizon is used thereby reducing the amount of computational effort and preview information. Finally, bilevel optimization problems typical of control-aware design optimization where the system design parameters are optimized in the outer loop and a discrete-time control trajectory is optimized in the inner loop will be considered. Noting similarity to MPC, convergence properties of inexact iterative solution schemes for such bilevel optimization problems can be studied using control-theoretic tools.

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An interaction-aware, monotone systems approach to reachability of neural feedback loops

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Abstract

In this talk, we propose a computationally efficient framework, based on interval analysis and mixed monotone systems theory, for rigorous reachability verification of nonlinear continuous-time dynamical systems with neural network controllers. Given a neural network, we use an existing verification algorithm to construct inclusion functions for its input-output behavior. Inspired by mixed monotone theory, we embed the closed-loop dynamics into a larger system using an inclusion function of the neural network and a decomposition function of the open-loop system. This embedding provides a scalable approach for safety analysis of the neural control loop while preserving the nonlinear structure of the system. We show that one can efficiently compute interval over-approximations of the reachable sets using a single trajectory of the embedding system. We design an algorithm to leverage this computational advantage through partitioning strategies, improving our reachable set estimates while balancing its runtime with tunable parameters.

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Stat-duality based method for rapid solution of high-dimensional first-order HJ PDE problems with low-dimensional nonlinearities

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Abstract

The curse-of-dimensionality for solution of HJ PDE problems is an artificial impediment induced by classical methods, which induce complexity by the imposition of a grid over the domain. The true difficulty is the curse-of-complexity. This may be heuristically imagined as a mapping from problem complexity to computational cost. Standard linear-quadratic control problems are of relatively negligible complexity, and may be solved via Riccati equations. Unfortunately, the panoply of nonlinear control problems, and their associated HJ PDE problems, does not allow for such approaches. Methods developed over the past twenty years have made massive progress against the curse-of-dimensionality, but limits, albeit more distant, nonetheless persist.

A single nonlinearity in the dynamics or the cost of an otherwise linear-quadratic problem generally affects the behavior of the value function (i.e., the HJ PDE problem solution) over all dimensions yielding non-quadratic behavior in all of the arguments. We refer to nonlinearities in the dynamics and/or cost that may be expressed as functions on a low-dimensional subspace of the domain, and in the case of the dynamics instantaneously affecting only a low-dimensional subspace of the range, as "low-dimensional" nonlinearities. If there is a low-dimensional nonlinearity within a high-dimensional HJ PDE, the problem complexity almost entirely reflects only the complexity of the low-dimensional nonlinearity.

Here, stat-duality is employed to move all nonlinearities into a single function within the running cost. This is a function only of newly introduced control processes that are introduced via the stat-duality. These new control processes have dimension equal to that of the low-dimensional nonlinearity, say dimension m. All remaining elements of the problem are linear-quadratic. Solution of the problem is reduced to that of a nonlinear staticization control problem of dimension m, essentially independent of the original space dimension. In particular, the staticization problem may be solved independently at any desired point in the original state space.

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New results on convergent spectral decomposition of dynamic systems

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Abstract

In this talk, we present new results on provably convergent singular value decomposition (SVD) of total derivative operators corresponding to dynamic systems. Dynamic systems are modeled as total derivative operators that operate on reproducing kernel Hilbert spaces (RKHSs). The resulting total derivative operators are shown to be compact provided the domain and the range RKHSs are selected carefully. Compactness is used to construct a novel sequence of finite rank operators that converges, in norm, to the total derivative operator. The finite rank operators are shown to admit SVDs that are easily computed given sample trajectories of the underlying dynamical system. Compactness is further exploited to show convergence of the singular values and the right and left singular functions of the finite rank operators to those of the total derivative operator. Finally, the convergent SVDs are utilized to construct estimates of the vector field that models the system. The estimated vector fields are shown to be provably convergent, uniformly on compact sets. Extensions to systems with control and to partially unknown systems are also discussed.

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Operator Decompositions for Inverse Problems

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Abstract

Dynamic Mode Decompositions (DMD) have grown in popularity over the past decade, starting with Schmidt and Mezic, and continuing with Brunton, Kawahara, and others. It is an approach to determining a model the state of an unknown dynamical system from observed snapshots of the system. The modern approach to DMD is to view it through the lens of the Koopman operator, which casts a nonlinear dynamical system over a finite dimensional state space into a linear operator over an infinite dimensional space, where approximations of the Koopman operator can then be transformed into estimations of the unknown dynamics.

The purpose of this talk is to present DMD as a special case of a new approach to inverse problems. Where typical inverse problems are of the form $Bf = h$, where f is an unknown vector in a Banach space, h is a vector of measurements, and B is an system operator that relates the measurements to the unkown, DMD casts the unknown f within the Koopman operator, and information about the unkown f is extracted through the interaction of the Koopman operator with observables.

Different operators are appropriate for different dynamics or other inverse problems. We will discuss a gamut of different operators and observables, talk about convergence guarantees, and possible new directions for nonlinear approximation theory.

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Duality theory for nonlinear filtering

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Abstract

There is a fundamental dual relationship between estimation and control. The dual relationship is expressed in two interrelated manners: (i) Duality between observability and controllability; and (ii) Duality between optimal estimation (filtering) and optimal control. The second item means expressing one type of problem as another type of problem. In our work, the main interest is to convert a filtering problem into a control problem.

In my talk, I will describe an extension of the duality theory to hidden Markov models (HMM) with white noise observations. I will quickly review the classical Kalman's duality (taught as part of any introductory course in Linear Systems Theory) and discuss the difficulty in extending the duality to nonlinear stochastic systems (HMMs). After describing the extension, I will close my talk with an application of the duality theory for the purposes of nonlinear filter stability analysis.

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Model-Based Machine Learning Methods for Optimal Feedback Control

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Abstract

The lack of effective computational algorithms has created a significant bottleneck in optimal feedback control design, primarily due to the curse of dimensionality encountered when solving Hamilton-Jacobi-Bellman (HJB) equations. In this talk, we will first provide a brief overview of our work in designing a data-driven approach for optimal feedback control of high-dimensional systems. Our approach entails the integration of physics-informed neural network training and theory-guided neural network feedback control structures, effectively combining the knowledge of control system with the computational power of deep neural networks. Then we will focus on our recent work on neural network approximation theory for compositional functions. We will introduce compositional features that play a pivotal role in determining the complexity and error upper bound of neural network approximations for high-dimensional functions and dynamical systems. Theorems on the neural network complexity in the approximation of trajectories of ODEs, optimal control laws, and Lyapunov functions will be introduced. The results shed light on the reason why using neural network approximation helps to avoid the curse of dimensionality. The research presented in this talk is a collaborative effort with Prof. Wei Kang at the Naval Postgraduate School.

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Simultaneous perception-action design for minimum sensing navigation

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Abstract

This YIP project has been concerned with the motion planning problem in Gaussian belief spaces for minimum sensing navigation. In our approach, we formulate a shortest path problem on a Gaussian belief space equipped with a quasi-pseudometric representing the minimum information gain required to steer the belief state. Over the last three years, sampling-based algorithms (namely RRT*- and PRM*-based schemes) have been developed and their effectiveness has been demonstrated experimentally. In our Year 3 effort, we theoretically proved the asymptotic optimality and improved the computational efficiency of the proposed class of algorithms.

In this presentation, I will also present a generalized framework for simultaneous perception-action design that extends the framework of Gaussian belief space planning we have been considering in this YIP project. In our new paper titled "Simultaneous perception-action design via invariant finite belief sets" published in Automatica, we formulate the minimum sensing navigation problem in discrete-state Markov decision processes. Our formulation differs from that of a partially observable Markov decision process, since the agent is free to synthesize not only its policy for action selection but also its belief-dependent observation function. To obtain a computationally tractable solution, we approximate the value function using a novel method of invariant finite belief sets, wherein the agent acts exclusively on a finite subset of the continuous belief space. We solve the approximate problem through value iteration in which a linear program is solved individually for each belief state in the set, in each iteration.

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Convexification of Motion Planning and Control through Liftings and Hypercomplex Numbers

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Abstract

The goal of the proposed research is the development of a rigorous mathematical and algorithmic framework to explore the role of parameterization in convexifying complex motion planning problems for controlled, autonomous, dynamical systems. Specifically, we will examine how novel parameterizations of control, configuration space, and logical constraints, can be effectively utilized to develop convex optimization-based solution methods and real-time executable algorithms for complex motion planning problems.

Over the last year, we have focused on several research directions in convexification of motion planning and control problems. A key technology needed for convexification-based motion control is the ability to solve finite dimensional convex optimization problems in real-time. To this end, we have explored adaptation of first-order projected gradient search algorithms of convex optimization by formulating novel schemes to accelerate these algorithms. Another key focus area is the incorporation of uncertainty into motion planning and control formulation and methods of convexification for analysis and synthesis. We first formulated deterministic state and control dependent uncertainties, in linearly constrained motion planning problems, and developed ways to convexify the resulting robust optimization problems. Then we have investigated joint control invariant funnel and trajectory computation problems within a convex optimization formulation. Currently we are focusing on stochastic versions of these problems as well as extension of coordination and trajectory design problems to Riemannian manifolds.

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Design of robust and accurate biosensing systems

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Abstract

Engineering biology promises revolutionary changes to the way we approach problems in areas ranging from energy and environment to health and medicine. In particular, engineering cells to concurrently sense multiple molecular species and compute a response based on these is going to be critical in a number of applications, including biosignature classification. In this project, we focus on the design of robust and accurate multi-input biosensors that compute the ratio between the levels of different molecular species. Despite tremendous progress in sensor design, capabilities for tracking the ratios of multiple biomarkers in a simple and deployable format have not been realized. Yet, ratiometric biomarker signatures carry key information about stress, fatigue, and cognitive overload in challenging environments. Additionally, although today we can, in principle, build complex genetic circuits comprising multiple genes, such circuits are often fragile and inaccurate.

In this project, we have designed a synthetic genetic circuit to perform ratiometric computation, robustly with respect to the cellular context. Specifically, we have used a feedforward network architecture that allows to achieve robustness of the output ratio computation to any changes in the intra-cellular environment that affect availability of cellular resources. We also demonstrate experimentally our ability to quantitatively tune the input/output function of the sensor based on mathematical model predictions. These predictions come from an ordinary differential equation (ODE) model that is simple enough for design. We obtained this convenient model following a model order reduction process, leveraging time scale separation, starting from a high dimensional partial differential equation (PDE) model that described the spatial dynamics of genetic circuits in bacterial cells. We used the robustness result of contraction theory to perform this reduction and to obtain approximation bounds.

Due to the COVID pandemic, we steered the project in a direction of practical relevance given the world-wide crisis situation. We therefore addressed the problem of rapid detection of airborne viruses, focusing on SARS-CoV2 as a specific case study. Sensing virus from the air is highly challenging due to the ultra-low virus concentrations found in air, which yet could make someone infected if inhaled. In order to overcome this problem and reach a useful detection sensitivity, we designed and built an RNA enrichment system that uses

*Supported by the Air Force Office of Scientific Research under the Award No: FA9550-20-1-0044, e-mail: ddv@mit.edu. Collaborators: Jorge Chavez, AFRL, Svetlana Harbaugh, AFRL electrokinetics to concentrate femtogram-scale RNA in a small volume. We assembled the whole process in our lab, including air sampling through electrostatic precipitation, RNA extraction, enrichment, and detection via RT-qPCR. Our sensitivity is in the low range of what encountered for COVID or influenza in enclosed spaces and the sensing time is about one hour.

Data-guided Learning and Control of Higher Order Structures

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Abstract

The objective of this project is to develop a principled data-guided framework for learning and control of higher order structure, function, and dynamics. Underlying our framework are the concepts of tensor algebra, hypergraphs, and nonlinear dynamics and control. We are building our framework on biological data from cell and genome dynamics, anticipating direct applications in cell reprogramming and wound healing. This biology-inspired data-guided framework, beyond having a high impact on the human genome, is generally applicable to other complex systems including supply chain networks, opinion dynamics, synchronization and disease spread.

We will present an extension of the matrix Kronecker product to tensor Kronecker product, and present several new algebraic, spectral, structural, and tensor decomposition properties associated with the product. We also apply the tensor Kronecker product for defining Kronecker hypergraphs which provides a systematic and scalable framework for composing networks. We further introduce the concept of observability of hypergraphs and discuss an associated computational framework based on polynomial dynamical system and nonlinear observability concepts. We also highlight the development of our hypergraph toolbox, where we are integrating various hypergraph related computational techniques into a comprehensive implementation. Finally, we conclude with description of ongoing work on data collection for capturing genomic form and function and genome-wide imaging across a tissue, and applications of our theoretical and computational hypergraph framework for data-guided learning and control.

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Functional and Distributional Control of Ensemble Systems using Moment Kernel Machines

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Abstract

Applications concerning with the control of large ensembles of systems have emerged in pervasive areas of science and technology and have been driving the advancement of theory of ensemble control. In this talk, problems of functional and distributional control of ensemble systems with respect to different types of aggregated measurements related to system labels will be discussed. A moment kernel machine (MKM) transforming these challenging infinite-dimensional control problems to a dual space of the state space will be presented. The proposed moment transform quantizes ensemble systems and reveals distinctive structures which enable tractable systems-theoretic analysis, control design, and dynamic learning. The generalization and connection of the MKM to information geometry, optimal transport, data-driven control, and control on fibered manifolds will also be illustrated.

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Distinguished Sets of Lie Algebras and Their Applications

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Abstract

We call a finite, spanning set of a semi-simple real Lie algebra a *distinguished set* if it satisfies the following property: The Lie bracket of any two elements out of the set is, up to some constant, another element in the set; conversely, for any element in the set, there are two elements out of the set whose Lie bracket is, up to some constant, the given element. In this talk, we will first show that every semi-simple real Lie algebra has a distinguished set, and then describe how these sets can be used for designing continuum ensembles of nonholonomic systems with provable guarantees on both controllability and observability.

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Optimal Transport with Sign-indefinite Structure: A nonlinear Sinkhorn algorithm & its application to gene networks

Allen Tannenbaum and Tryphon T. Georgiou *
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Abstract

In this year's review we will present a recent development on an inverse problem that pertains to the structure of networks with promotion/inhibition links. The motivation for this work stems from gene regulatory networks and the need for an in depth analysis of their functionality to predict clinical response to drugs.

We introduce a new formalism for inverse problems to estimate network parameters for graphs that encode promotion/inhibition relation between nodes, based on expression (invariant distributions) at the node sites. Thus, we seek a suitable coupling between nodes which, in contrast to standard theory, it is now sign-indefinite. The sign assigned to relations quantifies promotion/inhibition strength between nodes. To this end we introduce a formalism where the transition mechanism (transition probabilities) between nodes is not restricted to be non-negative. Early proponents of expanding the model of probability to allow "negative probabilities" were notable scientists, as Paul Dirac and Richard Feynman.

We develop a generalization of the classical Sinkhorn iteration for indefinite transport. Our algorithm shares with the classical Sinkhorn algorithm the iterative nature in correcting a prior successively along different directions, until convergence to a posterior that explains the data. However, while at each iteration the Sinkorn algorithm amounts to scaling rows or columns, the scaling in our algorithm distinguishes the sign of the prior and applies multiplication or division, accordingly. It is similarly efficient to the classical Sinkhorn and dispays the same numerical convergence properties. It reduces to the classical Sinkhorn when the prior is sign definite.

We note that in modern applications of Optimal Mass Transport, the classical Sinkhorn algorithm is typically the computational tool of choice. The original conception of what came to be known as the Sinkhorn argorithm goes back to Erwin Schrodinger in 1931/32, and was studied by R. Fortet shortly afterwards. Richard Sinkhorn's contribution in the 1960's, in Statistics, established the algorithm as a standard numerical tool. Our version that aims at sign-indefinite priors represents a non-linear generalization of the classical version of the algorithm.

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Distributed asynchronous non-convex optimization: Fundamental limits of convergence rates

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Abstract

Non-convex optimization algorithms arise throughout learning and autonomy, and algorithms to solve such problems are often decentralized, whether deliberately to accelerate computations or out of necessity through having many interacting autonomous agents. These decentralized computations are often asynchronous, simply because synchronization is sometimes difficult or impossible to achieve. For total asynchrony — that is, with delays in communications and computations that are finite but unbounded — it is known that it is faster to compute asynchronously than it is to wait to synchronize. For partial asynchrony — that is, with delays that are bounded — we provide a complementary result that shows that, under certain mild conditions, it is faster for a network of agents to stop and synchronize rather than operating asynchronously. Thus, it is desirable to stop and synchronize if possible, even if it is difficult to do so. This result applies to cases in which the timing of all agents' communications and computations can be tightly controlled. This talk will present this result in detail, as well as other future plans for the development of novel optimization algorithms for non-convex problems.

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Resilience and guaranteed task completion for partially unknown nonlinear control systems

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Abstract

The ability of a system to correctly respond to a sudden adverse event is critical for high-level autonomy in complex, changing, or remote environments. By assuming continuing structural knowledge about the system, classical methods of adaptive or robust control largely attempt to design control laws which enable the system to complete its original task even after an adverse event. However, catastrophic events such as physical system damage may render the original task impossible to complete. In other words, any control law that attempts to complete the task is doomed to be unsuccessful. Thus, an autonomous planner should recognize the task as impossible to complete, propose an alternative that can be completed given the current knowledge, and formulate a control law that drives the system to complete this new task. To do so, we develop a twin framework of resilience and guaranteed performance. Combining methods of optimal control, online learning, and reachability analysis, our approach offers an ability to compute a set of tasks completable under all system dynamics consistent with the planner's partial knowledge and use online learning methods to design an appropriate control law.

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Dynamical Theory on Efficacy of Reservoir Computing

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Abstract

Reservoir Computing (RC) is an approach to constructing and training artificial neural networks that specifically targets time series data of deterministic dynamical systems. A successfully trained RC is a surrogate model constructed from measurement data. Like many other machine learning based approaches, RC is completely data driven, black-box, and generally successful when tasked at predicting time series data. Unlike its peers, however, the RC approach shows astounding ability to predict chaotic systems, while also boasting short training times (due to its linear readout) and small memory footprint. In this presentation, we attempt to peel back the black-box nature of RC using dynamical systems theory, particularly that of contraction and generalized synchronization. We show that entire classes of Recurrent Neural Networks (RNN) will exhibit a unique and stable generalized synchronization with the measured dynamical system. After training, this RNN is then 'converted' to a single-layer feedforward network with a linear readout. We argue that this feedforward network can and will learn dynamics that are topologically conjugate to the measured data.

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Topology in optimization, global stabilization and system equivalence

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Abstract

We review some of our recent findings on topological issues in optimization, stabilization and system equivalence. In particular, we present results on the qualitative behavior of gradient flows of Morse functions initialized near maxima, and its genericity, necessary conditions for global stabilization using arbitrary continuous feedback defined over arbitrary fibre bundles and, if time permits, new topological obstructions to system equivalence.

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Dynamic operads

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Abstract

I'll discuss our abstract compositional approach to composable and adaptive dynamical systems. In particular, I'll explain how a compositional theory of classical mechanics is almost the same as that of deep learning; the difference is only in the skew homomorphism $T^*M \to TM$, which for mechanics is given by a Poisson structure and for neural networks is given by a Riemannian structure. The compositional structure of a trainable neural network made of many interacting neurons is replaced piece-by-piece with that of a Hamiltonian physical system made of many interacting subsystems.

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Effective Whitney Stratification of Real Algebraic Varieties

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Abstract

We describe an algorithm to compute Whitney stratifications of real algebraic varieties. The basic idea is to first stratify the complexified version of the given real variety using conormal techniques, and then to show that the resulting stratifications admit a description using only real polynomials. This method also extends to stratification problems involving certain basic semialgebraic sets as well as certain algebraic maps. One of the map stratification algorithms described here yields a new method for solving the real root classification problem.

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Mathematically Justified Computational Platform for Nonlinear Dynamics

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Abstract

We present our current progress on the development of a *combinatorial topological, computationally efficient, theory of dynamics.* The broader goal consists of at least three overlapping parts: theory, applicability, and applications. However, we focus on applicability, that is, solving the mathematical, algorithmic, and computational challenges that allow the theory to be used to solve problems of practical interest. Here we present developments related to the following two aims: *Develop Surrogate Models of Dynamics* and *Develop Computationally Efficient Combinatorial Representations of Dynamics.* We will discuss combinatorial and topological methods to characterize dynamics and present the following instantiations of this approach: (1) The use of a Gaussian process as a surrogate model to characterize the dynamics of data and obtain confidence levels on this characterization; (2) A combinatorial description of dynamics of networks (DSGRN) that, via order theory and algebraic topology, can characterize the dynamics over all of parameter space.

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Uncertainty-Aware Guidance for Target Tracking subject to Intermittent Measurements using Motion Model Learning

Andres Pulido[1], Kyle Volle[2], Zachary I. Bell[3], Prashant Ganesh[4] and Jane Shin[1]

Abstract

This work presents a novel guidance law for target tracking applications where the target motion model is unknown and sensor measurements are intermittent. The target motion model is trained based on previous measurements and used in the prediction step of a particle filter estimating the target state. The information-driven guidance law calculates the best next goal position by the maximum expected entropy reduction of target state. A simulation and hardware experiments with a quadcopter and a turtlebot were performed to demonstrate the presented guidance law outperforms two baseline guidance methods.

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The Geometry of Hybrid Dynamical Systems: From Intrinsic Properties to Robust Hybrid Geometric Control

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Abstract

Many control problems are difficult to solve due to topological obstructions that are intrinsic to the system being controlled. Such obstructions emerge in most autonomous vehicles problems. In particular, achieving robust global asymptotic stability of the attitude of a rigid body is rife with topological difficulty stemming from the very structure of the rigid body state space, which, unavoidable, includes the special orthogonal group of order three. Although geometric controllers can be designed to avoid singularities associated with a local chart, designing a smooth global controller on a compact manifold or a compact Lie group is nontrivial. Fortunately, it is possible to achieve global and robust asymptotic stability using hybrid feedback controllers. However, most hybrid control algorithms in the literature embed the ambient manifold (or space) in Euclidean space, namely, they rely on coordinates. In fact, the design of hybrid controllers that are geometric – in the sense that they are coordinate free – has not been explored much, with only a few special cases of manifolds treated thus far. Arguably, one reason for such lack of development is perhaps the fact that most theories for the study of hybrid dynamical systems focus on hybrid systems written on specific coordinates. In this project, we propose to fill this gap by generating tools that exploit the natural dynamics and geometry of the system to be controlled, and lead to i) methods for the study of geometry and intrinsic properties of hybrid dynamical systems, given rise to a new framework that we refer to as *geometric hybrid dynamical systems*, and ii) tools for the design of geometric hybrid control algorithms assuring that the desired properties hold with robustness to uncertainty. This talk will provide an overview of this project, by introducing the problems being studied and by outlining initial results and ideas.

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Multi-channel time domains and clustering protocols for large-scale interconnections of hybrid systems

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Abstract

Hybrid systems combine continuous change and instantaneous change. Over the last two decades, significant progress has been made on stability theory and control design for, or using, hybrid systems. Recently, there has been a new push to better understand how to model and describe large-scale interconnections of hybrid systems. For this purpose, this talk introduces the notions of multi-channel hybrid time domains and clustering protocols. It then shows how these notions facilitate working with large-scale interconnections of hybrid systems. A distributed, hybrid average consensus algorithm is used to illustrate the new concepts.

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Averaging Tools for a Class of Stochastic Hybrid Dynamical Systems with Multi-Time Scale Flows

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Abstract

We present new tools for studying deterministic and stochastic hybrid dynamical systems that exhibit multi-time scale behaviors in the continuous-time dynamics of the system. The tools rely on Lyapunov-like functions that are suitable for the study of stability and/or recurrence of sets in singularly perturbed hybrid dynamical systems, as well as in systems that incorporate flow maps having well-defined average mappings with respect to a fast-varying state. Given that for stochastic hybrid systems the notion of semi-global stability can be ill-posed, we focus on a class of systems for which global stability and recurrence results are feasible under suitable smoothness assumptions on the dynamics. By introducing and studying the average stochastic hybrid dynamics of the original system, and by leveraging the existence of suitable Lyapunov-Foster functions for this system, we derive stability and/or recurrence results for the original stochastic hybrid dynamics. The proposed tools are then used for the analysis and synthesis of adaptive parameter estimation algorithms, and also for the study of deterministic hybrid algorithms operating in unknown/adversarial environments. We finish by discussing new architectures of averaging-based (deterministic) hybrid algorithms suitable for the solution of model-free optimization and stabilization problems that do not admit non-hybrid solutions.

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Probabilistic Invariance and Data-Driven Reachability for Gaussian Process State Space Models

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Abstract

Gaussian process state space models provide a link between traditional control methods that rely on established state models and purely data-based approaches that lack prior knowledge of the model. These models enable us to represent a nominal model as the mean of the process, with the variance diminishing as additional data is acquired. In this presentation, we provide safety guarantees for Gaussian process state space models in the form of probabilistic invariant sets, where the state trajectory is guaranteed to lie within an invariant set for all time with a particular probability. We then present a method for designing state-feedback controllers that maximize the probability of remaining within the probabilistic invariant set subject to input and state constraints. Lastly, we compute finite-horizon forward reachable sets for Gaussian process state space models, providing exact probability measures for state trajectories of arbitrary length. We will present examples that demonstrate the power of this approach, such as providing highly non-convex reachable sets and detecting holes in the reachable set.

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Piecewise-deterministic Markov processes: abrupt context changes and structured uncertainty

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Abstract

Piecewise-Deterministic Markov Processes (PDMPs) can be interpreted as an unusual type of hybrid systems, in which the discrete component of the state switches through an (inhomogeneous) continuous-in-time Markov chain. Such mode-switches do not instantaneously affect the continuous component of the state, which has deterministic (mode-dependent) dynamics in between the switch events. I will describe the general framework of PDMPs, focusing on their usefulness in modeling and the associated computational challenges. This will be illustrated by applications in ecology and surveillance-evasion. Joint work with Marissa Gee and former REU students at Cornell University.

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Topological Methods for Assured Transitions in Hybrid Systems

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Abstract

A critical and ubiquitous challenge for real-life implementation of high-level plans with low-level (hybrid) controllers is that of ensuring transitions from one domain of the state space to another despite significant state-estimation error originating from model- and state-uncertainties.

For highly symmetric transition boundaries such as spheres and hyperplanes, the knee-jerk approach works well: one plans for the system state to hit anywhere within a prescribed error ball contained in the target region. In the generic case, however, this approach proves to be overly restrictive by far, contributing to the brittleness of the overall hybrid system. Instead, intuitively, one needs to aim at a much larger error ball situated behind the transition boundary—a notion formalized by topological transition guarantees (TTGs).

In a nutshell, given an initial estimated position $p \in \mathbb{R}^d$ and estimation error bound ϱ, a transition to the goal $\mathcal{F}$ is sought by finding reachable points q such that the projected error ball about q upon arrival is separated from p within the error cone, by $\partial\mathcal{F}$. An optimal q is one for which ϱ is maximized.

In this talk, we will formally introduce TTGs with some preliminary work, discuss their basic properties and related problems, and survey applications we will tackle in this project.

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Hybrid Dynamics - Deconstruction and Aggregation: an overview of the project.

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Abstract

I will present the key ideas behind the HyDDRA project: (soft) departure from analysis-driven paradigm; firm reliance on algebraic tools (via algebraic topology and category theory tools); validation of the abstract ideas in domain-specific models.

Robustness with respect to compositions, invariants as analytic tools and descriptors, rules of and obstacles of compositions, and formal verification, - those are stepping stones of the project, highlighted in the consequent talks.

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On invariants, composition, and networks of hybrid systems

Paulo Taduada *
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Abstract

In this talk we will revisit classical notions of invariants and composition for dynamical and control systems towards with the goal of motivating a suitable generalization to networks of hybrid systems. We will regard invariants as abstractions of systems, we will argue that for open systems we must allow "invariants" to vary, and we will explore the role of interconnection as a control mechanism. Our ultimate objective is to distill the knowledge that exists today, fragment across different disciplines, into a unifying framework for the modeling and design of hybrid control systems. Lyapunov theory that can be deployed in practice.

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Toward a Toolbox (and its Use) for Systems ID of Nonlinear Hybrid Dynamical Systems from Data

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Abstract

Agile behavior in animals presents a gold mine of nonlinear hybrid dynamical systems whose modeling, understanding and reuse holds equal interest for organismal biology and robotics. Bernstein's century old fascination with the organization of animal sensorimotor systems heralded decades of subsequent effort to extract the putative low-dimensional structure of purposive mobile manipulation. That slowly accelerating stream of high dimensional kinematic data now flows in ever-greater torrents through increasingly automated motion capture pipelines. The actual extraction of useful motor modules from this flood of data can be rendered in the language of classical dynamical systems theory by recourse to the longstanding traditions of dimension collapse, formalized in terms of normally hyperbolic invariant manifolds (NHAIM). However, that classical formalism holds only partial sway in the face of the making and breaking of limb contacts that play an essential role in the organization and efficacy of animals' and robots' negotiation of their environment.

Our ongoing work aims to build a hybrid extension to a recently initiated computational pipeline for extracting classical NHAIMs from high dimensional kinematic motion capture data. This talk will review the existing pipeline before turning to work in progress targeting its extension to hybrid dynamical systems. Next, attention turns to examples of the use of such hybrid NHAIMs in robotics based upon previous, human-extracted examples of bioinspired restriction dynamics. The talk will conclude with some speculative remarks bearing upon the interaction of these local geometric methods with global topological methods (presently under investigation by Baryshnikov and colleagues) for systems identification of highly nonlinear hybrid dynamical systems from empirical datasets.

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A topological view of design for multi-agent hybrid systems

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Abstract

In this talk, we will discuss design and analysis problems for open hybrid systems using the lens of algebraic topology with the aim of developing algorithms and new types of lower-bounds. First, on the problem of state estimation over finite capacity channels, we will discuss several proposed definitions of topological entropy and recently established relationships among them for switched systems and systems with uncertain inputs.

Next, we will discuss ongoing explorations on how the notions of topological complexity could provide lower-bounds for hybrid multi-agent planning problems. It is well-known that control policies for multi-agent planning problems may have to be hybrid or discontinuous, to meet safety and other logical constraints.

While the complexity of such control problems have been abstractly related to the notions of topological complexity and perplexity, these results have not been concretized and systematized to derive effective lower-bounds for multi-agent planning problems with perception constraints. We will also discuss the roadmap for how these results can be embodied in a hybrid multi-agent design tool for design and analysis.

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A Categorical Perspective on Lyapunov

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Abstract

Lyapunov-based methods provide a powerful means of encoding invariants in systems. Classically, as inspired by the conservation of energy in a mechanical system, Lyapunov functions certify stability via "energy-like" functions. More recently, this has led to the synthesis of wide-classes of controllers for nonlinear and hybrid systems via control Lyapunov functions (CLFs)—the existence of a CLF is necessary and sufficient for smooth stabilization. Control barrier functions (CBFs) extend this approach to safety encoded by forward set invariance—these are again necessary and sufficient. Both CLFs and CBFs have proven useful in practice: a wide range of robotic systems have benefited from these theoretic methods implemented in practice. This talk outlines the first steps of framing Lyapunov theory in a categorical context, i.e., capturing the essential components of Lypunov-like functions for general systems with appropriate categorical structures. This will be motivated by robotic systems—including those with underlying hybrid system models—with a view toward developing a categorical approach to Lyapunov theory that can be deployed in practice.

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Hybrid dynamical systems with slow and fast time-variation and mode switching

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Abstract

For hybrid/switched systems that feature slow parameter drift and infrequent mode transitions, there are several results (including some recent ones by the speaker) showing that stability of the hybrid system is inherited from that of the individual modes, i.e. compositionally robust. On the other hand, for very fast parameter variations and very frequent mode transitions, averaging methods allow us to approximate the hybrid system by an auxiliary averaged system, whose stability informs (with some caveats) stability of the original system. This talk will focus on some ongoing efforts to develop a combination of these two approaches, enabling us to treat hybrid systems as "evolving on three time scales."

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Unified Framework for Invariance and Composition of Open Hybrid Dynamical Systems

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Abstract

The first part of this talk presents the overview of the project to establish a unified framework for the invariance and composition of open hybrid systems by exploiting their geometric and topological structures. This will be achieved by multidisciplinary collaborative efforts in dynamical systems and theoretical computer science, which are focused on three research thrusts: (i) discovery of geometric and topological invariance; (ii) investigating properties of invariance in open, uncertain environments and composition; (iii) extension to large-scale composition and formal verification. The research objectives of each topic are presented with the plan for synergistic integration and verification.

Next, focusing on the openness with respect to uncertainties, we present the current efforts to analyze stochastic hybrid systems. In particular, a computational framework is presented to propagate a probability density function of hybrid states over the flows of a generalized stochastic hybrid system, and further, it is extended to Bayesian estimation and mean field game.

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Geometry, Topology, and Symmetry of Open Smooth and Hybrid Systems

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Abstract

This talk covers two topics in geometry, topology, and symmetry of open systems.

(i) Exploring Symmetry Breaking Impacts: Symmetries are important in the study of dynamical systems as they can provide a recipe for finding invariants. In the context of impact systems, an invariant is produced when the impact is internal. External impacts, however, break this symmetry and can be used to generate non-trivial holonomy which is useful for control applications.

(ii) Open Classical and Quantum Systems: We discuss methods for describing a classical or quantum system interacting smoothly with its environment. In particular we discuss double bracket and other classical notions of nonlinear Rayleigh dissipation. We also discuss metriplectic systems in various settings and relate this to double bracket dissipation and to quantum dissipation. In the quantum setting we consider Lindblad equations and their analysis with applications to controllability and the description of stable asymptotic orbits.

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Interaction Networks, Homological Dynamics, and Control

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Abstract

For multiscale systems, e.g., systems biology, ecology, etc., models often take the form of interaction networks (IN). The nodes in the IN indicate species. A directed edge from node n to node m indicates that x_n, the quantity of species associated with node n, directly influences the rate of production of x_m. We assume that the dynamics associated with an IN can be modeled by an ordinary differential equation, but because these are multiscale systems we do not assume that the nonlinearities that govern the dynamics can be derived from first principles.

Two fundamental questions that we are interested in addressing are as follows.

I. (Analysis) Consider an IN.
   - What dynamic behaviors (e.g., fixed points, periodic orbits, chaotic dynamics, multistability) or sequence of dynamic behaviors can this IN exhibit?
   - How robust/fragile is this behavior to perturbation?

II. (Design) Consider a desired dynamic behavior or sequence of dynamic behaviors.
   - What INs can produce these behaviors?
   - What INs can produce these desired behaviors under a given set of perturbations?

We will outline a novel approach, based on the following four constructs, to address these questions.

1. A combinatorial representation of parameter space.
2. A combinatorial representation of dynamics.
3. A homological characterization of dynamics.

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4. Organization of the information of items 1 - 3 via cellular sheaves.

The development of these constructs represents ongoing work, but in this talk we discuss the theory, algorithms and code that have been developed and describe our current vision for the challenges that remain.

As a simple example to explain the relevance of and interrelations between these constructs we will consider the problem of identifying when an IN can act as a hysteretic switch. We will also propose methods to quantify how robust the switch behavior of an IN is with respect to perturbations.

Modeling, Control, and Trajectory Optimization by Exploiting Lie Group Symmetry

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Abstract

The kinodynamic motion planning or trajectory optimization is fundamental in robotics. The nonconvexity caused by rigid body systems makes it hard for gradient-based methods to find the globally optimal solution. However, the nonconvexity can be coordinate-dependent and resolved via convex relaxation. I will discuss how we formulate the discrete-time planning problem of the rigid body as a Polynomial Optimization Problem (POP) via Lie group variational integrator. We leverage Lasserre's hierarchy of moment relaxation to obtain the globally optimal solution via Semidefinite Programming (SDP). In addition, I will present recent results on modeling and control of rigid body systems by exploiting symmetry. The goal is to illustrate a picture of current progress for rigid body systems and to motivate the proposed work in this project by contrasting the shortcomings.

In the end, I will conclude with a discussion of how these ideas provide the foundations of the proposed work for modeling and learning for high-dimensional open hybrid dynamical systems.

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Real-Time Verification of High-Dimensional Systems via Composition of Reachable Sets

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Abstract

Traditional approaches to perform verification of dynamical systems rely on constructing reachable sets. However, building these reachable sets for high dimensional systems has proven extremely challenging as the complexity of constructing these sets scales exponentially with the size of the state space. These difficulties are compounded for systems undergoing contact. This talk will present preliminary results on efficiently constructing overapproximations to these reachable sets for high dimensional robotic systems that are described as branched kinematic trees. This talk will illustrate how this construction can be efficiently represented within an online optimization framework to perform provably safe motion planning. The talk will conclude with a discussion about how this project will explore how to extend this formulation to high-dimensional dynamical systems undergoing contact.

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Compositional Reactive Planning for Complex Tasks using Topological Invariants of Strategy Spaces

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Abstract

Problems in distributed control of autonomous mobile agent systems (MAS) possess an underlying hybrid structure whose modes are indexed by the admissible communication protocols and the connectivity structure they impose (e.g. connected components). Such hybrid structures may often be naturally regarded as open parallel compositions of simpler systems: 'open', since real-time decisions are required on the appropriate operational mode; 'parallel composition' since the MAS behavior in each mode may often be broken up as an interconnected system of smaller MASs of the same kind. In addition, autonomy requires resilience to agent loss, a capability to assimilate new members, and, more generally, adaptation of the communication protocols to changes in the state of the MAS in relation to the task at hand. All of these features indicate that the design of autonomous MAS may benefit from a category-theoretic viewpoint of their compositional nature.

In this talk, I will outline an approach to leveraging this kind of compositionality in MAS controller design. A fundamental challenge lies with formalizing tasks and generating computationally tractable certificates of task achievability. Recalling Erdmann's construction of "strategy spaces" for open finite transition systems, using which he characterized the solvability of navigation problems in such systems using the homotopy type of the corresponding strategy spaces, I propose a focus on establishing a categorical framework for open hybrid dynamical systems, for which putative extensions of Erdmann's construction of a strategy space will be functorial. Constructing such strategy spaces will enable the use of their homotopy type as certificates of task feasibility, while the functoriality of the construction will facilitate the study of such certificates for composed systems, regarded as functions of the certificates produced by their components.

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