Imaging and Non-Imaging Polarimeters

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ASL Members, Collaborators, Funding

- Postdoctoral Fellows
  - Oscar Rodríguez (NUI-Galway)
  - Tingkui Mu (Xi'An Jiaotong)
- Graduate Students
  - Andrey Alenin
  - Israel Vaughn
  - Adoum Mahamat
  - Eric Ramon
  - Anael Guilmo
  - Mohan Xu
  - Brian Redman
  - Jefrey Wilhite
- Recent Group Alumni
  - Zhipeng Wang (SSC/NASA)
  - Sergio Johnson (RMS)
  - Miguel Piñeros (Schlumberger)
  - Miena Armanious (Qualcom)
  - Gabriel Birch (Sandia)
  - Charles LaCasse (Sandia)
  - Sean Keller (RMS)
  - Wiley Black (Control Vision)
- UA Collaborators
  - Clara Curiel (AZ Cancer Center)
  - Jim Schwiegerling (OSC)
  - Russell Chipman (OSC)
  - Rick Ziolkowski (ECE)
  - John Reagan (ECE)
  - Liz Ritchie (ATMO)
  - Chris Walker (Steward)
- Recent External Collaborators
  - NC State (Mike Kudenov)
  - ASR Corporation (ABQ)
  - AFRL/RYJT (Sensors – Dayton)
  - AFRL/RDHP (DE – ABQ)
  - Advanced Optical Technologies (ABQ)
  - Polaris Sensor Technologies (Huntsville)
  - Eureka Aerospace (Pasadena)
  - K&A Wireless (ABQ)
  - Schlumberger Water Services (Tucson)
  - Sunkist Corp. (Ontario, CA)
  - Control Vision, Inc (Tucson)
  - ITT/Exelis (Dayton)

Synergistic Projects

Defense-Funded Projects Directly Related to AFOSR Project

- AFRL/RYJT, IR Polarimetry for Human Signatures Detection (2010 - 2011)
- NASA JPL SURP Program, Time-Modulated Polarimeters (2010 - 2011)
- USA NVESD SBIR (sub to AOT), Partial Mueller Polarimeters (2010 – 2013)
- MDA SBIR (sub to Polaris), Microgrid Polarimeters for Seeker Applications (2010 – 2013)
- AFRL/RYJT, Active polarimeters for target detection (2012 – 2013)
- Sandia, Polarization Tags (2013 – 2014)

Other Projects Leveraging AFOSR Results

- Partial Mueller Polarimeters for Skin Cancer Monitoring (Clara Curiel) 2010 - 2013
- Polarization sensors for citrus grading systems (Sunkist Citrus) 2012 - 2014
- Snapshot spectrally modulated imaging polarimeters (NA-22) 2014 – 2017

Research Thrusts of Current Award

- Modulated Polarimeter Reconstruction (Charles LaCasse, Russell Chipman, Israel Vaughn)
- Channeled Polarimeters (Andrey Alenin)
- Maximizing Information Bandwidth in Modulated Polarimeters (Israel Vaughn, Oscar Rodríguez)
- Partial Mueller Polarimeters (Andrey Alenin, Israel Vaughn, Adoum Mahamat)
- DURIP Laboratory and Field Polarimeters (Israel Vaughn, Mohan Xu)
- Microgrid Polarimeter Calibration and Analysis (Wiley Black, Brad Ratliff, Charles LaCasse)
- Coherence and its Relationship with Polarization (Oscar Rodríguez)

Generalized Channeled Polarimetry (Alenin)

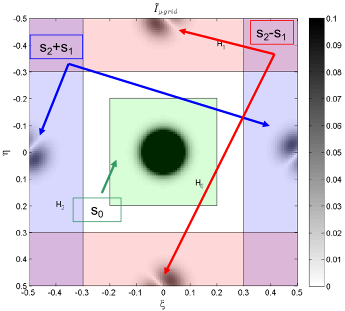
Oka, Dereniak, Kudenov, and their collaborators have popularized the class of "snapshot" polarimeters based on channeled concepts.

Tyo's group has shown that Division of Time and Division of Focal Plane (microgrid) polarimeters are really just channeled devices.

It's also possible to use related strategies that use "random codes," which are just another form of channel

Common modulation schemes include:

- Rotating retarder or PEM-based (time modulated)
- Microgrids (space modulated)
- Birefringent prisms (space modulated)
- High-order retarders (spectrally modulated)
- Combinations thereof



Design Automation Process

Every modulation will perform a convolution in the corresponding frequency domain

- With arbitrary modulation dimensions, we might need a N-dimensional convolution
- If successive modulations are within the same dimension, they can be combined

The entire channeled modulation can be represented by combining individual dimensions

$$\underline{\mathbf{q}}_{\{\tau,\omega,\xi,\eta\};m_{ij}} = \underline{\mathbf{q}}_{\{\tau\};m_{ij}} \otimes \underline{\mathbf{q}}_{\{\omega\};m_{ij}} \otimes \underline{\mathbf{q}}_{\{\xi\};m_{ij}} \otimes \underline{\mathbf{q}}_{\{\eta\};m_{ij}}$$

If modulations are interleaved, it may be easier to deal with convolutions directly

$$\underline{\mathbf{q}}_{\xi_{e_1}/\eta_{e_2}/\xi_{e_3}/\eta_{e_4};m_{ij}} = \mathrm{vec}\left(\underline{\mathbf{q}}_{\xi_{e_1};m_{ij}} * \underline{\mathbf{q}}_{\eta_{e_2};m_{ij}} * \underline{\mathbf{q}}_{\xi_{e_3};m_{ij}} * \underline{\mathbf{q}}_{\eta_{e_4};m_{ij}}\right)$$

Similarly, we can express the construction of those channels in terms of PSA and PSG modulations:

$$\underline{\mathbf{q}}_{m_{ij}} = \mathrm{vec}\left(\underline{\mathbf{q}}_{g_i} * \underline{\mathbf{q}}_{a_j}\right) \rightarrow \underline{\underline{\mathbf{D}}} = \underline{\mathbf{A}}\,\underline{\mathbf{G}}^T \rightarrow \mathcal{F}\left\{\underline{\underline{\mathbf{D}}}\right\} = \mathcal{F}\{\underline{\mathbf{A}}\} * \mathcal{F}\{\underline{\mathbf{G}}\}^T$$

Combining sixteen of those vectors will produce the corresponding $\underline{\underline{\mathbf{Q}}}$ matrix,

$$\underline{\underline{\mathbf{Q}}} = \left(\begin{array}{cccccc} \underline{\mathbf{q}}_{\{\tau,\omega,\xi,\eta\};m_{00}} & \cdots & \underline{\mathbf{q}}_{\{\tau,\omega,\xi,\eta\};m_{03}} & \underline{\mathbf{q}}_{\{\tau,\omega,\xi,\eta\};m_{10}} & \cdots & \underline{\mathbf{q}}_{\{\tau,\omega,\xi,\eta\};m_{33}} \end{array}\right)$$

Spatially Channeled Polarimeter - I[?]

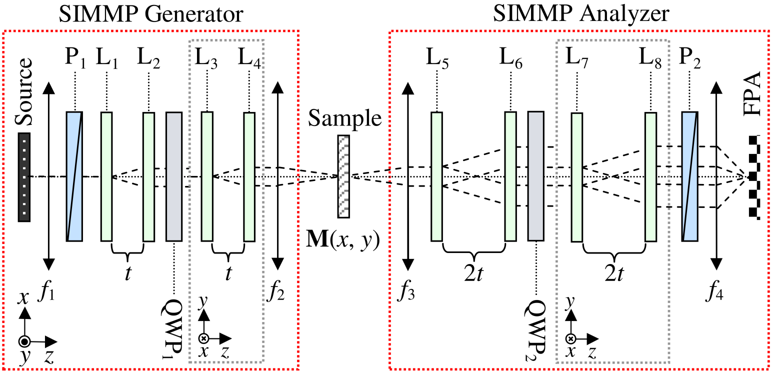
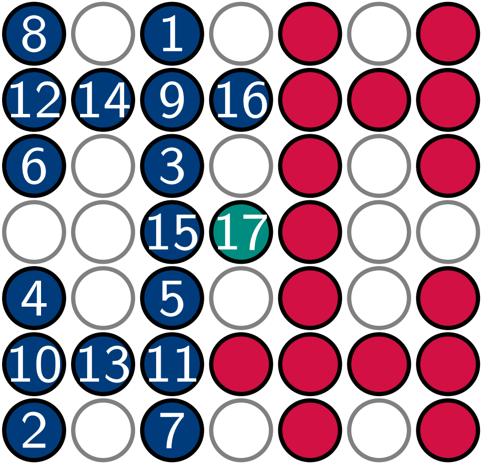


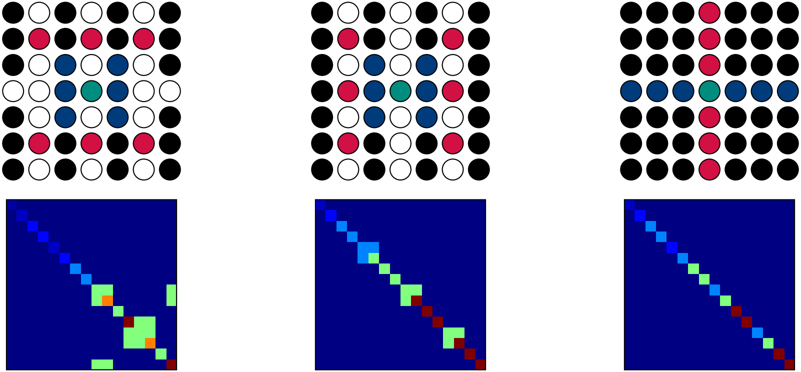
Figure: SIMMP optical configuration. PGs L_1, L_2, L_5 and L_6 shear the beam along x while L_3, L_4, L_7 and L_8 shear along y. P_1 and P_2 are linear polarizers at 45° while two quarter wave-plates, QWP_1 and QWP_2, have fast axes oriented at 45° and 0°, respectively. All PGs have identical grating periods Λ and the generator's and analyzer's PGs are separated by a distance t and $2t$, respectively.

Spatially Channeled Polarimeter - II

$$
\begin{aligned}
I(x,y) = &|A_1|\cos[\kappa\alpha(1x+3y)+A_1^a] \\
&+|A_2|\cos[\kappa\alpha(3x-3y)+A_2^a] \\
&+|A_3|\cos[\kappa\alpha(1x+1y)+A_3^a] \\
&+|A_4|\cos[\kappa\alpha(3x-1y)+A_4^a] \\
&+|A_5|\cos[\kappa\alpha(1x-1y)+A_5^a] \\
&+|A_6|\cos[\kappa\alpha(3x+1y)+A_6^a] \\
&+|A_7|\cos[\kappa\alpha(1x-3y)+A_7^a] \\
&+|A_8|\cos[\kappa\alpha(3x+3y)+A_8^a] \\
&+|A_9|\cos[\kappa\alpha(1x+2y)+A_9^a] \\
&+|A_{10}|\cos[\kappa\alpha(3x-2y)+A_{10}^a] \\
&+|A_{11}|\cos[\kappa\alpha(1x-2y)+A_{11}^a] \\
&+|A_{12}|\cos[\kappa\alpha(3x+2y)+A_{12}^a] \\
&+|A_{13}|\cos[\kappa\alpha(2x-2y)+A_{13}^a] \\
&+|A_{14}|\cos[\kappa\alpha(2x+2y)+A_{14}^a] \\
&+|A_{15}|\cos[\kappa\alpha(1x)+A_{15}^a] \\
&+|A_{16}|\cos[\kappa\alpha(2y)+A_{16}^a]+A_{17}
\end{aligned}
$$



Spatially Channeled Polarimeter - IV



(a) $g_x = g_y = 1$, $a_x = a_y = 2$, $x/y/y/x$, 33/49 channels used, EWV = 209

(b) $g_x = g_y = 1$, $a_x = a_y = 2$, $x/y/x/y$, 35/49 channels used, EWV = 151

(c) $g_1 = 2g_2 = a_1 = 2a_2 = 2$, $x/x/y/y$, 49/49 channels used, EWV = 121

Figure: Top row shows the $\xi - \eta$ plane of channels constructed by the particular configuration. The bottom row shows $\underline{\underline{\mathbf{Q}}}^{+}\underline{\underline{\mathbf{Q}}}^{+\dagger}$, the product which contains the Mueller element reconstruction variances within its diagonal.

Spatially Channeled Polarimeter - V

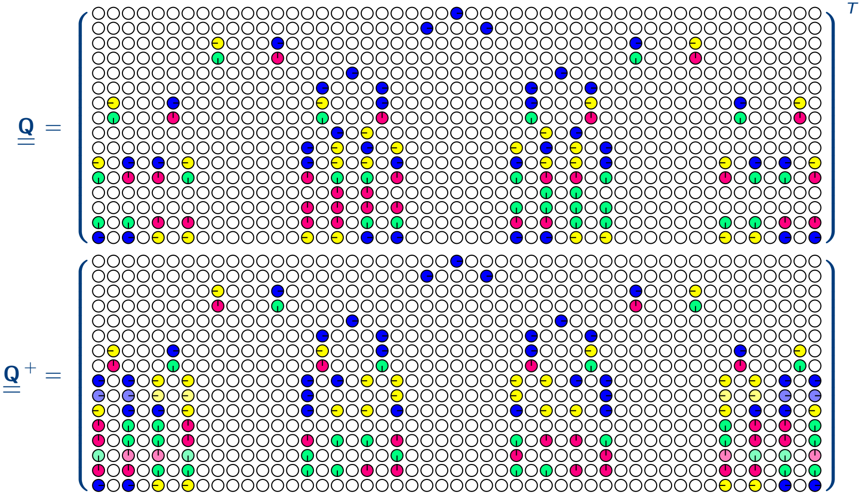


Figure: Kudenov's original polarimeter, $\mathrm{EWV} = 209$

Spatially Channeled Polarimeter - V

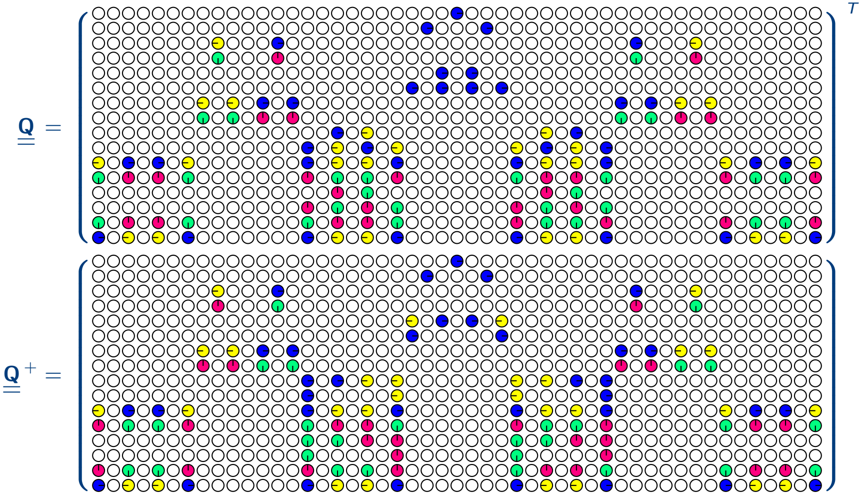
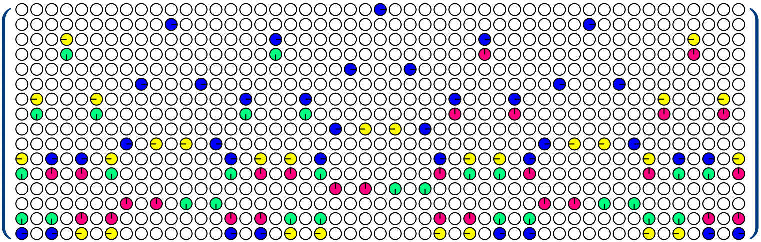


Figure: Asymmetrical modulation polarimeter, $\mathrm{EWV} = 151$

Spatially Channeled Polarimeter - V

$\underline{\underline{\mathbf{Q}}} =$



T

$\underline{\underline{\mathbf{Q}}}^{+} =$

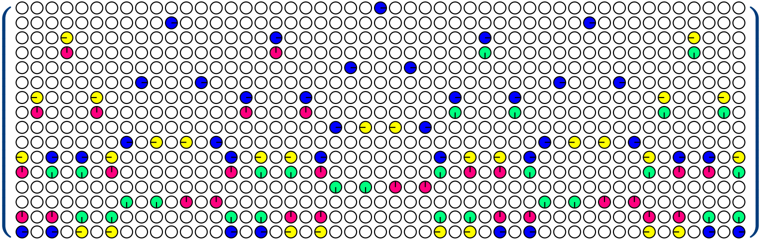


Figure: Orthogonalized modulation polarimeter, $\mathrm{EWV} = 121$

Partial Mueller Polarimeters (Alenin, Vaughn)

- Knowledge of object's Mueller matrix can provide discrimination ability
- Measuring the full Mueller matrix requires at least 16 measurements
- It is possible that some Mueller elements are more important than others

Solution:

- We want to find a system that provides
  - Minimum number of measurements
  - Maximum relevance of measurements
  - Ability to determine the best set of measurements to capture a specific subset of the Mueller matrix

Starting Point

$$\underline{\mathbf{I}} = \underline{\underline{\mathbf{W}}}\,\underline{\mathbf{M}}$$

- $\underline{\underline{\mathbf{W}}}$ is a $N \times 16$ matrix, where N is the number of $\underline{\mathbf{A}} - \underline{\mathbf{G}}$ pairs
- If we have additive noise, i.e. $\underline{\mathbf{g}} = \underline{\mathbf{f}} + \underline{\mathbf{n}}$, then the reconstruction will be:

$$\underline{\underline{\mathbf{W}}}^{+}\underline{\mathbf{g}} = \underline{\underline{\mathbf{W}}}^{+}\underline{\mathbf{f}} + \underline{\underline{\mathbf{W}}}^{+}\underline{\mathbf{n}}$$

  The pseudo inverse solves for the least squares fit of a given linear system
- In general, $\mathrm{tr}(\underline{\underline{\mathbf{W}}}^{+}\underline{\underline{\mathbf{W}}}) = \mathrm{rank}(\underline{\underline{\mathbf{W}}})$
- If $\underline{\underline{\mathbf{W}}}$ is full rank, then $\underline{\underline{\mathbf{W}}}^{+}\underline{\underline{\mathbf{W}}} = \mathbb{I}_{16\times 16}$

Taking the pseudo inverse

Let's consider the different ways to calculate the pseudo inverse:

- The default:

$$\underline{\underline{\mathbf{W}}}^{+} = (\underline{\underline{\mathbf{W}}}^{\mathrm{T}}\underline{\underline{\mathbf{W}}})^{-1}\underline{\underline{\mathbf{W}}}^{\mathrm{T}} = (\underline{\underline{\mathbf{W}}}^{\mathrm{T}}\underline{\underline{\mathbf{W}}})\backslash\underline{\underline{\mathbf{W}}}^{\mathrm{T}}$$

- SVD decomposition:

$$\underline{\underline{\mathbf{W}}} = \underline{\underline{\mathbf{U}}}\,\underline{\underline{\Sigma}}\,\underline{\underline{\mathbf{V}}}^{\mathrm{T}} \longrightarrow \underline{\underline{\mathbf{W}}}^{+} = \underline{\underline{\mathbf{V}}}\,\underline{\underline{\Sigma}}^{+}\underline{\underline{\mathbf{U}}}^{\mathrm{T}},$$

  where
  - $\underline{\underline{\mathbf{U}}}$ is an $N \times N$ unitary matrix containing left singular vectors (columns)
  - $\underline{\underline{\Sigma}}$ is an $N \times 16$ diagonal matrix containing singular values
  - $\underline{\underline{\mathbf{V}}}$ is an 16×16 unitary matrix containing right singular vectors (columns)

Non-full Rank $\underline{\underline{\mathbf{W}}}$

- If $\underline{\underline{\mathbf{W}}}$ is not full rank, the SVD inverse creates a maximally orthogonal matrix
- Instead of looking at the rank, let's look at how it is distributed
- The vector, $\mathrm{diag}(\underline{\underline{\mathbf{W}}}^{+}\underline{\underline{\mathbf{W}}})$, contains the fraction of "energy" that is preserved for each Mueller element within the system
- For clarity, we can refer to $\underline{\underline{\mathbf{B}}}$, the Mueller matrix form of $\underline{\mathbf{B}}'$:

$$\underline{\mathbf{B}}' = \mathrm{vec}(\underline{\underline{\mathbf{B}}}) = \mathrm{diag}(\underline{\underline{\mathbf{W}}}^{+}\underline{\underline{\mathbf{W}}}).$$

SVD structure - I

The core multiplication can be rewritten as:

$$\underline{\underline{\mathbf{W}}}^+\underline{\underline{\mathbf{W}}} = \underline{\underline{\mathbf{V}}}\,\underline{\underline{\Sigma}}^+\underline{\underline{\mathbf{U}}}^{\mathrm{T}}\underline{\underline{\mathbf{U}}}\,\underline{\underline{\Sigma}}\,\underline{\underline{\mathbf{V}}}^{\mathrm{T}} = \underline{\underline{\mathbf{V}}}\,\underline{\underline{\Sigma}}^+\underline{\underline{\Sigma}}\,\underline{\underline{\mathbf{V}}}^{\mathrm{T}},$$

where

$$\underline{\underline{\Sigma}} = \begin{bmatrix} \sigma_1 & 0 & \cdots & 0 & \cdots & 0 \\ 0 & \sigma_2 & \cdots & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma_N & \cdots & 0 \end{bmatrix}$$

$$\underline{\underline{\Sigma}}^+ = \begin{bmatrix} \frac{1}{\sigma_1} & 0 & \cdots & 0 & \cdots & 0 \\ 0 & \frac{1}{\sigma_2} & \cdots & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{1}{\sigma_N} & \cdots & 0 \end{bmatrix}^{\mathrm{T}}$$

SVD structure - II

$$\underline{\underline{\Sigma}}^{+}\underline{\underline{\Sigma}} = \begin{bmatrix} 1 & 0 & \cdots & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & \cdots & 0 \end{bmatrix}$$

- The multiplication reveals a sub-identity matrix, $\mathbb{I}_{N \times N}$
- This has the effect of cropping the right singular matrix to only contain columns with nonzero singular values, i.e. $\underline{\underline{\mathbf{V}}}' = \underline{\underline{\mathbf{V}}}(:, 1 : N)$

$$\underline{\underline{\mathbf{W}}}^{+}\underline{\underline{\mathbf{W}}} = \underline{\underline{\mathbf{V}}}'\underline{\underline{\mathbf{V}}}'^{\mathrm{T}}$$

The calculation reduces to:

$$\underline{\mathbf{B}}' = \mathrm{vec}(\underline{\underline{\mathbf{B}}}) = \mathrm{diag}(\underline{\underline{\mathbf{V}}}'\underline{\underline{\mathbf{V}}}'^{\mathrm{T}}).$$

Example - I

Suppose we want to measure the diagonal of the Mueller matrix

$$\underline{\underline{\mathbf{B}}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Using the four-blocks would mean that we need to perform twelve measurements:

$$\underline{\underline{\mathbf{A}}} \Rightarrow \frac{1}{2} \begin{bmatrix} \rightarrow & \rightarrow & \uparrow & \uparrow & \nearrow & \nearrow & \nwarrow & \nwarrow & \circlearrowright & \circlearrowright & \circlearrowleft & \circlearrowleft \end{bmatrix}$$

$$\underline{\underline{\mathbf{G}}} \Rightarrow \begin{bmatrix} \rightarrow & \uparrow & \rightarrow & \uparrow & \nearrow & \nwarrow & \nearrow & \nwarrow & \circlearrowright & \circlearrowleft & \circlearrowright & \circlearrowleft \end{bmatrix}$$

or equivalently in $\underline{\underline{\mathbf{B}}}$ notation, we combine three sub- $\underline{\underline{\mathbf{B}}}$ s:

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \& \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \& \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

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$$\underline{\underline{\mathbf{G}}} \Rightarrow \begin{bmatrix} \rightarrow & \uparrow & \rightarrow & \uparrow & \nearrow & \nwarrow & \nearrow & \nwarrow & \circlearrowright & \circlearrowleft & \circlearrowright & \circlearrowleft \end{bmatrix}$$

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$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \& \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \& \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

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Using the four-blocks would mean that we need to perform twelve measurements:

$$\underline{\underline{\mathbf{A}}} \Rightarrow \frac{1}{2}\left[\begin{matrix} \rightarrow & \rightarrow & \uparrow & \uparrow & \nearrow & \nearrow & \nwarrow & \nwarrow & \circlearrowright & \circlearrowright & \circlearrowleft & \circlearrowleft \end{matrix} \right]$$

$$\underline{\underline{\mathbf{G}}} \Rightarrow \left[\begin{matrix} \rightarrow & \uparrow & \rightarrow & \uparrow & \nearrow & \nwarrow & \nearrow & \nwarrow & \circlearrowright & \circlearrowleft & \circlearrowright & \circlearrowleft \end{matrix} \right]$$

or equivalently in $\underline{\underline{\mathbf{B}}}$ notation, we combine three sub-$\underline{\underline{\mathbf{B}}}$s:

$$\underline{\underline{\mathbf{B}}}_{\text{total}} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

Example - I

Suppose we want to measure the diagonal of the Mueller matrix

$$\underline{\underline{\mathbf{B}}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Using the four-blocks would mean that we need to perform twelve measurements:

$$\begin{array}{lll} \underline{\underline{\mathbf{A}}} & \Rightarrow \frac{1}{2} & \left[\begin{array}{cccccccccccc} \rightarrow & \rightarrow & \uparrow & \uparrow & \nearrow & & \nwarrow & \nwarrow & \circlearrowright & & \circlearrowleft & \circlearrowleft \end{array} \right] \\ \underline{\underline{\mathbf{G}}} & \Rightarrow & \left[\begin{array}{cccccccccccc} \rightarrow & \uparrow & \rightarrow & \uparrow & \nearrow & & \nearrow & \nwarrow & \circlearrowright & & \circlearrowright & \circlearrowleft \end{array} \right] \end{array}$$

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or equivalently in $\underline{\underline{\mathbf{B}}}$ notation, we combine three sub- $\underline{\underline{\mathbf{B}}}$ s:

$$\underline{\underline{\mathbf{B}}}_{\text{total}} = \begin{bmatrix} 1 & 1 & 0.5 & 0.5 \\ 1 & 1 & 0 & 0 \\ 0.5 & 0 & 1 & 0 \\ 0.5 & 0 & 0 & 1 \end{bmatrix}$$

Example - II

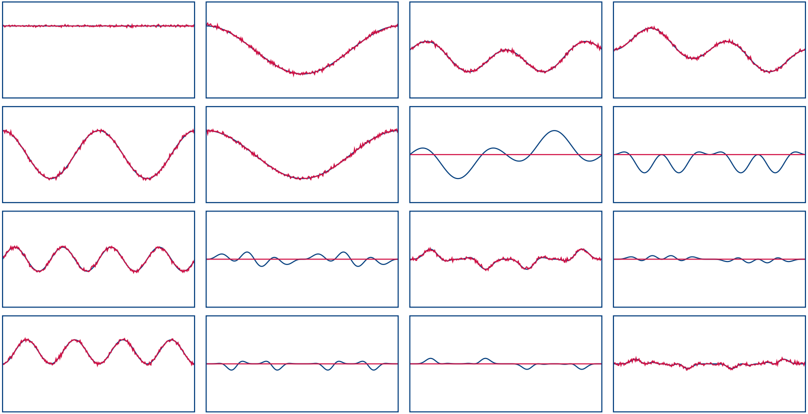


Figure: Reconstruction from twelve measurements

Example - II

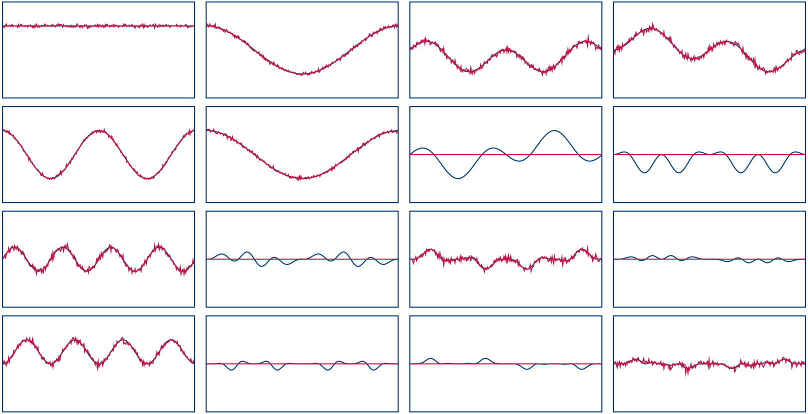


Figure: Reconstruction from ten measurements

Example - II

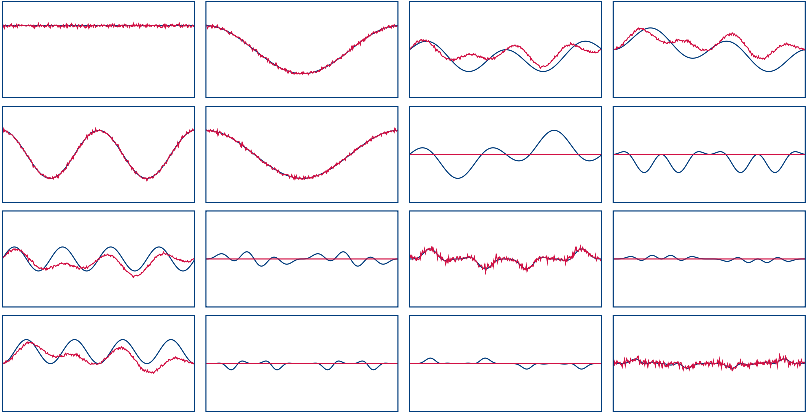


Figure: Reconstruction from eight measurements

Example - III

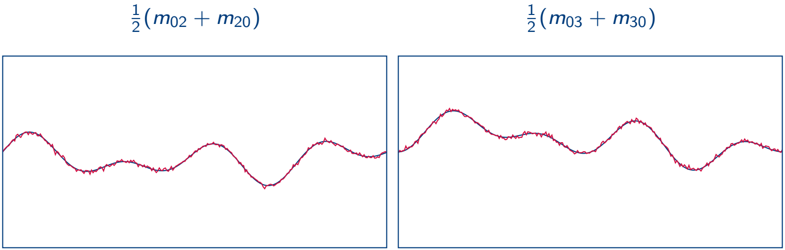


Figure: Linear combinations reconstruction in eight measurement case

DURIP Polarimeters (Vaughn)

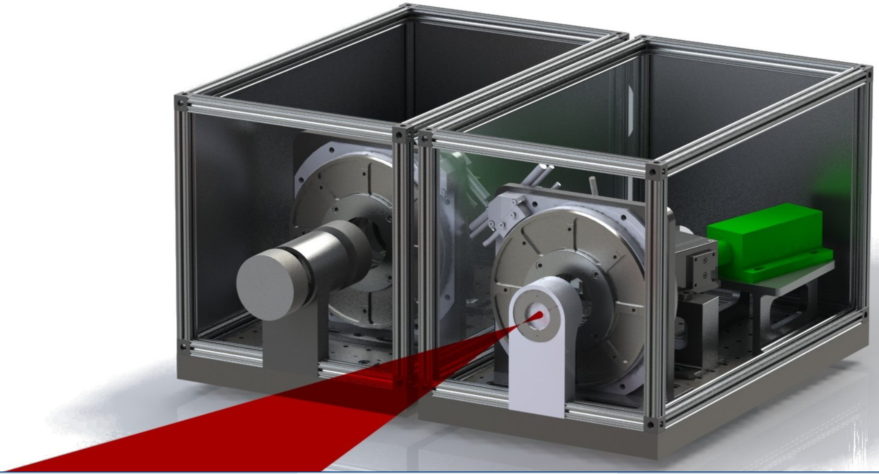
In recent years, our lab has made several theoretical discoveries about the physics and image reconstruction of imaging polarimeters. These theoretical models need to be validated.

Additionally, our lab has been involved in several classification tasks.

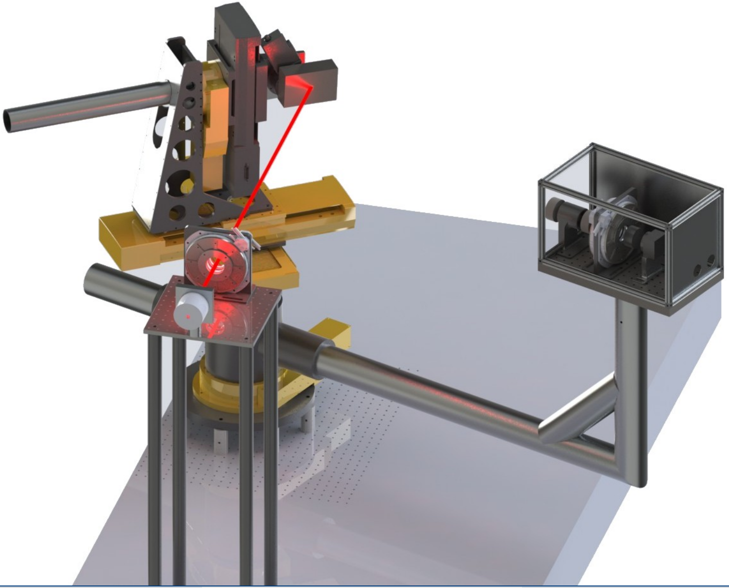
Our lab was in need of an instrument which would meet the following requirements:

- Validation of various theoretical developments (VCPol, Modulated Polarimeters, noise effects, etc.)
- Data collection for task specific detection and pMMP design
- Validation of pMMP methods and task specific discrimination
- Fully polarized BRDF (pBRDF) measurements
- Remote sensing capability

Portable Instrument - Design



Scatterometer

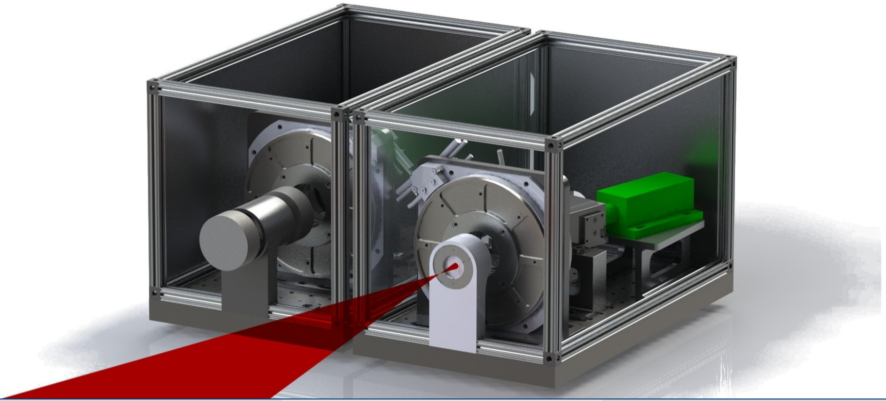


Portable Instrument - Current Status

- Channel structure for portable instrument research ongoing, good candidates for optimized bandwidth found
- Control and acquisition software in process
- Calibration in process
- Channel structure testing and validation will occur over the next 6 months
- Task based imaging statistics on orange texture to begin after completion of calibration and control software
- Other validation tasks can also begin after completion of calibration and control software

Portable Instrument Overview

- Receiver imaging package (PSA) designed for spectral range of $630 - 835nm$
- Transmitter (PSG) currently utilizes a laser source at $671nm$
- Tripod mounted for potential real-time testing/validation and data acquisition
- One of the first portable Mueller matrix instruments

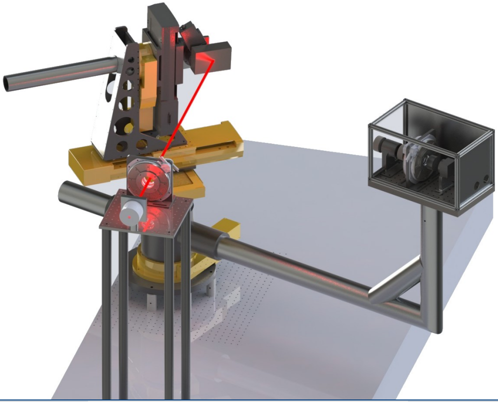


Portable Instrument Detail

- PSA utilizes micropolarizer array camera with $9\mu m$ square pixels, silicon sensor
- PSA $\mathrm{f}/\# = 2.4$
- PSG utilizes $200mW$ laser source at $671nm$
- Speckle not an issue with laser source, ~ 21 speckles per pixel at $671nm$
- PSA and PSG each use two retarders, each retarder can be rotated up to $1000rpm$
- PSG source matched to FOV of PSA when object is $> 50m$
- Retarders located in collimated optical spaces
- Instrument is a spatio-temporally modulated Mueller matrix imaging polarimeter
- Retains portable Stokes polarimetric capability in a smaller package

Scatterometer Overview

- Utilizes portable receiver imaging package + separate monochromator ($430 - 1200nm$) for PSG
- Designed to operate in transmission or reflection
- Positioning resolution of 0.25°, potentially much lower due to imaging optics resolution in receiver
- 6-axis robotic sample arm allows for full pBRDF measurement
- Designed for a sample weight of up to $2kg$.



Optimization of DURIP Channel Structure

The portable DURIP instrument has a channel structure which is constrained by:

- The structure of the micropolarizer array
- Time modulation of rotating linear retarders, limited to 2 in each of the PSG and PSA

$$\mathbf{R}(\nu,\delta;t)=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & \cos^2\frac{\delta}{2}+\sin^2\frac{\delta}{2}\cos(8\pi\nu t) & \sin^2\frac{\delta}{2}\sin(8\pi\nu t) & -\sin(4\pi\nu t)\sin\delta\\ 0 & \sin^2\frac{\delta}{2}\sin(8\pi\nu t) & \cos^2\frac{\delta}{2}-\sin^2\frac{\delta}{2}\cos(8\pi\nu t) & \cos(4\pi\nu t)\sin\delta\\ 0 & \sin(4\pi\nu t)\sin\delta & -\cos(4\pi\nu t)\sin\delta & \cos\delta\end{bmatrix}$$

$$\mathbf{p}(x;y)=\frac{1}{2}\begin{bmatrix}1\\ \frac{\cos\pi x+\cos\pi y}{2}\\ \frac{\cos\pi x-\cos\pi y}{2}\\ 0\end{bmatrix}$$

Optimization of DURIP Channel Structure

Consequence : modulation functions are separable, i.e.,

$$f(x, y, t) = f(x, y)f(t).$$

Implies bandwidth optimization can be realized from channel cancellation, but *not from ideal delta function placement*.

Optimization of DURIP Channel Structure

The portable instrument has the following optimization parameters:

- Normalized (to some maximum frequency) rotation rates, $\nu_1, \nu_2, \nu_3, \nu_4$
- Offset angles between the fast axes of the retarders
- Retardances, $\delta_1, \delta_2, \delta_3, \delta_4$

There is no spatial parameter because the micropolarizer camera is fixed and expensive.

Note that changing the retardances is something that is not too expensive compared with the total cost of the instrument, but the other parameters are free.

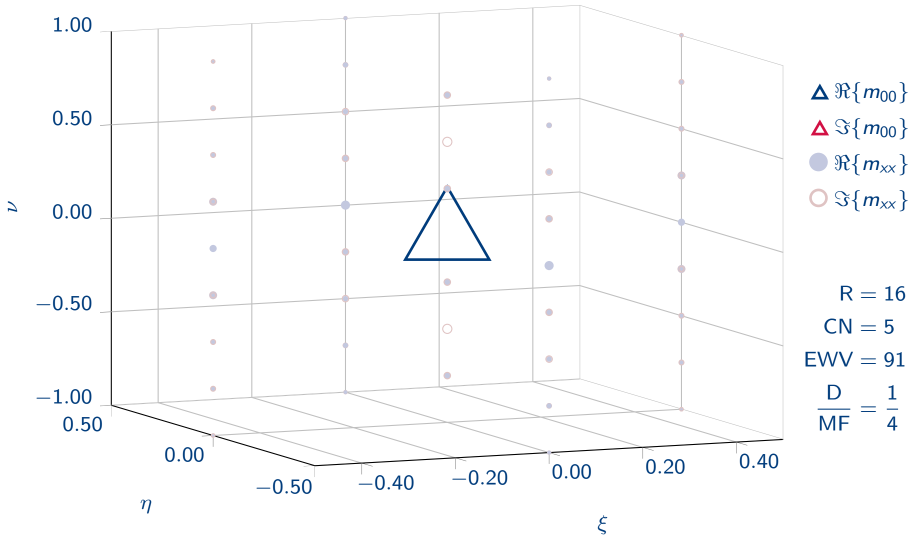
Channel Structure - Notation

- Retardances are denoted by $\delta_1, \delta_2, \delta_3, \delta_4$
- Relative rotation frequencies are denoted by $\nu_1, \nu_2, \nu_3, \nu_4$ and are constrained to be in $[-1, 1]$
- $\frac{\mathrm{D}}{\mathrm{MF}}$ denotes the normalized minimum temporal channel frequency distance (i.e. between channels), divided by the maximum global channel frequency
- CN and EWV are the condition number and the equal weighted variance, respectively, for reconstructability of some specified rank, R

For context a typical DRR Mueller matrix polarimeter has a maximum possible $\frac{\mathrm{D}}{\mathrm{MF}} = \frac{1}{12}$

Channel Structure - Four Retarders

$$\delta_1 = \pi, \delta_2 = \frac{\pi}{2}, \delta_3 = \pi - \arccos\frac{1}{\sqrt{3}}, \delta_4 = \pi, \nu_1 = \frac{1}{2}, \nu_2 = -1, \nu_3 = 1, \nu_4 = \frac{1}{2}$$



R = 16

CN = 5

EWV = 91

$$\frac{\text{D}}{\text{MF}} = \frac{1}{4}$$

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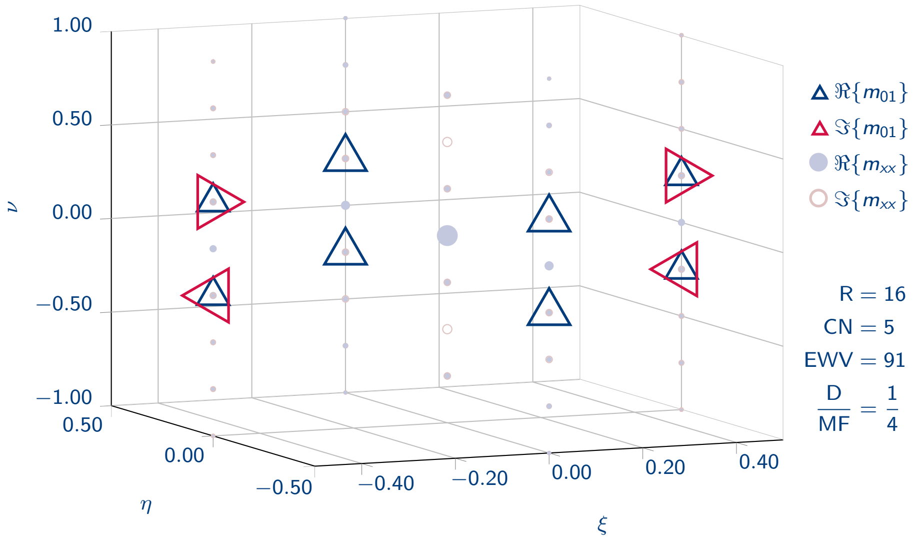

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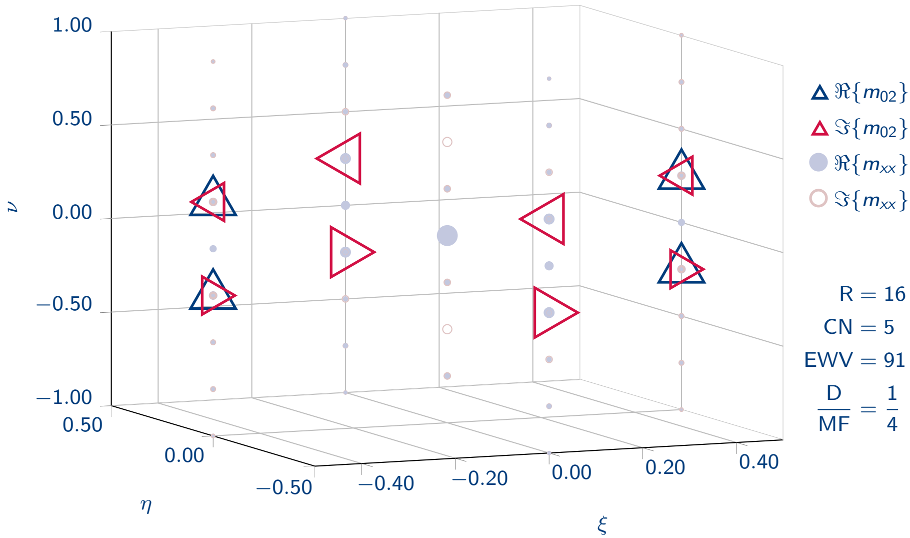

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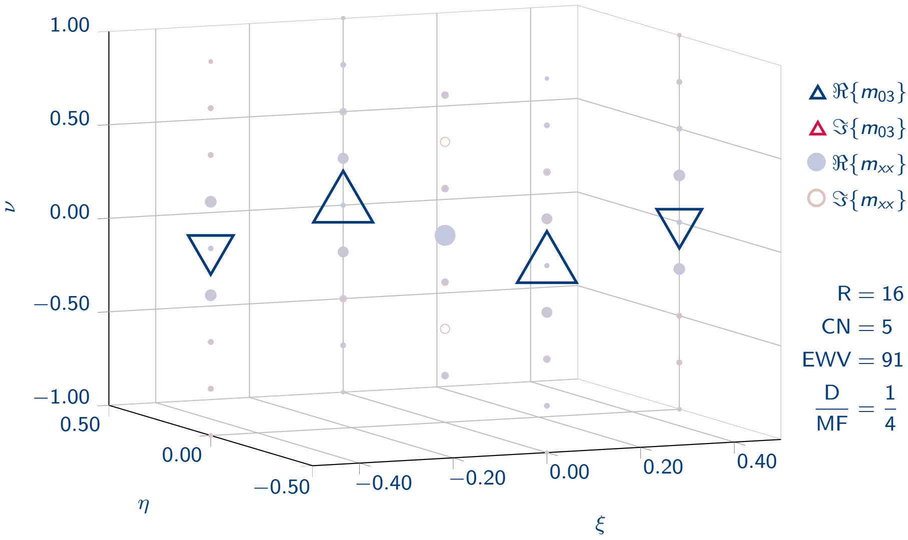

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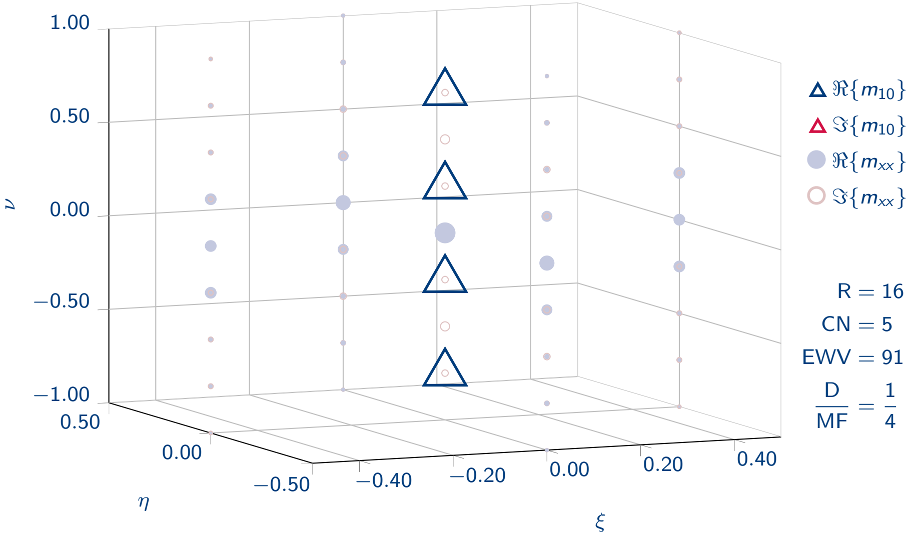

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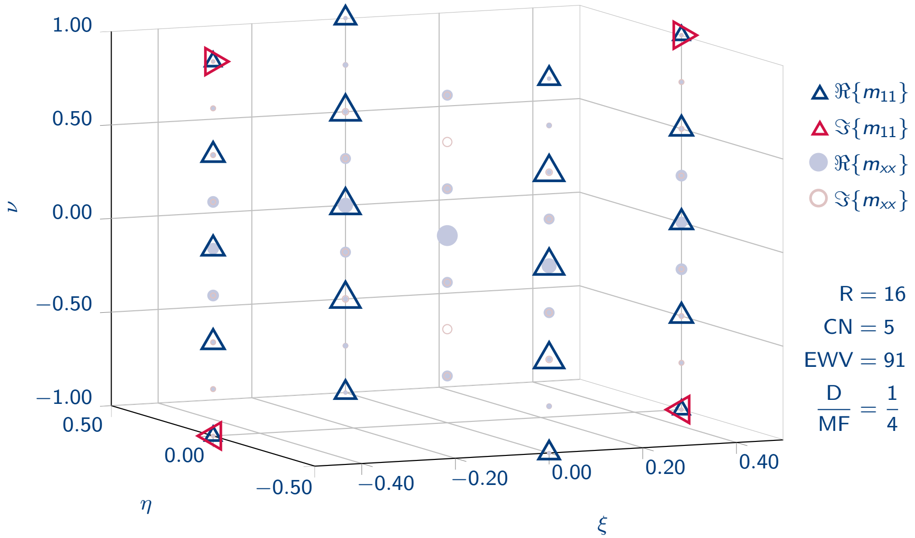

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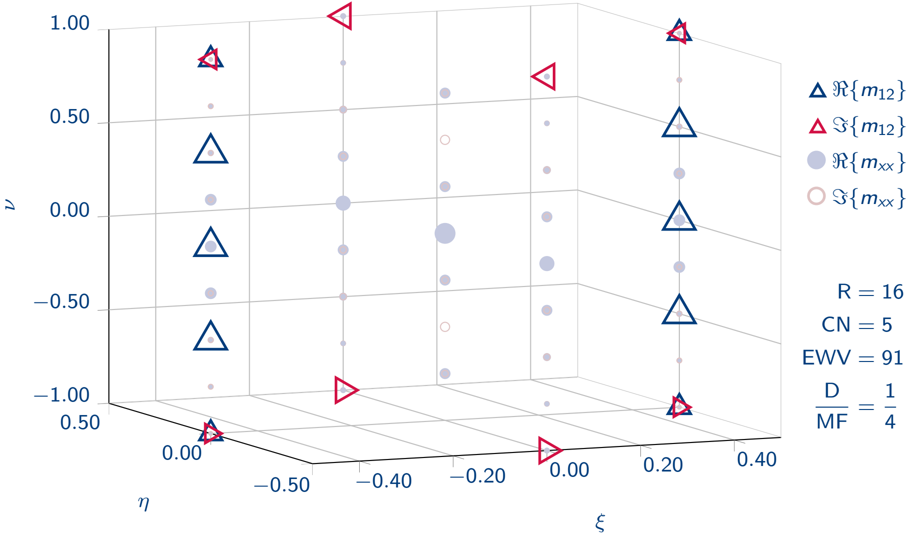

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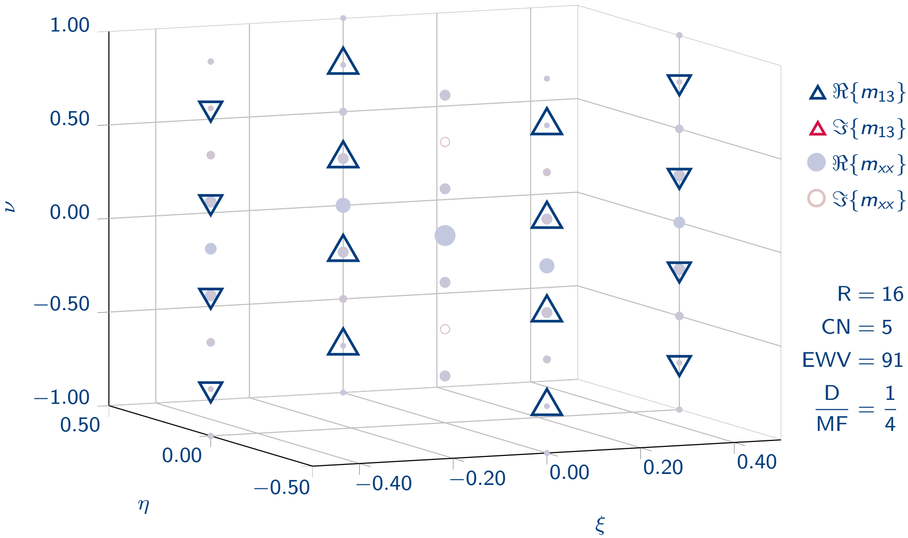

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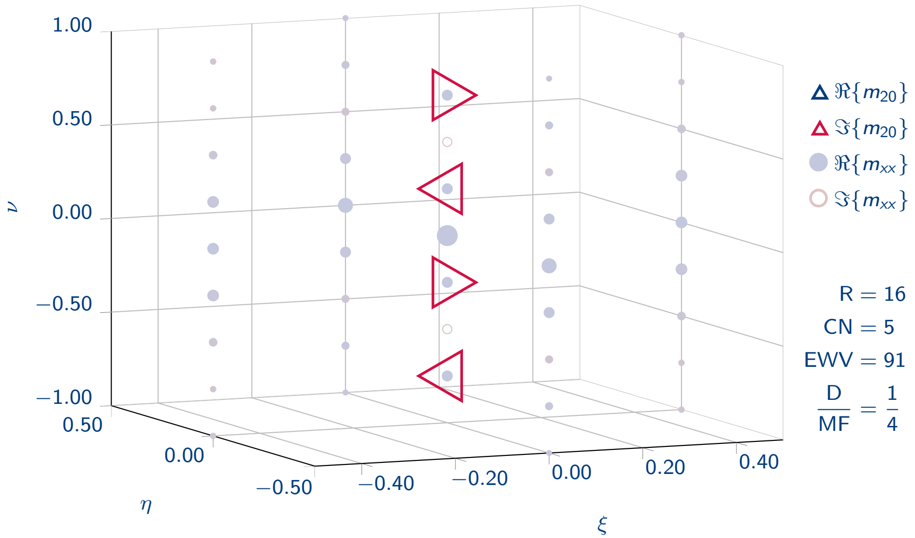

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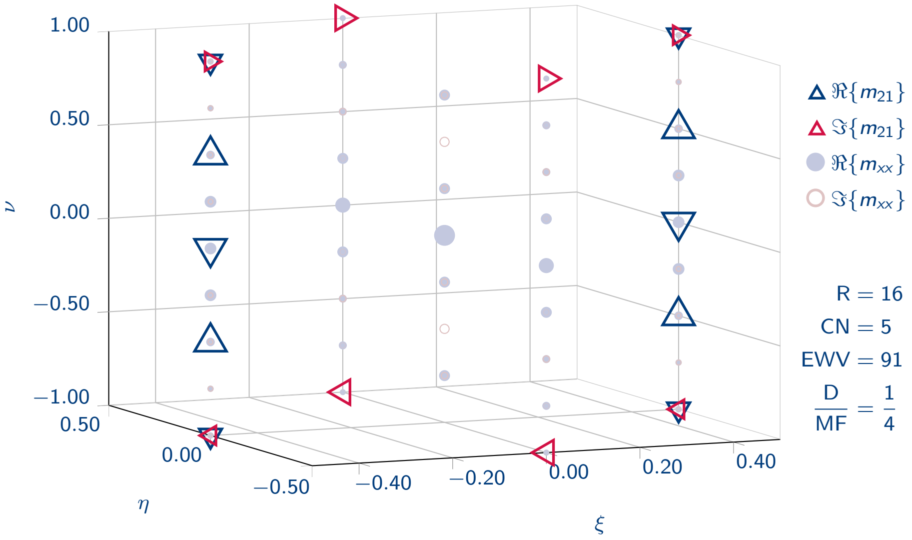

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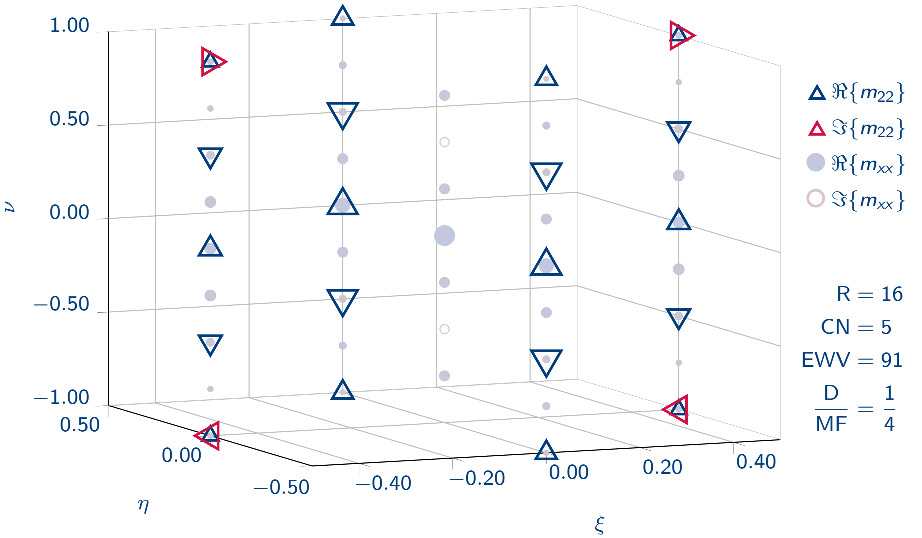

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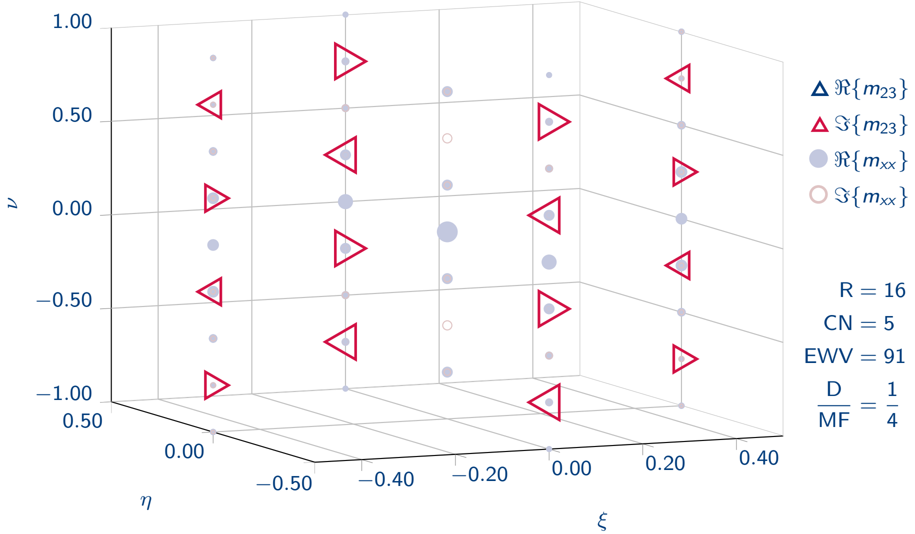

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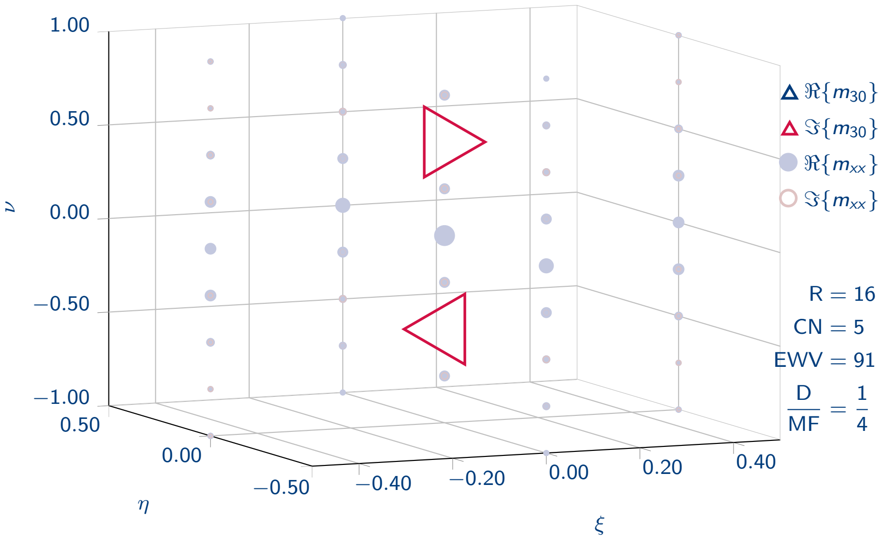

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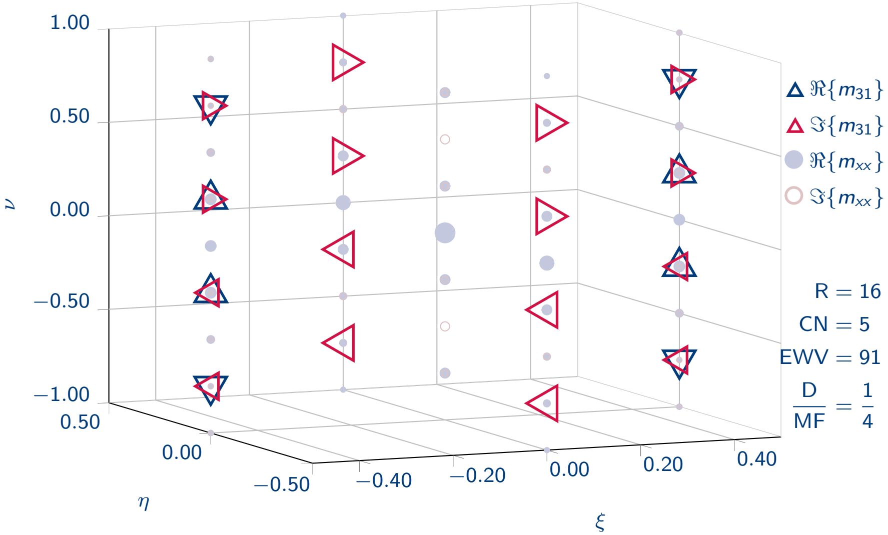

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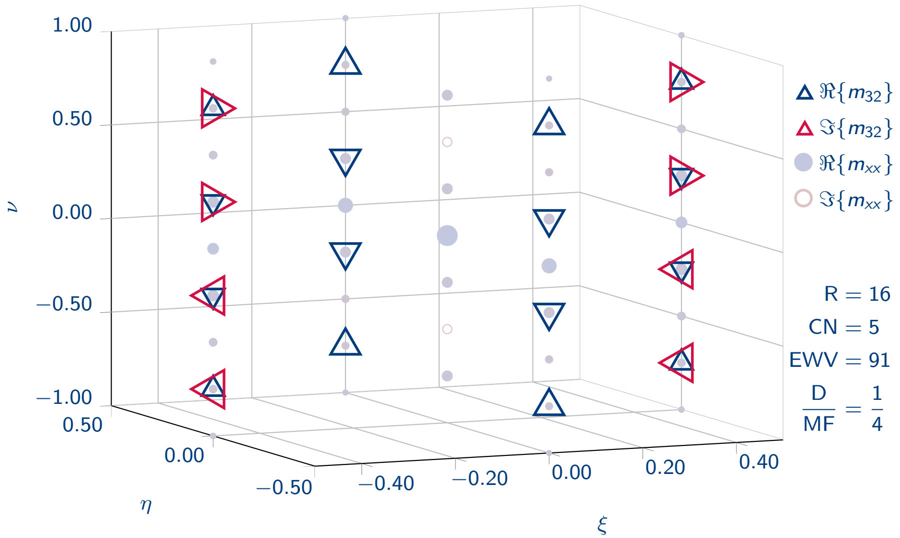

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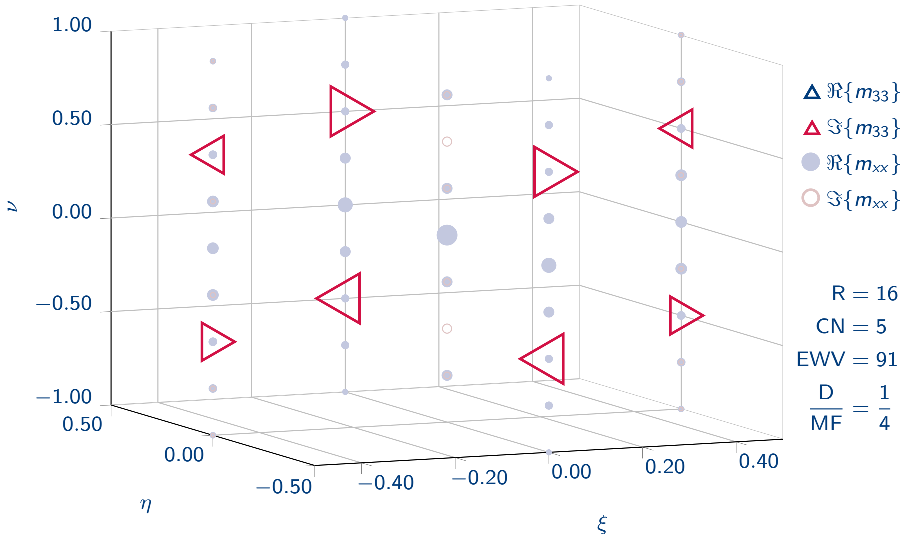

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Channel Structure - Summary

- 4 retarder + micropolarizer achieves best performance, better CN, EWV, and *threefold temporal bandwidth increase* over a typical DRR polarimeter
- 2 retarder + micropolarizer simpler mechanically, but only 50% temporal bandwidth improvement over a typical DRR polarimeter
- Research still ongoing for this particular type of channel structure, better results may surface

Publications Funded by AFOSR

- W. T. Black and J. S. Tyo, "Improving Feedback-integrated Scene Cancellation Nonuniformity Correction Through Optimal Selection of Available Camera Motion," accepted for publication in *J. Electronic Imaging*, September 2014
- A. Alenin, J. S. Tyo, "Generalized Channeled Polarimetry," *JOSA A* **31**:1013 – 1022 (2014)
- W. T. Black and J. S. Tyo, "Frequency-Independent Scene Cancellation Nonuniformity Correction Technique," *J. Electronic Imaging*, **23**:023005 (2014)
- O. G. Rodriguez-Herrera and J. S. Tyo, "Generalized van Cittert-Zernike theorem for the cross-spectral density matrix of quasi-homogeneous electromagnetic sources," *JOSA A*, **29**:1939 – 1947 (2012)
- C. F. LaCasse, J. S. Tyo, and R. A. Chipman, "The Role of the Null Space in Modulated Polarimeters," *Opt. Lett.* **37**: 1097 – 1099 (2012)
- C. F. LaCasse, R. A. Chipman, and J. S. Tyo, "Band limited reconstruction in modulated polarimeters," *Optics Express* **19**: 14976 – 14989 (2011)
- F. Goudail and J. S. Tyo, "When is polarimetric imaging preferable to intensity imaging for target detection?" *JOSA A* **28**:46 – 53 (2011)
- J. S. Tyo, Z. Wang, S. J. Johnson, and B. G. Hoover, "Design and optimization of partial Mueller matrix polarimeters," *Appl. Opt.* **49**: 2326 – 2333 (2010)
- A. Alenin and J. S. Tyo, "The Role of Additive Decompositions in Mueller Polarimetry," to be submitted to *JOSA A*
- C. F. LaCasse, J. S. Tyo, R. A. Chipman, "Wiener Filtering in Modulated Polarimeters," to be submitted to *Optics Express*
- W. T. Black and J. S. Tyo, "Scene Based NUC for Microgrid Polarimeters," to be submitted to *Applied Optics*

Questions?