Applications of quantum probability theory to human-machine communication networks (FA9550-20-1-0027

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AFOSR Program Review:
Computational Cognition and Machine Intelligence Program
(October 7, 2020, Zoom)

Applications of quantum probability theory to human-machine communication networks

Busemeyer, IU; Wang, OSU, Balakrishnan, MUST

Objective:

- Develop and test quantum probability theory for human communication networks.
- Apply and test theories with dynamic information flows in larger networks.

Approach:

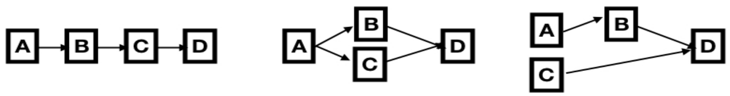
- Mathematical development
- Computational modeling and empirical testing

DoD Benefits:

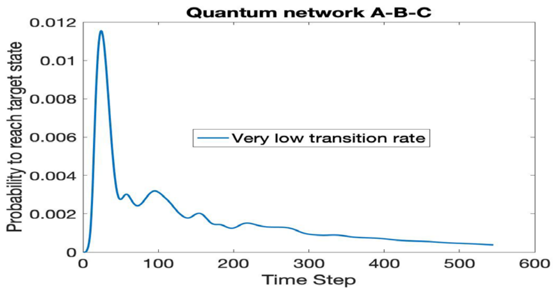
- Predict human-machine team performance interactions in networks
- optimize, and control networks for field tasks with limited time and resource

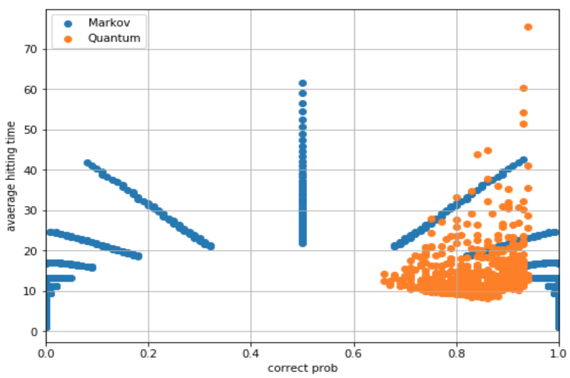
Progress: (Only 6 months into new grant)

- Developed quantum dynamic models for searching and communicating in networks
- Compared search and communication rates in quantum and Markov networks



Different types of command control communication networks





Quantum walks systematically detect communication at correct target faster than Markov walks

List of New Project Goals

1. The first goal is aimed to investigate small human network communications (like chains of command). We plan to develop and test quantum probability theory for human communication networks by generalizing our previous theoretical developments for dynamic and strategic decisions to human communication through networks.
2. Experimentally test the new theory with human data using human teams communicating sequentially through networks to make decisions.
3. Compare the predictive accuracies of the new theories to traditional theories.
4. Apply and test theories with dynamic information flows in larger networks. Theories from the first objective will be applied to larger networks and used to investigate dynamic network flows with interacting human and artificial agents.
5. Ultimate goal is to predict, optimize, and control human-machine team networks for field tasks with limited time and resource

Progress Towards New Goals

(only 6 months into new grant)

At this early stage we have mainly worked on theory and computation

1. Developed applications of quantum-Markov open systems for describing dynamics of evidence accumulation
2. Compared speed of “opinion consensus” for quantum versus Markov communication networks on one dimensional linear type lattices with asymmetric transition rates
3. Started developing theory for building quantum networks with asymmetric transition rates for networks on small but arbitrary graphs

Recent Additions to Previous Grant

Applications of Quantum Theory to Strategic Decision Making

1. Wang, Busemeyer & deBuys submitted a new manuscript to a special issue on "Extending Rationality" to appear in *Topics in Cognitive Science* titled "Beliefs, actions, and rationality in strategical decisions."

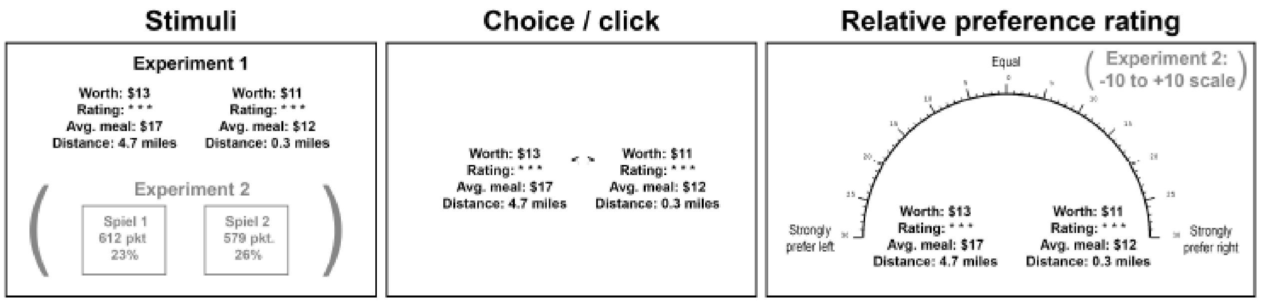
This reports our work applying quantum models to account for interference effects of predictions on actions in social-economic type games.

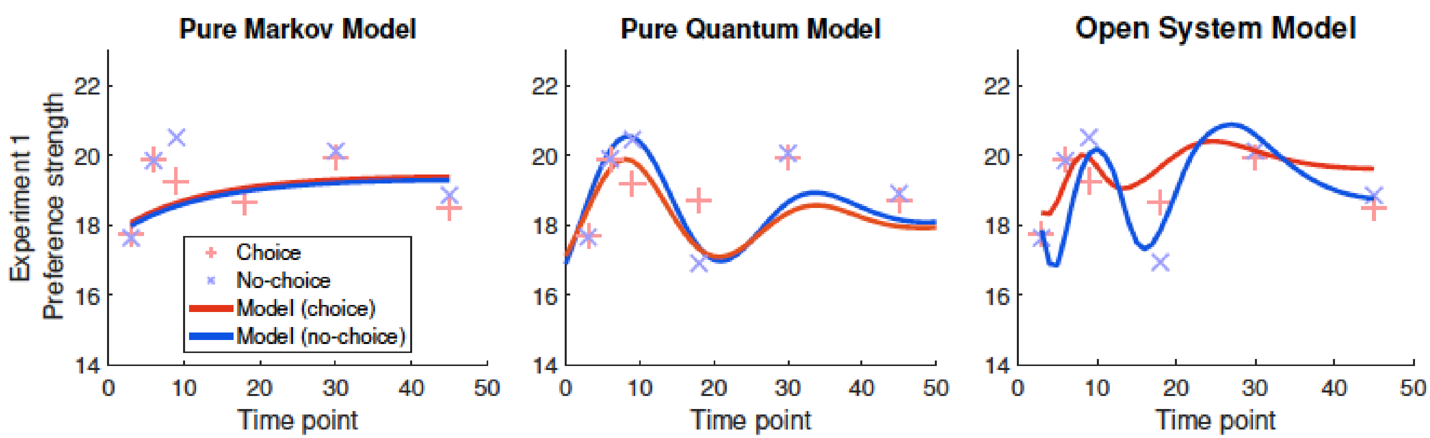
2. Zhang, Balakrishnan & Busemeyer are preparing a new manuscript to submit to *IEEE Transactions on man, systems, cybernetics* titled "Strategic Driver-Assist Systems to Mitigate Inattention in Drivers with Open-Quantum Cognition Models"

We developed a computational model for a drive-assist car problem within a game-theoretic setting. An artificial car-agent assesses the road condition, and the human driver seeks to drive safely using the car-agent's advice. We model the decisions of the human to follow or not the car-agent's advice using a quantum open system. At the same time, the car-agent learns the parameters of the human's decision system to optimize advice to the human. Numerical results are presented to illustrate results of both simultaneous and subgame perfect equilibria.

1. New Research on Quantum-Markov Open System Dynamics

Kvam, P. D., Busemeyer, J. R., & Pleskac, T. J. (2020, submitted) Temporal oscillations in preference strength: Evidence for an open system model of constructed preference. Under review at *Scientific Reports*





Quantum-Markov Open System Model

Busemeyer, J. R., Zhang, Q., Balakrishnan, S. N., Wang, Z. (2020) Application of Quantum—Markov Open System Models to Human Cognition and Decision. *Entropy,* 22, 990; e22090990

Open systems master equation

$$\frac{d}{dt}\rho(t) = -i\cdot(1-w)\cdot[H,\rho(t)] + w\cdot\sum_{i,j}\gamma_{ij}\cdot\left(\left(L_{ij}\cdot\rho(t)\cdot L_{ij}^{\dagger}\right) - .5\cdot\{(L_{ij}^{\dagger}\cdot L_{ij}),\rho(t)\}\right).$$

Density Matrix

Quantum Dynamics

Lindblad (Markov) Dynamics

$$\rho = \sum p_i\left(\psi_i\cdot\psi^{\dagger}{}_i\right)$$

Density contains both epistemic (p_i) and ontic ψ_i uncertainty

$$L_{ij} = \left|i\right\rangle\left\langle j\right|$$

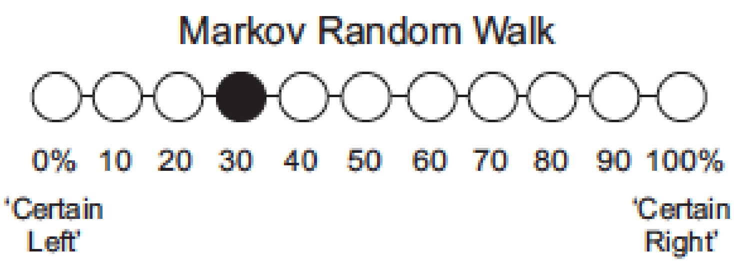
Transition operator from state j to state i

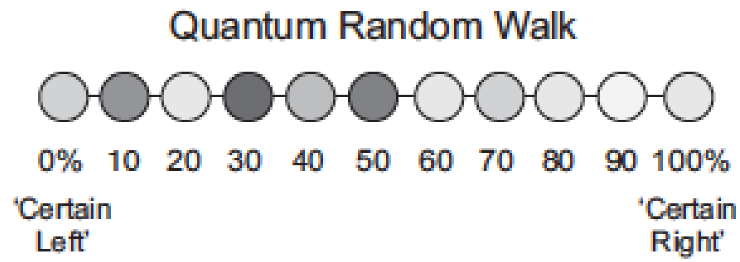
$$\gamma_{ij} =$$

Probability to transit from state j to state i

2. New research comparing classical versus quantum walks in social communication networks

- Several articles have established quadratic and sometimes exponential speed up in network search using quantum walks compared to Markov walks.
- These models could also be used to understand opinion or consensus reaching produced by communication in social networks
- Previous work comparing quantum and Markov models has been limited to symmetric/undirected walks
- We have started comparing quantum and Markov walks using asymmetric/directed walks on a line
- Our work uses the continuous time quantum walk. The direction of evolution is produced by the choice of potential function on the diagonal of the Hamiltonian





$$K = \begin{bmatrix} -\alpha & \beta & & & & & 0 \\ \alpha & -(\alpha+\beta) & \beta & & & & \\ & \alpha & -(\alpha+\beta) & \ddots & & & \\ & & \alpha & \ddots & \beta & & \\ & & & \ddots & -(\alpha+\beta) & \beta & \\ & & & & \alpha & -(\alpha+\beta) & \beta \\ 0 & & & & & \alpha & -\beta \end{bmatrix}$$

$$H = \begin{bmatrix} 1\cdot\beta & \sigma & & & & & 0 \\ \sigma & 2\cdot\beta & \sigma & & & & \\ & \sigma & 3\cdot\beta & \ddots & & & \\ & & \sigma & \ddots & \sigma & & \\ & & & \ddots & (N-2)\,\beta & \sigma & \\ & & & & \sigma & (N-1)\,\beta & \sigma \\ 0 & & & & & \sigma & N\cdot\beta \end{bmatrix}$$

$$P(\tau) = e^{\tau \cdot K}$$

$$U(\tau) = e^{-i\cdot\tau\cdot H}$$

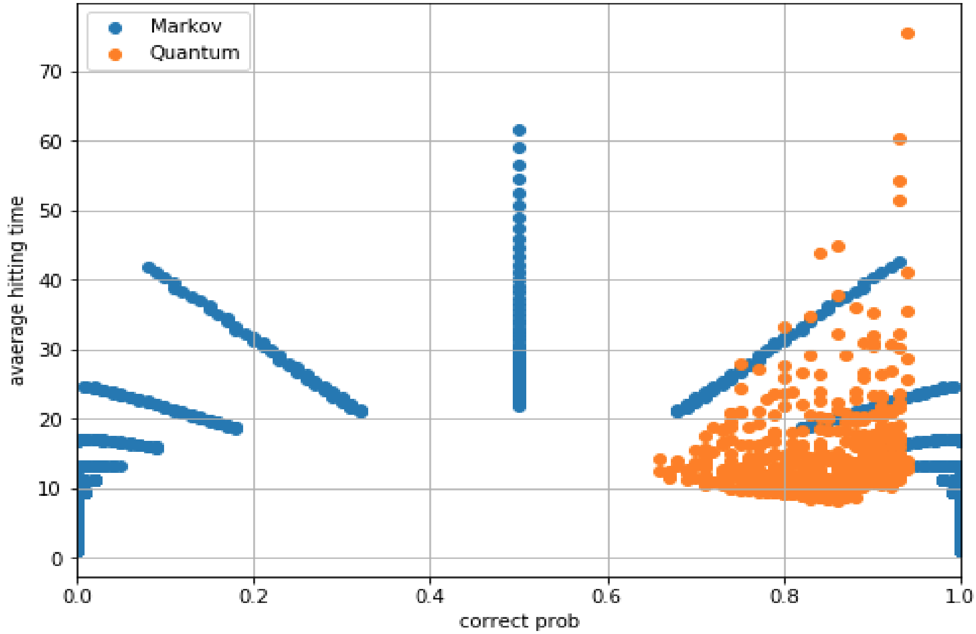
$$p_{error}(n+1) = \|(M_I \cdot P(\tau)) \cdot (M_C \cdot P(\tau))^n \cdot \phi_0\|^1$$
$$p_{correct}(n+1) = \|(M_G \cdot P(\tau)) \cdot (M_C \cdot P(\tau))^n \cdot \phi_0\|^1$$

$$p_{error}(n+1) = \|(M_I \cdot U(\tau)) \cdot (M_C \cdot U(\tau))^n \cdot \psi_0\|^2$$
$$p_{error}(n+1) = \|(M_G \cdot U(\tau)) \cdot (M_C \cdot U(\tau))^n \cdot \psi_0\|^2 .$$

Mean Detection Time as a function of probability of detecting correct alternative for Markov and quantum models

(computations done by Adam Huang, UG, Carleton University)

N = 101 states.
Varying parameters of each model across a wide range of values.
Each point represents one combination of model parameters.

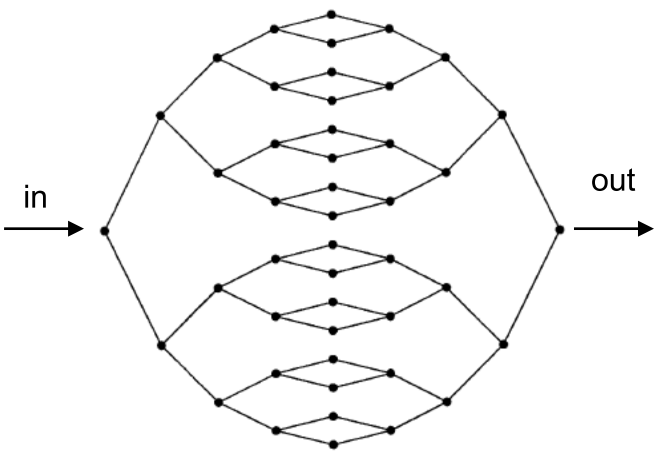


Generally, the mean time to detect for the quantum model is shorter than the Markov model, equating probability of detecting the correct target

Quantum Walks on Small graphical networks

Past work with quantum models on graphs assumed symmetric/undirected walk

(e.g., Childs, AM, Farhi, E & Gutmann, S (2002) An Example of the Difference Between Quantum and Classical Random Walks. *Quantum Information Processing, Vol. 1*, Nos. 1/2)



given by the quantum Hamiltonian with matrix elements[1]

$$\langle a|H|b\rangle = M_{ab}$$

$$M_{ab} = \begin{cases} -\gamma & a \neq b, a \text{ and } b \text{ connected by an edge} \\ 0 & a \neq b, a \text{ and } b \text{ not connected} \\ k\gamma & a = b, k \text{ is the valence of vertex } a \end{cases}$$

We are developing quantum walks on graphs with asymmetric rates of transitions using control U gates

Suppose $A1, A2, A3$ are agents and each agent may or may not be infected (by an idea) and one agent can spread the infection to another, but also an agent can remove an infection. $A1$ interacts with $A2$ and $A2$ interacts with $A3$. $0 =$ not infected, $1 =$ infected. $z = A3$ state, $y = A2$ state, $x = A1$ state.

$$A1 \longleftrightarrow A2 \longleftrightarrow A3$$

$$\begin{array}{ccc} A3 & A2 & A1 \\ z & y & x \end{array}$$

Directed Markov model

In column

$$T = \begin{array}{cc} & \begin{array}{cccccccc} 000 & 001 & 010 & 011 & 100 & 101 & 110 & 111 \end{array} \\ \begin{array}{c} 000 \\ 001 \\ 010 \\ 011 \\ 100 \\ 101 \\ 110 \\ 111 \end{array} & \begin{bmatrix} q_0 & p & p & 0 & p & 0 & 0 & 0 \\ (1-q_0)/3 & q & 0 & p & 0 & p/2 & 0 & 0 \\ (1-q_0)/3 & 0 & q & 0 & 0 & 0 & 0 & 0 \\ 0 & r & r/2 & q & 0 & 0 & 0 & (1-q_1)/3 \\ (1-q_0)/3 & 0 & 0 & 0 & q & p/2 & p & 0 \\ 0 & 0 & 0 & 0 & 0 & q & 0 & (1-q_1)/3 \\ 0 & 0 & r/2 & 0 & r & 0 & q & (1-q_1)/3 \\ 0 & 0 & 0 & r & 0 & r & r & q_1 \end{bmatrix} \end{array}$$

Out row

Transition probabilities are not symmetric

Quantum Model with directed walk

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 000 & 001 & 010 & 011 & 100 & 101 & 110 & 111 \end{bmatrix}$$

$$\alpha_C = \left[\begin{array}{cccccccc} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$
$$\alpha_T = \left[\begin{array}{cccccccc} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right].$$

$$U = \begin{bmatrix} U_1 & & & \\ & U_2 & & \\ & & \ddots & \\ & & & U_8 \end{bmatrix}.$$

$$M_0 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, M_1 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$
$$M_s = diag\left[\begin{array}{cccccccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right], M_c = I_8 - M_s.$$

$$U_\theta = \begin{bmatrix} cos(\theta) & -sin(\theta) \\ sin(\theta) & cos(\theta) \end{bmatrix}$$

U1, 000

$$\begin{aligned} U_X &= I_4 \otimes U_{\theta x} \\ U_Y &= I_2 \otimes U_{\theta y} \otimes I_2 \\ U_Z &= U_{\theta z} \otimes I_4 \\ U_1 &= U_z \cdot U_y \cdot U_x \end{aligned}$$

U2, 001

$$\begin{aligned} U_Y &= I_2 \otimes (I_2 \otimes M_0 + U_{\theta y} \otimes M_1) \\ U_X &= I_2 \otimes (M_0 \otimes U_{\theta x} + M_1 \otimes I_2) \\ U_2 &= U_X \cdot U_Y \end{aligned}$$

U3, 010

$$\begin{aligned} U_X &= I_2 \otimes (M_0 \otimes I_2 + M_1 \otimes U_{\theta x}) \\ U_Z &= (I_2 \otimes M_0 + U_{\theta z} \otimes M_1) \otimes I_2 \\ U_{Yx} &= I_2 \otimes (U_{\theta yx} \otimes M_0 + I_2 \otimes M_1) \\ U_{Yz} &= (M_0 \otimes U_{\theta yz} + M_1 \otimes I_2) \otimes I_2 \\ U_3 &= U_{Yx} \cdot U_{Yz} \cdot U_Z \cdot U_X \end{aligned}$$

U4, 011

$$\begin{aligned} U_Y &= (M_0 \otimes U_{\theta y} + M_1 \otimes I_2) \otimes I_2 \\ U_Z &= (I_2 \otimes M_0 + U_{\theta z} \otimes M_1) \otimes I_2 \\ U_4 &= U_Y \cdot U_Z \end{aligned}$$

U5, 100

$$\begin{aligned} U_Y &= (M_0 \otimes I_2 + M_1 \otimes U_{\theta y}) \otimes I_2 \\ U_Z &= (U_{\theta z} \otimes M_0 + I_2 \otimes M_1) \otimes I_2 \\ U_5 &= U_Z \cdot U_Y \end{aligned}$$

U6, 101

$$\begin{aligned} U_X &= I_2 \otimes (I_2 \otimes M_0 + U_{\theta x} \otimes M_1) \\ U_Z &= (M_0 \otimes I_2 + M_1 \otimes U_{\theta z}) \otimes I_2 \\ U_{Xy} &= I_2 \otimes (M_0 \otimes U_{\theta xy} + M_1 \otimes I_2) \\ U_{Zy} &= (U_{\theta zy} \otimes M_0 + I_2 \otimes M_1) \otimes I_2 \\ U_6 &= U_{Zy} \cdot U_{Xy} \cdot U_Z \cdot U_X \end{aligned}$$

U7, 110

$$\begin{aligned} U_X &= I_2 \otimes (M_0 \otimes I_2 + M_1 \otimes U_{\theta x}) \\ U_Y &= I_2 \otimes (U_{\theta y} \otimes M_0 + I_2 \otimes M_1) \\ U_7 &= U_Y \cdot U_X \end{aligned}$$

U8, 111

$$U_8 = I_8$$

Recursion

$$\psi_0 = \alpha_C \otimes \alpha_T$$
$$\rho_0 = \psi_0 \cdot \psi_0^\dagger$$

First step

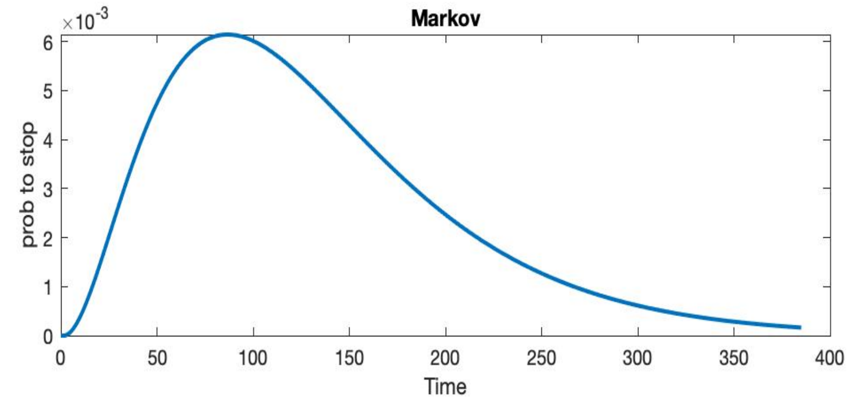
$$\rho_T(1) = Tr_C\left(U \cdot \rho_0 \cdot U^\dagger\right)$$
$$p_c(1) = Tr\left(M_c \cdot \rho_T(1) \cdot M_c\right)$$
$$\rho_c(1) = \frac{M_c \cdot \rho_T(1) \cdot M_c}{p_c(1)}$$

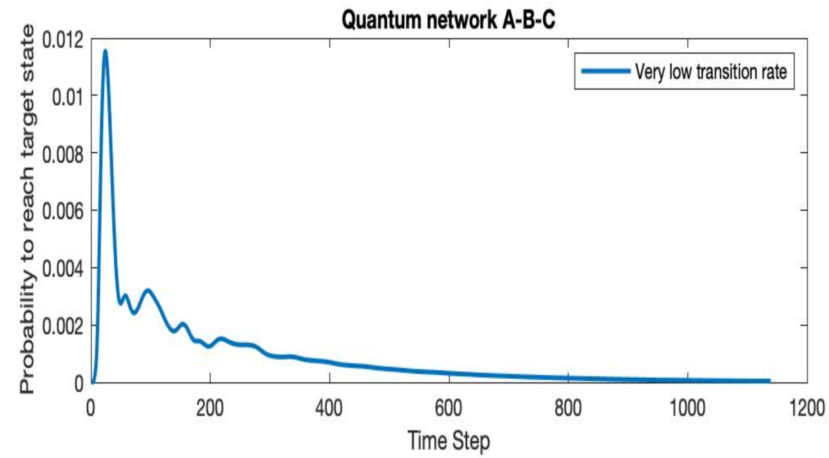
$$P_c(1) = p_c(1)$$
$$P_s(1) = 1 - p_c(1).$$

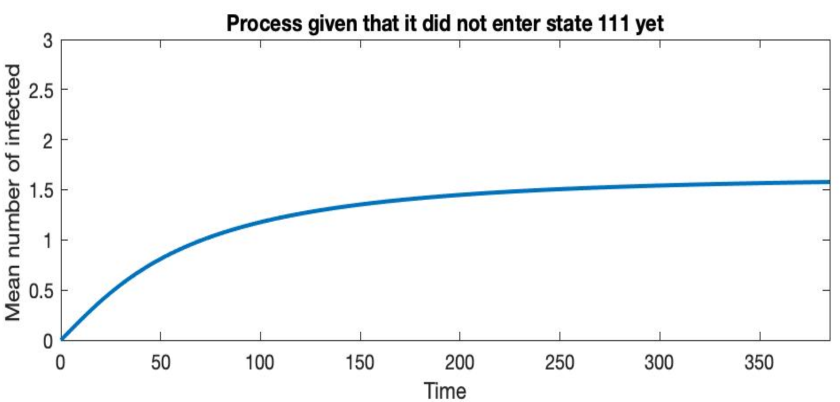
For $t > 1$,while $\sum_{\tau=1}^{t} P_s(\tau) < 1$, using lifting operation

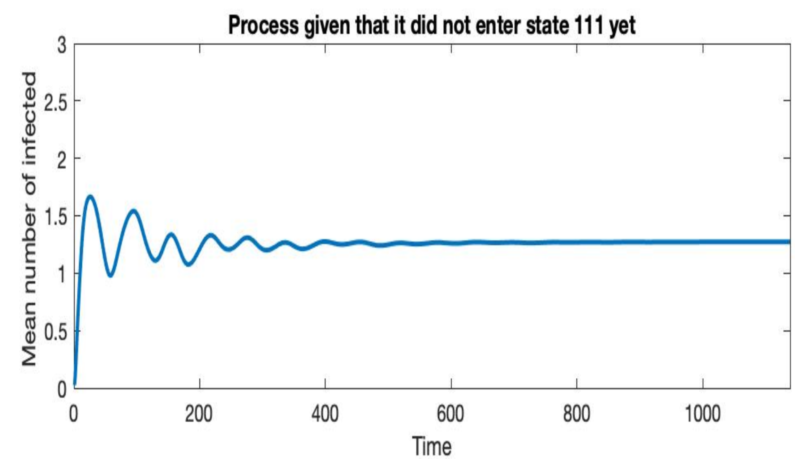
$$\rho(t) = \rho_c(t-1) \otimes \rho_c(t-1)$$
$$\rho_T(t) = Tr_C\left(U \cdot \rho(t-1) \cdot U^\dagger\right)$$
$$p_s(t) = Tr\left(M_s \cdot \rho_T(t) \cdot M_s\right)$$
$$p_c(t) = Tr\left(M_c \cdot \rho_T(t) \cdot M_c\right)$$
$$\rho_c(t) = \frac{M_s \cdot \rho_T(t) \cdot M_s}{p_s(t)}$$

$$P_c(t) = P_c(t-1) \cdot p_c(t)$$
$$P_s(t) = P_c(t-1) \cdot p_s(t).$$


Markov
×10^-3
prob to stop
0
1
2
3
4
5
6
0
50
100
150
200
250
300
350
400
Time


Quantum network A-B-C
Very low transition rate
Probability to reach target state
0
0.002
0.004
0.006
0.008
0.01
0.012
0
200
400
600
800
1000
1200
Time Step


Process given that it did not enter state 111 yet
Mean number of infected
0
0.5
1
1.5
2
2.5
3
0
50
100
150
200
250
300
350
Time


Process given that it did not enter state 111 yet
Mean number of infected
0
0.5
1
1.5
2
2.5
3
0
200
400
600
800
1000
Time

List of Publications, Awards, Honors, etc. Attributed to the Grant

1. Busemeyer, J. R., Zhang, Q., Balakrishnan, S.N., & Wang, Z. (2020) Application of Quantum-Markov Open System Models to Human Cognition and Decision. *Entropy*, 22, 990; e22090990
2. Busemeyer, J. R., Kvam, P. D., & Pleskac, T. J. (2020) Comparing Markov Decision Models with Quantum Decision Models. *WIREs Cognitive Science*. e1576
3. Broekaert, J. B., Busemeyer, J. R., and Pothos, E. M. (2020) The Disjunction Effect in two-stage simulated gambles. An experimental study and comparison of a heuristic logistic, Markov and quantum-like model. *Cognitive Psychology*. *117*,
4. Busemeyer, J. R. & Wang, Z. (2019) Hilbert space multidimensional modeling of continuous measurements. *Philosophical Transactions A*, 377(2157), 20190142.
5. Busemeyer, J. R., Kvam, P. D., & Pleskac, T. J. (2019) Markov versus quantum dynamic models of belief change during evidence monitoring. *Scientific Reports, 9,* 18025
6. Busemeyer, J. R. & Pothos, Z. (to appear). Quantum Models of Cognition. R. Sun (Ed) *The Cambridge Handbook on Computational Cognitive Sciences*. Cambridge University Press
7. Rajagopal, K., Zhang, Q., Balakrishnan, S. N., Fakhari, P., & Busemeyer, J. R. (to appear). Quantum amplitude amplification for reinforcement learning. In K. G. Vamvoudakis (Ed) *Handbook on Reinforcement Learning and Control.* Springer Studies in Systems, Decision and Control
8. Busemeyer, J. R. & Wang, Z. (2019). Introduction to Hilbert space multi-dimensional modeling. In Aerts, D., Khrennikov, A. I., Melucci, M., & Toni, B. (Eds.). (2019). Quantum-Like Models for Information Retrieval and Decision-Making. Springer Books