Validation of the Slepian Approach to Truncation-Error Reduction in Spherical Near-Field Scanning[+]

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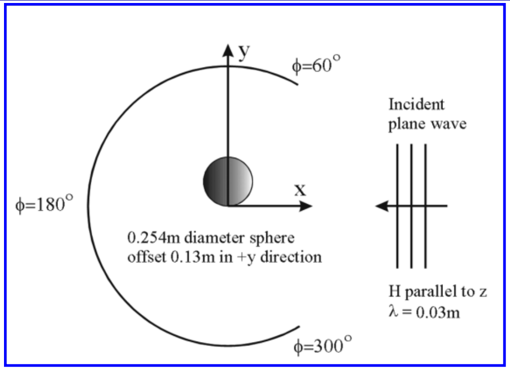
Kristopher T. Kim
Sensors Directorate
Air Force Research Laboratory
Wright-Patterson AFB, OH

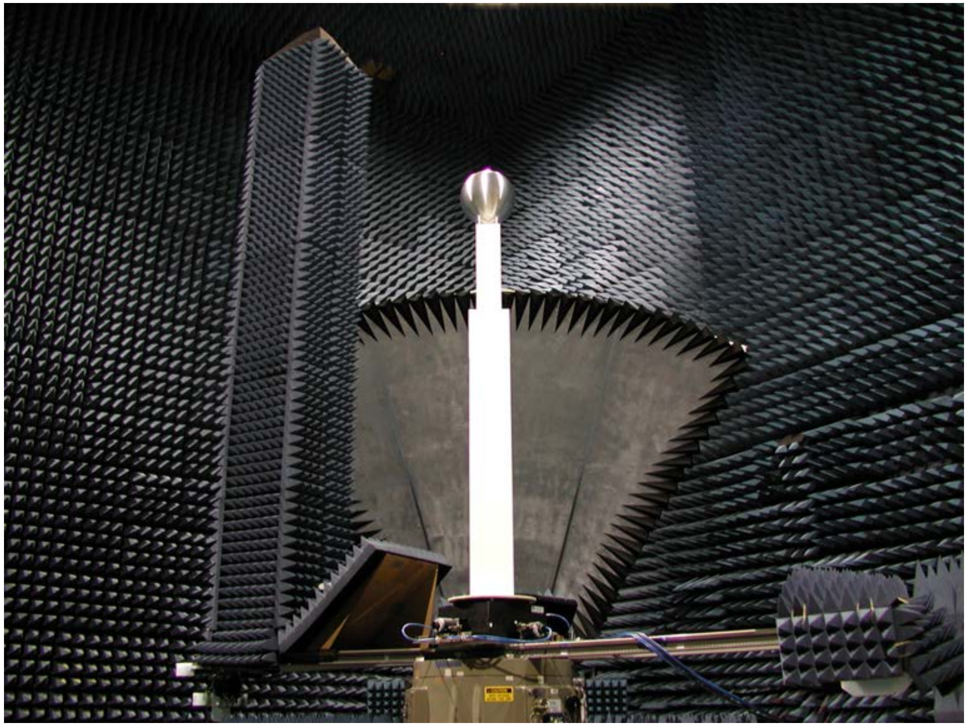
+ Supported in part by the Air Force Office of Scientific Research

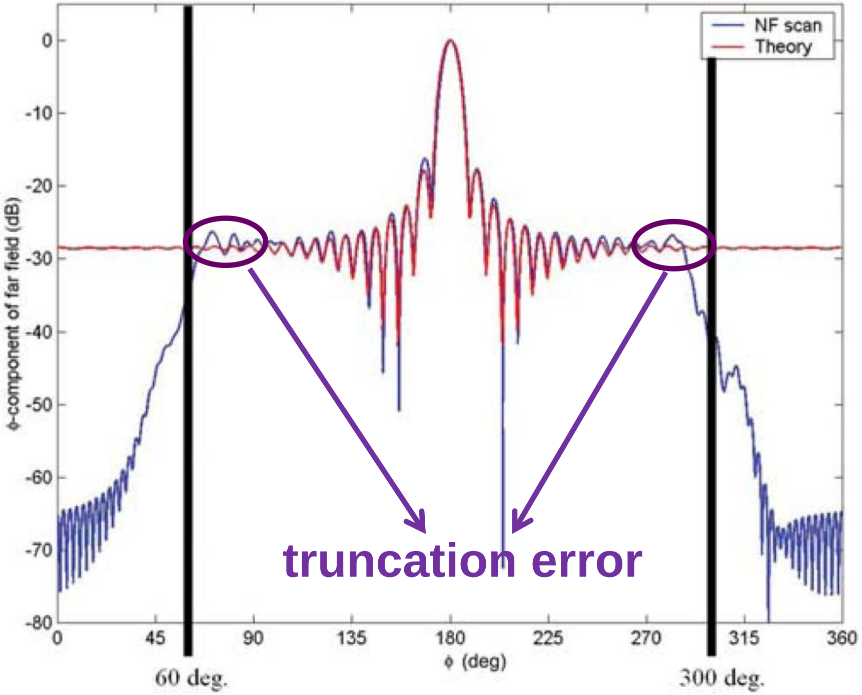
Motivation:
Truncation Error in Near-Field Scanning

Bistatic RCS of an offset 10” PEC sphere at 10 GHz

- $60^{\circ} \leq \varphi \leq 300^{\circ}, \ \Delta\varphi = 1.5^{\circ}$ (161 φ samples)
- $-1.26\text{m} \leq z \leq 1.26\text{m}, \ \Delta z = 0.015\text{m} = \lambda/2$ (168 z samples)
- scan radius $= 0.82\text{m} = 27.3\lambda$







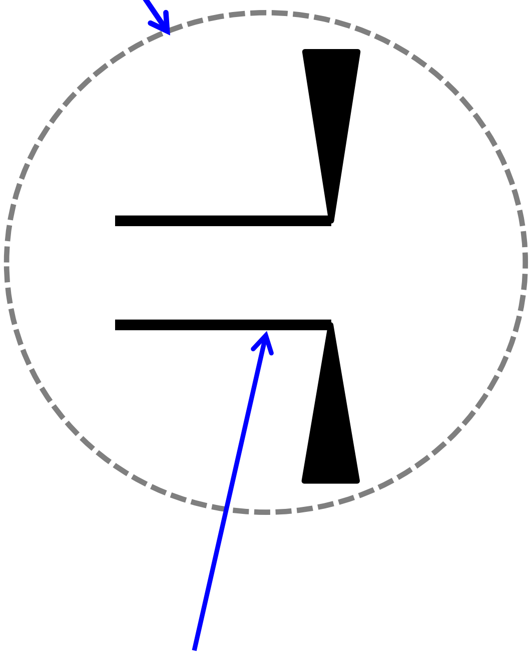
R. Marr, *et al.*, “Bistatic RCS Calculations from Cylindrical Near-Field Measurements—Part II: Experiment,” *IEEE Trans. Antennas Propag.*, vol. 54, no 12, pp. 3857-3864, 2006

Outline

- Classical Spherical NF-to-FF Transformation
  - Expansion of NF scan data in terms of classical transverse vector spherical harmonics (TVSH)
  - Why do truncation errors occur in classical NF-to-FF transformation?
- Slepian TVSH
  - Numerical properties of Slepian TVSH
- Description of scan data from Prof. S. Pivnenko of TUD
- NF-to-FF transformation using Slepian TVSH
  - Truncation-error reduction for various truncation geometries
- Summary

Classical Spherical NF-to-FF Transformation:
Expansion of scan data in terms of TVSH

Expansion of the transverse radiated/scattered field*



$$\overline{E}_t(\overline{r}) = E_\theta(r,\theta,\phi)\,\hat{\theta} + E_\phi(r,\theta,\phi)\,\hat{\phi}$$
$$= \sum_{l=1}^{L}\sum_{m=-l}^{l}\left[b_{l,m}^{(1)}\,f_l^{(1)}(kr)\,\overline{X}_{l,m}^{(1)}(\theta,\phi) + b_{l,m}^{(2)}\,f_l^{(2)}(kr)\,\overline{X}_{l,m}^{(2)}(\theta,\phi)\right]$$
$$= \sum_{j=1}^{2}\sum_{l=1}^{L}\sum_{m=-l}^{l}\left[b_{l,m}^{(j)}\,f_l^{(j)}(kr)\,\overline{X}_{l,m}^{(j)}\right]$$

where:

$$f_l^{(1)}(z) = h_l(z) = \text{spherical Hankel function of order } l$$
$$f_l^{(2)}(z) = \frac{d}{zdz} z h_l(z)$$
$$\overline{X}_{l,m}^{(1)}(\theta,\phi) = \frac{1}{i\sqrt{l(l+1)}}\,\overline{r}\times\nabla Y_{l,m}(\theta,\phi)$$
$$\overline{X}_{l,m}^{(2)}(\theta,\phi) = \hat{r}\times\overline{X}_{l,m}^{(1)}$$
$$Y_{l,m}(\theta,\phi) = \text{spherical harmonics of degree } l \text{ and order } m$$
$$L = \text{int}(kr_o) + n_o$$

*J.D. Jackson, "Classical Electrodynamics," 3rd ed. 1999.
*M.H. Francis and R.C. Wittmann, "Near-Field Scanning Measurement: Theory and Practice," in *Modern Antenna Handbook*, C.A. Balanis, Editor, 2008.

Classical Spherical NF-to-FF Transformation:

Why do truncation errors occur in classical NF-to-FF transformation?

- When the scan samples, $\overline{E}_t(a,\theta,\phi)$, are collected over the entire spherical surface of radius a, $\Omega = \{0 \leq \theta \leq \pi, 0 \leq \phi \leq 2\pi\}$, $b_{l,m}^{(i)}$ can be determined using the orthogonality relation, $\int_\Omega \overline{X}_{l,m}^{(i)}(\theta,\phi) \cdot \overline{X}_{l',m'}^{(j)\,*}(\theta,\phi)\, d\Omega = \delta_{i,j}\delta_{l,l'}\delta_{m,m'}$:

$$b_{l,m}^{(i)} = \frac{1}{f_l^{(i)}(ka)} \int_\Omega \overline{E}_t(a,\theta,\phi) \cdot \overline{X}_{l,m}^{(i)\,*}(\theta,\phi)\, d\Omega, \quad i = 1,2$$

- However, when $\overline{E}_t(a,\theta,\phi)$ are collected over a truncated spherical surface, $\Omega_t = \{\theta_1 \leq \theta \leq \theta_2, \phi_1 \leq \phi \leq \phi_2\}$, $b_{l,m}^{(i)}$ are estimated using the truncated NF samples

$$\tilde{b}_{l,m}^{(i)} = \frac{1}{f_l^{(i)}(ka)} \int_{\Omega_t} \overline{E}_t(a,\theta,\phi) \cdot \overline{X}_{l,m}^{(i)\,*}(\theta,\phi)\, d\Omega, \quad i = 1,2$$

- Since $\tilde{b}_{l,m}^{(i)} \neq b_{l,m}^{(i)}$, $\tilde{\overline{E}}_t(\infty,\theta,\phi)$ computed from the $\tilde{b}_{l,m}^{(i)}$ do not agree with the true $\overline{E}_t(\infty,\theta,\phi)$ over the entire Ω_t; they diverge around the boundary of Ω_t.

- Thus, it is always necessary to collect NF samples, $\overline{E}_t(a,\theta,\phi)$, over a range of angles wider than the range of angles over which $\overline{E}_t(\infty,\theta,\phi)$ need to be computed, resulting in increased data collection time and higher measurement cost

- It can be shown that the extent of the truncation error critically depends on the scan radius, a (Hansen, et al: 2005, Kim:2010, Kim:2011)

Slepian Transverse Vector Spherical Harmonics

- Note that $\int_{\Omega_t} \overline{X}_{l,m}^{(i)}(\theta,\phi) \cdot \overline{X}_{l',m'}^{(j)}{}^{*}(\theta,\phi)\, d\Omega \neq \delta_{i,j}\,\delta_{l,l'}\,\delta_{m,m'}, \quad i,j = 1,2.$

- We want to construct the *Slepian* transverse vector spherical harmonics (TVSH), $\overline{Z}_{l,m}^{(1)}(\theta,\phi)$ and $\overline{Z}_{l,m}^{(2)}(\theta,\phi)$, from the classical TVSH, $\overline{X}_{l,m}^{(1)}(\theta,\phi)$ and $\overline{X}_{l,m}^{(2)}(\theta,\phi)$, for a given truncated spherical surface, Ω_t, such that

$$\int_{\Omega_t} \overline{Z}_{l,m}^{(i)}(\theta,\phi) \cdot \overline{Z}_{l',m'}^{(j)}{}^{*}(\theta,\phi)\, d\Omega = \lambda_{i,l,m}\,\delta_{i,j}\,\delta_{l,l'}\,\delta_{m,m'}, \quad i,j = 1,2.$$

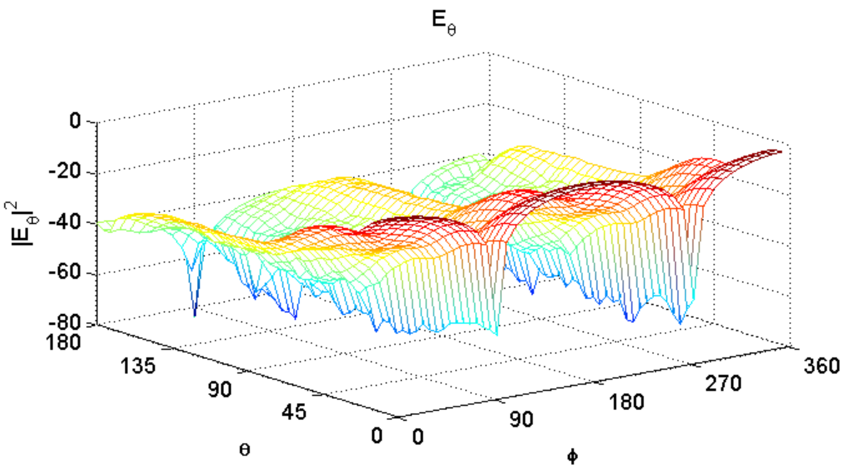
- Truncation errors can be avoided, if $\overline{E}_t(a,\theta,\phi)$ can be expanded in terms of $\overline{Z}_{l,m}^{(i)}(\theta,\phi)$.

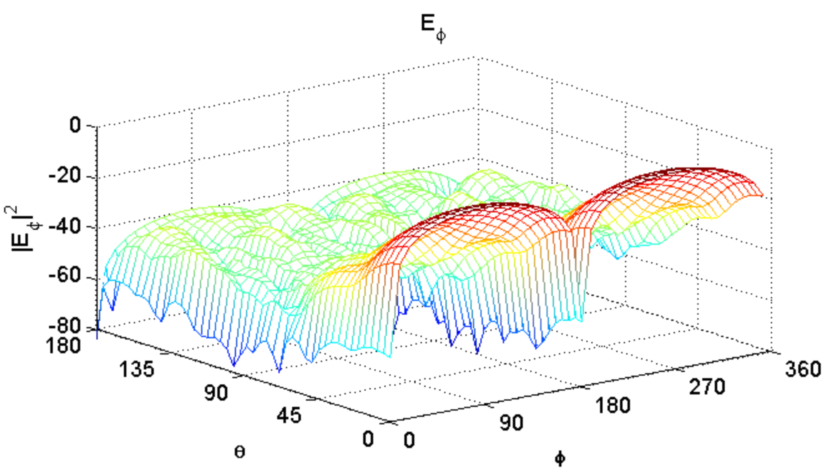
- We showed in (*) the following:
  1. how to construct $\overline{Z}_{l,m}^{(1)}(\theta,\phi)$ and $\overline{Z}_{l,m}^{(2)}(\theta,\phi)$ from $\overline{X}_{l,m}^{(1)}(\theta,\phi)$ and $\overline{X}_{l,m}^{(2)}(\theta,\phi)$
  2. their mathematical/numerical properties
  3. how to perform NF-to-FF transformation with $\overline{Z}_{l,m}^{(1)}(\theta,\phi)$ and $\overline{Z}_{l,m}^{(2)}(\theta,\phi)$
  4. validation of the Slepian NF-to-FF transformation using synthetic NF data

(*) K.T. Kim, "Slepian Transverse Vector Spherical Harmonics and Their Application to Near-Field Scanning", Proceedings of the IEEE Antennas and Propagation Society International Conference, Spokane, WA, 2010.

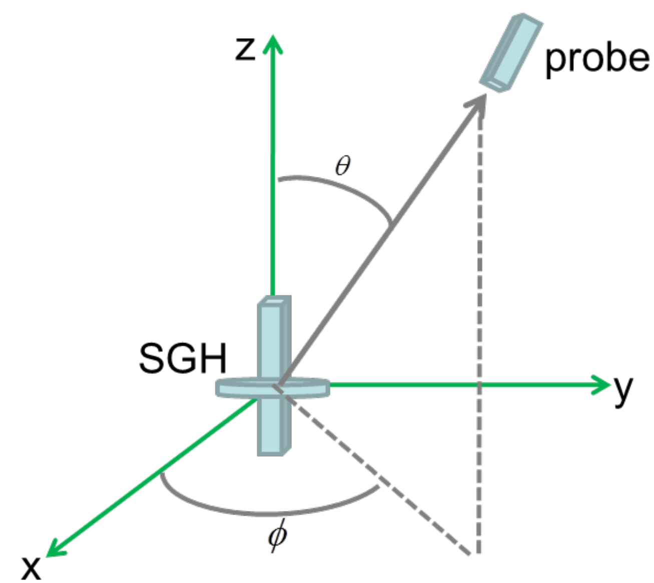
TUD Scan Data* (1)

Scan data at 2.2 GHz (scan radius = 6 m):





DUT: SA 12-1.7 Standard Gain Horn
λ=0.136 m
$\Delta\theta = 4^o$, $\Delta\varphi = 9^o$ (46 x 41 samples)
minimum sphere radius ≈ 1.5 λ
FF distance ≈ 2.5 m

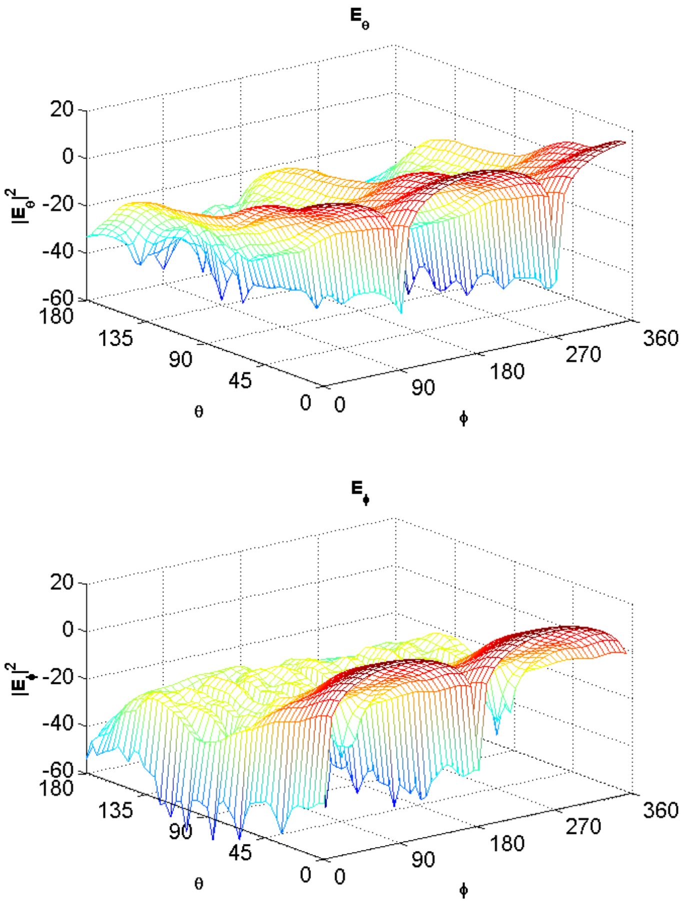


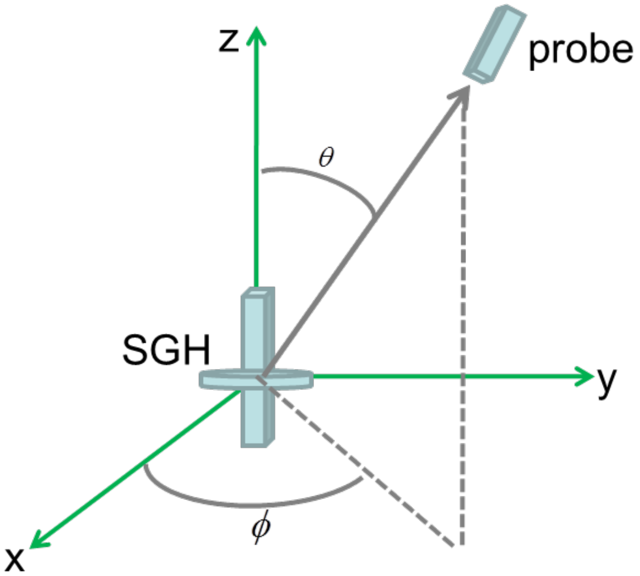
scan geometry

* Scan data provided by Prof. S Pivnenko of TUD

TUD Scan Data (2)

"FF-to-NF" transformed data (scan radius = 0.6 m):



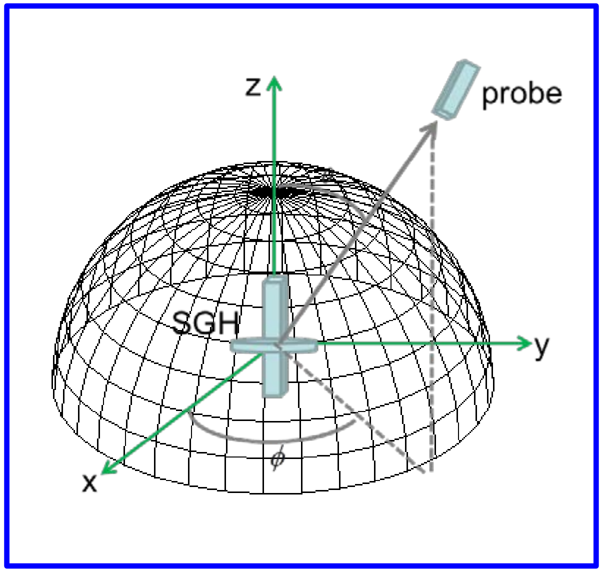


scan geometry

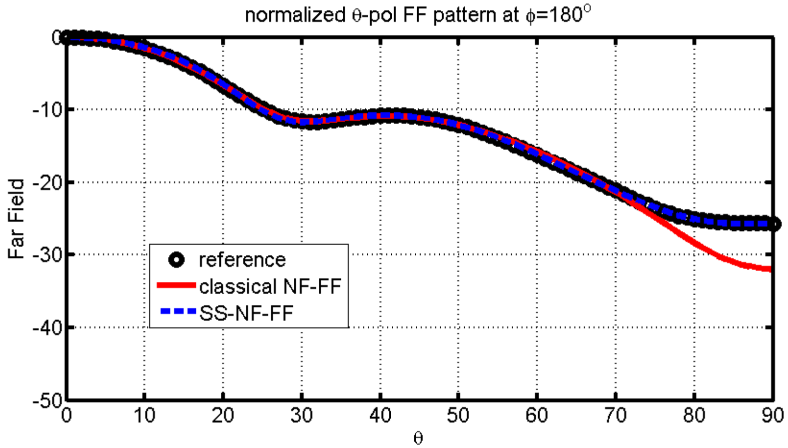
NF-to-FF Transformation using Slepian TVSH (1)

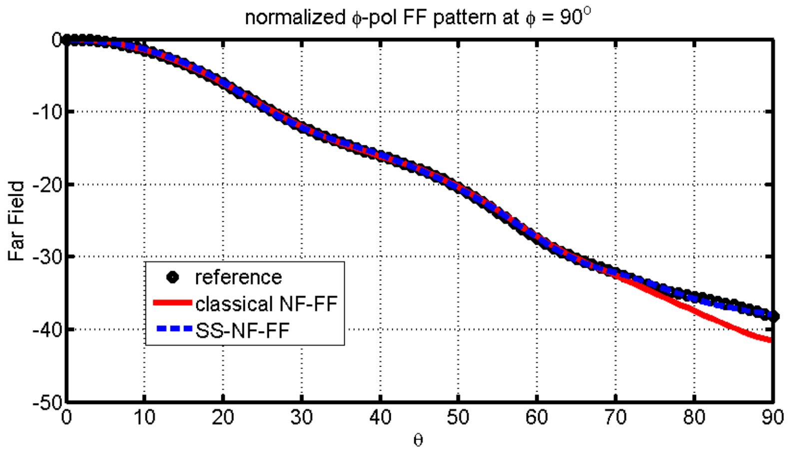
FF reconstruction from hemispherical scan data:

$$0^\circ \le \theta \le 90^\circ, 0^\circ \le \phi \le 360^\circ$$



scan geometry



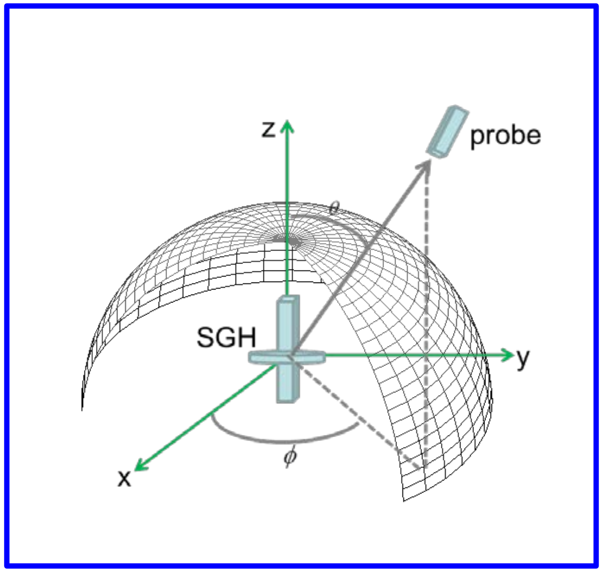


reconstructed far fields

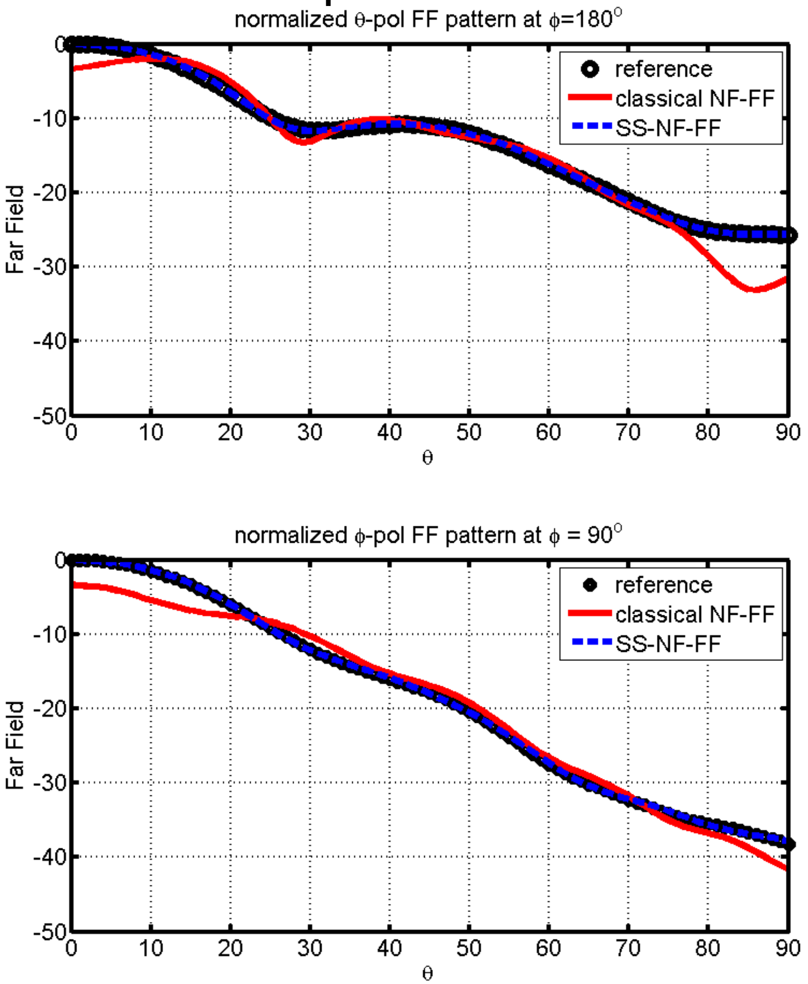
NF-to-FF Transformation using Slepian TVSH (2)

FF reconstruction from scan data truncated in θ and φ:

$$0^\circ \leq \theta \leq 90^\circ, 60^\circ \leq \phi \leq 300^\circ$$



scan geometry

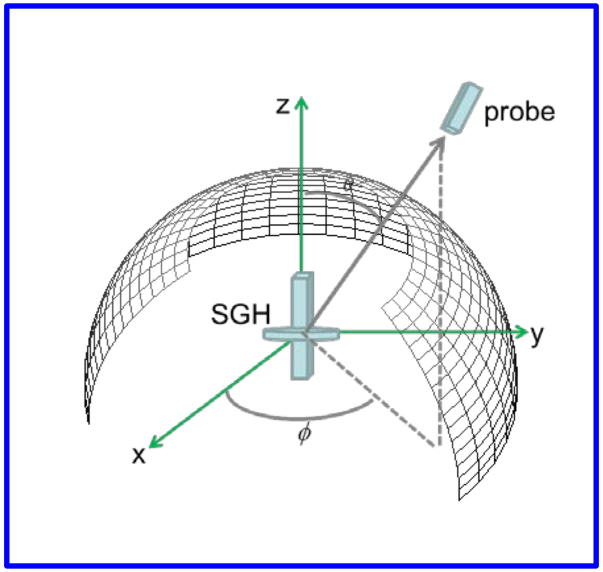


reconstructed far fields

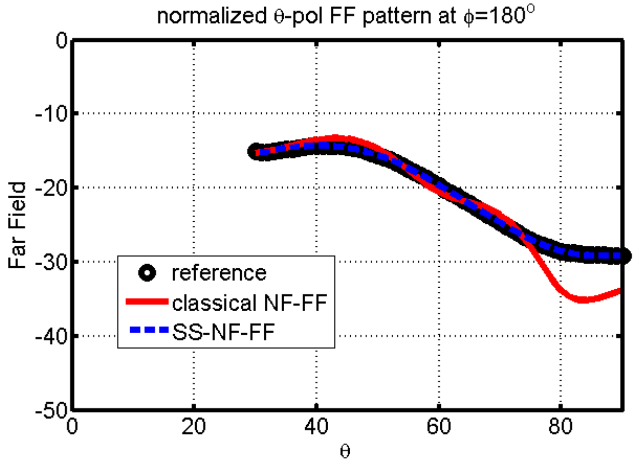
NF-to-FF Transformation using Slepian TVSH (3)

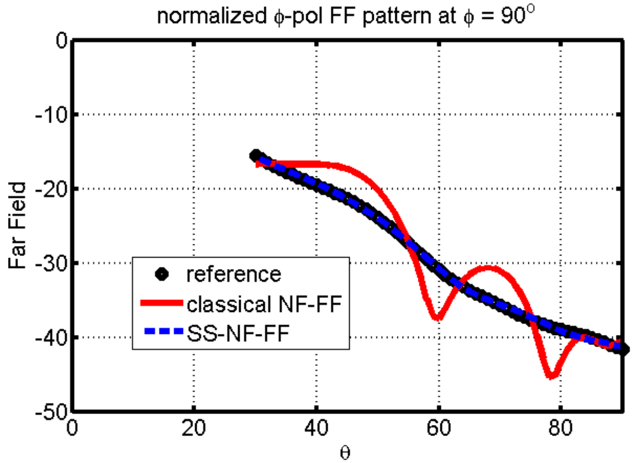
FF reconstruction from scan data truncated in θ and φ:

$$30^\circ \le \theta \le 90^\circ, 60^\circ \le \phi \le 300^\circ$$



scan geometry





reconstructed far fields

Summary

- Traditional NF-FF transformation based on the classical transverse vector spherical harmonics (TVSH) leads to truncation errors, resulting in increased NF collection time and higher measurement cost
  - Extents of truncation errors depend on scan radius
  - The classical TVSH are not orthogonal over a truncated spherical surface, Ω_t
- The Slepian TVSH can be constructed for a given Ω_t as a linear transformation of the classical TVSH.
  - The Slepian TVSH are orthogonal over Ω_t with the orthogonality constants equal to their concentrations over Ω_t
- NF-FF transformation using Slepian TVSH significantly reduces truncation errors