



# PECASE: MULTI-SCALE EXPERIMENTS AND MODELING IN WALL TURBULENCE

**AFOSR # FA9550-09-1-0701 (P.M. J. Schmisser)**

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**Ati Sharma**

*University of Sheffield  
U.K.*

# OBJECTIVES

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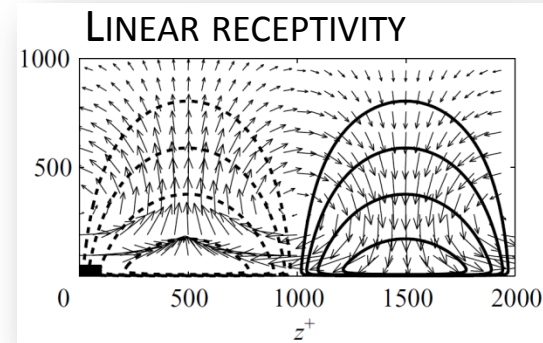
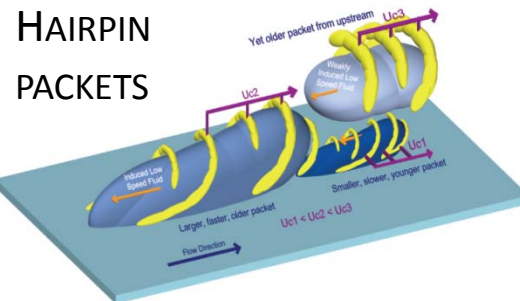
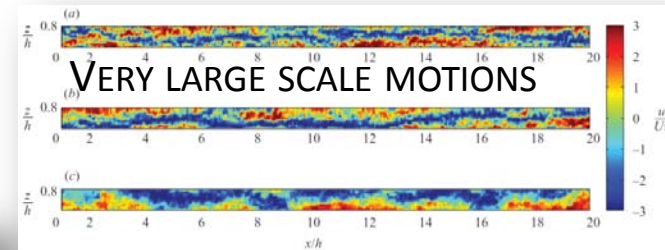
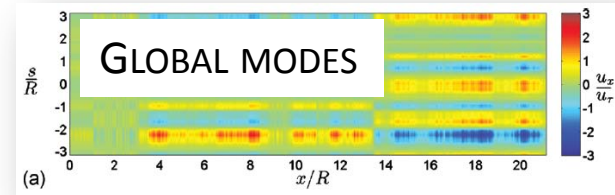
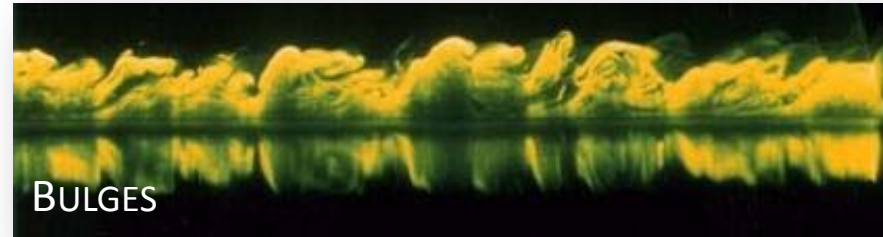
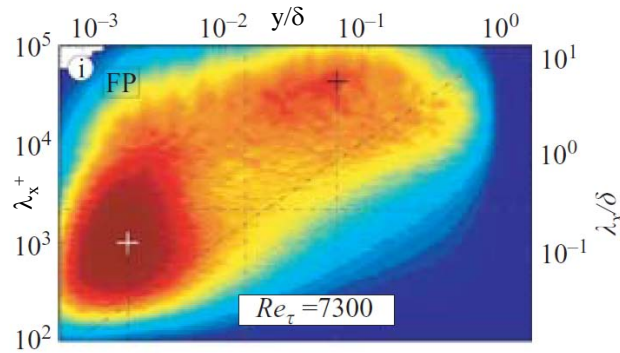
**Experimental & analytical investigation into building blocks of wall turbulence**

**Attempt to unify statistical and spectral understanding via explicit description of turbulent flow field in physical  $(x,y,z)$  and spectral  $(k,n,\omega)$  space**

- Formulation of a multi-scale model of wall turbulence
  - singular value decomposition of the linear operator governing a canonical wall-bounded turbulent flow (*McKeon & Sharma, 2010*)
- Experimental investigation of spatio-temporal spectrum
  - TRPIV (*LeHew et al, 2011*)
- Investigation of response of ZPGTBL to “dynamic” roughness
  - Spatially-impulsive, time dependent wall perturbation (*Jacobi & McKeon, 2011*)
- Maintenance of the mean velocity profile
  - Two-dimensional, three-component, 2D/3C, model (*Bourguignon & McKeon, 2011*)

# TOWARDS THE BUILDING BLOCKS OF TURBULENCE

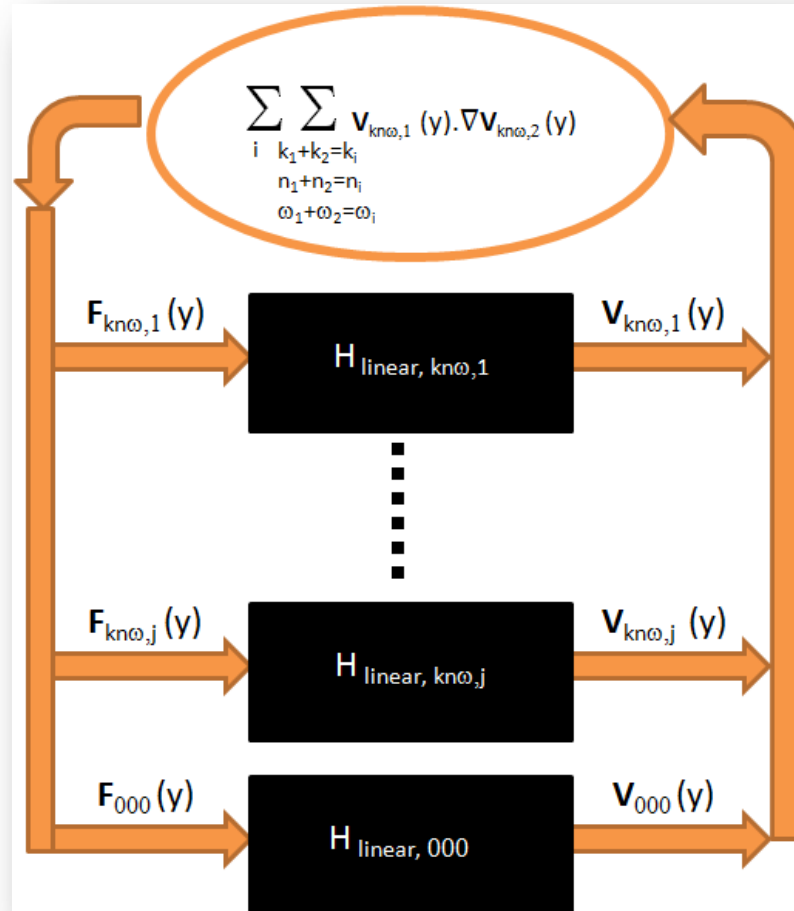
## STATISTICS & SPECTRA



*“So, actually the elephant has all the features you mentioned...”*

**Credits, CW from top left**  
 Hutchins & Marusic  
 Gad-el-Hak  
 Hellstroem, Sinha & Smits  
 Monty, Stewart, Williams & Chong  
 Adrian, Meinhardt & Tomkins  
 Del Alamo & Jimenez

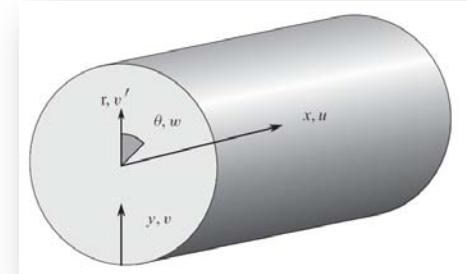
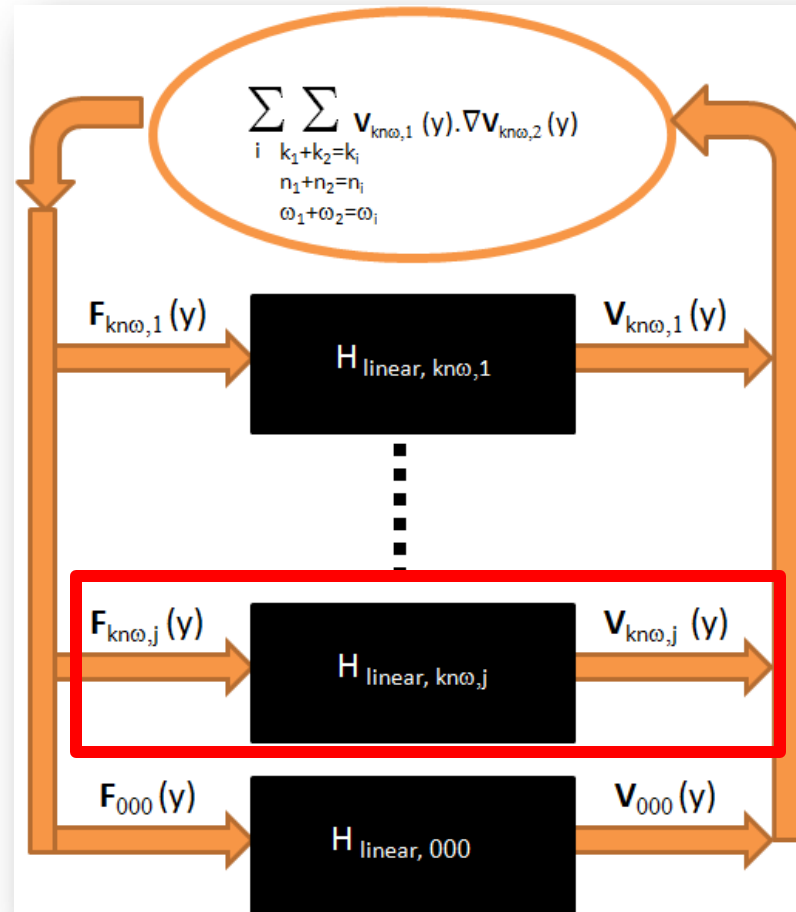
# OBJECTIVES –CARTOON FORM



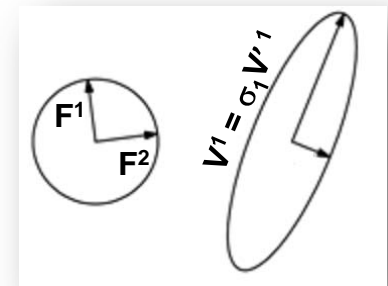
Assume propagating modes with  $(k, n, \omega)$  in a divergence-free basis

$$v(r, x, \theta, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} c_{mkn\omega} \xi_m(r) e^{i(\omega t - kx - n\theta)} dk d\omega \quad v_{kn\omega} = (v, e^{i(\omega t - kx - n\theta)})$$

# WORK TO DATE: I



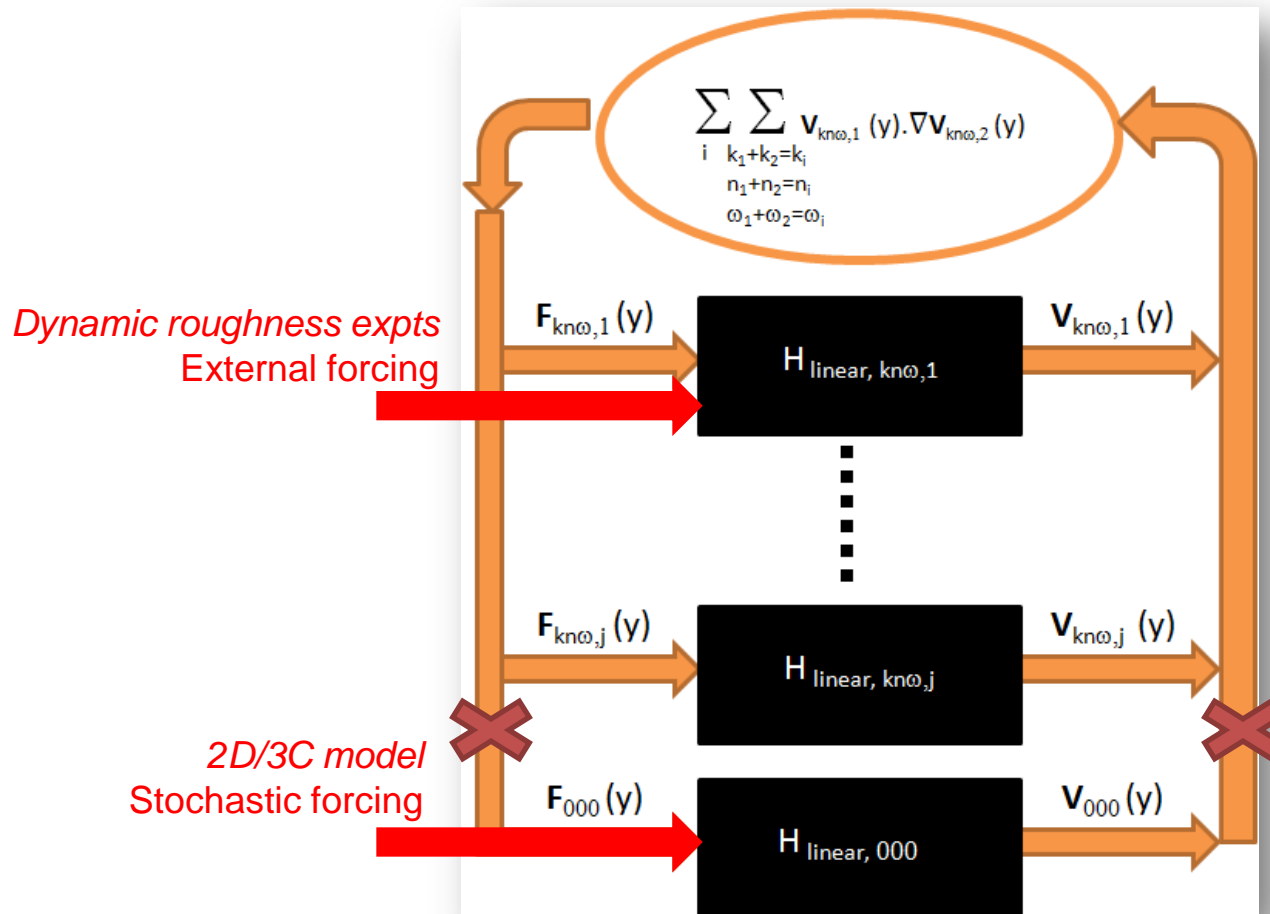
*Identify  $F(y)$  which leads to largest amplification – assume this appears in and dominates real flow*



Assume propagating modes with  $(k, n, \omega)$  in a divergence-free basis

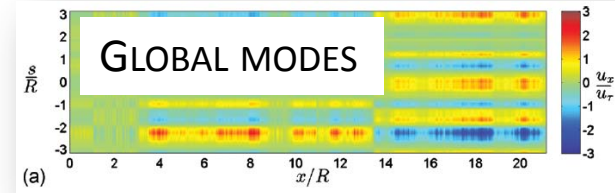
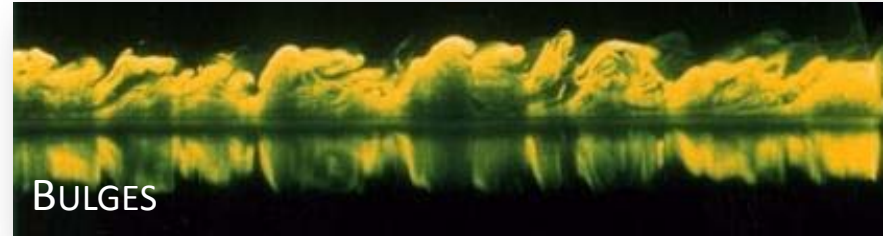
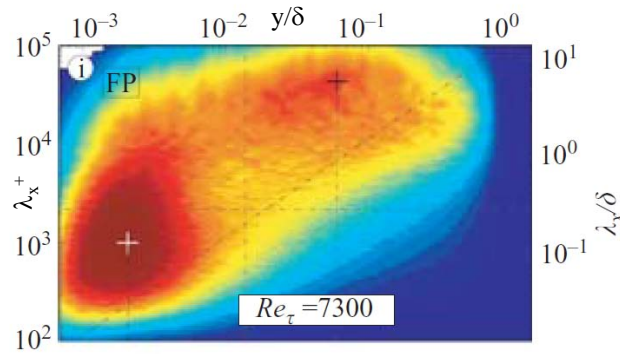
$$v(r, x, \theta, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} c_{mkn\omega} \xi_m(r) e^{i(\omega t - kx - n\theta)} dk d\omega \quad v_{kn\omega} = (v, e^{i(\omega t - kx - n\theta)})$$

# WORK TO DATE: II

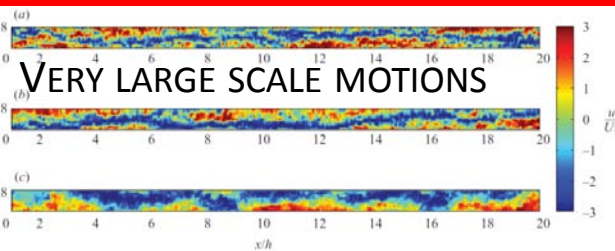
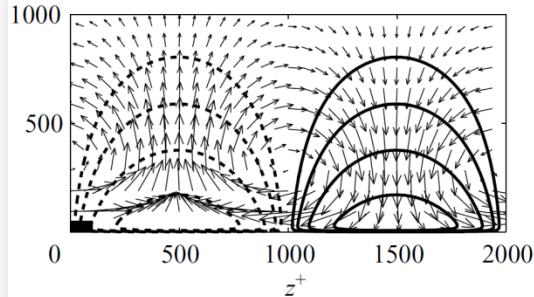


# TOWARDS THE BUILDING BLOCKS OF TURBULENCE

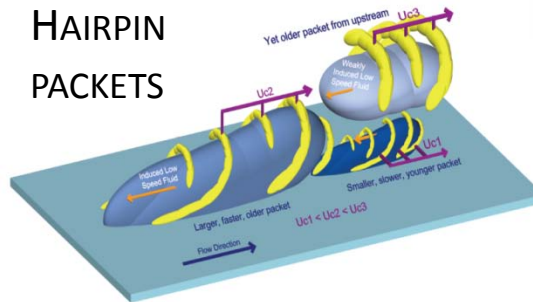
## STATISTICS & SPECTRA



## LINEAR RECEPTIVITY



## HAIRPIN PACKETS



*“So, actually the elephant has all the features you mentioned...”*

**Credits, CW from top left**  
 Hutchins & Marusic  
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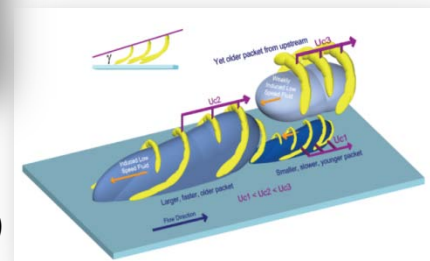


# BACKGROUND: VORTICAL STRUCTURE IN WALL TURBULENCE

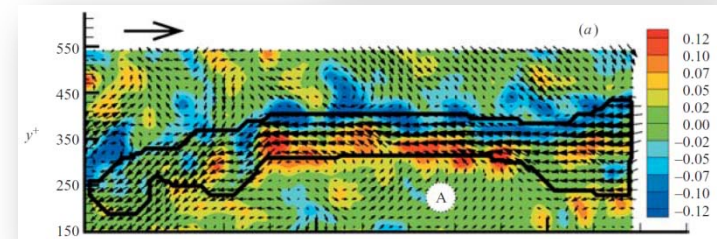
- Hairpin vortices and the attached eddy model (Theodorsen, 1952; Townsend, 1976; Perry & Chong, 1982, et al)
- Vortex packets, constant momentum zones, alignment of vortices along internal shear layers (Adrian, Meinhart & Tomkins, 2000)
- Packet alignment in wall-parallel planes (Ganapathisubramani, Longmire & Marusic, 2003; Tomkins & Adrian, 2003)
- Prograde and retrograde vortices (Falco, 1977; Carlier & Stanislas, 2005; Natrajan, Wu & Christensen, 2007)



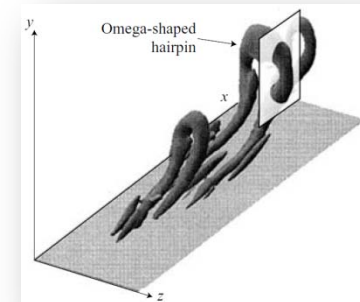
*Theodorsen (1952)*



*Adrian et al (2000)*



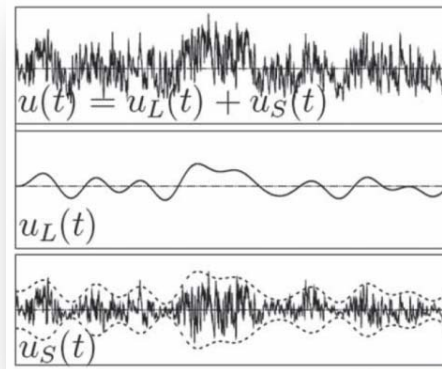
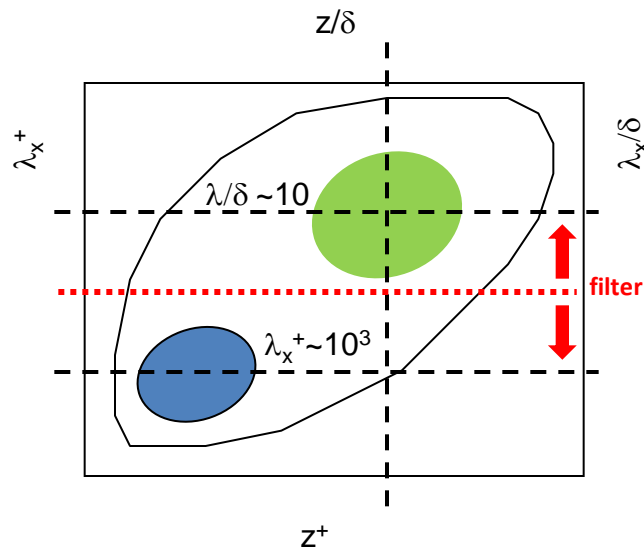
*Ganapathisubramani et al (2003)*



*Natrajan et al (2007)*

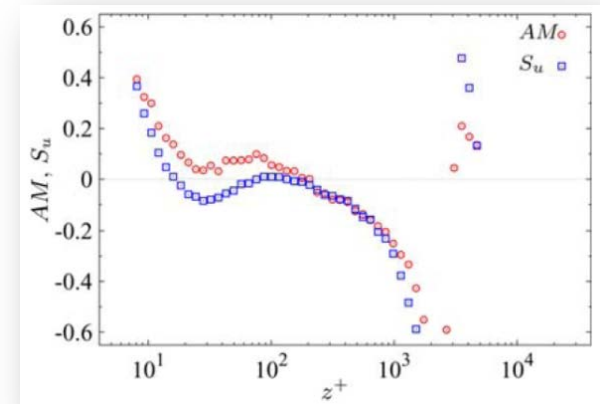
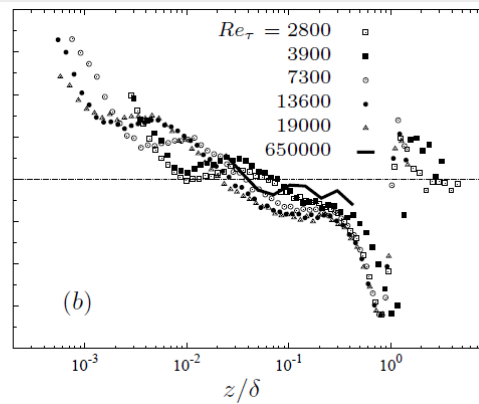
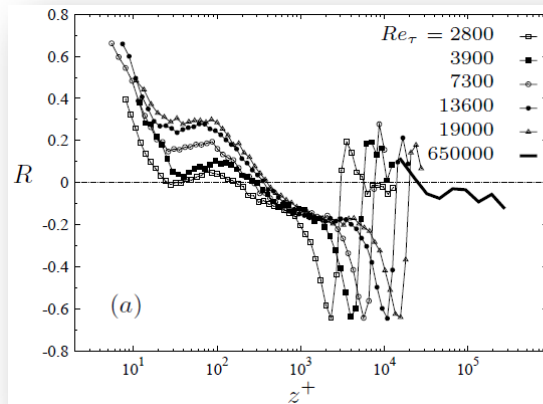


# BACKGROUND: AMPLITUDE MODULATION AND SKEWNESS



$$AM(z^+) = \frac{\overline{u_L^+ E_L(u_S^+)}}{\sqrt{\overline{u_L^{+2}}} \sqrt{\overline{E_L(u_S^+)^2}}}$$

$$\hat{S}_u(y) = \frac{\langle u^3 \rangle}{\langle u^2 \rangle^{3/2}}$$



Mathis, Hutchins & Marusic, *J. Fluid Mech.* 2009

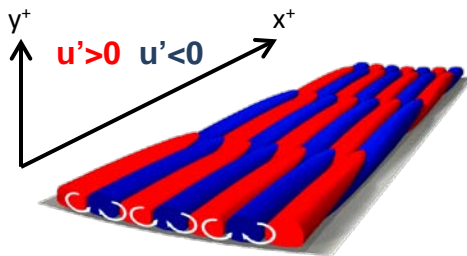
Marusic, Mathis & Hutchins, *Science* 2010

Mathis, Marusic, Hutchins & Sreenivasan, *Phys. Fluids* 2011

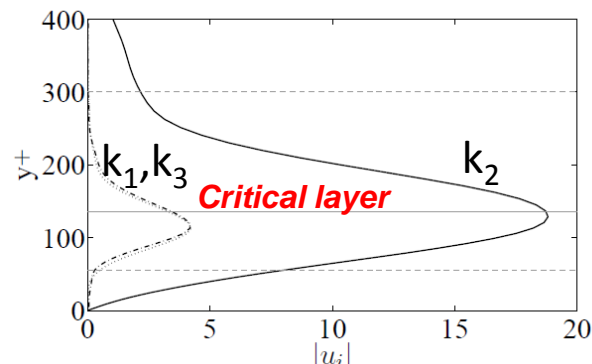
# SHAPE OF THE RESPONSE MODES

$$v(r, x, \theta, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} c_{mkn\omega} \xi_m(r) e^{i(\omega t - kx - n\theta)} dk d\omega$$

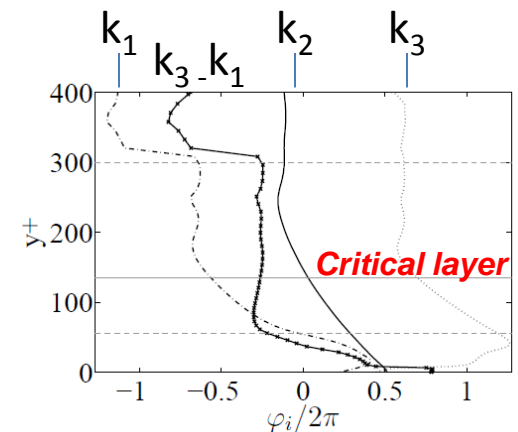
- Select three velocity response modes,  $\mathbf{v}_i$ , and linearly superpose
- Wavenumber/frequency combinations are triadically consistent:  
 $k_3 = k_1 + k_2$ ,  $n_3 = n_1 + n_2$ ,  $c_3 = c_1 + c_2$ ,
- “Ideal”/persistent combination: all modes have the same convection velocity



Sample mode shape



Amplitude distributions



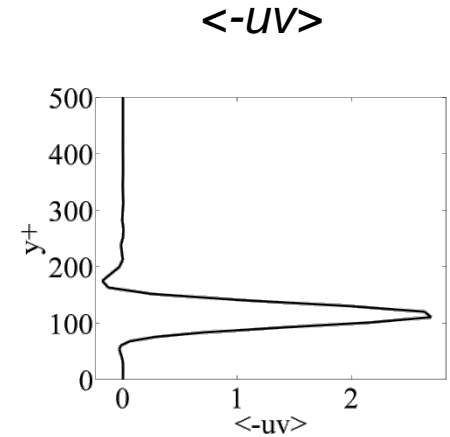
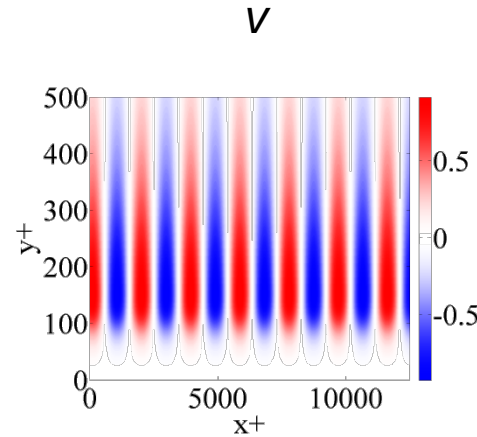
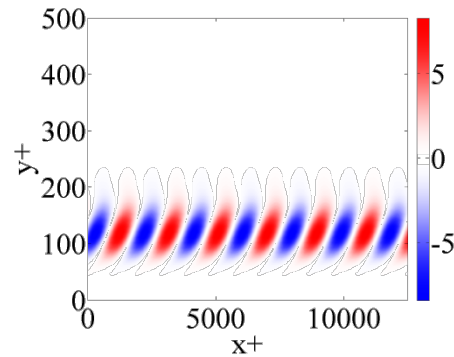
Phase distributions

# SHAPE OF THE RESPONSE MODES

$(k, n, c, A)$

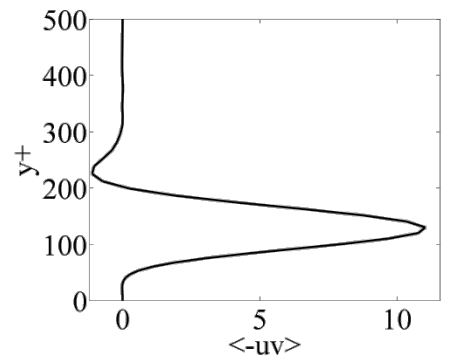
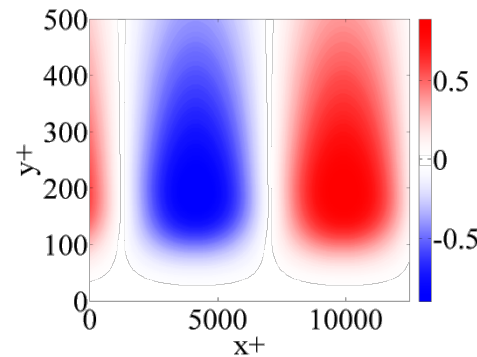
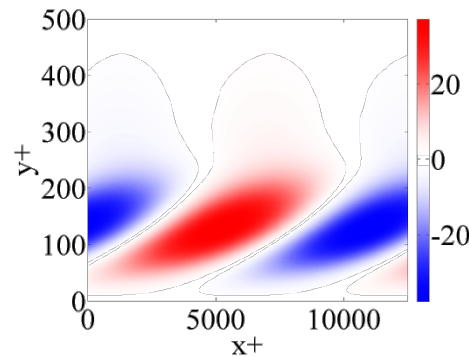
$k_1$

$(+/- 6, +/- 6, 2/3, -1.00i)$



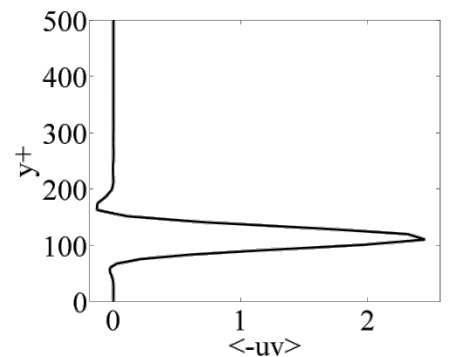
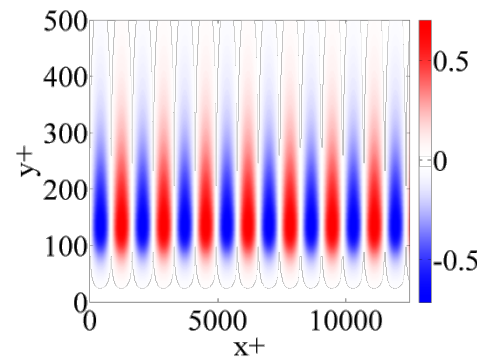
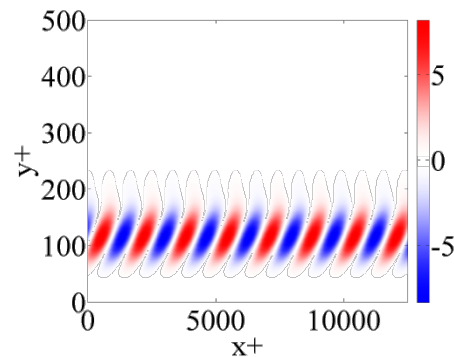
$k_2$

$(+/- 1, +/- 6, 2/3, 4.50)$



$k_3$

$(+/- 7, +/- 6, 2/3, 0.83i)$

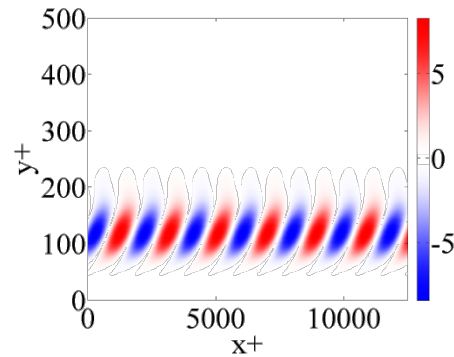


# STRUCTURE FROM RESPONSE MODES

$(k, n, c, A)$

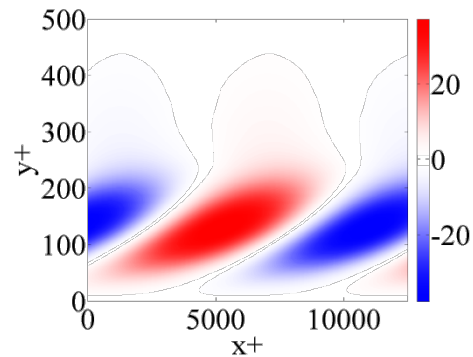
$k_1$

$(+/- 6, +/- 6, 2/3, -1.00i)$



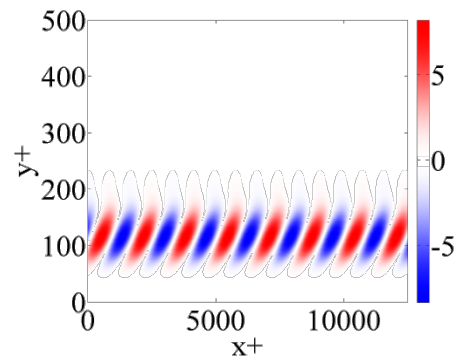
$k_2$

$(+/- 1, +/- 6, 2/3, 4.50)$

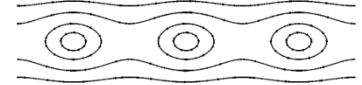
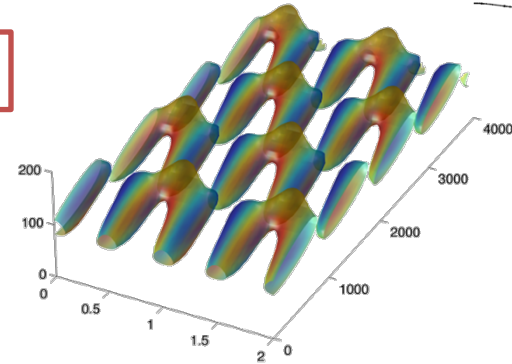


$k_3$

$(+/- 7, +/- 6, 2/3, 0.83i)$



$k_1$

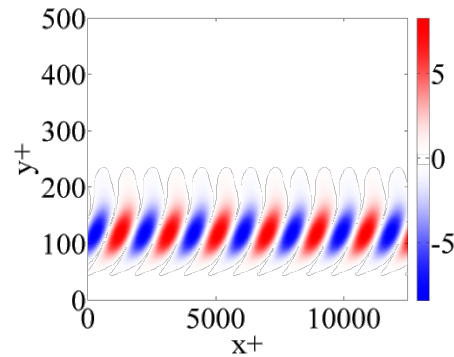


# STRUCTURE FROM RESPONSE MODES

$(k, n, c, A)$

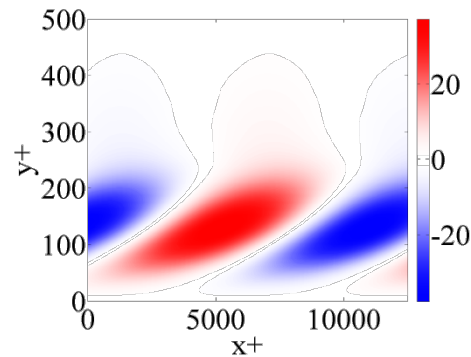
$k_1$

$(\pm 6, \pm 6, 2/3, -1.00i)$



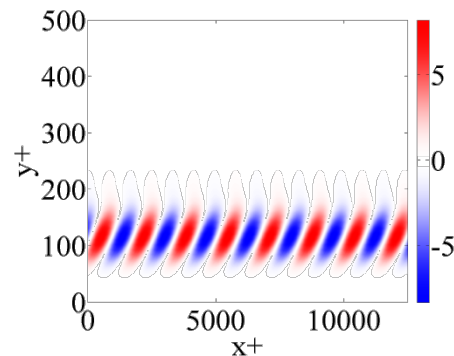
$k_2$

$(\pm 1, \pm 6, 2/3, 4.50)$

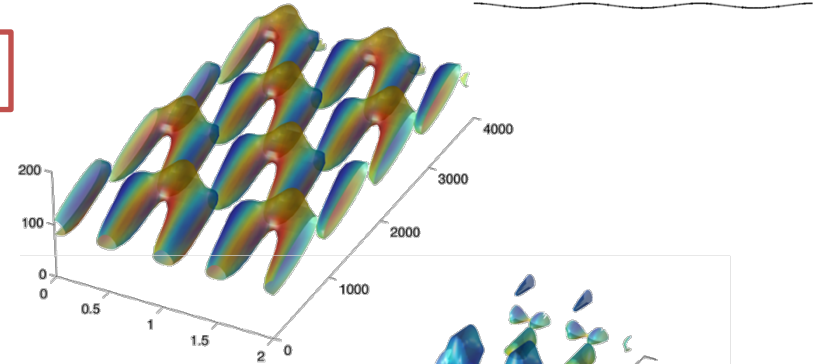


$k_3$

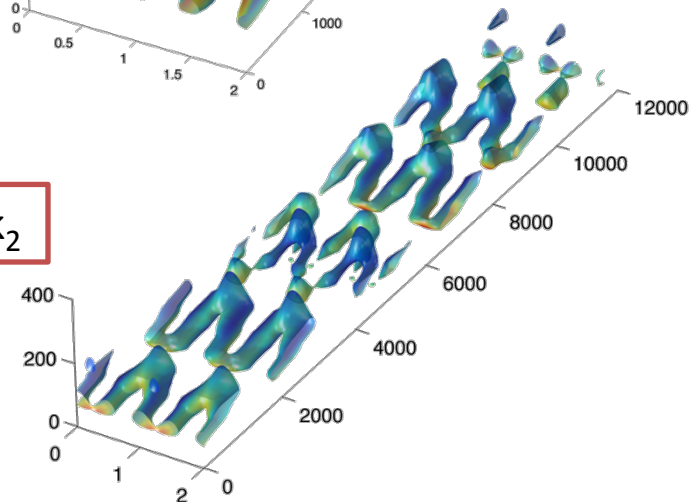
$(\pm 7, \pm 6, 2/3, 0.83i)$



$k_1$



$k_1 + k_2$



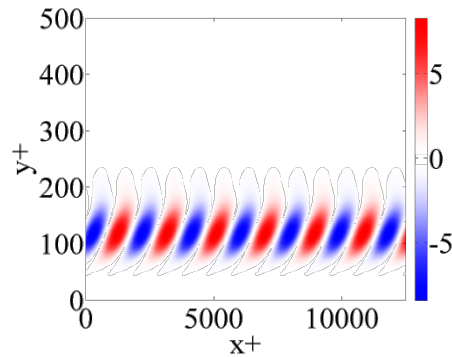
# STRUCTURE FROM RESPONSE MODES

$(k, n, c, A)$

$u$

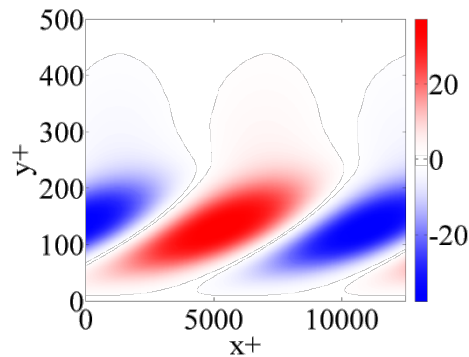
$k_1$

$(\pm 6, \pm 6, 2/3, -1.00i)$



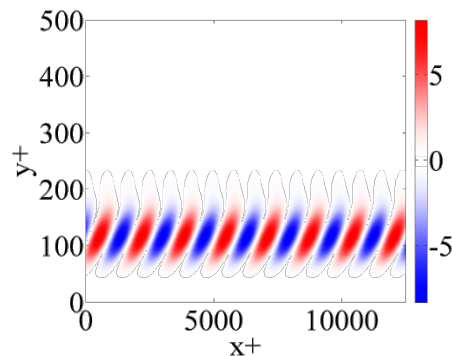
$k_2$

$(\pm 1, \pm 6, 2/3, 4.50)$

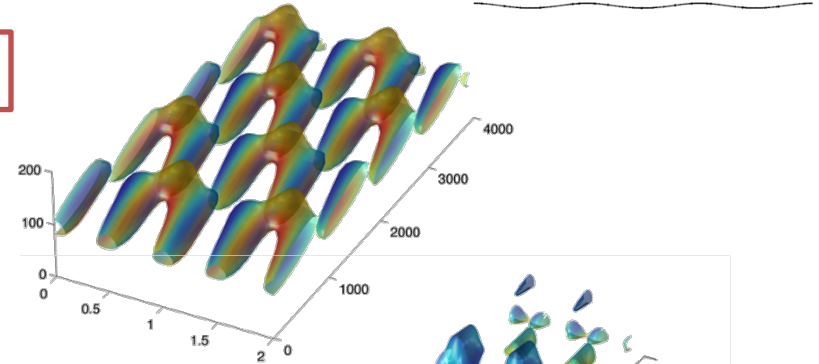


$k_3$

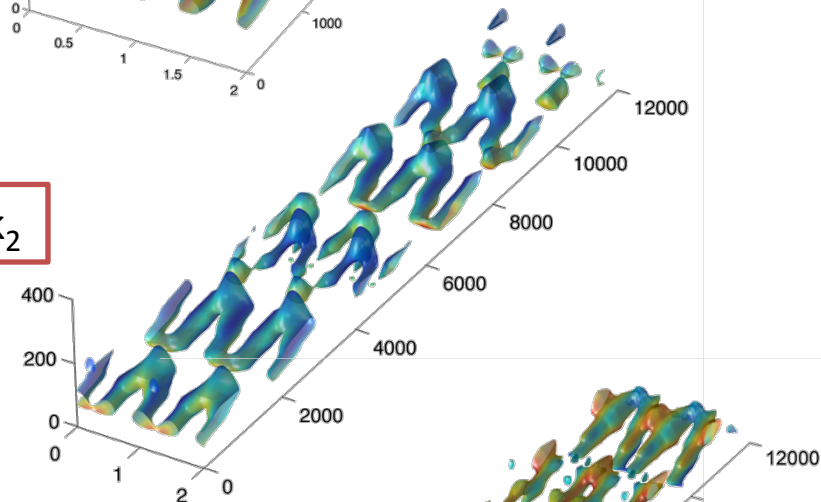
$(\pm 7, \pm 6, 2/3, 0.83i)$



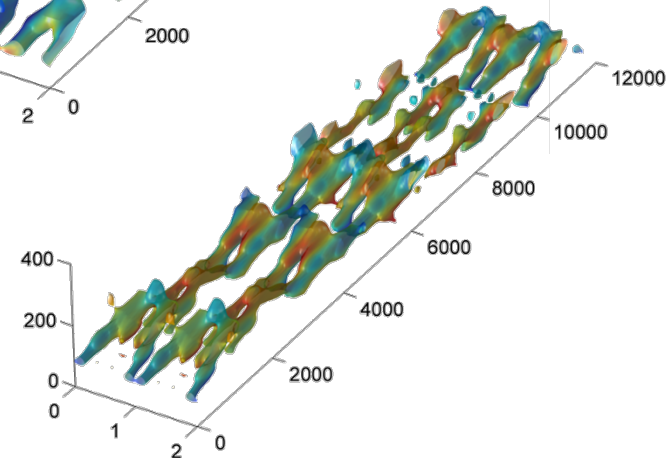
$k_1$



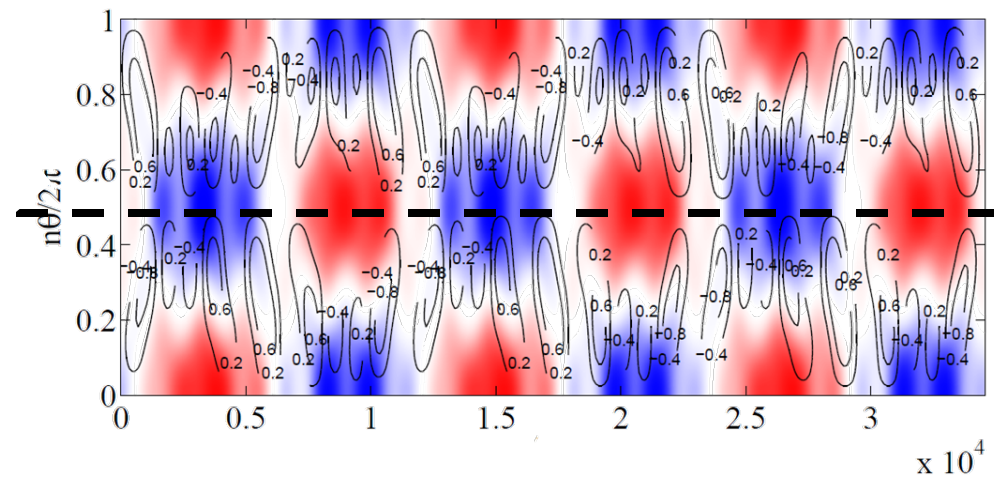
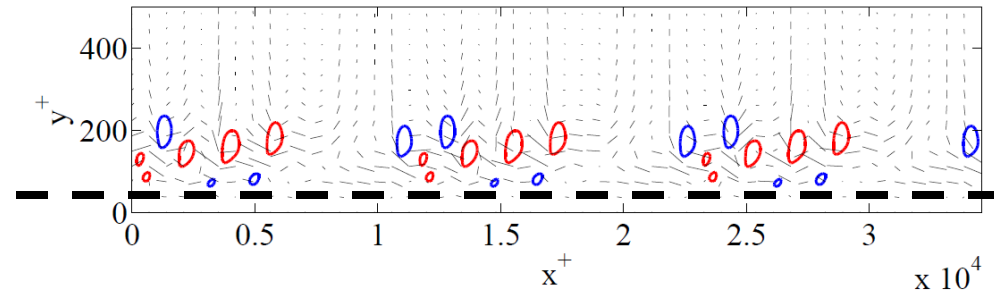
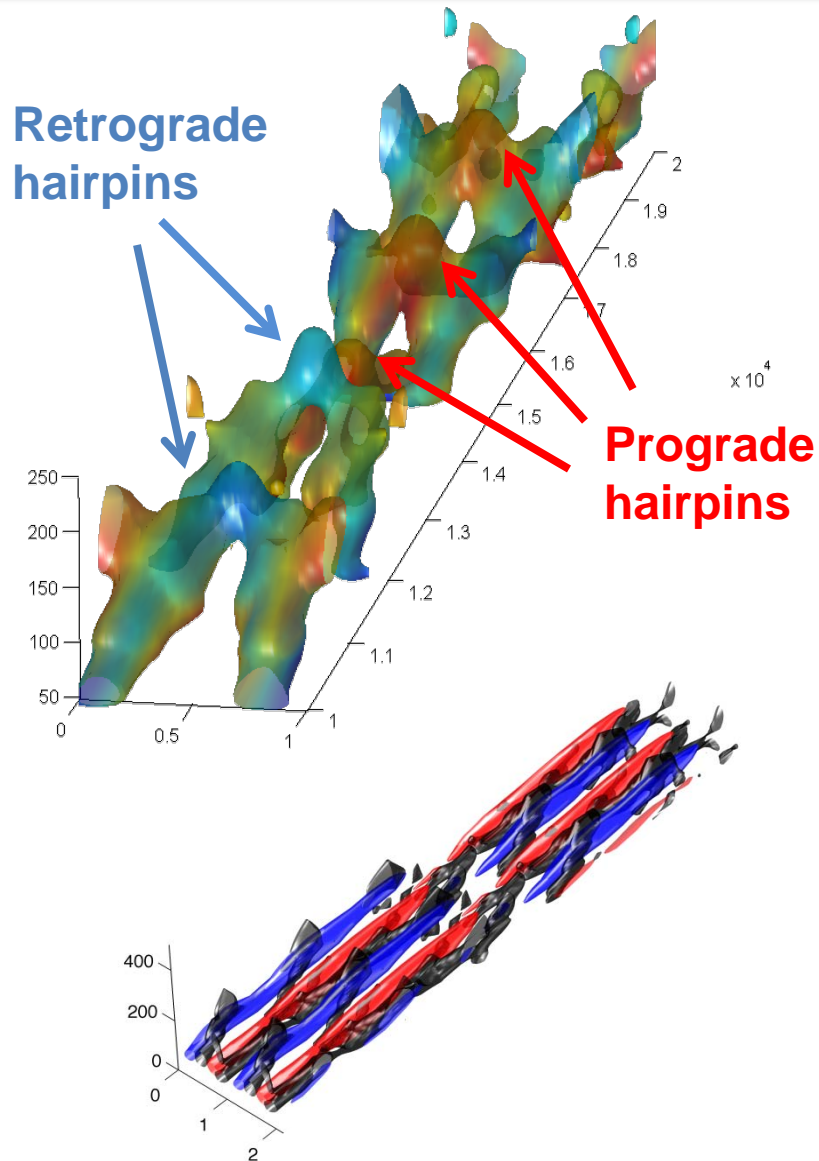
$k_1 + k_2$



$k_1 + k_2 + k_3$

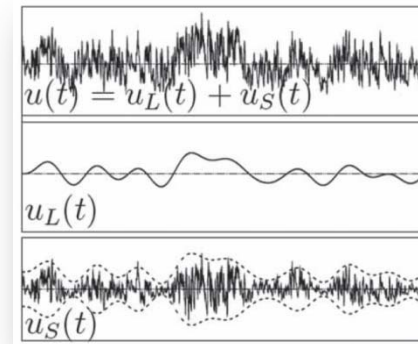
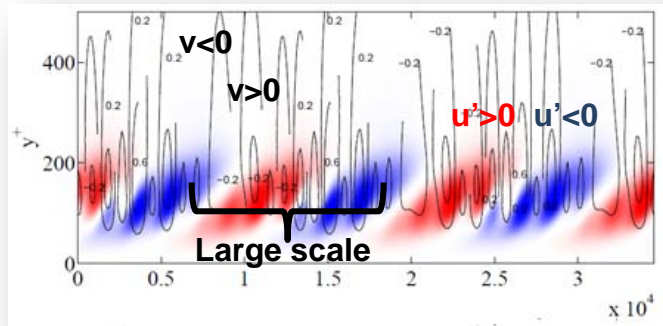


# MODULATING PACKET





# STATISTICS FROM THE MODULATING PACKET



**u and v “amplitude modulated” by large scale**

*Marusic, Mathis & Hutchins, Science 2010*

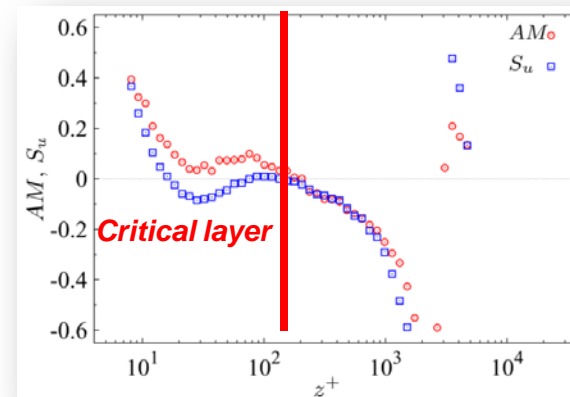
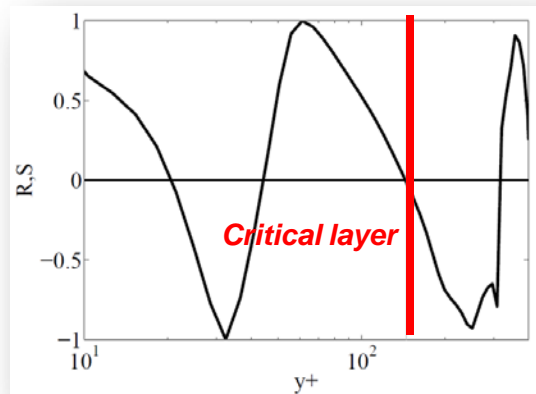
**Amplitude modulation coefficient**

$$R = \frac{(\langle A_2^2(y) \rangle \langle A_{env}^2(y) \rangle)^{1/2} \cos [\varphi_2(y) - \Delta\varphi(y)]}{(\langle A_2^2(y) \rangle \langle A_{env}^2(y) \rangle)^{1/2}} = \cos [\varphi_2(y) + \varphi_1 - \varphi_3(y)]$$

**Velocity skewness**

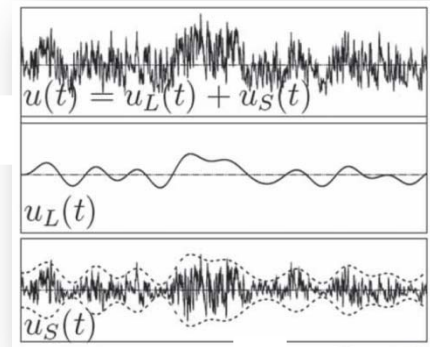
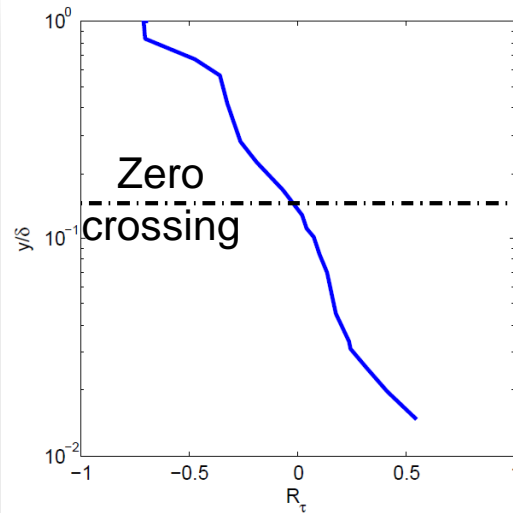
$$\hat{S}_u(y) = \frac{\langle u^3 \rangle}{\langle u^2 \rangle^{3/2}} = \frac{2A_1 A_2 A_3 \cos [\varphi_2(y) + \varphi_1(y) - \varphi_3(y)]}{(A_1^2 + A_2^2 + A_3^2)^{3/2}}$$

$$S = \cos [\varphi_2(y) + \varphi_1(y) - \varphi_3(y)]$$



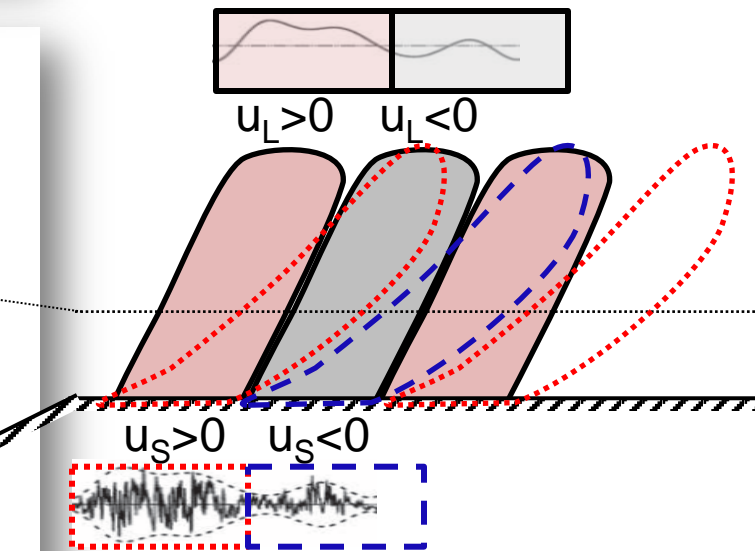
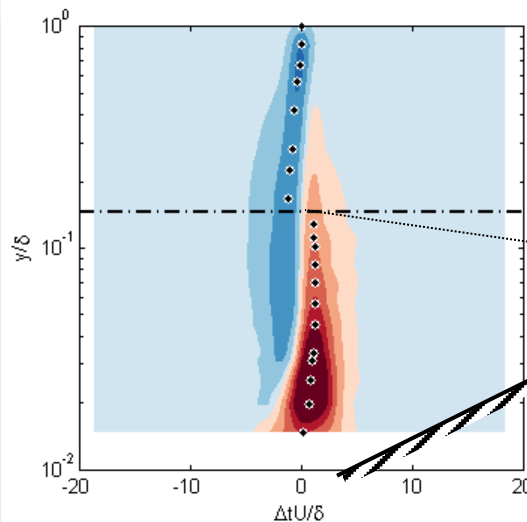
# AMPLITUDE MODULATION IN THE UNPERTURBED TBL

$$R_\tau = \frac{\langle u_L u_s \rangle}{\langle (u_L^2)^{1/2} \rangle \langle (u_s^2)^{1/2} \rangle}$$



Marusic, Mathis & Hutchins, Science 2010

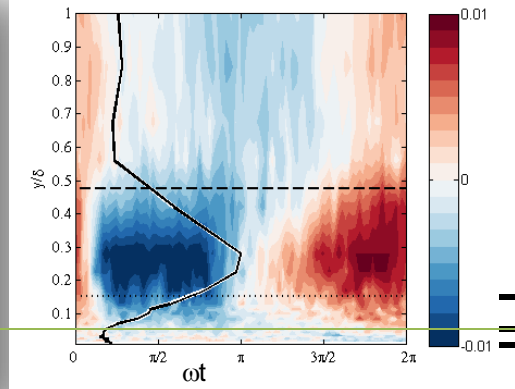
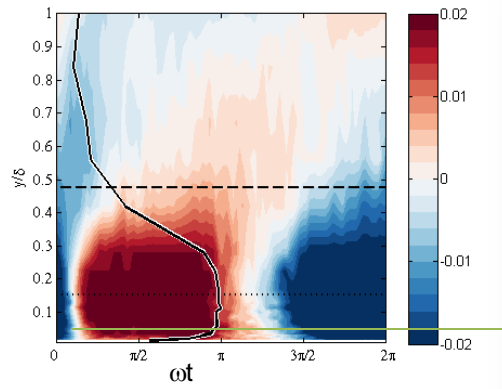
$$R_\tau(\Delta t) = \frac{\langle u_L(t + \Delta t) u_s(t) \rangle}{\langle (u_L^2)^{1/2} \rangle \langle (u_s^2)^{1/2} \rangle}$$



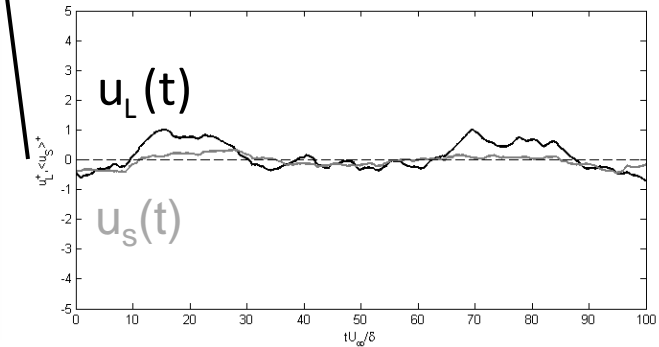
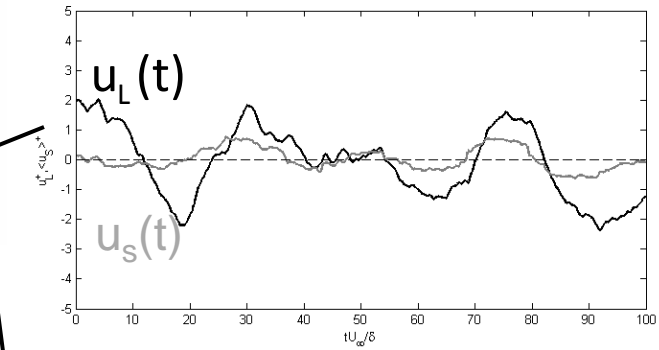
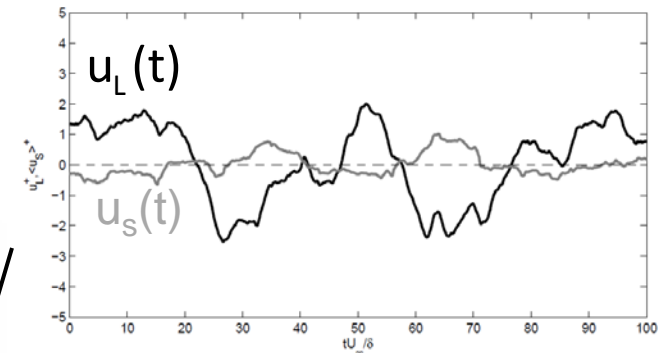
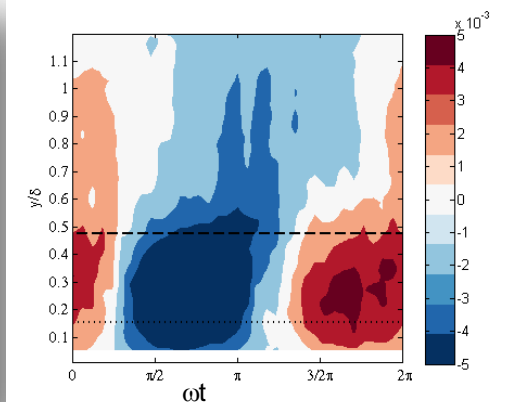
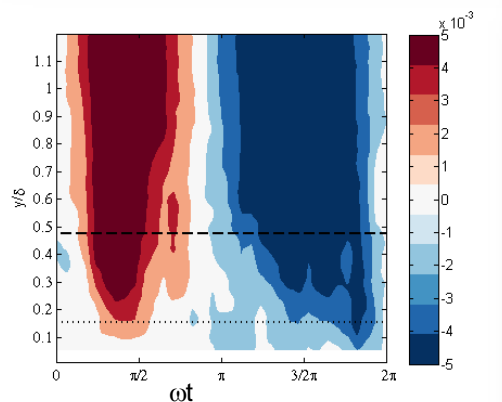
# AMPLITUDE MODULATION IN THE PERTURBED TBL

$$u_i(y, t) = \bar{U}_i(y) + \tilde{u}_i(y, t) + u_i'(y, t)$$

STREAMWISE



WALL-NORMAL



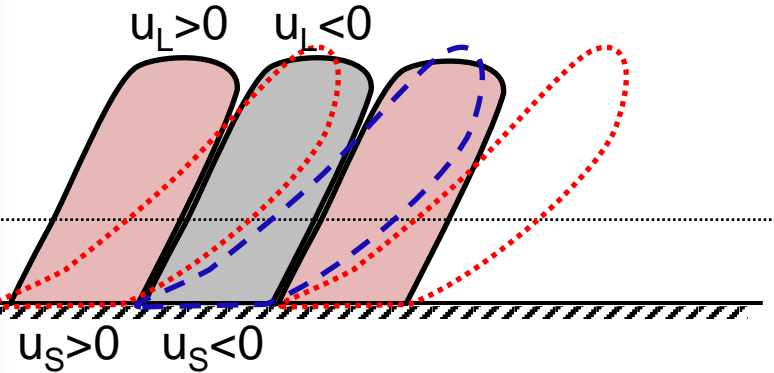
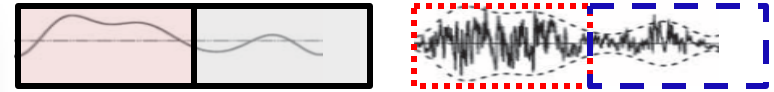
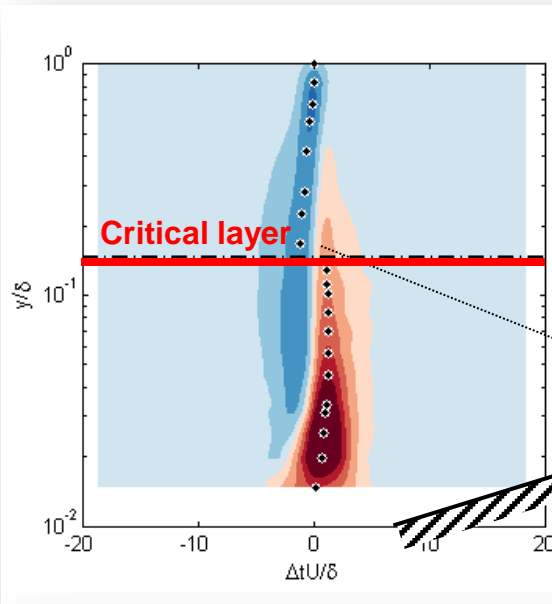
$$x = 3\delta_0$$

$$\hat{u}_i(y, \omega t) = u'_{i,rms}(y, \omega t) - \overline{u'_{i,rms}(y)}$$

# MODULATION EFFECT OF THE ARTIFICIAL LARGE SCALE

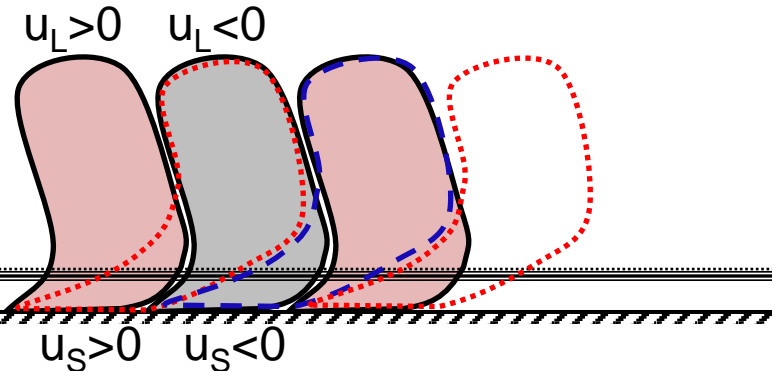
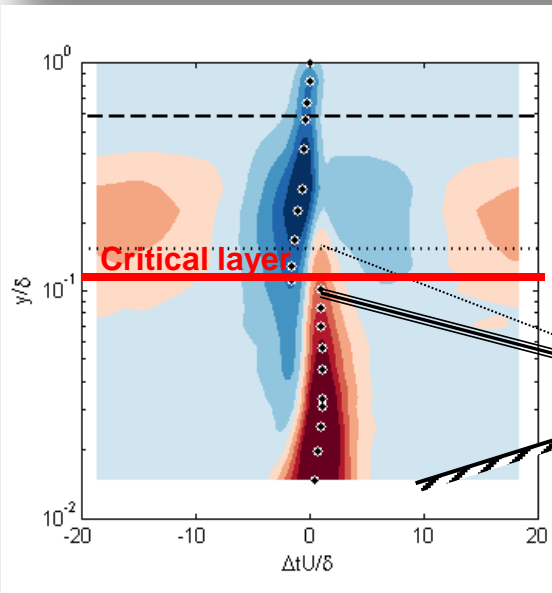
## UNPERTURBED

Zero crossing

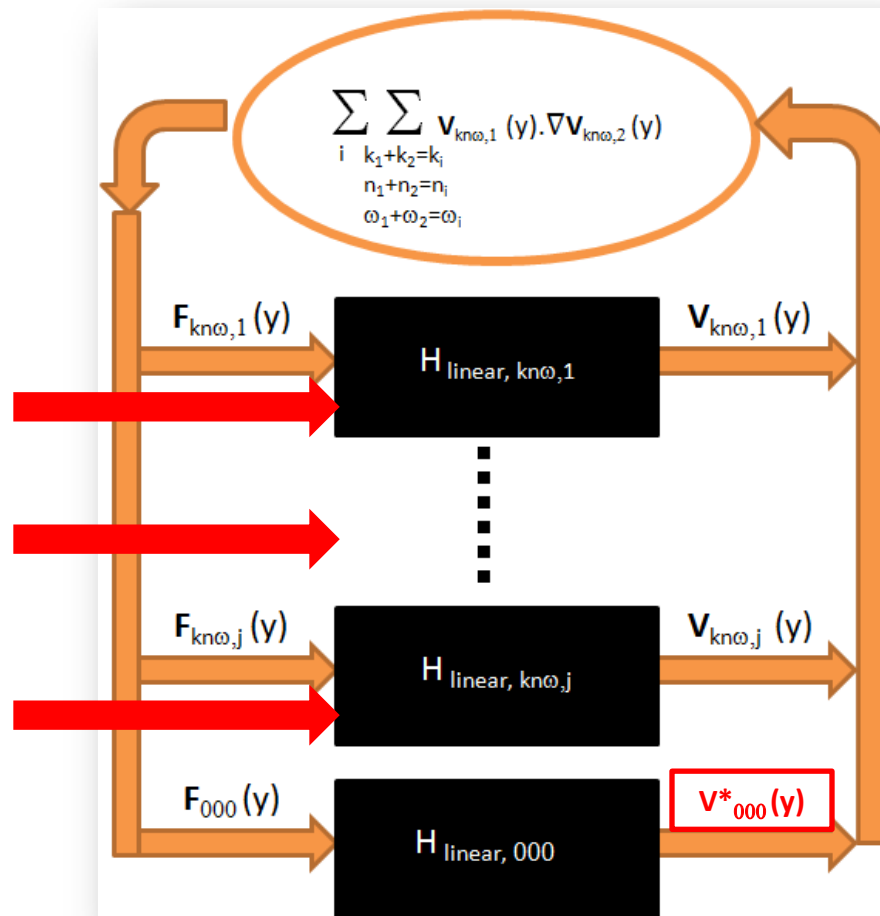


## DYNAMICALLY PERTURBED

Zero crossing



# A BLUEPRINT FOR CONTROLLING TURBULENCE



# CONCLUSIONS

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- Critical layer framework of McKeon & Sharma (2010) extended from reconstruction of significant features of statistics of wall turbulence
  - Nonlinearity retained (and supports mean velocity profile)
  - Provides basis for coherence in  $y$
  - Prediction of hairpin vortices from decomposition of Navier-Stokes equations at each wavenumber-frequency combination
- Proposed new kernel of turbulence
  - Three mode representation with appropriate relative phases (fixed to agree with experiment at the VLSM critical layer)
  - Leads to recognizable hairpin packet structure
  - Captures skewness and amplitude modulation behavior (these are *linear* phenomena)
  - Consistent with attached eddy hypothesis
  - Importance of critical layer proven by dynamic roughness and predictable (not shown)
- The response of the TBL to a synthetic large scale via a spatial impulse of dynamic roughness (last two years' reviews) confirms results
  - Coupling of external forcing to the most amplified modes
  - Suggests method can be used for control of near-wall structure

# OUTSTANDING CHALLENGES

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- Phase...
- Need to bridge the gap between theoretical and experimental studies
- Extension of model: other classical results using essentially linear model?
- “Close the loop”: determine correct nonlinear forcing for model to support correct mean velocity profile (currently assumed)?
  - Success will provide informed approach to control
  - Restores the difficulty of nonlinearity in the problem
- Coupling of dynamic roughness actuation with amplified modes demonstrated but what is best (practical) way of exciting them?
- Accurate values of shear stress required to make any scaling arguments for controlled/manipulated cases
- Low order model?