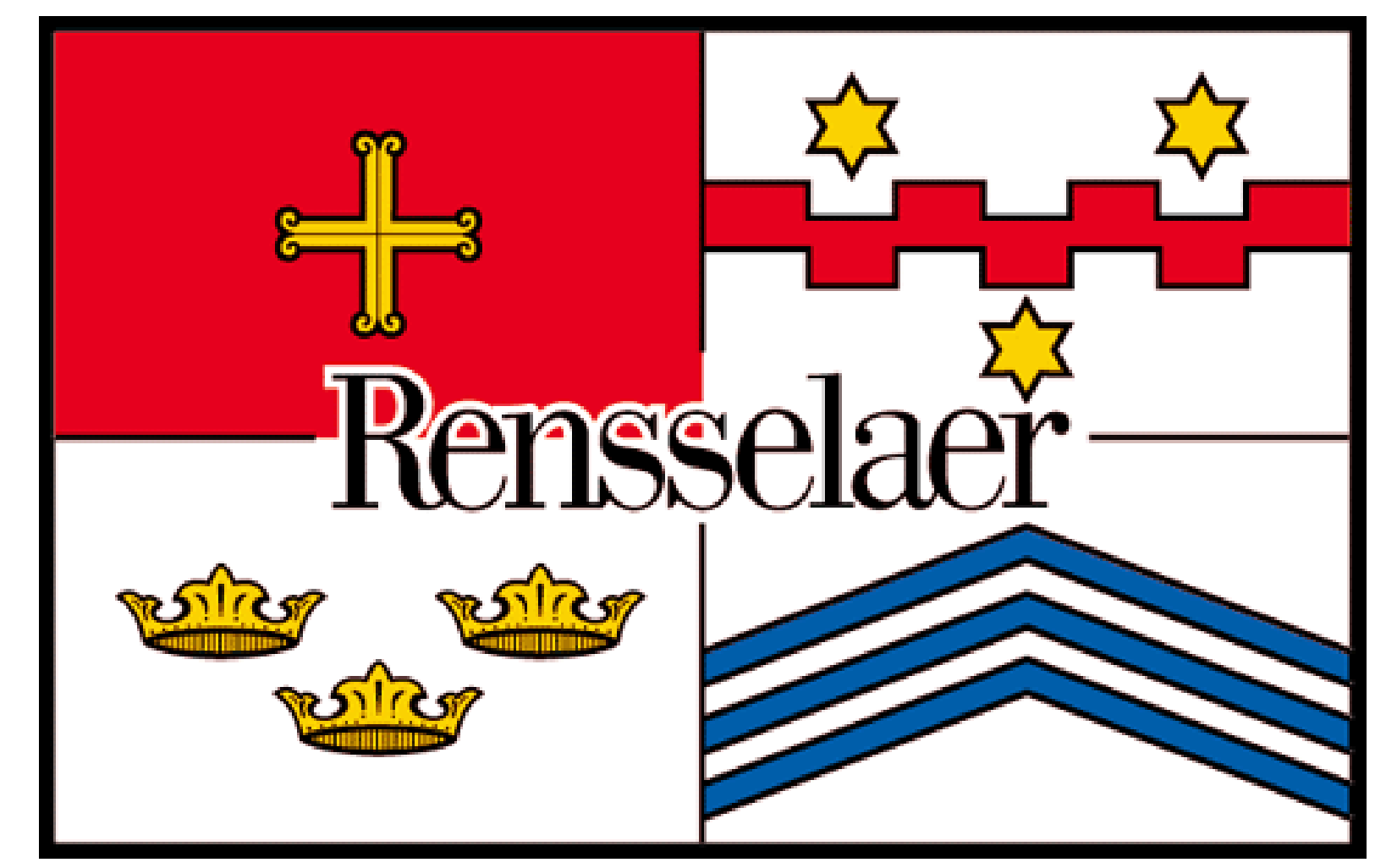


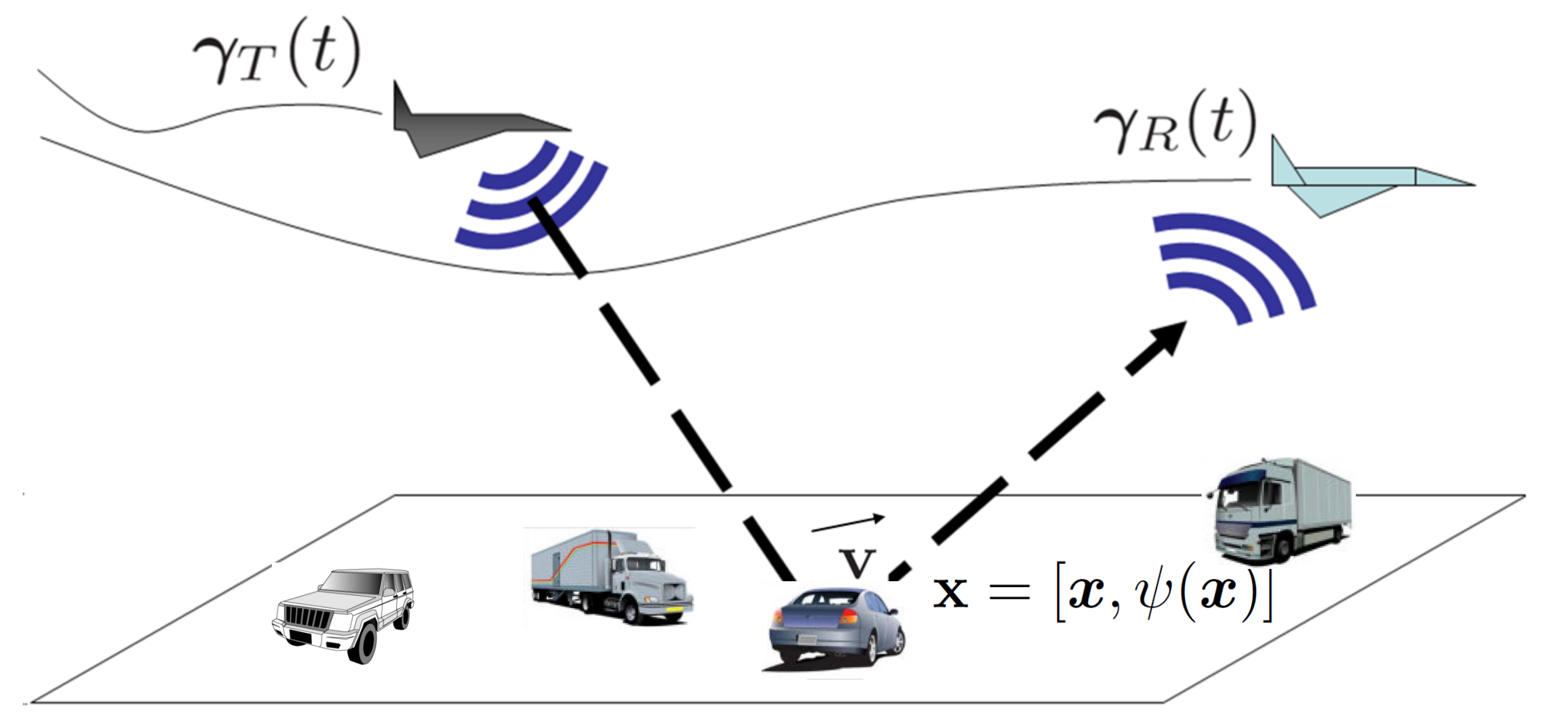
SAR Imaging of Multiple Ground Moving Targets Using Ultranarrow-Band Continuous Wave (CW) SAR

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Summary: We present a novel method of imaging multiple moving targets using a SAR system transmitting ultranarrowband continuous waveforms. Our method comprises of a new forward model that relates the velocity as well as reflectivity information at each location to a correlated received signal; and a novel image formation method based on filtered-backprojection and image-contrast optimization. *The method results in well-focused reflectivity images of moving targets and their velocity estimates regardless of the target location, speed, and velocity direction.*



RECEIVED SIGNAL

Received Signal for a time interval starting at $t=s$:

$$r(t+s) = \int \frac{\ddot{p}(\alpha t - \tau + s)q(\mathbf{x}, \mathbf{v})}{(4\pi)^2 |\gamma_R(s) - (\mathbf{x} + \mathbf{v}s)| |(\mathbf{x} + \mathbf{v}s) - \gamma_T(s)|} d\mathbf{x} d\mathbf{v}$$

$\gamma_R(s)$: Receiver trajectory

$p(t)$: Transmitted waveform

$\gamma_T(s)$: Transmitter trajectory

Velocity of the target

$$q(\mathbf{x}, \mathbf{v}) = \rho(\mathbf{x})\delta(\mathbf{v} - \mathbf{v}_\mathbf{x})$$

$$\mathbf{x} = [\mathbf{x}, \psi(\mathbf{x})]$$

$$\mathbf{v} = [\mathbf{v}, \nabla_{\mathbf{x}}\psi(\mathbf{x}) \cdot \mathbf{v}]$$

Surface topography

Phase-space reflectivity function of Moving Scene

$$\alpha = \frac{1 - \widehat{\gamma_R(s) - (\mathbf{x} + \mathbf{v}s)} \cdot \dot{\gamma_R(s)}/c_0}{1 + \widehat{\gamma_T(s) - (\mathbf{x} + \mathbf{v}s)} \cdot \dot{\gamma_T(s)}/c_0} \cdot \frac{1 + \widehat{\gamma_T(s) - (\mathbf{x} + \mathbf{v}s)} \cdot \mathbf{v}/c_0}{1 - \widehat{\gamma_R(s) - (\mathbf{x} + \mathbf{v}s)} \cdot \mathbf{v}/c_0}$$

Time dilatation factor

$\widehat{\mathbf{x}} = \mathbf{x}/|\mathbf{x}|$: Unit vector in the direction of \mathbf{x}

FORWARD MODEL

Correlation of the received signal with a scaled version of the transmitted signal over a finite window:

$$d(s, \mu) = \int r(t+s)p^*(\mu t)\phi(t)dt$$

scale factor
windowing function

$$d(s, \mu) \approx \mathcal{F}[q](s, \mu) := \int e^{-i\phi(t, \mathbf{x}, \mathbf{v}, s, \mu)} A(t, \mathbf{x}, \mathbf{v}, s, \mu) q(\mathbf{x}, \mathbf{v}) d\mathbf{x} d\mathbf{v} dt$$

Phase: $\phi(t, \mathbf{x}, \mathbf{v}, s, \mu) = 2\pi f_0 t[(\mu - 1) + f_d(s, \mathbf{x}, \mathbf{v})/f_0]$

$$f_d(s, \mathbf{x}, \mathbf{v}) = \frac{f_0}{c_0} [\gamma_T(s) - (\mathbf{x} + \mathbf{v}s) \cdot (\dot{\gamma}_T(s) - \mathbf{v}) + \gamma_R(s) - (\mathbf{x} + \mathbf{v}s) \cdot (\dot{\gamma}_R(s) - \mathbf{v})]$$

Bistatic Doppler frequency for moving targets

Amplitude: $A(t, \mathbf{x}, \mathbf{v}, s, \mu) = \frac{\tilde{p}(t - \tau + s)\tilde{p}^*(t)e^{i\omega_0(s-\tau)}\omega_0^4}{(4\pi)^2 G_{TR}(s, \mathbf{x}, \mathbf{v})}$

Product of Geometrical spreading factor

IMAGE FORMATION

$$d(s, \mu) \xrightarrow{?} q(\mathbf{x}, \mathbf{v})$$

Two variable dependent

Four dimensional space

1. Filtered backprojection of the data onto bistatic iso-Doppler contour in position space for a fixed hypothetical velocity:

\mathbf{v}_h -reflectivity image

$$q_{\mathbf{v}_h}(z) := \mathcal{K}_{\mathbf{v}_h}[d](z)$$

$$= \int e^{i\phi(t, z, \mathbf{v}_h, s, \mu)} Q_{\mathbf{v}_h}(z, t, s) d(s, \mu) dt ds d\mu$$

Choose the filter such that leading order term of the point spread function is Dirac Delta

$$(t, s) \rightarrow \xi = t 2\pi \nabla_z f_d(s, z, \mathbf{v}_h) \quad Q_{\mathbf{v}_h}(z, \xi) = \frac{\chi_{\Omega_{\mathbf{v}_h, z}}}{\eta(z, \mathbf{v}_h, \xi)} \frac{A^*(z, \mathbf{v}_h, \xi)}{|A(z, \mathbf{v}_h, \xi)|^2}$$

determinant of the Jacobian of $c.v.$

2. Contrast-maximization based Velocity determination:

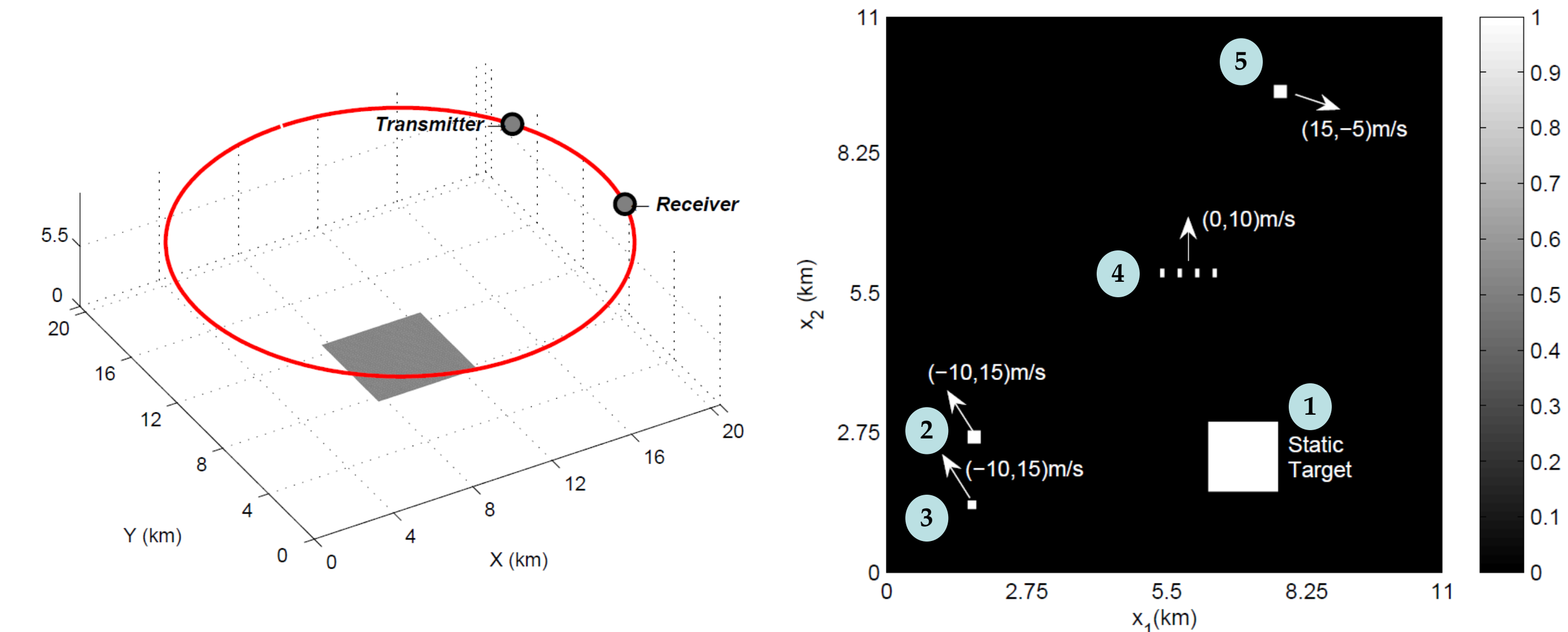
Localizing the local maxima in the contrast-image

$$I(\mathbf{v}_h) = \mathcal{C}[q_{\mathbf{v}_h}]$$

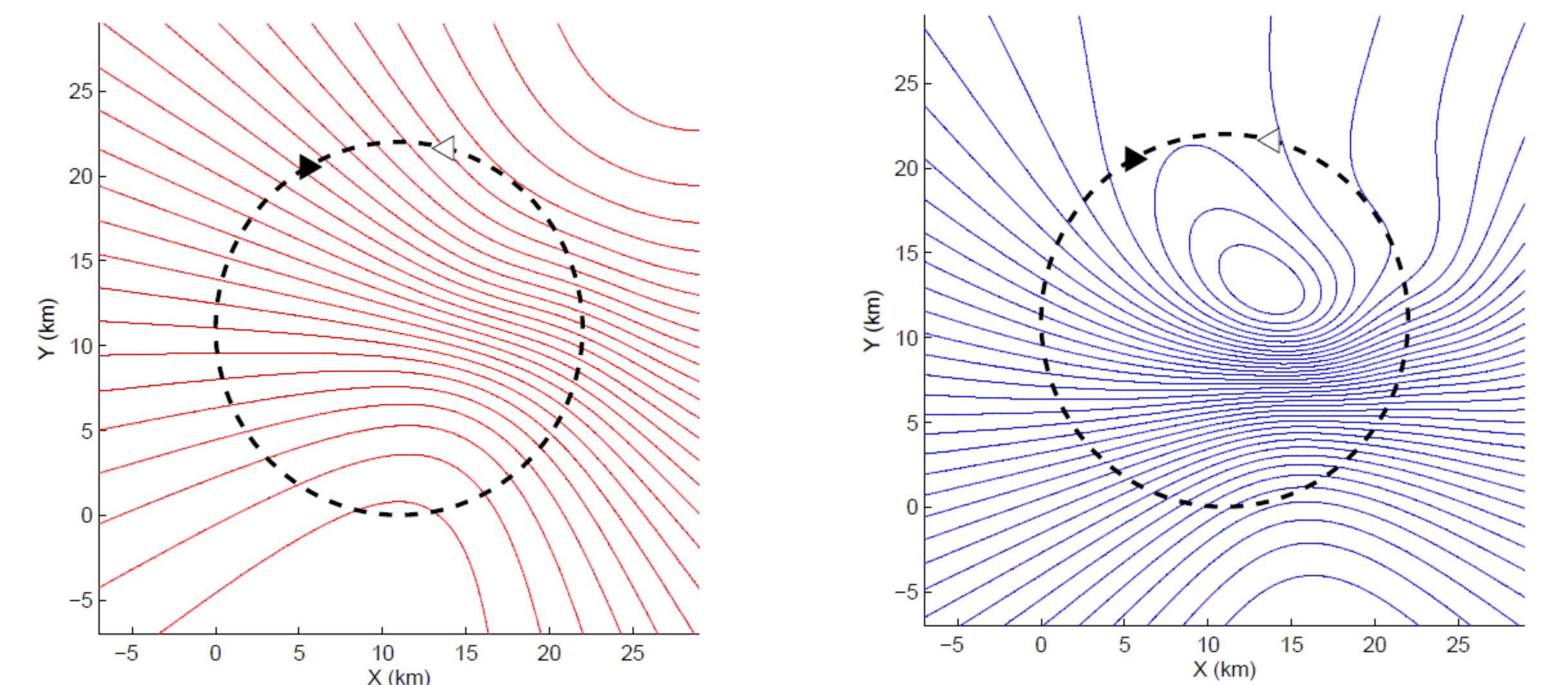
Image-contrast operator

NUMERICAL RESULTS

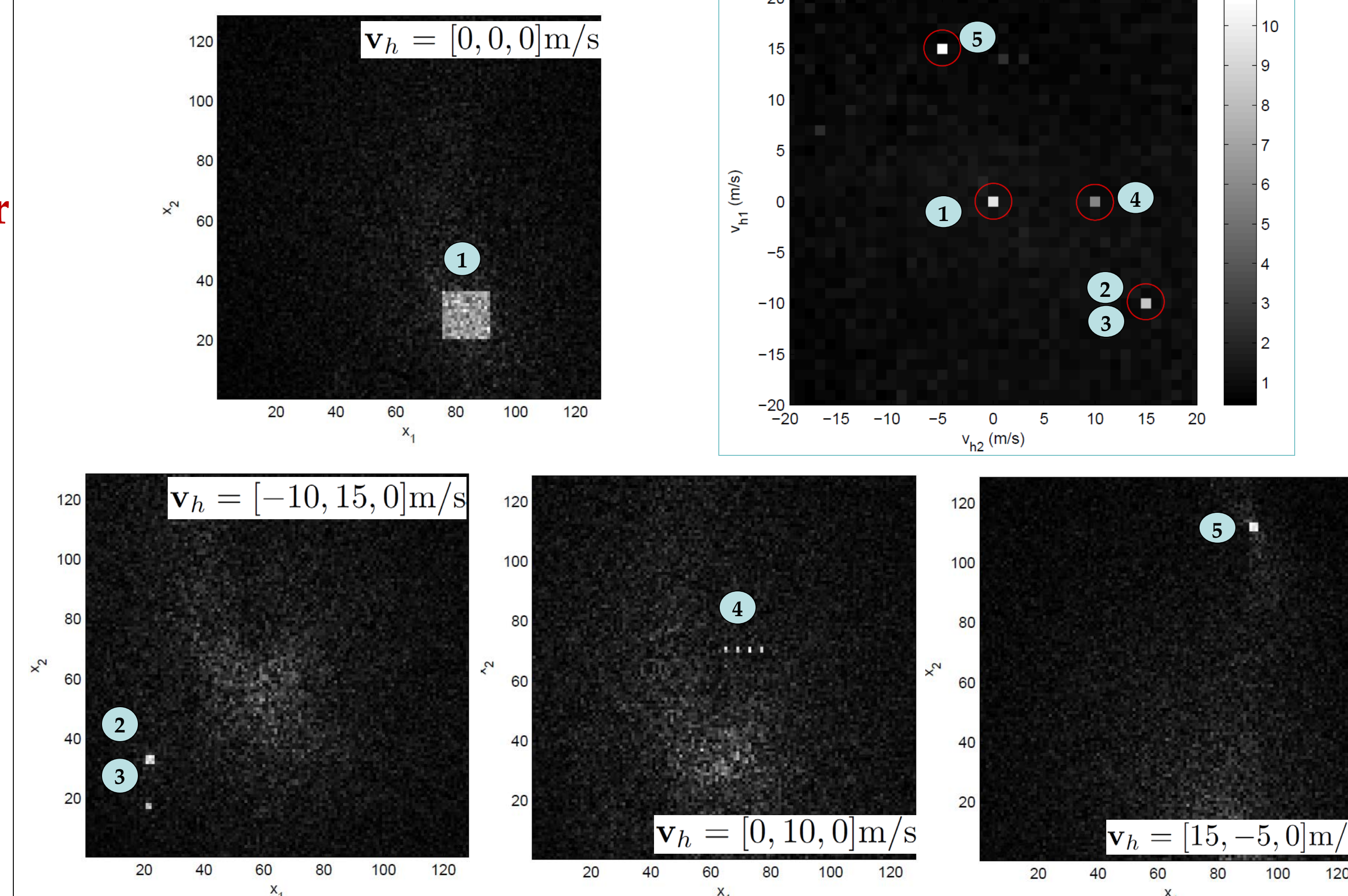
flight trajectory and moving target scene set-ups



Position-space bistatic iso-Doppler AND iso-Doppler-rate contours:



FBP Reconstructions



Contrast-image

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