Advancing the Science of Information

Multiple Hypothesis Tracking for Space Surveillance

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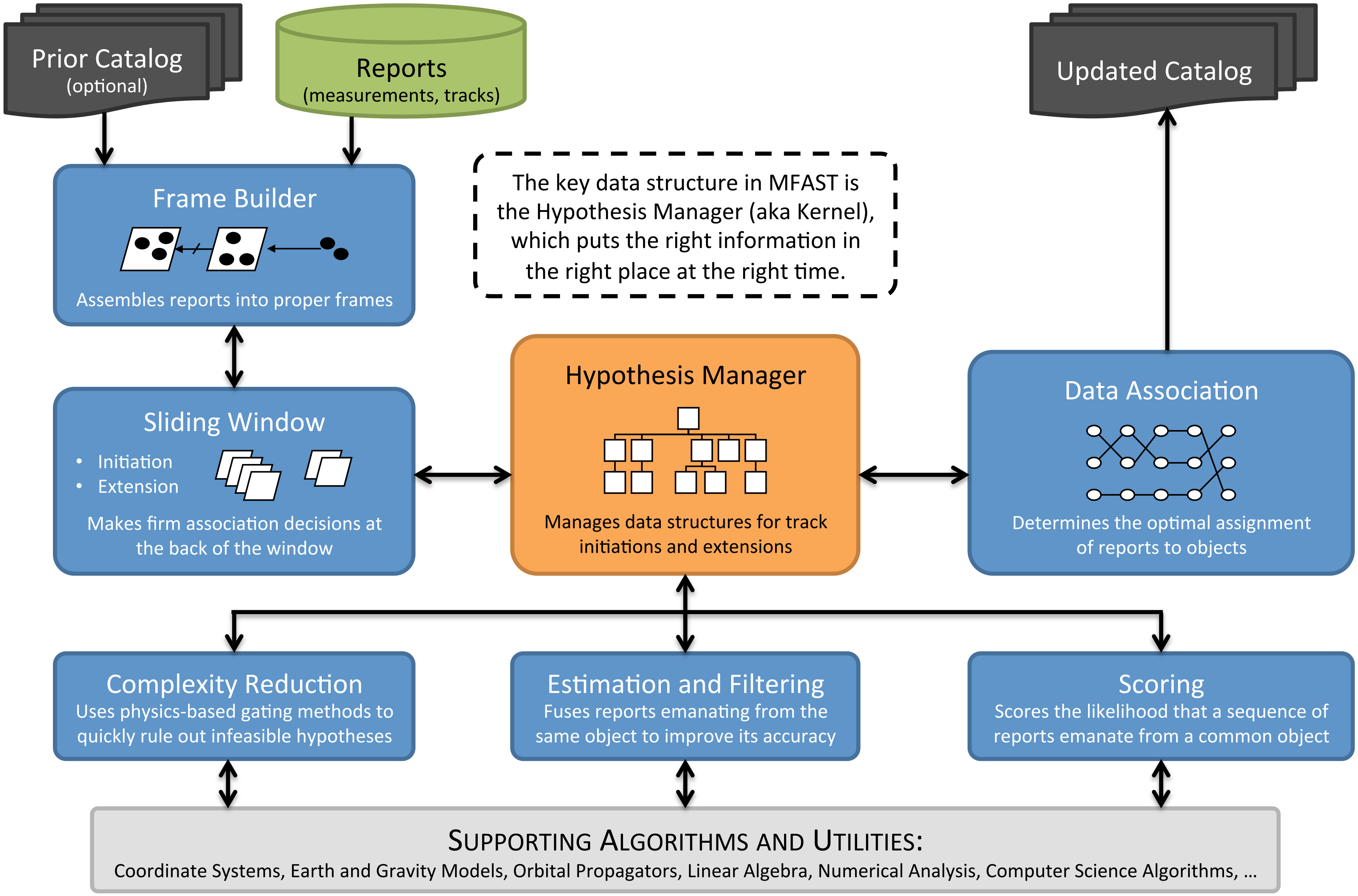
Algorithm Architecture for Multiple Frame Assignment Space Tracker (MFAST)



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Introduction to Multiple Target Tracking

Data Association: Given a set of sensor reports or detections of a collection of objects, determine which reports emanate from which objects and which are unrelated.

Nonlinear Estimation and Fusion: Given a sequence of reports emanating from the same object, use fusion and estimation techniques to improve the estimate of the state.

Multiple target tracking methods divide into single and multiple frame methods. The most successful of the multiple frame methods are **Multiple Hypothesis Tracking (MHT)** and **Multiple Frame Assignments (MFA)**. **MFA** is an optimization based MHT that formulates the data association problem as a multidimensional assignment problem.

The strength of MHT/MFA derives from its ability to change past decisions or, equivalently, to delay difficult decisions until more information is available.

A **goal** of this research program has been to develop **advanced algorithms** in support of the development of a **MFA Space Tracker (MFAST)** for space surveillance and to use it to resolve **Uncorrelated Tracks (UCTs)**.

Basic Research [1/2]

Three Optimization Based Formulations of the Data Association Problem in our Multiple Hypothesis Tracker (MHT) called MFAST

- Current: Multi-Dimensional Assignment (MDA) problem formulation the data association problem in MHT.
- Improved: multi-arc, multi-assign, multi-dimensional assignment (M3DA) Problem formulation of MHT. (Improves on MDA formulation.)
- *New:* Generalized MHT for UCT processing, breakups, clusters, and catalog maintenance.
- Uncertainty in the Association Process
  - The problem is to identify orbit segments with relatively pure data associations (say > 95%) in support of object identification.
  - Traditional methods include K-Best or Markov Chain Monte Carlo (MCMC).
  - *New:* Correspondence method improves quality of K-Best or MCMC by an order of magnitude in fractions of a second and identifies orbit segments that are relatively pure.

Sensor Resolution: Merged Measurement Algorithms

- *New:* System/network level algorithm determines likely candidates; multi-assignment in M3DA facilitates assignments.
- At the sensor level,
  - Decompose pixel-clusters consistent with established tracks.
  - Enhanced Resolution Algorithm based on GMRES algorithm followed by EM algorithm to generate measurements.

Cluster Tracking (In Progress)

- Clustering methods for LEO breakups and GEO clusters
- Continuous transition from cluster tracking to individual tracking object tracking

Basic Research [2/2]

Complexity Reduction

- Dual Pane Sliding Window implementation of MHT
- Gating algorithms remove highly unlikely strings of reports prior to association. Examples include dynamic, multi-frame, filter prediction, and likelihood ratio gates.
- Track hypothesis pruning is based on K-Best or *Arc Conditioning*.
- *New:* Snippets (EO tracks), which are sets of angle and angle rates derived from angle only data, are used in gating, but not estimation. **Reduces complexity from O(N^4) to O(N^2.3)**.

Treatment of Biases at the System Level

- JABE Algorithm for track to track biases is based on branch and bound and A*-search. (Appropriate for matching EO orbits and radar orbits.)
- Consider Analysis and Schmidt Filter treat residual biases between calibrations.
- Bias estimation.

Uncertainty Quantification (Initiated under an AFOSR STTR)

- *New* Nonlinear Filter: The GVM filter and comparisons to UKF, EKF, and Gaussian Sums.
- *New:* Transformation of uncertainty necessary to support mission areas: conjunction assessments, data association, anomaly detection, sensor tasking and scheduling
- *New* Orbital Propagator: GL-IRK methods for propagation of orbits and their uncertainty

New: Anomaly detection and maneuver reconstruction.

New: Metrics for uncertainty realism

UCTs processing using radar and EO sensors

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PROPAGATION OF UNCERTAINTY IN THE STATE OF AN OBJECT

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Overview

We have implemented some existing algorithms and developed new ones for the propagation of uncertainty through nonlinear transformations and in time, e.g.,

- Unscented transform and filter
- Extended Kalman filter
- Particle filter
- Gauss von Mises filter
- Mixtures of Gaussians or GVMs

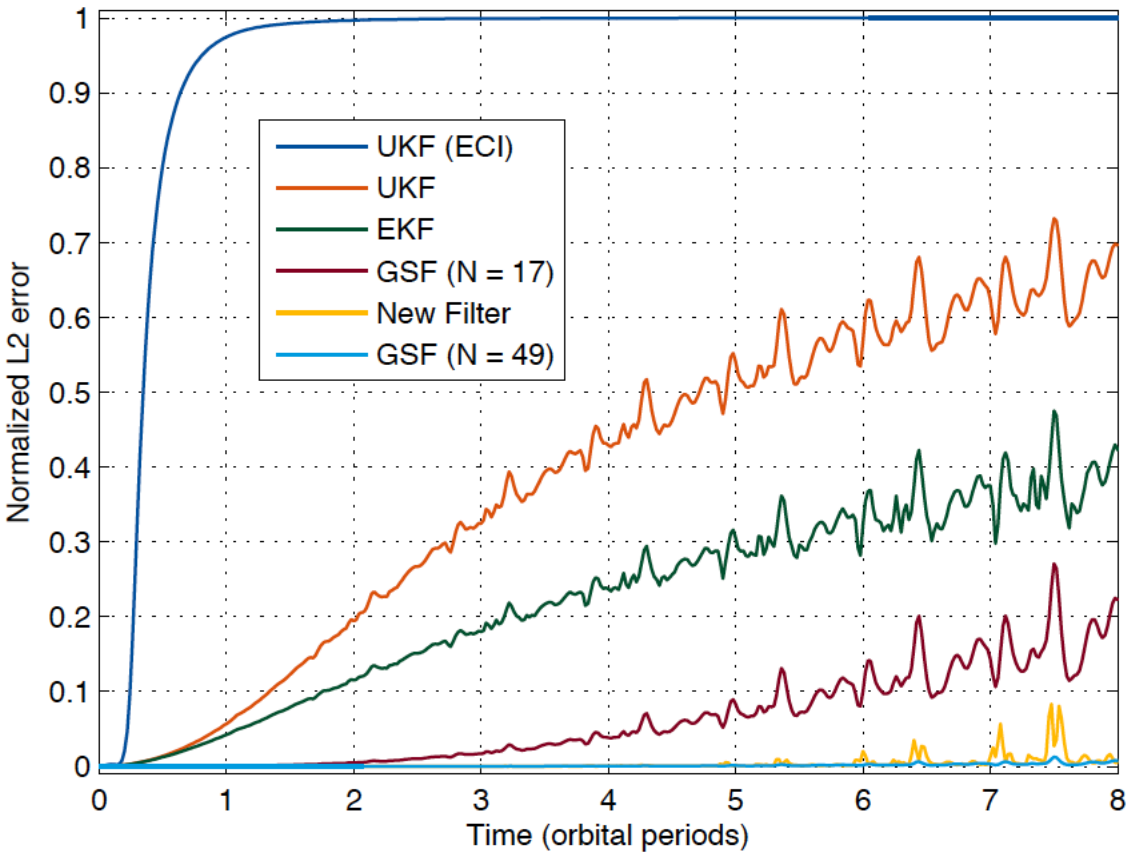
in different coordinate systems, e.g.,

- Cartesian
- Orbital elements (e.g., Equinoctial Elements)

in support of several space missions, e.g.,

- conjunction assessments,
- sensor resource management,
- data association for orbit or catalogue maintence,
- anomaly detection.

Comparison of the GVM, EKF, UKF, and Gaussian Sum Filters



Initial state: circular, non-inclined orbit in LEO with $\mu_a = 7136.635$ km

Initial uncertainty:

$\sigma_a = 20$ km

$\sigma_h = \sigma_k = \sigma_p = \sigma_q = 10^{-3}$

$\sigma_\ell = 36''$

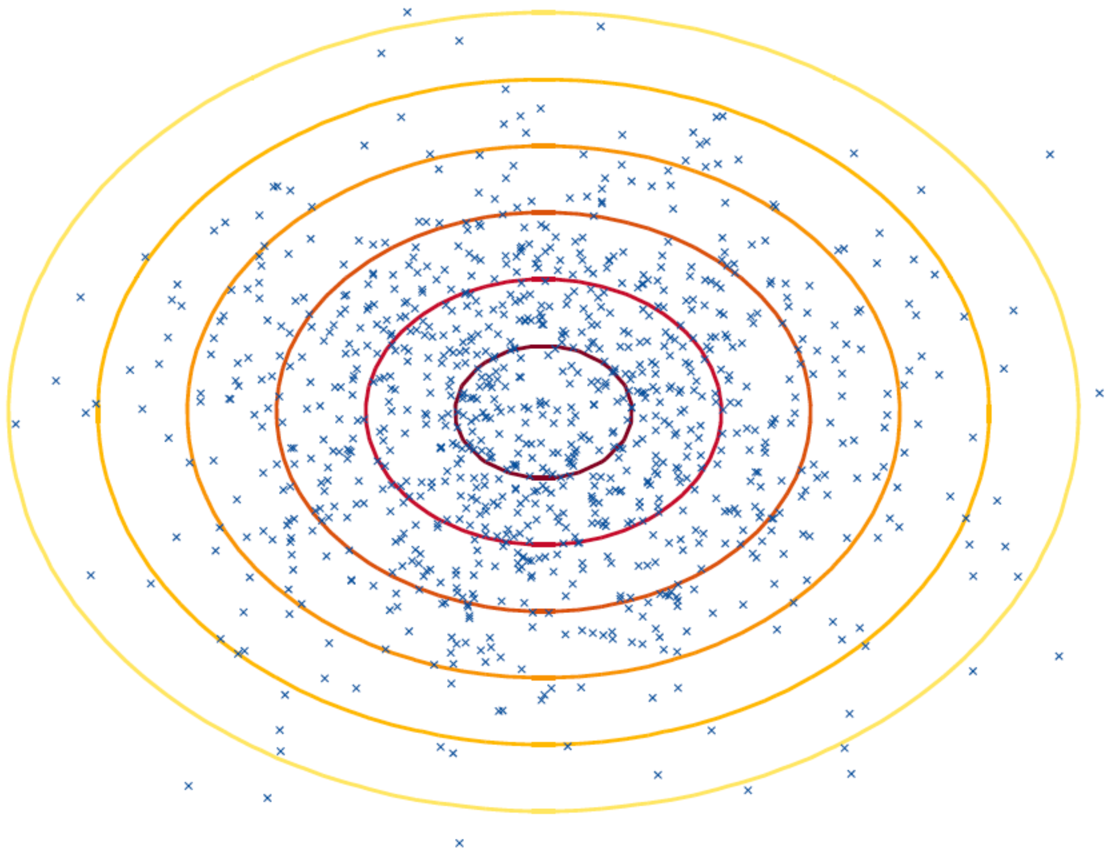
Normalized L_2 error:

$$\bar{L}_2 = \frac{\|p_{\text{approx}} - p_{\text{exact}}\|^2_{L_2}}{\|p_{\text{approx}}\|^2_{L_2} + \|p_{\text{exact}}\|^2_{L_2}}$$

The **new GVM filter**

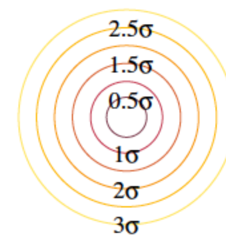
1. Achieves accuracy comparable to a $N = 17$ or $N = 49$ term Gaussian sum;
2. Maintains a proper characterization of the uncertainty for up to 8 times as long as the EKF/UKF.

Comparison of the GVM filter with the EKF and UKF



Predicted uncertainty after
$t = 0$ orbital periods

× particles



levels curves in the semi-major axis a and mean longitude ℓ coordinates

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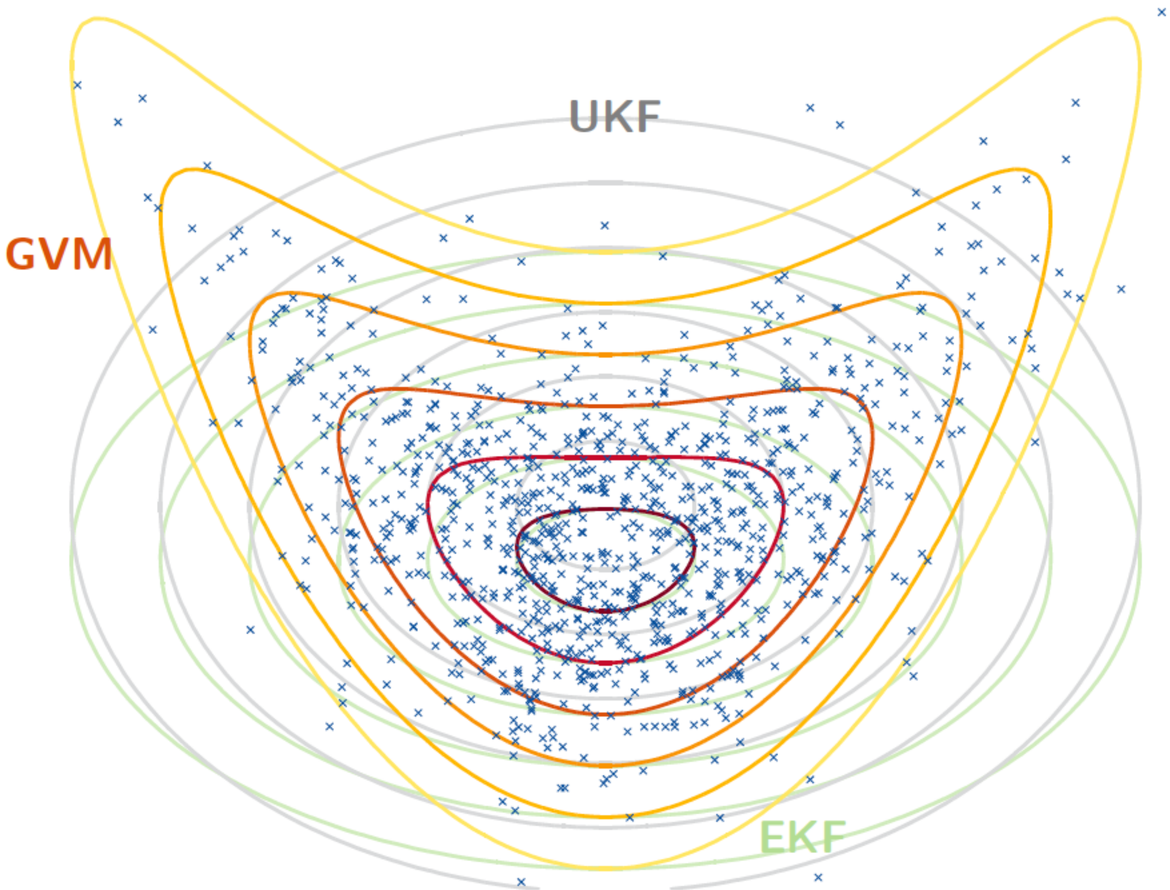
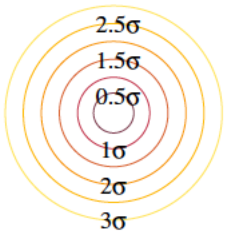


Predicted uncertainty after
$t = 1$ orbital periods

× particles



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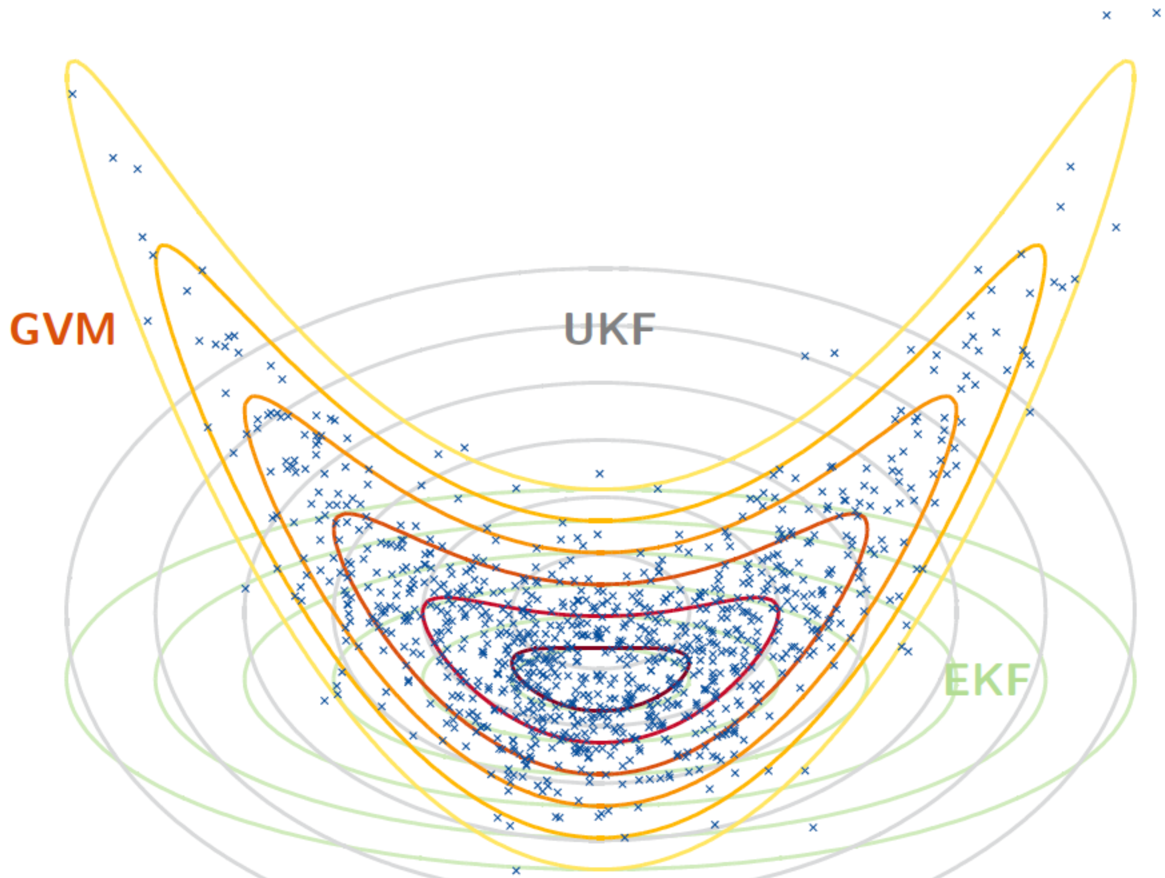
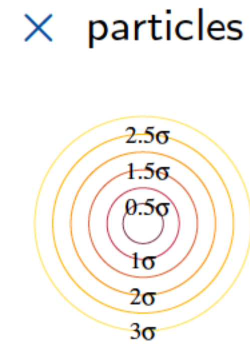


Predicted uncertainty after
$t = 2$ orbital periods



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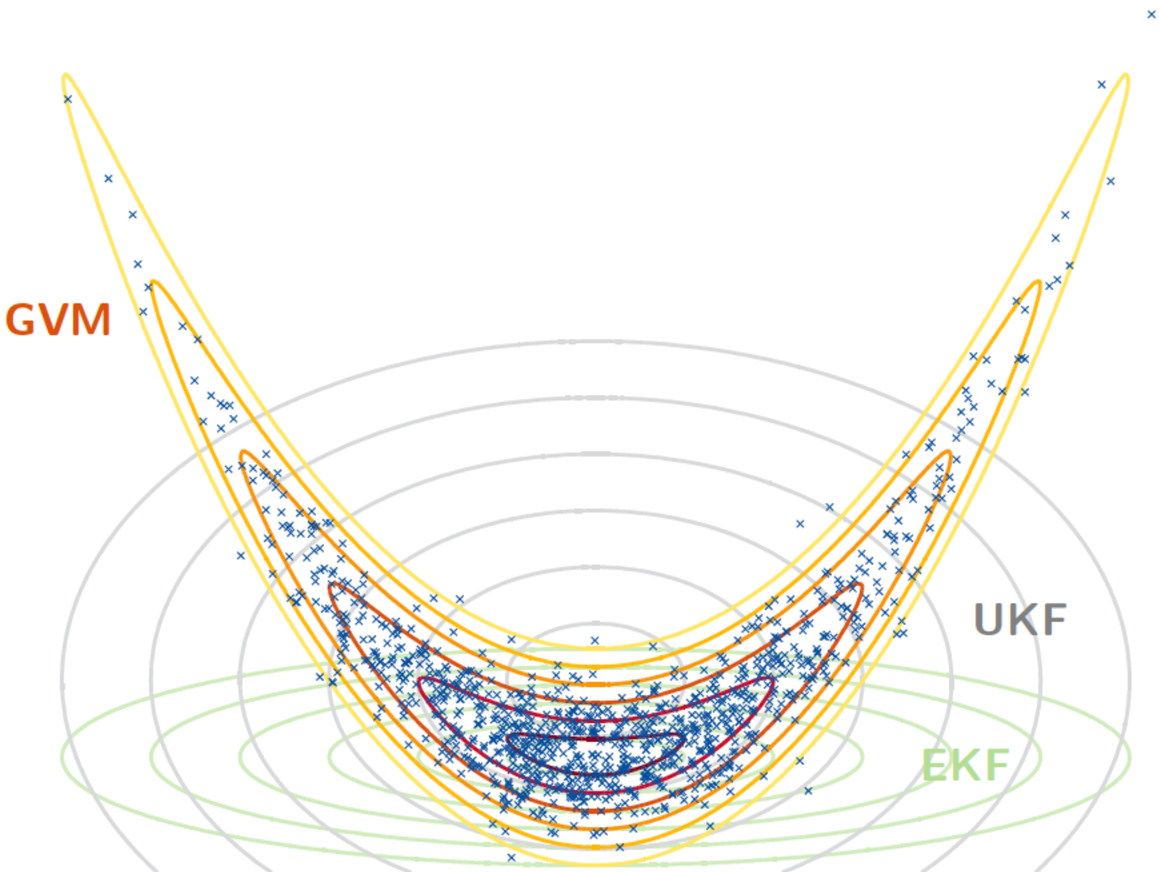
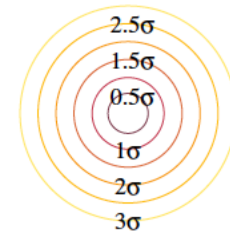


Predicted uncertainty after

$t = 4$ orbital periods

× particles



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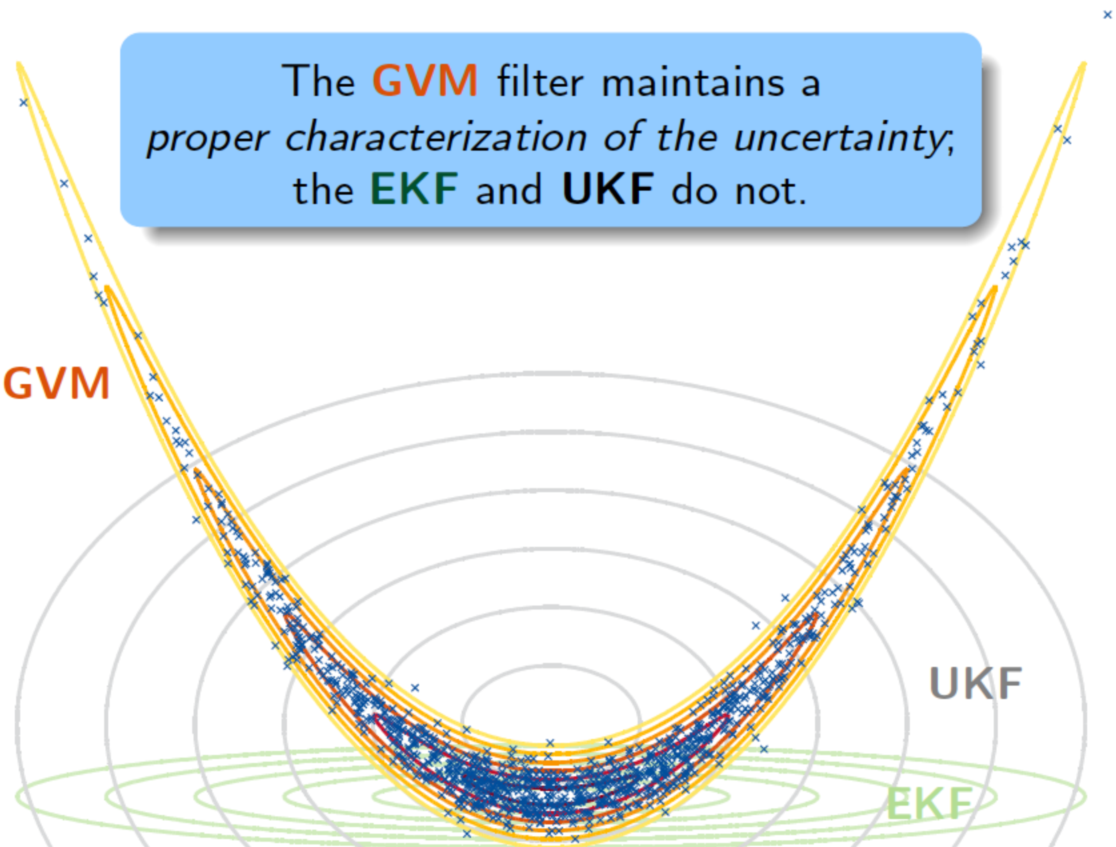
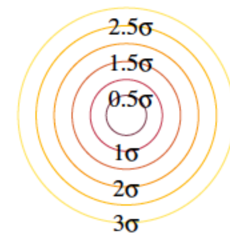
Comparison of the GVM filter with the EKF and UKF



Predicted uncertainty after
$t = 8$ orbital periods

× particles



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$\sigma_\ell = 36''$

The Gauss von Mises PDF and Filter

The Gauss von Mises (GVM) PDF was developed specifically for orbital element spaces. (One can transform the GVM to a Cartesian space using a Gaussian sum representation.)

- Provides both covariance and uncertainty realism by modeling higher-order cumulants (e.g., skewness and kurtosis) beyond a state and covariance.
- Reduces to a Gaussian for a subset of the parameter space.
- The Gauss von Mises (GVM) filter provides improved covariance and uncertainty realism at no additional computational cost compared to traditional methods for uncertainty propagation.
- The GVM maintains uncertainty realism up to eight (8) times longer than the UKF/EKF.
- Can be extended to a mixture filter, to provide improved accuracy in extreme cases at over a 95% reduced cost compared to Gaussian mixtures.

- Recent Reference: J. T. Horwood and A. B. Poore, “Gauss von Mises distribution for improved uncertainty realism in space situational awareness,” *SIAM Journal of Uncertainty Quantification*, vol. 2, pp. 276-304, 2014.

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Orbit and Uncertainty Propagation

Salient Features of Numerica's Implicit Runge-Kutta (IRK) Propagator

- The Method
  - Based on collocation methods using any set of orthogonal polynomials
  - Settled on Gauss Legendre orthogonal polynomials for the reasons below.
- Numerical properties
  - A-stable: can be applied to problems with multiple time scales
  - Super-convergent: no method converges faster
  - Parallelizable: can exploit advanced computing architectures

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Implicit Runge-Kutta (IRK) Propagator

How it works:

User specifies the desired accuracy (relative and absolute errors) of the solution per time step. Then, the algorithm adaptively selects

- **Stepsizes up to 0.8 of an orbit** (from a convergence proof),
- **Number of stages** (collocation points/nodes),
- **Convergence criteria** for the iterative solution of the system of nonlinear equations (relative and absolute errors),
- **Order of the gravity model.**

to achieve (reliability and) the prescribed accuracy in the solution at a minimum computational cost.

Why it works for astrodynamics:

- Convergence proof has been established for the system of nonlinear equations solved each step for astrodynamics. Domain of contraction is almost "global".
- Good initial approximations to solution provided by classical astrodynamics, e.g., Keplerian dynamics or by nearby solutions in the case of an ensemble propagation.

Ensemble Propagation

Traditional algorithms propagate each orbit, individually, within an ensemble of states. The GL-IRK method facilitates the propagation of nearby orbits together, thereby reducing the cost of uncertainty propagation.

An efficiency breakthrough: **propagation of a covariance** costs about the same as that of one to two orbit propagations using existing methods such as Dormand-Prince 8(7).

Some Computational Results

- Even before potential parallelization, the new GL-IRK based propagator significantly reduces the computational cost of orbit and uncertainty propagation by 70-90% depending on the regimes of space.

Orbit Type	a (km)	e	i ($^\circ$)	Ω ($^\circ$)	ω ($^\circ$)	M ($^\circ$)	Orbits	**Savings**
LEO	6640	0.0095	72.9	116	57.7	105	3	**72-80%**
LEO	6640	0.0095	72.9	116	57.7	105	30	**72-80%**
LEO	7878	0	30.0	137	0	36.0	3	**70-77%**
LEO	7878	0	30.0	137	0	36.0	30	**79-81%**
MEO	25508	0.0023	65.9	358	343	107	3	**86-87%**
MEO	25508	0.0023	65.9	358	343	107	30	**90-91%**
GEO	42164	0	0	0	0	250	3	**85-86%**
GEO	42164	0	0	0	0	250	30	**82-89%**
GEO	42164	0.0005	14	18	333	26.5	3	**82-87%**
GEO	42164	0.0005	14	18	333	26.5	30	**84-86%**
HELO	26628	0.7416	63.4	120	261	144	3	**74-81%**
HELO	26628	0.7416	63.4	120	261	144	30	**81-82%**

Savings based on runtime comparison to Dormand-Prince 8(7) from netlib.org for the propagation of 13 sigma points.
See Aristoff, Horwood & Poore, *J. Celestial Mech. and Dynamical Astronomy* (2014) for further details.

**Moreover, the propagator worked out-of-the-box on live UCT data.
No tuning was needed!**

Data Association for a Generalized MHT

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Characteristics of SSA Data Association Problem

The central problem in multi-object tracking is the **data association problem** of partitioning the sensor reports into tracks and unrelated reports.

- For space objects, we have good dynamical models, but we need a better characterization of the uncertainty in the model dynamics.
- Sparsity of the data (detections) presents major challenges.
- Sensor reports have poorly characterized uncertainties.
- In a multi-sensor environment, biases tend to be a major issue but these may be hidden somewhat in older sensors.
- UCTs are detected when sensors are tasked to view catalogued objects.

- Maneuvers can undermine the dynamical models.

Desired Features of a Generalized MHT

- A frameless MHT that can efficiently process days and weeks of sensor reports to determine good orbits from UCTs.
- The ability to incorporate frames as desired.
- A cluster tracker that can spawn reliable new orbits as they separate from the cluster in support of LEO breakups and GEO clusters. (NASA requires the individual objects as soon as available for conjunction assessments.)
- Uncertainty quantification of the association process.
- New types of assignments such as merges, spawns, and cluster assignments not present in the current MFA formulation.
- Provide a framework for a joint association and bias estimation (JABE).

- Preserve the framework for current MDA based association algorithms, i.e., be backward compatible with current MHT association algorithms.

The Group Assignment Problem

Group Assignment Problem (GAP)

Minimize $\sum_{(i,j)\in A} c_{ij} x_{ij}$

Subject to (1) $\sum_{(i,j)\in A} m_{ik} x_{ij} \le 1, \quad (k \in K),$

(2) $\sum_{(i,j)\in A} n_{jl} x_{ij} \le 1, \quad (l \in L),$

(3) $x_{ij} \in \{0,1\}.$

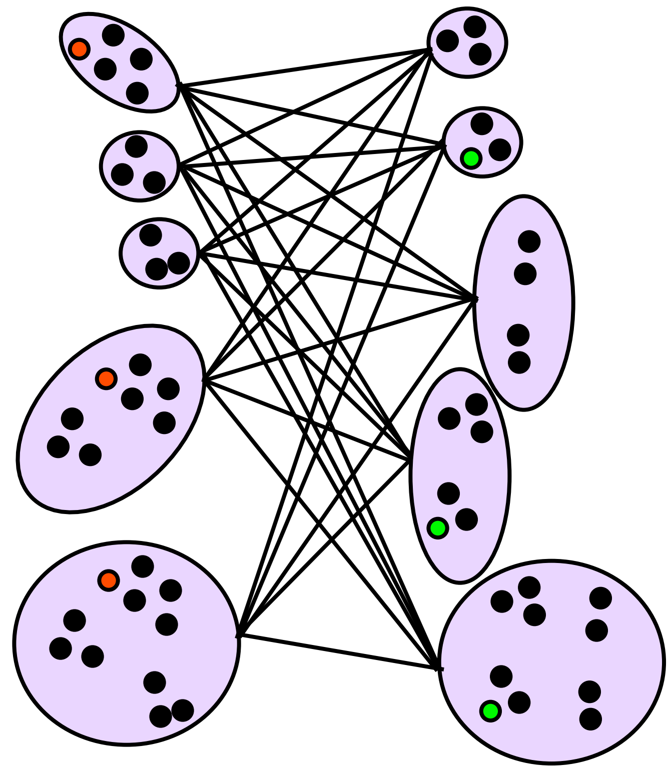
where the indicator functions are defined by

(4) $m_{ik} = 1$ if $k \in K_i$, 0 otherwise;

(5) $n_{jl} = 1$ if $l \in L_j$, 0 otherwise;

$\{K_i\}$ is a set covering of K;

$\{L_j\}$ is a set covering of L.



The group assignment problem is an assignment formulation of a generalized MHT capable of treating traditional MHT, cluster tracking, transition from cluster to MHT tracking, genealogy, and a host of new measurement types, e.g., spawn, merge, cluster assignments.

GA subsumes the MDA.

Variations are fundamental to sensor and communications resource management either as a deterministic problem or as a rollout policy for a stochastic dynamic programming approach.

UCTS: SOME COMPUTATIONAL FINDINGS

AFOSR Sponsored Research Program and AF STTR/SBIRs

AFOSR sponsored research is to develop longer term SSA capabilities; however, we needed an environment

- to evaluate new algorithms, and
- to demonstrate capability.

With AF SBIR and AFOSR STTR support, we have over the last 3 years developed a Python tracker with C++ components called *Multiple Frame Assignment Space Tracker* (MFAST)

Thanks to a recent AF Phase II SBIR (Dr. Alok Das, WPAFB), MFAST is being transitioned to Dahlgren to process live UCT data.

- MFAST is now fully automated and processing real EO and radar UCT data at the SSAL to produce new candidate orbits. Just last week, MFAST generated new UCT orbits that that were then sent to Dahlgren for verification.
- MFAST will be updated with new or improved capabilities as they mature.
- MFAST is being re-written in C++ for delivery to Dahlgren in April, 2015.

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Comments on Simulated and Real Data

We have processed several data sets through MFAST and plan regression testing.

- 2004 SSN (radar and optical)
- Ibex SST ModSim (GEO and mixed regimes)
- JMS NumVal (ongoing)
- Historical breakups
- Live UCT data in the SSAL environment.

Using 2004 SSN data as a baseline, we observe

- 99% correctness for radar UCTs.
- 95% correctness for frequently observed objects by EO sensors.
- Cross-tags are non-existent except for infrequently seen objects.

For live UCT data,

- If you look only for GEO orbits (with the EO sensors), you will miss most of the orbits from whence the UCTs come.
- There are many HEO orbits seen by both radar and EO sensors. One can correlate and fuse many of the common objects. JABE is particularly appropriate for this matching.

GENERAL CONCLUSIONS

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Concluding Comments

We have made much progress in developing future capabilities for MFAST to resolve UCTs using multi-sensor data.

MFAST is now running in the SSAL and processing live UCT data to produce UCT orbits.

A summary of the many algorithm components that are necessary for a future MFAST for space surveillance is provided on Slides 4-5.

Future Challenges and Opportunities

- Treatment of (sensor and dynamic model) biases
- Covariance/Uncertainty realism of sensor reports
- Achievement of uncertainty realism in the state of RSOs.
- A generalized MHT for space surveillance needs. The group assignment problem is designed to address this need.
- Sensor resolution issues
- Interactions with and support of the different mission areas.
- High performance computing

BACKUPS

Some Definitions

LEO-1: Low-Earth Orbits (Perigee < 500 km) are defined as RSOs with a Period < 225 minutes, perigee height < 500 km, and eccentricity < 0.25.

LEO-2 : Low-Earth Orbits (Perigee > 500 km) are defined as RSOs with a period < 225 minutes, perigee height > to 500 km, and eccentricity < 0.25.

HEO: Highly Elliptical Orbits are defined as all RSOs with 225 minutes < period < 1300 minutes and eccentricity > 0.25.

MEO: Medium-Earth Orbits are defined as RSOs with 225 minutes < period <1300 minutes and eccentricity < 0.25.

GEO: Geosynchronous Earth Orbits are defined as RSOs with 1300 minutes < period <1800 minutes, Inclination < 35 degrees, eccentricity < 0.25.

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MANEUVER DETECTION AND RECONSTRUCTION

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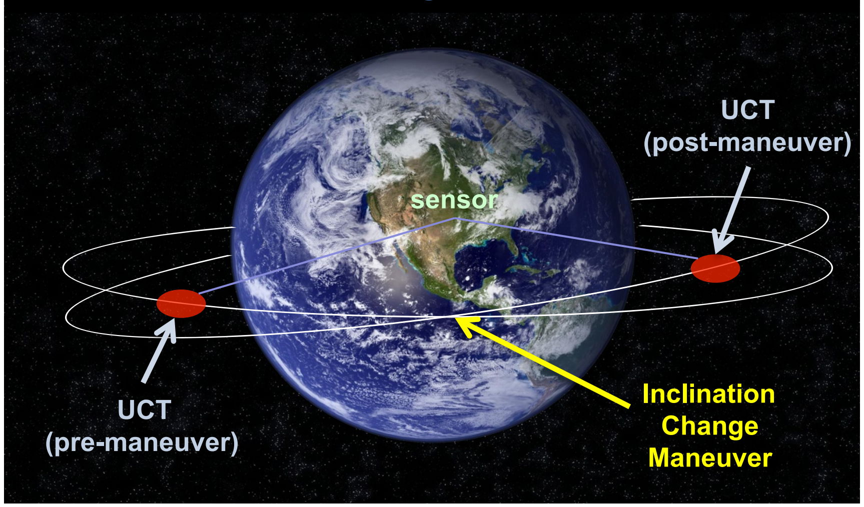
Maneuver Detection and Reconstruction

Can we detect, reconstruct, and validate the unseen maneuver, given the two UCTs?



Example: inclination change detection

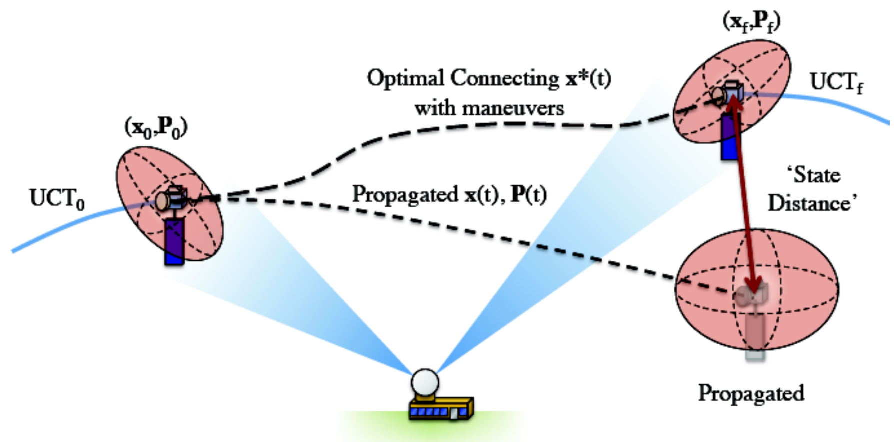


Illustration Courtesy of Dan Scheeres, University of Colorado, Boulder

Traditional methods

- Use multiple motion models
- Suited for continuous updates through maneuver

Impacted SSA Functions:

- UCT Resolution
- Space Catalog Maintenance
- Data/Track Association

Numerica's method

- Can treat sparse scenarios with unseen maneuvers
- Reconstructs fuel-optimal maneuvers
- Uses efficient optimal control framework
- Exploits fuel-usage characteristics of orbital maneuvers to estimate maneuver probabilities
- Rigorously treats uncertainty in track states
- Can be incorporated within MHT framework

Goals, Assumptions, Approach, Key Findings

- **Goals:**
  - Compute fuel-optimal control cost and profile to connect disparate orbital states
  - Account for uncertainties in the orbital states, and compute probability distribution of the optimal control cost
  - Formulate hypothesis test for maneuver feasibility and UCT correlation based on fuel cost

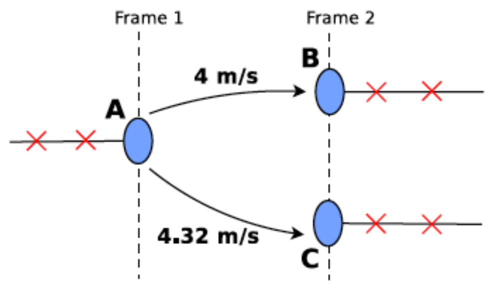
- **Key assumptions:**
  - Most orbital maneuvers are fuel-optimal
  - Given a choice of different track association hypotheses, the one with the lowest fuel cost is the correct one

- **Approach:** To achieve accurate and robust maneuver detection and validation, we developed an optimal control framework that employs the total velocity increment ΔV as a cost functional to determine if a realistic, feasible maneuver possibly connects two broken tracks/UCTs
  - Why ΔV: (1) It provides an accurate estimate of fuel cost (in comparison to other metrics such as energy), 2) it is catalogued for many satellites and maneuver types, and (3) We have overcome the issues related to numerical singularities presented by this cost functional

- **Key findings:**
  - Uncertain boundary conditions can be accounted for via the Unscented Transform
  - A hypothesis test for scoring association hypotheses based on maneuver probabilities can be formulated
  - It is possible to reconstruct a maneuver and classify its type even if it is not seen directly

Example UCT Association Scenario

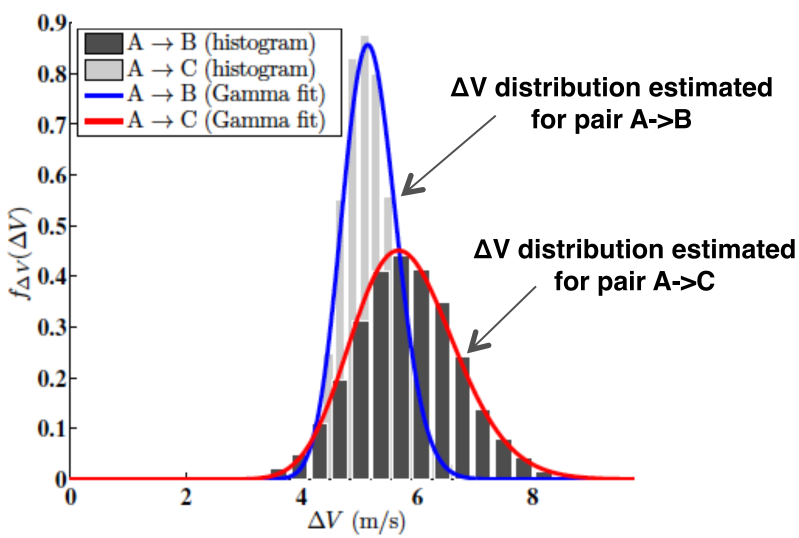
- A truth object generates a **pre-maneuver UCT** (UCT A in frame 1 below)
- The object conducts a ΔV=4 m/s ***inclination change maneuver*** and generates a ***post-maneuver UCT*** (B in frame 2)
- Another UCT (C) in frame 2 belongs to a different truth object (true ΔV distance from UCT A is about 4.32 m/s)



A ⟶ B : True Association

A ⟶ C : False Association

ΔV probability density estimates:



Association decision:

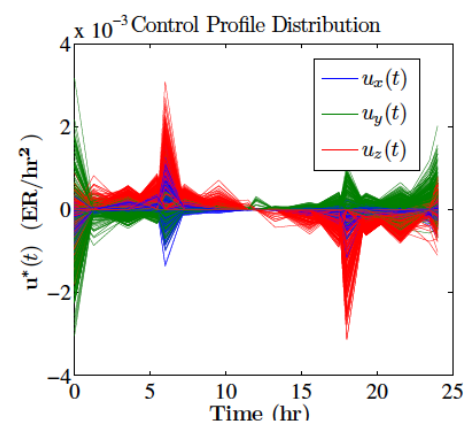
$$P_{A\to B}(\Delta V \leqslant \Delta V^{\mathrm{th}})$$

is greater than

$$P_{A\to C}(\Delta V \leqslant \Delta V^{\mathrm{th}})$$

Hence,
Pair (A → B) wins

Reconstructed maneuver profile distribution for pair (A → B):



Transitions

MFAST to Dahlgren to process and resolve live UCTs

Nonlinear filtering and estimation work to AFSC/A9

PUBLICATIONS

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Publications [1/3]

1. J. T. Horwood, N. D. Aragon, and A. B. Poore. Sliding window batch estimation filtering for enhanced anomaly detection and uncorrelated track resolution. In Proceedings of the 21st AAS/AIAA Space Flight Mechanics Meeting, New Orleans, LA, February 2011. Paper AAS-11-152

2. J. T. Horwood and A. B. Poore. Adaptive Gaussian sum filters for space surveillance. IEEE Transactions on Automatic Control, 56(8):1777–1790, 2011

3. J. T. Horwood and A. B. Poore. Uncertainty management and multiple hypothesis tracking for uncorrelated track mitigation. In 2011 Joint Meeting of the Military Sensing Symposia (MSS) Specialty Groups on National Symposium on Sensor and Data Fusion (NSSDF), Washington, DC, October

2011, NSSDF.

4. J. T. Horwood, N. D. Aragon, and A. B. Poore. Gaussian sum filters for space surveillance: theory and simulations. Journal of Guidance, Control, and Dynamics, 34(6):1839–1851, November-December 2011

5. P. D. Nielsen, K. T. Alfriend, M. J. Bloomfield, J. T. Emmert, Y. Guo, T. D. Maclay, J. G. Miller, R. F. Morris, A. B. Poore, R. P. Russell, D. G. Saari, D. J. Scheeres, W. P. Schonberg, and R. Sridharan. Continuing Kepler's Quest: Assessing Air Force Space Command's Astrodynamics Standards, NationalAcademies Press, Washington, DC, 2012

6. J. M. Aristoff and A. B. Poore. Implicit Runge-Kutta methods for orbit propagation. In Proceedings of the 2012 AAS/AIAA Astrodynamics Specialist Conference, Minneapolis, MN, August 2012

7. J. M. Aristoff and A. B. Poore. Implicit Runge-Kutta methods for uncertainty propagation. In Proceedings of the 2012 Advanced Maui Optical and Space Surveillance Technologies Conference, Wailea, HI, September 2012

8. J. T. Horwood and A. B. Poore. Orbital state uncertainty realism. In Proceedings of the 2012 Advanced Maui Optical and Space Surveillance Technologies Conference, Wailea, HI, September 2012

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